MeijerG1

Notations

Traditional name

Generalized Meijer G-function

Traditional notation

$$G_{p,q}^{m,n}\left(z,\, r \,\middle|\, \begin{array}{c} a_1,\, \ldots,\, a_n,\, a_{n+1},\, \ldots,\, a_p \\ b_1,\, \ldots,\, b_m,\, b_{m+1},\, \ldots,\, b_q \end{array}\right)$$

Mathematica StandardForm notation

MeijerG$[\{\{a_1, \ldots, a_n\}, \{a_{n+1}, .., a_p\}\}, \{\{b_1, \ldots, b_m\}, \{b_{m+1}, \ldots, b_q\}\}, z, r]$

Primary definition

07.35.02.0001.01

$$G_{p,q}^{m,n}\left(z,\, r \,\middle|\, \begin{array}{c} a_1,\, \ldots,\, a_n,\, a_{n+1},\, \ldots,\, a_p \\ b_1,\, \ldots,\, b_m,\, b_{m+1},\, \ldots,\, b_q \end{array}\right) = \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\left(\prod_{k=1}^{m}\Gamma(b_k+s)\right)\prod_{k=1}^{n}\Gamma(1-a_k-s)}{\left(\prod_{k=n+1}^{p}\Gamma(a_k+s)\right)\prod_{k=m+1}^{q}\Gamma(1-b_k-s)}\, z^{-\frac{s}{r}}\, ds\, /;$$

$$r \in \mathbb{R} \land r \neq 0 \land m \in \mathbb{N} \land n \in \mathbb{N} \land p \in \mathbb{N} \land q \in \mathbb{N} \land m \leq q \land n \leq p$$

The infinite contour of integration $\mathcal{L}$ separates the poles of $\Gamma(1-a_k-s)$at $s = 1 - a_k + j$, $j \in \mathbb{N}$ from the poles of $\Gamma(b_i + s)$ at $s = -b_i - l$, $l \in \mathbb{N}$. Such a contour always exists in the cases $a_k - b_i - 1 \notin \mathbb{N}$.

There are three possibilities for the contour $\mathcal{L}$:

(i) $\mathcal{L}$ runs from γ-$i\,\infty$ to γ+$i\,\infty$ (where $\text{Im}(\gamma) = 0$) so that all poles of $\Gamma(b_i + s)$, $i = 1, \ldots, m$, are to the left, and all the poles of $\Gamma(1 - a_i - s)$, $i = 1, \ldots, n$, to the right, of $\mathcal{L}$.
This contour can be a straight line $(\gamma - i\,\infty,\ \gamma + i\,\infty)$ if $\text{Re}(b_i - a_k) > -1$ (then $-\text{Re}(b_i) < \gamma < 1 - \text{Re}(a_k)$). (In this case the integral converges if $p + q < 2\,(m + n)$, $|\text{Arg}(z)| < \left(m + n - \frac{p+q}{2}\right)\pi$. If $m + n - \frac{p+q}{2} = 0$, then z must be real and positive and additional condition $(q - p)\,\gamma + \text{Re}(\mu) < 0$, $\mu = \sum_{l=1}^{q} b_l - \sum_{k=1}^{p} a_k + \frac{p-q}{2} + 1$, should be added.)

(ii) $\mathcal{L}$ is a left loop, starting and ending at -∞ and encircling all poles of $\Gamma(b_i + s)$, $i = 1, \ldots, m$, once in the positive direction, but none of the poles of $\Gamma(1 - a_i - s)$, $i = 1, \ldots, n$.
(In this case the integral converges if $q \geq 1$ and either $q > p$ or $q = p$ and $|z| < 1$ or $q = p$ and $|z| = 1$and $m + n - \frac{p+q}{2} \geq 0$ and $\text{Re}(\mu) < 0$.)

(iii) $\mathcal{L}$ is a right loop, starting and ending at $+\infty$ and encircling all poles of $\Gamma(1-a_i-s)$, $i=1, \ldots, n$, once in the negative direction, but none of the poles of $\Gamma(b_i+s)$, $i=1, \ldots, m$.
(In this case the integral converges if $p \geq 1$ and either $p>q$ or $p=q$ and $|z|>1$ or $q=p$ and $|z| == 1$and $m+n-\frac{p+q}{2} \geq 0$ and $\operatorname{Re}(\mu)<0$.)

Specific values

Specialized values

General case with restrictions on z and r

07.35.03.0001.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) == G_{p,q}^{m,n}\left(z^{1/r} \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) /; r \geq 1 \bigvee r<-1 \bigvee -\pi r < \arg(z) \leq \pi r$$

07.35.03.0002.01

$$G_{p,q}^{m,n}\left(z, 1 \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) == G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$$

07.35.03.0003.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) ==$$
$$G_{p r, q r}^{m r, n r}\left(r^{r(p-q)} z \,\middle|\, \begin{matrix} \frac{a_1}{r}, \ldots, \frac{a_1+r-1}{r}, \ldots, \frac{a_n}{r}, \ldots, \frac{a_n+r-1}{r}, \frac{a_{n+1}}{r}, \ldots, \frac{a_{n+1}+r-1}{r}, \ldots, \frac{a_p}{r}, \ldots, \frac{a_p+r-1}{r} \\ \frac{b_1}{r}, \ldots, \frac{b_1+r-1}{r}, \ldots, \frac{b_m}{r}, \ldots, \frac{b_m+r-1}{r}, \frac{b_{m+1}}{r}, \ldots, \frac{b_{m+1}+r-1}{r}, \ldots, \frac{b_q}{r}, \ldots, \frac{b_q+r-1}{r} \end{matrix}\right) /;$$
$$\xi == \left(\frac{p+q}{2}-m-n\right)(r-1) \bigwedge \theta == \frac{p-q}{2} - \sum_{k=1}^{p} a_k + \sum_{k=1}^{q} b_k + 1 \bigwedge r \in \mathbb{N}^+$$

Case $\{m, n, p, q\} = \{0, 1, 2, 0\}$

07.35.03.0004.01

$$G_{2,0}^{0,1}\left(z, \frac{1}{2} \,\middle|\, a, c\right) == \left(\frac{1}{z}\right)^{a-c} z^{2(a-1)} J_{c-a}\left(\frac{2}{z}\right)$$

07.35.03.0005.01

$$G_{2,0}^{0,1}\left(z, \frac{1}{2} \,\middle|\, a, a+\frac{1}{2}\right) == \frac{z^{2a-1}}{\sqrt{\pi}} \sin\left(\frac{2}{z}\right)$$

07.35.03.0006.01

$$G_{2,0}^{0,1}\left(z, \frac{1}{2} \,\middle|\, a, a-\frac{1}{2}\right) == \frac{z^{2(a-1)}}{\sqrt{\pi}} \cos\left(\frac{2}{z}\right)$$

Case $\{m, n, p, q\} = \{0, 1, 3, 1\}$

07.35.03.0007.01

$$G_{3,1}^{0,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a, c, a-\frac{1}{2} \\ a-\frac{1}{2} \end{matrix}\right) == \frac{z^{a+c-2}}{\pi} I_{c-a}\left(\frac{2}{z}\right)$$

07.35.03.0008.01

$$G_{3,1}^{0,1}\left(z, \frac{1}{4} \,\middle|\, \begin{matrix} a, a-\frac{1}{2}, a+\frac{1}{4} \\ a-\frac{3}{4} \end{matrix}\right) = \frac{(1-i)\, z^{4a-3}}{\pi}\, C\left(\frac{1+i}{z} \sqrt{\frac{2}{\pi}}\right)$$

07.35.03.0009.01

$$G_{3,1}^{0,1}\left(z, \frac{1}{4} \,\middle|\, \begin{matrix} a, a-\frac{1}{2}, a+\frac{1}{4} \\ a-\frac{3}{4} \end{matrix}\right) = \frac{z^{4a-3}}{2\pi}\left(\operatorname{erf}\left(\frac{\sqrt{2}}{z}\right) + \operatorname{erfi}\left(\frac{\sqrt{2}}{z}\right)\right)$$

07.35.03.0010.01

$$G_{3,1}^{0,1}\left(z, \frac{1}{4} \,\middle|\, \begin{matrix} a, a+\frac{1}{2}, a+\frac{3}{4} \\ a-\frac{1}{4} \end{matrix}\right) = -\frac{(1+i)\, z^{4a-1}}{\pi}\, S\left(\frac{1+i}{z} \sqrt{\frac{2}{\pi}}\right)$$

07.35.03.0011.01

$$G_{3,1}^{0,1}\left(z, \frac{1}{4} \,\middle|\, \begin{matrix} a, a+\frac{1}{2}, a+\frac{3}{4} \\ a-\frac{1}{4} \end{matrix}\right) = \frac{z^{4a-1}}{2\pi}\left(\operatorname{erfi}\left(\frac{\sqrt{2}}{z}\right) - \operatorname{erf}\left(\frac{\sqrt{2}}{z}\right)\right)$$

Case $\{m, n, p, q\} = \{0, 2, 2, 0\}$

07.35.03.0012.01

$$G_{2,0}^{0,2}\left(z, \frac{1}{2} \,\middle|\, a, c\right) = \frac{\pi \csc((c-a)\pi)}{z^2}\left(z^{2c}\left(\frac{1}{z}\right)^{c-a} I_{a-c}\left(\frac{2}{z}\right) - z^{2a}\left(\frac{1}{z}\right)^{a-c} I_{c-a}\left(\frac{2}{z}\right)\right)$$

07.35.03.0013.01

$$G_{2,0}^{0,2}\left(z, \frac{1}{2} \,\middle|\, a, c\right) = 2\, z^{a+c-2} K_{a-c}\left(\frac{2}{z}\right) /;\ z \notin (-\infty, 0)$$

07.35.03.0014.01

$$G_{2,0}^{0,2}\left(z, \frac{1}{3} \,\middle|\, a, a+\frac{1}{3}\right) = 2\sqrt[6]{3}\, \pi\, z^{3a-2} \operatorname{Ai}\left(\frac{3^{2/3}}{z}\right)$$

07.35.03.0015.01

$$G_{2,0}^{0,2}\left(z, \frac{1}{3} \,\middle|\, a, a+\frac{2}{3}\right) = -\frac{2\pi z^{3a-1}}{\sqrt[6]{3}} \operatorname{Ai}'\left(\frac{3^{2/3}}{z}\right)$$

Case $\{m, n, p, q\} = \{0, 2, 2, 1\}$

07.35.03.0016.01

$$G_{2,1}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ b \end{matrix}\right) = 4^{a-b} z^{2a-1} \exp\left(-\frac{1}{z^2}\right) H_{2b-2a}\left(\frac{1}{z}\right)$$

07.35.03.0017.01

$$G_{2,1}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a-\frac{1}{2} \end{matrix}\right) = \sqrt{\pi}\, z^{2a-1} \operatorname{erfc}\left(\frac{1}{z}\right)$$

07.35.03.0018.01

$$G_{2,1}^{0,2}\left(z, \frac{2}{3} \,\middle|\, \begin{matrix} a, a+\frac{2}{3} \\ a-\frac{1}{6} \end{matrix}\right) = 2^{2/3} \sqrt[6]{3} \sqrt{\pi}\, z^{\frac{3a-1}{2}} \exp\left(-\frac{1}{2 z^{3/2}}\right) \operatorname{Ai}\left(\frac{3^{2/3}}{2\sqrt[3]{2}\, z}\right)$$

07.35.03.0019.01

$$G_{2,1}^{0,2}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{4}{3} \\ a+\frac{1}{6} \end{array}\right) = -\frac{2\sqrt[3]{2}\sqrt{\pi}}{\sqrt[6]{3}} z^{\frac{3a+1}{2}} \exp\left(-\frac{1}{2z^{3/2}}\right) \mathrm{Ai}'\left(\frac{3^{2/3}}{2\sqrt[3]{2}\,z}\right)$$

Case $\{m, n, p, q\} = \{0, 2, 2, 2\}$

07.35.03.0020.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, c \\ b, b+\frac{1}{2} \end{array}\right) = \frac{\Gamma(2b-2c+2)\,\theta(|z|-1)}{\Gamma\left(a-2b+c-\frac{1}{2}\right)(2(a-2b+c-1))_{2b-2c+1}} \left(1-\frac{1}{z^2}\right)^{a-2b+c-\frac{3}{2}} z^{2a-2b+2c-3}\, C_{2b-2c+1}^{a-2b+c-1}(z)$$

07.35.03.0021.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, c \\ b, b+c-a \end{array}\right) = \frac{\Gamma(b-c+1)\,\theta(|z|-1)}{\Gamma(2a-b-c)} z^{2a-b+c-2} \left(1-\frac{1}{z^2}\right)^{2a-2b-1} C_{b-c}^{a-b}\left(\frac{z^2+1}{2z}\right)$$

07.35.03.0022.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, c \end{array}\right) = \frac{\Gamma(2c-2a+1)}{\Gamma\left(2a-b-c+\frac{1}{2}\right)(2(2a-b-c))_{2c-2a}} \left(1-\frac{1}{z^2}\right)^{2a-b-c-\frac{1}{2}} z^{2a-1}\, C_{2c-2a}^{2a-b-c}\left(\frac{1}{z}\right) \theta(|z|-1)$$

07.35.03.0023.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b-1 \end{array}\right) = \frac{\theta(|z|-1)\,2}{\sqrt{\pi}\,(2b-2a+1)} \sqrt{1-\frac{1}{z^2}}\, z^{2a-1}\, U_{2b-2a}\left(\frac{1}{z}\right)$$

07.35.03.0024.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b-\frac{1}{2} \end{array}\right) = z^{2a-1}\, P_{2a-2b-1}\left(\frac{1}{z}\right) \theta(|z|-1)$$

07.35.03.0025.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b \end{array}\right) = \frac{\theta(|z|-1)}{\sqrt{\pi}\sqrt{1-\frac{1}{z^2}}} z^{2a-1}\, T_{2a-2b}\left(\frac{1}{z}\right)$$

07.35.03.0026.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b \end{array}\right) = \frac{(a-b)\,\theta(|z|-1)}{\sqrt{\pi}\sqrt{1-\frac{1}{z^2}}} z^{2a-1}\, C_{2a-2b}^{(0)}\left(\frac{1}{z}\right)$$

07.35.03.0027.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+1 \\ b, b+\frac{1}{2} \end{array}\right) = \frac{z^{2b-1}\,\theta(|z|-1)}{\sqrt{\pi}\sqrt{1-\frac{1}{z^2}}} T_{2a-2b-1}(z)$$

07.35.03.0028.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+\frac{3}{2} \\ b, b+\frac{1}{2} \end{array}\right) = z^{2b}\, P_{2a-2b-2}(z)\, \theta(|z|-1)$$

07.35.03.0029.01

$$G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+2 \\ b, b+\frac{1}{2} \end{array}\right) = \frac{\theta(|z|-1)}{(b-a+1)\sqrt{\pi}} z^{2b+1} \sqrt{1-\frac{1}{z^2}}\, U_{2b-2a+1}(z)$$

Case $\{m, n, p, q\} = \{0, 2, 3, 1\}$

07.35.03.0030.01

$$G_{3,1}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, c, a+\frac{1}{2} \\ a+\frac{1}{2} \end{array}\right) = z^{a+c-2}\, Y_{a-c}\left(\frac{2}{z}\right)$$

07.35.03.0031.01

$$G_{3,1}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a, a-\frac{1}{2} \\ a-1 \end{array}\right) = -\frac{2 z^{2a-2}}{\sqrt{\pi}} \operatorname{Ci}\left(\frac{2}{z}\right) /;\, z \notin (-\infty, 0)$$

Case $\{m, n, p, q\} = \{0, 2, 3, 2\}$

07.35.03.0032.01

$$G_{3,2}^{0,2}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{4}{3}, a+\frac{5}{3} \\ a+\frac{1}{6}, a+\frac{5}{3} \end{array}\right) = -\frac{\sqrt[3]{2}}{\sqrt[6]{3}\, \sqrt{\pi}} z^{\frac{3a+1}{2}} \exp\left(\frac{1}{2 z^{3/2}}\right) \operatorname{Bi}'\left(\frac{3^{2/3}}{2 \sqrt[3]{2}\, z}\right)$$

Case $\{m, n, p, q\} = \{0, 2, 4, 2\}$

07.35.03.0033.01

$$G_{4,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a, a-\frac{1}{2}, a-\frac{1}{2} \\ a-1, a-\frac{1}{2} \end{array}\right) = -\frac{2 z^{2a-2}}{\pi^{3/2}} \operatorname{Chi}\left(\frac{2}{z}\right) /;\, z \notin (-\infty, 0)$$

07.35.03.0034.01

$$G_{4,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a, a+\frac{1}{2}, a+\frac{1}{2} \\ a-\frac{1}{2}, a \end{array}\right) = \frac{2 z^{2a-1}}{\pi^{3/2}} \operatorname{Shi}\left(\frac{2}{z}\right)$$

07.35.03.0035.01

$$G_{4,2}^{0,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a-\frac{1}{3}, a+\frac{1}{6} \\ a-\frac{1}{3}, a+\frac{1}{6} \end{array}\right) = \frac{\sqrt[6]{3}}{2\pi} z^{3a-2} \operatorname{Bi}\left(\frac{3^{2/3}}{z}\right)$$

07.35.03.0036.01

$$G_{4,2}^{0,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a-\frac{1}{2}, a-\frac{1}{6} \\ a-\frac{7}{12}, a-\frac{1}{12} \end{array}\right) = \frac{1}{\pi} \sqrt[6]{\frac{3}{2}}\, z^{3a-2} \cosh\left(\frac{1}{z^{3/2}}\right) \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

07.35.03.0037.01

$$G_{4,2}^{0,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a+\frac{1}{2}, a+\frac{5}{6} \\ a-\frac{1}{12}, a+\frac{5}{12} \end{array}\right) = -\frac{1}{\pi} \sqrt[6]{\frac{3}{2}}\, z^{3a-\frac{1}{2}} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \sinh\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0038.01

$$G_{4,2}^{0,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, a+\frac{1}{3}, a+\frac{5}{6} \\ a+\frac{1}{3}, a+\frac{5}{6} \end{array}\right) = -\frac{1}{2\sqrt[6]{3}\, \pi} z^{3a-1} \operatorname{Bi}'\left(\frac{3^{2/3}}{z}\right)$$

Case $\{m, n, p, q\} = \{0, 3, 3, 1\}$

07.35.03.0039.01

$$G_{3,1}^{0,3}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{3}, a+\frac{2}{3} \\ a-\frac{1}{6} \end{matrix}\right) = 2\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2} z^{3a-1} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)^2 \,/;\, z \notin (-\infty, 0)$$

07.35.03.0040.01

$$G_{3,1}^{0,3}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{3}, a+\frac{2}{3} \\ a+\frac{1}{6} \end{matrix}\right) = -4\, \pi^{3/2} z^{3a-1} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) /;\, z \notin (-\infty, 0)$$

07.35.03.0041.01

$$G_{3,1}^{0,3}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{2}{3}, a+\frac{4}{3} \\ a+\frac{1}{6} \end{matrix}\right) = 4 \sqrt[3]{\frac{2}{3}}\, \pi^{3/2} z^{3a+1} \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)^2 \,/;\, z \notin (-\infty, 0)$$

Case $\{m, n, p, q\} = \{0, 4, 4, 2\}$

07.35.03.0042.01

$$G_{4,2}^{0,4}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{3}, 2b-a+\frac{2}{3}, 2b-a+1 \\ b, b+\frac{1}{2} \end{matrix}\right) = 2 \sqrt[3]{2} \sqrt[6]{3} \sqrt{\pi}\, z^{3b-1} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) K_{2b-2a+\frac{2}{3}}\left(\frac{1}{z^{3/2}}\right) /;\, \operatorname{Re}(z) > 0$$

Case $\{m, n, p, q\} = \{1, 0, 0, 2\}$

07.35.03.0043.01

$$G_{0,2}^{1,0}\left(z, \frac{1}{2} \,\middle|\, b, c\right) = z^{b+c} J_{b-c}(2z)$$

07.35.03.0044.01

$$G_{0,2}^{1,0}\left(z, \frac{1}{2} \,\middle|\, b, b-\frac{1}{2}\right) = \frac{z^{2b-1}}{\sqrt{\pi}} \sin(2z)$$

07.35.03.0045.01

$$G_{0,2}^{1,0}\left(z, \frac{1}{2} \,\middle|\, b, b+\frac{1}{2}\right) = \frac{z^{2b}}{\sqrt{\pi}} \cos(2z)$$

Case $\{m, n, p, q\} = \{1, 0, 0, 4\}$

07.35.03.0157.01

$$G_{0,4}^{1,0}\left(z, \frac{1}{4} \,\middle|\, b, b+\frac{1}{4}, b+\frac{1}{2}, b+\frac{3}{4}\right) = \frac{z^{4\left(b+\frac{1}{4}\right)-1} \cos\left(2\sqrt{2}\, z\right) \cosh\left(2\sqrt{2}\, z\right)}{\sqrt{2}\, \pi^{3/2}}$$

Case $\{m, n, p, q\} = \{1, 0, 1, 3\}$

07.35.03.0158.01

$$G_{1,3}^{1,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a \\ b, a-\frac{1}{2}, 2a-b-1 \end{matrix}\right) = \frac{z^{2a-1} \sin((a-b)\pi)}{\sqrt{\pi}} I_{b-a+\frac{1}{2}}(z)^2$$

07.35.03.0046.01

$$G_{1,3}^{1,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{2}, a, b \end{array}\right) = \frac{z^{a+b-\frac{1}{2}}}{\pi} I_{a-b-\frac{1}{2}}(2z)$$

07.35.03.0047.01

$$G_{1,3}^{1,0}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a \\ a-\frac{3}{4}, a-1, a-\frac{1}{4} \end{array}\right) = \frac{1-i}{\pi} z^{4(a-1)} C\left((1+i)\sqrt{\frac{2}{\pi}} z\right)$$

07.35.03.0048.01

$$G_{1,3}^{1,0}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a \\ a-\frac{3}{4}, a-1, a-\frac{1}{4} \end{array}\right) = \frac{z^{4(a-1)}}{2\pi} \left(\operatorname{erf}\left(\sqrt{2}\,\sqrt[4]{z}\right) + \operatorname{erfi}\left(\sqrt{2}\,\sqrt[4]{z}\right)\right)$$

07.35.03.0049.01

$$G_{1,3}^{1,0}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{4}, a-1, a-\frac{3}{4} \end{array}\right) = -\frac{1+i}{\pi} z^{4(a-1)} S\left((1+i)\sqrt{\frac{2}{\pi}} z\right)$$

07.35.03.0050.01

$$G_{1,3}^{1,0}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{4}, a-1, a-\frac{3}{4} \end{array}\right) = \frac{z^{4(a-1)}}{2\pi} \left(\operatorname{erfi}\left(\sqrt{2}\, z\right) - \operatorname{erf}\left(\sqrt{2}\, z\right)\right)$$

Case $\{m, n, p, q\} = \{1, 1, 1, 2\}$

07.35.03.0051.01

$$G_{1,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{2}, a-1 \end{array}\right) = \sqrt{\pi}\, z^{2(a-1)} \operatorname{erf}(z)$$

Case $\{m, n, p, q\} = \{1, 1, 1, 3\}$

07.35.03.0052.01

$$G_{1,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a \\ a, a-\frac{1}{2}, b \end{array}\right) = z^{a+b-\frac{1}{2}} \boldsymbol{H}_{a-b-\frac{1}{2}}(2z)$$

07.35.03.0053.01

$$G_{1,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{2}, a-1, a-1 \end{array}\right) = \frac{2 z^{2(a-1)}}{\sqrt{\pi}} \operatorname{Si}(2z)$$

07.35.03.0159.01

$$G_{1,3}^{1,1}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a \\ a-\frac{3}{4}, a-1, a-\frac{1}{4} \end{array}\right) = 2 z^{4a-4} C\left(\frac{2z}{\sqrt{\pi}}\right)$$

07.35.03.0160.01

$$G_{1,3}^{1,1}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{4}, a-1, a-\frac{3}{4} \end{array}\right) = 2 z^{4a-4} S\left(\frac{2z}{\sqrt{\pi}}\right)$$

Case $\{m, n, p, q\} = \{1, 1, 2, 1\}$

07.35.03.0054.01

$$G_{2,1}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ a-\frac{1}{2} \end{array}\right) = \sqrt{\pi}\, z^{2a-1} \operatorname{erf}\left(\frac{1}{z}\right)$$

Case $\{m, n, p, q\} = \{1, 1, 2, 4\}$

07.35.03.0055.01

$$G_{2,4}^{1,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, b \\ a, b, a-\frac{1}{2}, 2b-a-\frac{1}{2} \end{array}\right) = \frac{z^{2b-1}\sin((b-a)\pi)}{\pi} \boldsymbol{L}_{2a-2b}(2z)$$

Case $\{m, n, p, q\} = \{1, 1, 3, 1\}$

07.35.03.0056.01

$$G_{3,1}^{1,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, c \\ a \end{array}\right) = z^{a+c-\frac{3}{2}} \boldsymbol{H}_{c-a-\frac{1}{2}}\left(\frac{2}{z}\right)$$

07.35.03.0057.01

$$G_{3,1}^{1,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, a+\frac{1}{2} \\ a-\frac{1}{2} \end{array}\right) = \frac{2z^{2a-1}}{\sqrt{\pi}} \operatorname{Si}\left(\frac{2}{z}\right)$$

Case $\{m, n, p, q\} = \{1, 1, 4, 2\}$

07.35.03.0058.01

$$G_{4,2}^{1,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, b, a+\frac{1}{2}, 2b-a+\frac{1}{2} \\ a, b \end{array}\right) = \frac{z^{2b-1}\sin((a-b)\pi)}{\pi} \boldsymbol{L}_{2b-2a}\left(\frac{2}{z}\right) /; z \notin (-\infty, 0)$$

Case $\{m, n, p, q\} = \{1, 2, 2, 1\}$

07.35.03.0059.01

$$G_{2,1}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b \end{array}\right) = 2\sqrt{\pi}\ \Gamma(2b-2a+1) z^{2a-1} H_{2a-2b-1}\left(\frac{1}{z}\right)$$

07.35.03.0060.01

$$G_{2,1}^{1,2}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{2}{3} \\ a-\frac{1}{6} \end{array}\right) = 2\, 2^{2/3} \sqrt[6]{3}\ \pi^{3/2} z^{\frac{3a-1}{2}} \exp\left(\frac{1}{2z^{3/2}}\right) \operatorname{Ai}\left(\frac{3^{2/3}}{2\sqrt[3]{2}\ z}\right)$$

07.35.03.0061.01

$$G_{2,1}^{1,2}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{4}{3} \\ a+\frac{1}{6} \end{array}\right) = \frac{4\sqrt[3]{2}\ \pi^{3/2}}{\sqrt[6]{3}} z^{\frac{3a+1}{2}} \exp\left(\frac{1}{2z^{3/2}}\right) \operatorname{Ai}'\left(\frac{3^{2/3}}{2\sqrt[3]{2}\ z}\right)$$

Case $\{m, n, p, q\} = \{1, 2, 2, 2\}$

07.35.03.0062.01

$$G_{2,2}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a \\ a-\frac{1}{2}, a-1 \end{array}\right) = 2\sqrt{\pi}\ z^{2(a-1)} \sinh^{-1}(z)$$

07.35.03.0063.01

$$G_{2,2}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a \\ a, a-\frac{1}{2} \end{array}\right) = \frac{2z^{2a-1}\sinh^{-1}(z)}{\sqrt{\pi}\ \sqrt{z^2+1}}$$

07.35.03.0064.01

$$G_{2,2}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ a, a-\frac{1}{2} \end{array}\right) = 2z^{2a-1}\tan^{-1}(z)$$

Case $\{m, n, p, q\} = \{1, 2, 3, 1\}$

07.35.03.0065.01

$$G_{3,1}^{1,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a+\frac{2}{3} \\ a+\frac{1}{6} \end{array}\right) = -2\sqrt{\pi}\, z^{3a-1}\left(2\pi \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)+1\right) /;\, z \notin (-\infty, 0)$$

07.35.03.0066.01

$$G_{3,1}^{1,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a+\frac{2}{3} \\ a+\frac{1}{6} \end{array}\right) = -2\sqrt{\pi}\, z^{3a-1}\left(2\pi \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)-1\right) /;\, z \notin (-\infty, 0)$$

07.35.03.0067.01

$$G_{3,1}^{1,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a+\frac{2}{3} \\ a+\frac{1}{6} \end{array}\right) = -2\pi^{3/2} z^{3a-1}\left(\operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)+\operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\right) /;\, z \notin (-\infty, 0)$$

07.35.03.0068.01

$$G_{3,1}^{1,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, a+\frac{1}{3} \\ a-\frac{1}{6} \end{array}\right) = 2\, 2^{2/3}\sqrt[3]{3}\,\pi^{3/2} z^{3a-1} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) /;\, z \notin (-\infty, 0)$$

07.35.03.0069.01

$$G_{3,1}^{1,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{4}{3}, a+\frac{2}{3} \\ a+\frac{1}{6} \end{array}\right) = 4\sqrt[3]{\frac{2}{3}}\,\pi^{3/2} z^{3a+1} \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) /;\, z \notin (-\infty, 0)$$

Case $\{m, n, p, q\} = \{1, 2, 3, 2\}$

07.35.03.0070.01

$$G_{3,2}^{1,2}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, a+\frac{1}{3} \\ a-\frac{1}{6}, a+\frac{1}{3} \end{array}\right) = 2^{2/3}\sqrt[6]{3}\,\sqrt{\pi}\, z^{\frac{3a-1}{2}} \exp\left(-\frac{1}{2z^{3/2}}\right)\operatorname{Bi}\left(\frac{3^{2/3}}{2\sqrt[3]{2}\, z}\right)$$

07.35.03.0071.01

$$G_{3,2}^{1,2}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{4}{3}, a+\frac{5}{3} \\ a+\frac{1}{6}, a+\frac{5}{3} \end{array}\right) = \frac{2\sqrt[3]{2}\,\sqrt{\pi}}{\sqrt[6]{3}}\, z^{\frac{3a+1}{2}} \exp\left(-\frac{1}{2z^{3/2}}\right)\operatorname{Bi}'\left(\frac{3^{2/3}}{2\sqrt[3]{2}\, z}\right)$$

Case $\{m, n, p, q\} = \{1, 2, 3, 5\}$

07.35.03.0161.01

$$G_{3,5}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, b \\ b-\frac{1}{2}, c, b, 2a-b+\frac{1}{2}, 2a-c \end{array}\right) = \frac{z^{2a}}{\sqrt{\pi}} I_{b-c-\frac{1}{2}}(z)\, I_{-2a+b+c-\frac{1}{2}}(z)$$

07.35.03.0162.01

$$G_{3,5}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, b \\ b-\frac{1}{2}, b, b, \frac{b}{2}, \frac{b+1}{2} \end{array}\right) = \frac{2^{2a-b-1} z^{2b-1}\,\Gamma(2b-2a)}{\pi^{3/2}\,\Gamma(b)} \left({}_1F_1(2b-2a; b; -2z) + {}_1F_1(2b-2a; b; 2z)\right)$$

07.35.03.0163.01

$$G_{3,5}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, b \\ b-\frac{1}{2}, b-1, b, \frac{b-1}{2}, \frac{b}{2} \end{array}\right) = -\frac{2^{2a-b} z^{2b-2}\,\Gamma(-2a+2b-1)}{\pi^{3/2}\,\Gamma(b)} \left({}_1F_1(-2a+2b-1; b; -2z) - {}_1F_1(-2a+2b-1; b; 2z)\right)$$

Case $\{m, n, p, q\} = \{1, 2, 5, 3\}$

07.35.03.0072.01

$$G_{5,3}^{1,2}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{3}, a-\frac{1}{3}, a-\frac{1}{3}, a+\frac{1}{6} \\ a-\frac{1}{2}, a-\frac{1}{3}, a+\frac{1}{6} \end{matrix}\right) = \frac{1}{2\sqrt{\pi}} \sqrt[3]{\frac{3}{2}}\, z^{3a-2} \operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \left(\sqrt{3} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) + \operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\right)$$

07.35.03.0073.01

$$G_{5,3}^{1,2}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{3}, a-\frac{1}{3}, a+\frac{1}{6}, a+\frac{2}{3} \\ a-\frac{1}{6}, a-\frac{1}{3}, a+\frac{1}{6} \end{matrix}\right) = -\frac{1}{2\sqrt{\pi}} \sqrt[3]{\frac{3}{2}}\, z^{3a-1} \left(\sqrt{3} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) - \operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\right) \operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

07.35.03.0074.01

$$G_{5,3}^{1,2}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{2}{3}, a-\frac{1}{6}, a+\frac{1}{3}, a+\frac{4}{3} \\ a+\frac{1}{6}, a-\frac{1}{6}, a+\frac{1}{3} \end{matrix}\right) = \frac{z^{3a+1}}{2^{2/3}\sqrt[3]{3}\sqrt{\pi}} \operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \left(\sqrt{3} \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) + \operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\right)$$

07.35.03.0075.01

$$G_{5,3}^{1,2}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{2}{3}, a-\frac{2}{3}, a-\frac{1}{6}, a+\frac{1}{3} \\ a-\frac{1}{2}, a-\frac{1}{6}, a+\frac{1}{3} \end{matrix}\right) = \frac{z^{3a-1}}{2^{2/3}\sqrt[3]{3}\sqrt{\pi}} \operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \left(\operatorname{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) - \sqrt{3} \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\right)$$

Case $\{m, n, p, q\} = \{1, 3, 4, 2\}$

07.35.03.0076.01

$$G_{4,2}^{1,3}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{2}{3}, a+\frac{4}{3}, a+\frac{1}{3} \\ a+\frac{4}{3}, a+\frac{1}{6} \end{matrix}\right) = 4\pi^{3/2} z^{3a-1} \operatorname{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \operatorname{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

Case $\{m, n, p, q\} = \{2, 0, 0, 2\}$

07.35.03.0077.01

$$G_{0,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, b, c\right) = 2 z^{b+c} K_{c-b}(2z)$$

07.35.03.0164.01

$$G_{0,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, b, c\right) = \pi \csc(\pi(b-c))\, z^{b+c} \left(I_{c-b}(2z) - I_{b-c}(2z)\right)$$

07.35.03.0078.01

$$G_{0,2}^{2,0}\left(z, \frac{1}{3} \,\middle|\, b, b+\frac{1}{3}\right) = 2\sqrt[6]{3}\, \pi z^{3b} \operatorname{Ai}\left(3^{2/3} z\right)$$

07.35.03.0079.01

$$G_{0,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, b, b+\frac{1}{2}\right) = \sqrt{\pi}\, z^{2b} e^{-2z}$$

07.35.03.0080.01

$$G_{0,2}^{2,0}\left(z, \frac{1}{3} \,\middle|\, b, b+\frac{2}{3}\right) = -\frac{2\pi z^{3b}}{\sqrt[6]{3}} \operatorname{Ai}'\left(3^{2/3} z\right)$$

Case $\{m, n, p, q\} = \{2, 0, 1, 2\}$

07.35.03.0081.01

$$G_{1,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a \\ b, b+\frac{1}{2} \end{array}\right) = 2^{2a-2b-1} e^{-z^2} z^{2b} H_{2b-2a+1}(z)$$

07.35.03.0082.01

$$G_{1,2}^{2,0}\left(z, \frac{2}{3} \middle| \begin{array}{c} a \\ a-\frac{7}{6}, a+\frac{1}{6} \end{array}\right) = -\frac{2\sqrt[3]{2}\sqrt{\pi}}{\sqrt[6]{3}} z^{\frac{3a}{2}-\frac{7}{4}} \exp\left(-\frac{z^{3/2}}{2}\right) \mathrm{Ai}'\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

07.35.03.0083.01

$$G_{1,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a \\ a-1, a-\frac{1}{2} \end{array}\right) = \sqrt{\pi}\, z^{2(a-1)} \operatorname{erfc}(z)$$

07.35.03.0084.01

$$G_{1,2}^{2,0}\left(z, \frac{2}{3} \middle| \begin{array}{c} a \\ a-\frac{5}{6}, a-\frac{1}{6} \end{array}\right) = 2^{2/3} \sqrt[6]{3} \sqrt{\pi}\, z^{\frac{6a-5}{4}} \exp\left(-\frac{z^{3/2}}{2}\right) \mathrm{Ai}\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

Case $\{m, n, p, q\} = \{2, 0, 1, 3\}$

07.35.03.0085.01

$$G_{1,3}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a \\ b, a+\frac{1}{2}, a \end{array}\right) = z^{a+b+\frac{1}{2}} Y_{-a+b-\frac{1}{2}}(2z)$$

07.35.03.0086.01

$$G_{1,3}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a \\ a-1, a-1, a-\frac{1}{2} \end{array}\right) = -\frac{2z^{2a-2}}{\sqrt{\pi}} \operatorname{Ci}(2z)$$

Case $\{m, n, p, q\} = \{2, 0, 2, 2\}$

07.35.03.0087.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, c \\ b, b+\frac{1}{2} \end{array}\right) = \frac{\Gamma(2b-2c+2)\,\theta(1-|z|)}{\Gamma\left(a-2b+c-\frac{1}{2}\right)(2(a-2b+c-1))_{2b-2c+1}} z^{2b} \left(1-z^2\right)^{a-2b+c-\frac{3}{2}} C_{2b-2c+1}^{a-2b+c-1}(z)$$

07.35.03.0088.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, c \\ b, -a+b+c \end{array}\right) = \frac{\Gamma(b+c-2a+1)\,\theta(1-|z|)}{\Gamma(c-b)} z^{3b-2a+c} \left(1-z^2\right)^{2a-2b-1} C_{b+c-2a}^{a-b}\left(\frac{z^2+1}{2z}\right)$$

07.35.03.0089.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, c \end{array}\right) = \frac{\Gamma(2c-2a+1)\,\theta(1-|z|)}{\Gamma\left(2a-b-c+\frac{1}{2}\right)(4a-2(b+c))_{2c-2a}} z^{2(b+c-a)} \left(1-z^2\right)^{2a-b-c-\frac{1}{2}} C_{2c-2a}^{2a-b-c}\left(\frac{1}{z}\right)$$

07.35.03.0090.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b-1 \end{array}\right) = \frac{2\,\theta(1-|z|)}{(2b-2a+1)\sqrt{\pi}} z^{2a-2} \sqrt{1-z^2}\, U_{2b-2a}\left(\frac{1}{z}\right)$$

07.35.03.0091.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b-\frac{1}{2} \end{array}\right) = z^{2a-1} P_{2a-2b-1}\left(\frac{1}{z}\right) \theta(1-|z|)$$

07.35.03.0092.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b \end{array}\right) = \frac{z^{2a}\,\theta(1-|z|)}{\sqrt{\pi}\,\sqrt{1-z^2}}\, T_{2a-2b}\!\left(\frac{1}{z}\right)$$

07.35.03.0093.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+1 \\ b, b+\frac{1}{2} \end{array}\right) = \frac{\theta(1-|z|)}{\sqrt{\pi}\,\sqrt{1-z^2}}\, z^{2b}\, T_{2a-2b-1}(z) \;/;\; z \notin (-1, 0)$$

07.35.03.0094.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+1 \\ b, b+\frac{1}{2} \end{array}\right) = \frac{(2a-2b-1)\,\theta(1-|z|)\, z^{2b}}{2\sqrt{\pi}\,\sqrt{1-z^2}}\, C_{2a-2b-1}^{(0)}(z) \;/;\; z \notin (-1, 0)$$

07.35.03.0095.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+\frac{3}{2} \\ b, b+\frac{1}{2} \end{array}\right) = z^{2b}\, P_{2a-2b-2}(z)\, \theta(1-|z|) \;/;\; z \notin (-1, 0)$$

07.35.03.0096.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+2 \\ b, b+\frac{1}{2} \end{array}\right) = \frac{\theta(1-|z|)}{\sqrt{\pi}\,(b-a+1)}\, \sqrt{1-z^2}\, z^{2b}\, U_{2b-2a+1}(z) \;/;\; z \notin (-1, 0)$$

07.35.03.0097.01

$$G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+2 \\ b, b+\frac{1}{2} \end{array}\right) = \frac{\theta(1-|z|)}{(a-b-1)\sqrt{\pi}}\, z^{2b}\, \sqrt{1-z^2}\, U_{2a-2b-3}(z)$$

Case $\{m, n, p, q\} = \{2, 0, 2, 3\}$

07.35.03.0098.01

$$G_{2,3}^{2,0}\left(z, \frac{2}{3} \middle| \begin{array}{c} a, a+\frac{3}{2} \\ a+\frac{1}{3}, a+\frac{5}{3}, a \end{array}\right) = -\frac{\sqrt[3]{2}}{\sqrt[6]{3}\,\sqrt{\pi}}\, z^{\frac{3a+1}{2}} \exp\!\left(\frac{z^{3/2}}{2}\right) \mathrm{Bi}'\!\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

Case $\{m, n, p, q\} = \{2, 0, 2, 4\}$

07.35.03.0165.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b, a, a \end{array}\right) = \frac{z^{2a}}{2\sqrt{\pi}} \left(I_{a-b}(z)^2 + I_{b-a}(z)^2\right)$$

07.35.03.0166.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ 2a-b, b, a, a+\frac{1}{2} \end{array}\right) = \frac{z^{2a}}{2\pi} \left(I_{2b-2a}(2z) + I_{2a-2b}(2z)\right)$$

07.35.03.0167.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, 2a-b, 2a-b-\frac{1}{2}, b+\frac{1}{2} \end{array}\right) = \frac{1}{\sqrt{2}\,\pi}\, z^{2a-\frac{1}{2}} \left(I_{-2a+2b+\frac{1}{2}}(z)\cosh(z) + I_{2a-2b-\frac{1}{2}}(z)\sinh(z)\right)$$

07.35.03.0099.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ a-\frac{1}{2}, a-\frac{1}{2}, a, a \end{array}\right) = -\frac{2z^{2a-1}}{\pi^{3/2}}\, \mathrm{Chi}(2z)$$

07.35.03.0100.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a-\frac{1}{6}, a+\frac{1}{6}, a, a+\frac{1}{2} \end{matrix}\right) = \frac{\sqrt[6]{3}\, z^{3a-\frac{1}{2}}}{2\pi} \operatorname{Bi}\left(3^{2/3} z\right)$$

07.35.03.0101.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a, a, a-\frac{1}{2}, a-\frac{1}{2} \end{matrix}\right) = \frac{2\, z^{2a-1}}{\pi^{3/2}} \operatorname{Shi}(2z)$$

07.35.03.0102.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a+\frac{1}{6}, a+\frac{5}{6}, a, a+\frac{1}{2} \end{matrix}\right) = -\frac{z^{3a+\frac{1}{2}}}{2\sqrt[6]{3}\,\pi} \operatorname{Bi}'\left(3^{2/3} z\right)$$

07.35.03.0103.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a-\frac{5}{12}, a-\frac{1}{12}, a+\frac{5}{12}, a+\frac{1}{12} \end{matrix}\right) = \frac{1}{\pi} \sqrt[6]{\frac{3}{2}}\, z^{3a-\frac{5}{4}} \cosh\left(z^{3/2}\right) \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0104.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a-\frac{1}{6}, a+\frac{1}{6}, a, a+\frac{1}{2} \end{matrix}\right) = \frac{\sqrt[6]{3}}{2\pi}\, z^{-\frac{1}{2}+3a} \operatorname{Bi}\left(3^{2/3} z\right)$$

07.35.03.0105.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a+\frac{1}{12}, a+\frac{5}{12}, a-\frac{1}{12}, a-\frac{5}{12} \end{matrix}\right) = -\frac{1}{\pi} \sqrt[6]{\frac{3}{2}}\, z^{3a-\frac{5}{4}} \sinh\left(z^{3/2}\right) \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0106.01

$$G_{2,4}^{2,0}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a, a+\frac{1}{2} \\ a+\frac{1}{6}, a+\frac{5}{6}, a, a+\frac{1}{2} \end{matrix}\right) = -\frac{1}{2\sqrt[6]{3}\,\pi}\, z^{3a+\frac{1}{2}} \operatorname{Bi}'\left(3^{2/3} z\right)$$

Case $\{m, n, p, q\} = \{2, 1, 1, 2\}$

07.35.03.0107.01

$$G_{1,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a \\ b, b+\frac{1}{2} \end{matrix}\right) = 2\sqrt{\pi}\, z^{2b}\, \Gamma(2(b-a+1))\, H_{2(a-b-1)}(z)$$

07.35.03.0108.01

$$G_{1,2}^{2,1}\left(z, \frac{2}{3} \,\middle|\, \begin{matrix} a \\ a-\frac{7}{6}, a+\frac{1}{6} \end{matrix}\right) = \frac{4\sqrt[3]{2}}{\sqrt[6]{3}}\, \pi^{3/2}\, z^{\frac{3a}{2}-\frac{7}{4}} \exp\left(\frac{z^{3/2}}{2}\right) \operatorname{Ai}'\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

07.35.03.0109.01

$$G_{1,2}^{2,1}\left(z, \frac{2}{3} \,\middle|\, \begin{matrix} a \\ a-\frac{5}{6}, a-\frac{1}{6} \end{matrix}\right) = 2\, 2^{2/3} \sqrt[6]{3}\, \pi^{3/2}\, z^{\frac{6a-5}{4}} \exp\left(\frac{z^{3/2}}{2}\right) \operatorname{Ai}\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

Case $\{m, n, p, q\} = \{2, 1, 1, 3\}$

07.35.03.0110.01

$$G_{1,3}^{2,1}\left(z, \frac{1}{3} \,\middle|\, \begin{matrix} a \\ a-\frac{7}{6}, a+\frac{1}{6}, a-\frac{1}{2} \end{matrix}\right) = 4\sqrt[3]{\frac{2}{3}}\, \pi^{3/2}\, z^{3a-\frac{7}{2}} \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0111.01

$$G_{1,3}^{2,1}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{5}{6}, a-\frac{1}{6}, a-\frac{1}{2} \end{array}\right) = 2\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2} z^{3a-\frac{5}{2}} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0112.01

$$G_{1,3}^{2,1}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{6}, a+\frac{1}{6}, a-\frac{1}{2} \end{array}\right) = -2\sqrt{\pi}\, z^{3a-\frac{3}{2}} \left(2\pi \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) + 1\right)$$

07.35.03.0113.01

$$G_{1,3}^{2,1}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{6}, a+\frac{1}{6}, a-\frac{1}{2} \end{array}\right) = -2\sqrt{\pi}\, z^{3a-\frac{3}{2}} \left(2\pi \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) - 1\right)$$

07.35.03.0114.01

$$G_{1,3}^{2,1}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{6}, a+\frac{1}{6}, a-\frac{1}{2} \end{array}\right) = -2\pi^{3/2} z^{3a-\frac{3}{2}} \left(\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) + \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)\right)$$

Case $\{m, n, p, q\} = \{2, 1, 2, 2\}$

07.35.03.0115.01

$$G_{2,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ a-\frac{1}{2}, a-\frac{1}{2} \end{array}\right) = 2\sqrt{\pi}\, z^{2a-1} \operatorname{csch}^{-1}(z)$$

07.35.03.0116.01

$$G_{2,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ a, a-\frac{1}{2} \end{array}\right) = 2 z^{2a-1} \cot^{-1}(z)$$

07.35.03.0117.01

$$G_{2,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ a, a \end{array}\right) = \frac{2 z^{2a-1} \operatorname{csch}^{-1}(z)}{\sqrt{\pi} \sqrt{1+\frac{1}{z^2}}}$$

Case $\{m, n, p, q\} = \{2, 1, 2, 3\}$

07.35.03.0118.01

$$G_{2,3}^{2,1}\left(z, \frac{2}{3} \,\middle|\, \begin{array}{c} a, a-\frac{3}{2} \\ a-\frac{7}{6}, a+\frac{1}{6}, a-\frac{3}{2} \end{array}\right) = \frac{2\sqrt[3]{2}\sqrt{\pi}}{\sqrt[6]{3}} z^{\frac{3a}{2}-\frac{7}{4}} \exp\left(-\frac{z^{3/2}}{2}\right) \operatorname{Bi}'\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

07.35.03.0119.01

$$G_{2,3}^{2,1}\left(z, \frac{2}{3} \,\middle|\, \begin{array}{c} a, a-\frac{1}{2} \\ a-\frac{5}{6}, a-\frac{1}{6}, a-\frac{1}{2} \end{array}\right) = 2^{2/3} \sqrt[6]{3} \sqrt{\pi}\, z^{\frac{3a}{2}-\frac{5}{4}} \exp\left(-\frac{z^{3/2}}{2}\right) \operatorname{Bi}\left(\frac{3^{2/3} z}{2\sqrt[3]{2}}\right)$$

Case $\{m, n, p, q\} = \{2, 1, 3, 5\}$

07.35.03.0120.01

$$G_{3,5}^{2,1}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a, a-\frac{5}{6}, a-\frac{1}{3} \\ a-\frac{7}{6}, a-\frac{1}{2}, a-\frac{5}{6}, a-\frac{1}{3}, a+\frac{1}{6} \end{array}\right) = \frac{1}{2^{2/3}\sqrt[3]{3}\sqrt{\pi}} z^{3a-\frac{7}{2}} \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) \left(\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) - \sqrt{3} \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)\right)$$

07.35.03.0121.01

$$G_{3,5}^{2,1}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a-\frac{2}{3}, a-\frac{1}{6} \\ a-\frac{5}{6}, a-\frac{1}{2}, a-\frac{2}{3}, a-\frac{1}{6}, a-\frac{1}{6} \end{array}\right) = \frac{1}{2\sqrt{\pi}} \sqrt[3]{\frac{3}{2}}\, z^{3a-\frac{5}{2}} \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \left(\sqrt{3} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) + \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right)\right)$$

07.35.03.0122.01

$$G_{3,5}^{2,1}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a-\frac{1}{3}, a+\frac{1}{6} \\ a-\frac{1}{2}, a-\frac{1}{6}, a-\frac{5}{6}, a-\frac{1}{3}, a+\frac{1}{6} \end{array}\right) = \frac{1}{2\sqrt{\pi}} \sqrt[3]{\frac{3}{2}}\, z^{3a-\frac{5}{2}} \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \left(\operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) - \sqrt{3} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right)\right)$$

07.35.03.0123.01

$$G_{3,5}^{2,1}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a-\frac{1}{6}, a+\frac{1}{3} \\ a-\frac{1}{2}, a+\frac{1}{6}, a-\frac{7}{6}, a-\frac{1}{6}, a+\frac{1}{3} \end{array}\right) = \frac{1}{2^{2/3}\sqrt[3]{3}\sqrt{\pi}}\, z^{3a-\frac{7}{2}} \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) \left(\sqrt{3} \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) + \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)\right)$$

Case $\{m, n, p, q\} = \{2, 2, 2, 4\}$

07.35.03.0124.01

$$G_{2,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, b+\frac{1}{3}, 2a-b-\frac{1}{3}, 2a-b \end{array}\right) = 2\sqrt[3]{2}\sqrt[6]{3}\,\pi^{3/2} z^{3b} \left(z^{3/2}\right)^{2a-2b-\frac{1}{3}} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) I_{2b-2a+\frac{1}{3}}\left(z^{3/2}\right)$$

07.35.03.0125.01

$$G_{2,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ b, b+\frac{2}{3}, 2a-b-\frac{2}{3}, 2a-b \end{array}\right) = -\frac{2\, 2^{2/3}\pi^{3/2}}{\sqrt[6]{3}} z^{3b} \left(z^{3/2}\right)^{2a-2b-\frac{2}{3}} \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) I_{2b-2a+\frac{2}{3}}\left(z^{3/2}\right)$$

07.35.03.0126.01

$$G_{2,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ a-\frac{7}{12}, a+\frac{1}{12}, a-\frac{1}{12}, a+\frac{7}{12} \end{array}\right) = -4\sqrt[6]{\frac{2}{3}}\,\pi\, z^{3a-\frac{7}{4}} \cosh\left(z^{3/2}\right) \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0127.01

$$G_{2,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ a-\frac{1}{6}, a+\frac{1}{2}, a-\frac{1}{2}, a+\frac{1}{6} \end{array}\right) = 2\pi^{3/2} z^{3a-\frac{3}{2}} \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) \left(\sqrt{3} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) - \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right)\right)$$

07.35.03.0128.01

$$G_{2,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2} \\ a-\frac{1}{12}, a+\frac{7}{12}, a-\frac{7}{12}, a+\frac{1}{12} \end{array}\right) = -4\sqrt[6]{\frac{2}{3}}\,\pi\, z^{3a-\frac{7}{4}} \sinh\left(z^{3/2}\right) \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0129.01

$$G_{2,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{7}{6} \\ a, a+\frac{1}{3}, a+1, a+\frac{2}{3} \end{array}\right) = -2\, 2^{2/3}\sqrt[3]{3}\,\pi^{3/2} z^{3a+1} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

Case $\{m, n, p, q\} = \{2, 2, 3, 3\}$

07.35.03.0130.01

$$G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, 2b-a+1, a-\frac{1}{2} \\ b, b+\frac{1}{2}, a-\frac{1}{2} \end{array}\right) =$$

$$\frac{\sqrt{\pi}\, z^{2b}}{\sqrt{z^2+1}} \left(\tan((a-b)\pi) \sinh\left((2b-2a+1)\sinh^{-1}(z)\right) - \cot((a-b)\pi) \cosh\left((2b-2a+1)\sinh^{-1}(z)\right)\right)$$

07.35.03.0131.01

$$G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, b+\frac{1}{2} \\ b, 2a-b, b+\frac{1}{2} \end{array}\right) = \frac{\sqrt{\pi}\,\csc(2(a-b)\pi)\,z^{-2b}}{\sqrt{z^2+1}}\left(\sqrt{z^2+1}+1\right)^{-2(a+b)}\left(z^{4b}\left(\sqrt{z^2+1}+1\right)^{4a}-\cos(2(a-b)\pi)\,z^{4a}\left(\sqrt{z^2+1}+1\right)^{4b}\right)$$

07.35.03.0132.01

$$G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, a+\frac{1}{2} \\ a-\frac{1}{4}, a+\frac{1}{4}, a-\frac{1}{2} \end{array}\right) = 2\sqrt{2\pi}\, z^{2a-1}\cos^{-1}\left(\sqrt{z^2+1}-z\right) /; z \notin (-\infty, 0)$$

07.35.03.0133.01

$$G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} a, a+\frac{1}{2}, a+\frac{3}{4} \\ a-\frac{1}{4}, a+\frac{1}{4}, a-\frac{1}{4} \end{array}\right) = 2\sqrt{2\pi}\, z^{2a-\frac{1}{2}}\cos^{-1}\left(\frac{\sqrt{z^2+1}-1}{z}\right) /; z \notin (-\infty, 0)$$

Case $\{m, n, p, q\} = \{2, 2, 4, 2\}$

07.35.03.0134.01

$$G_{4,2}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, 2b-a+\frac{2}{3}, 2b-a+1 \\ b, b+\frac{1}{2} \end{array}\right) = 2\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}\left(\frac{1}{z^{3/2}}\right)^{2a-2b+\frac{1}{3}} z^{3a-\frac{1}{2}}\,\mathrm{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) I_{2b-2a+\frac{2}{3}}\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0135.01

$$G_{4,2}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, 2b-a+\frac{1}{3}, 2b-a+1 \\ b, b+\frac{1}{2} \end{array}\right) = -\frac{2\,2^{2/3}\,\pi^{3/2}}{\sqrt[6]{3}}\left(\frac{1}{z^{3/2}}\right)^{2a-2b-\frac{4}{3}} z^{3a-\frac{5}{2}}\,\mathrm{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) I_{2b-2a+\frac{1}{3}}\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0136.01

$$G_{4,2}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a-\frac{2}{3}, a-\frac{1}{3} \\ a-\frac{5}{6}, a+\frac{1}{3} \end{array}\right) = -2\,2^{2/3}\sqrt[3]{3}\,\pi^{3/2} z^{3a-3}\,\mathrm{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\mathrm{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

07.35.03.0137.01

$$G_{4,2}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, a-\frac{1}{2}, a+\frac{1}{6} \\ a-\frac{5}{12}, a+\frac{1}{12} \end{array}\right) = -4\sqrt[6]{\frac{2}{3}}\,\pi\, z^{3a-1}\cosh\left(\frac{1}{z^{3/2}}\right)\mathrm{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

07.35.03.0138.01

$$G_{4,2}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, a+\frac{1}{3}, a+1 \\ a, a+\frac{1}{2} \end{array}\right) = 2\pi^{3/2} z^{3a}\,\mathrm{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\left(\sqrt{3}\,\mathrm{Ai}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)-\mathrm{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)\right)$$

07.35.03.0139.01

$$G_{4,2}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, a+\frac{1}{2}, a+\frac{7}{6} \\ a+\frac{1}{12}, a+\frac{7}{12} \end{array}\right) = -4\pi\sqrt[6]{\frac{2}{3}}\, z^{3a+\frac{1}{2}}\sinh\left(\frac{1}{z^{3/2}}\right)\mathrm{Ai}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

Case $\{m, n, p, q\} = \{2, 2, 4, 6\}$

07.35.03.0140.01

$$G_{4,6}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2}, b+\frac{1}{6}, b+\frac{2}{3} \\ b, b+\frac{1}{3}, 2a-b, 2a-b-\frac{1}{3}, b+\frac{1}{6}, b+\frac{2}{3} \end{array}\right) = \frac{\sqrt[6]{3}}{2^{2/3}\sqrt{\pi}} z^{3b} \left(z^{3/2}\right)^{2a-2b-\frac{1}{3}} \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) I_{2b-2a+\frac{1}{3}}\left(z^{3/2}\right)$$

07.35.03.0141.01

$$G_{4,6}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2}, a-\frac{1}{4}, a+\frac{1}{4} \\ a-\frac{7}{12}, a+\frac{1}{12}, a+\frac{7}{12}, a+\frac{1}{4}, a-\frac{1}{12}, a-\frac{1}{4} \end{array}\right) = \frac{1}{\pi}\sqrt[6]{\frac{2}{3}}\, z^{3a-\frac{7}{4}} \cosh\left(z^{3/2}\right) \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0142.01

$$G_{4,6}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2}, a-\frac{1}{4}, a+\frac{1}{4} \\ a-\frac{5}{12}, a-\frac{1}{12}, a-\frac{1}{4}, a+\frac{1}{12}, a+\frac{1}{4}, a+\frac{5}{12} \end{array}\right) = \frac{1}{\pi}\sqrt[6]{\frac{3}{2}}\, z^{3a-\frac{5}{4}} \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \cosh\left(z^{3/2}\right)$$

07.35.03.0143.01

$$G_{4,6}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2}, a-\frac{1}{4}, a+\frac{1}{4} \\ a+\frac{1}{12}, a+\frac{5}{12}, a-\frac{5}{12}, a-\frac{1}{4}, a-\frac{1}{12}, a+\frac{1}{4} \end{array}\right) = -\frac{1}{\pi}\sqrt[6]{\frac{3}{2}}\, z^{3a-\frac{5}{4}} \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \sinh\left(z^{3/2}\right)$$

07.35.03.0144.01

$$G_{4,6}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2}, a+\frac{1}{4}, a+\frac{3}{4} \\ a-\frac{1}{12}, a+\frac{7}{12}, a+\frac{3}{4}, a+\frac{1}{4}, a+\frac{1}{12}, a-\frac{7}{12} \end{array}\right) = \frac{1}{\pi}\sqrt[6]{\frac{2}{3}}\, z^{3a-\frac{7}{4}} \sinh\left(z^{3/2}\right) \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

07.35.03.0145.01

$$G_{4,6}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{2}, b+\frac{1}{3}, b+\frac{5}{6} \\ b, b+\frac{2}{3}, b+\frac{1}{3}, b+\frac{5}{6}, 2a-b-\frac{2}{3}, 2a-b \end{array}\right) = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}} z^{3b} \left(z^{3/2}\right)^{2a-2b-\frac{2}{3}} \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z\right) I_{2b-2a+\frac{2}{3}}\left(z^{3/2}\right)$$

Case $\{m, n, p, q\} = \{2, 2, 6, 4\}$

07.35.03.0146.01

$$G_{6,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, b, b+\frac{1}{3}, a-\frac{1}{3}, a+\frac{1}{6} \\ \frac{3a+3b-2}{6}, \frac{3a+3b+1}{6}, a-\frac{1}{3}, a+\frac{1}{6} \end{array}\right) = \frac{\sqrt[6]{3}}{2^{2/3}\sqrt{\pi}} \left(\frac{1}{z^{3/2}}\right)^{a-b} z^{3a-2} \operatorname{Bi}\left(\frac{1}{z}\left(\frac{3}{2}\right)^{2/3}\right) I_{b-a}\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0147.01

$$G_{6,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{2}{3}, b, b+\frac{2}{3}, a-\frac{1}{6}, a+\frac{1}{3} \\ \frac{3a+3b-1}{6}, \frac{3a+3b+2}{6}, a-\frac{1}{6}, a+\frac{1}{3} \end{array}\right) = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}} \left(\frac{1}{z^{3/2}}\right)^{a-b} z^{3a-1} \operatorname{Bi}'\left(\frac{1}{z}\left(\frac{3}{2}\right)^{2/3}\right) I_{b-a}\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0148.01

$$G_{6,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a-\frac{1}{2}, a-\frac{1}{3}, a-\frac{1}{6}, a+\frac{1}{6} \\ a-\frac{7}{12}, a-\frac{1}{12}, a-\frac{1}{3}, a+\frac{1}{6} \end{array}\right) = \frac{1}{\pi}\sqrt[6]{\frac{3}{2}}\, z^{3a-2} \operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \cosh\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0149.01

$$G_{6,4}^{2,2}\left(z, \frac{1}{3} \middle| \begin{array}{c} a, a+\frac{1}{3}, a+\frac{1}{6}, a+\frac{1}{2}, a+\frac{2}{3}, a+\frac{5}{6} \\ a-\frac{1}{12}, a+\frac{5}{12}, a+\frac{1}{6}, a+\frac{2}{3} \end{array}\right) = -\frac{1}{\pi}\sqrt[6]{\frac{3}{2}}\, z^{3a-\frac{1}{2}} \operatorname{Bi}\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right) \sinh\left(\frac{1}{z^{3/2}}\right)$$

07.35.03.0150.01

$$G_{6,4}^{2,2}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a, a+\frac{2}{3}, a-\frac{1}{2}, a-\frac{1}{6}, a+\frac{1}{6}, a+\frac{1}{3} \\ a-\frac{5}{12}, a+\frac{1}{12}, a-\frac{1}{6}, a+\frac{1}{3} \end{array}\right) = \frac{1}{\pi} \sqrt[6]{\frac{2}{3}}\, z^{3a-1} \cosh\left(\frac{1}{z^{3/2}}\right) \mathrm{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

07.35.03.0151.01

$$G_{6,4}^{2,2}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a, a+\frac{2}{3}, a-\frac{1}{6}, a+\frac{1}{3}, a+\frac{1}{2}, a+\frac{7}{6} \\ a+\frac{1}{12}, a+\frac{7}{12}, a-\frac{1}{6}, a+\frac{1}{3} \end{array}\right) = \frac{1}{\pi} \sqrt[6]{\frac{2}{3}}\, z^{3a+\frac{1}{2}} \sinh\left(\frac{1}{z^{3/2}}\right) \mathrm{Bi}'\left(\frac{\left(\frac{3}{2}\right)^{2/3}}{z}\right)$$

Case $\{m, n, p, q\} = \{3, 0, 1, 3\}$

07.35.03.0152.01

$$G_{1,3}^{3,0}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{2}, a-\frac{7}{6}, a+\frac{1}{6} \end{array}\right) = 4 \sqrt[3]{\frac{2}{3}}\, \pi^{3/2} z^{3a-\frac{7}{2}} \mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)^2$$

07.35.03.0153.01

$$G_{1,3}^{3,0}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{5}{6}, a-\frac{1}{2}, a-\frac{1}{6} \end{array}\right) = 2\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2} z^{3a-\frac{5}{2}} \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right)^2$$

07.35.03.0154.01

$$G_{1,3}^{3,0}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{2}, a-\frac{1}{6}, a+\frac{1}{6} \end{array}\right) = -4 \pi^{3/2} z^{3a-\frac{3}{2}} \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

Case $\{m, n, p, q\} = \{3, 0, 2, 6\}$

07.35.03.0168.01

$$G_{2,6}^{3,0}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ 2a-b, b, b+\frac{1}{2}, a, a+\frac{1}{2}, 3b-2a \end{array}\right) = \frac{\sec(2(a-b)\pi)}{2\sqrt{\pi}} z^{4b} I_{4a-4b}\left(2\sqrt{2}\, z\right) \left(J_{4b-4a}\left(2\sqrt{2}\, z\right) + J_{4a-4b}\left(2\sqrt{2}\, z\right)\right)$$

07.35.03.0169.01

$$G_{2,6}^{3,0}\left(z, \frac{1}{4} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ 2a-b-\frac{1}{2}, b, b+\frac{1}{2}, a, a+\frac{1}{2}, -2a+3b+\frac{1}{2} \end{array}\right) = -\frac{\sec(2(a-b)\pi)}{2\sqrt{\pi}} z^{4b} I_{4a-4b-1}\left(2\sqrt{2}\, z\right) \left(J_{-4a+4b+1}\left(2\sqrt{2}\, z\right) - J_{4a-4b-1}\left(2\sqrt{2}\, z\right)\right)$$

Case $\{m, n, p, q\} = \{3, 1, 1, 3\}$

07.35.03.0170.01

$$G_{1,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a \\ a-\frac{1}{2}, b, 2a-b-1 \end{array}\right) = \frac{1}{2} \pi^{5/2} z^{2a-1} \csc^2((a-b)\pi) \left(J_{a-b-\frac{1}{2}}(z)\, J_{-a+b+\frac{1}{2}}(z) + Y_{a-b-\frac{1}{2}}(z)\, Y_{-a+b+\frac{1}{2}}(z)\right)$$

Case $\{m, n, p, q\} = \{3, 1, 2, 4\}$

07.35.03.0155.01

$$G_{2,4}^{3,1}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a, a+\frac{7}{6} \\ a, a+\frac{2}{3}, a+\frac{4}{3}, a+1 \end{array}\right) = 4 \pi^{3/2} z^{3a+2} \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) \mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z\right)$$

Case $\{m, n, p, q\} = \{4, 0, 2, 4\}$

07.35.03.0156.01

$$G_{2,4}^{4,0}\left(z, \frac{1}{3} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ b, b+\frac{1}{3}, 2a-b-\frac{1}{3}, 2a-b \end{array}\right) = 2\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}\, z^{3a-\frac{1}{2}} \operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z\right) K_{2b-2a+\frac{1}{3}}\left(z^{3/2}\right) /; \operatorname{Re}(z) > 0$$

07.35.03.0171.01

$$G_{2,4}^{4,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} a, a+\frac{1}{2} \\ b, b+\frac{2}{3}, 2a-b-\frac{2}{3}, 2a-b \end{array}\right) = -\frac{2\, 2^{2/3}\sqrt{\pi}}{\sqrt[6]{3}} z^{\frac{2}{3}(3a-1)} \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) K_{2a-2b-\frac{2}{3}}(z)$$

General characteristics

Branch cuts

Branch cut locations: When $p == q$, then there is a branch cut along $(-\infty, 0)$. If $p \neq q$, however, then the existence of the branch cut depends on the parameters of MeijerG; if the branch cut exists, it will be along $(-\infty, 0)$. In cases where $p == q$ and $m+n-p == 0$, there will be a jump discontinuity on the unit circle $|z| == 1$.

Sets of discontinuity

$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{array}{c} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array}\right)$ has discontinuity on the unit circle $|z| == 1$in the case $p == q == m + n$.

Series representations

Generalized power series

Expansions at $z == 0$

07.35.06.0001.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{array}{c} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array}\right) = \pi^{m-1} \sum_{k=1}^{m} \frac{\prod_{j=1}^{n} \Gamma\left(1-a_j+b_k\right)}{\prod_{\substack{j=1 \\ j\neq k}}^{m} \sin\left(\pi\left(b_j-b_k\right)\right) \prod_{j=n+1}^{p} \Gamma\left(a_j-b_k\right)} z^{\frac{b_k}{r}}$$

$${}_p\tilde{F}_{q-1}\left(1-a_1+b_k, \ldots, 1-a_p+b_k; 1-b_1+b_k, \ldots, 1-b_{k-1}+b_k, 1-b_{k+1}+b_k, \ldots, 1-b_q+b_k; (-1)^{p-m-n} z^{1/r}\right) /;$$
$$(p<q \lor p == q \land |z|<1) \land \forall_{\{j,k\},\{j,k\}\in\mathbb{Z}\land j\neq k\land 1\leq j\leq m\land 1\leq k\leq m} \left(b_j-b_k \notin \mathbb{Z}\right)$$

Expansions at $z == \infty$

07.35.06.0002.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{array}{c} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array}\right) = \pi^{n-1} \sum_{k=1}^{n} \frac{\prod_{j=1}^{m} \Gamma\left(-a_k+b_j+1\right)}{\prod_{\substack{j=1 \\ j\neq k}}^{n} \sin\left(\pi\left(a_k-a_j\right)\right) \prod_{j=m+1}^{q} \Gamma\left(a_k-b_j\right)} z^{\frac{a_k-1}{r}}$$

$${}_q\tilde{F}_{p-1}\left(1-a_k+b_1, \ldots, 1-a_k+b_q; 1-a_k+a_1, \ldots, 1-a_k+a_{k-1}, 1-a_k+a_{k+1}, \ldots, 1-a_k+a_p; \frac{(-1)^{q-m-n}}{z^{1/r}}\right) /;$$
$$(p>q \lor p == q \land |z|>1) \land \forall_{\{j,k\},\{j,k\}\in\mathbb{Z}\land j\neq k\land 1\leq j\leq n\land 1\leq k\leq n} \left(a_j-a_k \notin \mathbb{Z}\right)$$

Residue representations

07.35.06.0003.01

$$G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) = \sum_{k=1}^{m} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\prod_{k=1}^{m} \Gamma(s+b_k) \prod_{k=1}^{n} \Gamma(1-a_k-s)}{\prod_{k=n+1}^{p} \Gamma(s+a_k) \prod_{k=m+1}^{q} \Gamma(1-b_k-s)} z^{-\frac{s}{r}} \right) (-b_k - j) \;/;$$

$p < q \lor p = q \land |z| < 1$

07.35.06.0004.01

$$G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) = -\sum_{k=1}^{n} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\prod_{k=1}^{m} \Gamma(s+b_k) \prod_{k=1}^{n} \Gamma(1-a_k-s)}{\prod_{k=n+1}^{p} \Gamma(s+a_k) \prod_{k=m+1}^{q} \Gamma(1-b_k-s)} z^{-\frac{s}{r}} \right) (1 - a_k + j) \;/;$$

$p > q \lor p = q \land |z| > 1$

Integral representations

Contour integral representations

07.35.07.0001.01

$$G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) = \frac{1}{2 \pi i} \int_{\mathcal{L}} \frac{\left(\prod_{k=1}^{m} \Gamma(s+b_k)\right) \prod_{k=1}^{n} \Gamma(1-a_k-s)}{\left(\prod_{k=n+1}^{p} \Gamma(s+a_k)\right) \prod_{k=m+1}^{q} \Gamma(1-b_k-s)} z^{-\frac{s}{r}} \, ds \;/;$$

$r \in \mathbb{R} \land r \neq 0 \land m \in \mathbb{N} \land n \in \mathbb{N} \land p \in \mathbb{N} \land q \in \mathbb{N} \land m \leq q \land n \leq p$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.35.16.0001.01

$$G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} \alpha + a_1, \ldots, \alpha + a_n, \alpha + a_{n+1}, \ldots, \alpha + a_p \\ \alpha + b_1, \ldots, \alpha + b_m, \alpha + b_{m+1}, \ldots, \alpha + b_q \end{array} \right.\right) = z^{\alpha/r} \, G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right)$$

07.35.16.0002.01

$$G_{q,p}^{n,m}\left(\frac{1}{z}, r \left| \begin{array}{l} 1 - b_1, \ldots, 1 - b_m, 1 - b_{m+1}, \ldots, 1 - b_q \\ 1 - a_1, \ldots, 1 - a_n, 1 - a_{n+1}, \ldots, 1 - a_p \end{array} \right.\right) = G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) \;/;\; z \notin \mathbb{R}$$

07.35.16.0003.01

$$G_{p,q}^{m,n}\left(\frac{1}{z}, -r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) = G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) \;/;\; z \notin \mathbb{R}$$

07.35.16.0004.01

$$G_{q,p}^{n,m}\left(z, -r \left| \begin{array}{l} 1 - b_1, \ldots, 1 - b_m, 1 - b_{m+1}, \ldots, 1 - b_q \\ 1 - a_1, \ldots, 1 - a_n, 1 - a_{n+1}, \ldots, 1 - a_p \end{array} \right.\right) = G_{p,q}^{m,n}\left(z, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) \;/;\; z \notin \mathbb{R}$$

07.35.16.0005.01

$$G_{p,q}^{m,n}\left(z^t, r \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) = G_{p,q}^{m,n}\left(z, \frac{r}{t} \left| \begin{array}{l} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{array} \right.\right) \;/;\; z \notin \mathbb{R} \land t \in \mathbb{R} \land -1 < t \leq 1$$

Identities

Functional identities

07.35.17.0001.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = z^{-\frac{\alpha}{r}} G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} \alpha + a_1, \ldots, \alpha + a_n, \alpha + a_{n+1}, \ldots, \alpha + a_p \\ \alpha + b_1, \ldots, \alpha + b_m, \alpha + b_{m+1}, \ldots, \alpha + b_q \end{matrix}\right)$$

07.35.17.0002.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = G_{q,p}^{n,m}\left(\frac{1}{z}, r \,\middle|\, \begin{matrix} 1 - b_1, \ldots, 1 - b_m, 1 - b_{m+1}, \ldots, 1 - b_q \\ 1 - a_1, \ldots, 1 - a_n, 1 - a_{n+1}, \ldots, 1 - a_p \end{matrix}\right) /; z \notin \mathbb{R}$$

07.35.17.0004.01

$$G_{p,q}^{m,n}\left(z, -r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = G_{p,q}^{m,n}\left(\frac{1}{z}, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) /; z \notin \mathbb{R}$$

07.35.17.0005.01

$$G_{p,q}^{m,n}\left(z, -r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = G_{q,p}^{n,m}\left(z, r \,\middle|\, \begin{matrix} 1 - b_1, \ldots, 1 - b_m, 1 - b_{m+1}, \ldots, 1 - b_q \\ 1 - a_1, \ldots, 1 - a_n, 1 - a_{n+1}, \ldots, 1 - a_p \end{matrix}\right) /; z \notin \mathbb{R}$$

07.35.17.0003.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) =$$

$$(2\pi)^{\xi} r^{\theta} G_{p\,u,q\,u}^{m\,u,n\,u}\left(u^{r(p-q)} z, \frac{1}{v} \,\middle|\, \begin{matrix} \frac{a_1}{u}, \ldots, \frac{a_1+u-1}{u}, \ldots, \frac{a_n}{u}, \ldots, \frac{a_n+u-1}{u}, \frac{a_{n+1}}{u}, \ldots, \frac{a_{n+1}+u-1}{u}, \ldots, \frac{a_p}{u}, \ldots, \frac{a_p+u-1}{u} \\ \frac{b_1}{u}, \ldots, \frac{b_1+u-1}{u}, \ldots, \frac{b_m}{u}, \ldots, \frac{b_m+u-1}{u}, \frac{b_{m+1}}{u}, \ldots, \frac{b_{m+1}+u-1}{u}, \ldots, \frac{b_q}{u}, \ldots, \frac{b_q+u-1}{u} \end{matrix}\right) /;$$

$$r = \frac{u}{v} \bigwedge u \in \mathbb{N}^+ \bigwedge v \in \mathbb{N}^+ \bigwedge \xi = \left(\frac{p+q}{2} - m - n\right)(u-1) \bigwedge \theta = \frac{p-q}{2} - \sum_{k=1}^{p} a_k + \sum_{k=1}^{q} b_k + 1$$

Differentiation

Low-order differentiation

With respect to *z*

07.35.20.0001.01

$$\frac{\partial G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)}{\partial z} = \frac{1}{r} G_{p+1,q+1}^{m,n+1}\left(z, r \,\middle|\, \begin{matrix} -r, a_1 - r, \ldots, a_n - r, a_{n+1} - r, \ldots, a_p - r \\ b_1 - r, \ldots, b_m - r, 1 - r, b_{m+1} - r, \ldots, b_q - r \end{matrix}\right)$$

Symbolic differentiation

With respect to *z*

07.35.20.0002.02

$$\frac{\partial^u G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)}{\partial z^u} =$$

$$r^{-u} G_{p+u,q+u}^{m,n+u}\left(z, r \,\middle|\, \begin{matrix} -r, -2r, \ldots, -u\,r, a_1 - u\,r, \ldots, a_n - u\,r, a_{n+1} - u\,r, \ldots, a_p - u\,r \\ b_1 - u\,r, \ldots, b_m - u\,r, 1 - r, 1 - 2r, \ldots, 1 - u\,r, b_{m+1} - u\,r, \ldots, b_q - u\,r \end{matrix}\right) /; u \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *z*

07.35.20.0003.01

$$\frac{\partial^u G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)}{\partial z^u} = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma\left(1-\frac{s}{r}\right) \prod_{k=1}^{m} \Gamma(s+b_k) \prod_{k=1}^{n} \Gamma(1-a_k-s)}{\Gamma\left(1-\alpha-\frac{s}{r}\right) \prod_{k=n+1}^{p} \Gamma(s+a_k) \prod_{k=m+1}^{q} \Gamma(1-b_k-s)} z^{-\frac{s}{r}-\alpha} \, ds \,/;$$

$$r \in \mathbb{R} \land r \neq 0 \land m \in \mathbb{N} \land n \in \mathbb{N} \land p \in \mathbb{N} \land q \in \mathbb{N} \land m \leq q \land n \leq p$$

Integration

Indefinite integration

Involving only one direct function

07.35.21.0001.01

$$\int G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) dz = r\, G_{p+1,q+1}^{m,n+1}\left(z, r \,\middle|\, \begin{matrix} 1, r+a_1, \ldots, r+a_n, r+a_{n+1}, \ldots, r+a_p \\ r+b_1, \ldots, r+b_m, 0, r+b_{m+1}, \ldots, r+b_q \end{matrix}\right)$$

Involving one direct function and elementary functions

Involving power function

07.35.21.0002.01

$$\int z^{\alpha-1} G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) dz = r\, G_{p+1,q+1}^{m,n+1}\left(z, r \,\middle|\, \begin{matrix} 1, r\alpha+a_1, \ldots, r\alpha+a_n, r\alpha+a_{n+1}, \ldots, r\alpha+a_p \\ r\alpha+b_1, \ldots, r\alpha+b_m, 0, r\alpha+b_{m+1}, \ldots, r\alpha+b_q \end{matrix}\right)$$

Integral transforms

Mellin transforms

07.35.22.0001.01

$$\mathcal{M}_t\left[G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)\right](z) = \frac{r \prod_{k=1}^{m} \Gamma(r s+b_k) \prod_{k=1}^{n} \Gamma(1-a_k-r s)}{\prod_{k=n+1}^{p} \Gamma(r s+a_k) \prod_{k=m+1}^{q} \Gamma(1-b_k-r s)}$$

Representations through more general functions

Through hypergeometric functions

Hypergeometric functions are not more general than Meijer G function.

Involving ${}_p\tilde{F}_q$

07.35.26.0001.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = \pi^{m-1} \sum_{k=1}^{m} \frac{\prod_{j=1}^{n} \Gamma(-a_j + b_k + 1)}{\prod_{\substack{j=1 \\ j\neq k}}^{m} \sin(\pi (b_j - b_k)) \prod_{j=n+1}^{p} \Gamma(a_j - b_k)} z^{\frac{b_k}{r}}$$

$$_p\tilde{F}_{q-1}\left(1 - a_1 + b_k, \ldots, 1 - a_p + b_k; 1 - b_1 + b_k, \ldots, 1 - b_{k-1} + b_k, 1 - b_{k+1} + b_k, \ldots, 1 - b_q + b_k; (-1)^{p-m-n} z^{1/r}\right) /;$$
$$(p < q \lor p = q \land |z| < 1) \land \forall_{\{j,k\},\{j,k\}\in\mathbb{Z} \land j\neq k \land 1\leq j\leq m \land 1\leq k\leq m} (b_j - b_k \notin \mathbb{Z})$$

Involving $_pF_q$

07.35.26.0002.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = \sum_{k=1}^{m} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{m} \Gamma(b_j - b_k) \prod_{j=1}^{n} \Gamma(1 - a_j + b_k)}{\prod_{j=n+1}^{p} \Gamma(a_j - b_k) \prod_{j=m+1}^{q} \Gamma(1 - b_j + b_k)} z^{\frac{b_k}{r}}$$

$$_pF_{q-1}\left(1 - a_1 + b_k, \ldots, 1 - a_p + b_k; 1 - b_1 + b_k, \ldots, 1 - b_{k-1} + b_k, 1 - b_{k+1} + b_k, \ldots, 1 - b_q + b_k; (-1)^{p-m-n} z^{1/r}\right) /;$$
$$(p < q \lor p = q \land |z| < 1) \land \forall_{\{j,k\},\{j,k\}\in\mathbb{Z} \land j\neq k \land 1\leq j\leq m \land 1\leq k\leq m} (b_j - b_k \notin \mathbb{Z})$$

Through Fox *H*

07.35.26.0003.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = r\, H_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} \{a_1, r\}, \ldots, \{a_n, r\}, \{a_{n+1}, r\}, \ldots, \{a_p, r\} \\ \{b_1, r\}, \ldots, \{b_m, r\}, \{b_{m+1}, r\}, \ldots, \{b_q, r\} \end{matrix}\right)$$

Through Meijer *G*

Classical cases

Classical Meijer G function is particular case of Meijer G function with parameter r.

07.35.26.0004.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = G_{p,q}^{m,n}\left(z^{1/r} \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) /; r \geq 1 \lor r < -1 \lor -\pi r < \arg(z) \leq \pi r$$

07.35.26.0005.01

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) =$$

$$G_{p r,q r}^{m r,n r}\left(r^{r(p-q)} z \,\middle|\, \begin{matrix} \frac{a_1}{r}, \ldots, \frac{a_1+r-1}{r}, \ldots, \frac{a_n}{r}, \ldots, \frac{a_n+r-1}{r}, \frac{a_{n+1}}{r}, \ldots, \frac{a_{n+1}+r-1}{r}, \ldots, \frac{a_p}{r}, \ldots, \frac{a_p+r-1}{r} \\ \frac{b_1}{r}, \ldots, \frac{b_1+r-1}{r}, \ldots, \frac{b_m}{r}, \ldots, \frac{b_m+r-1}{r}, \frac{b_{m+1}}{r}, \ldots, \frac{b_{m+1}+r-1}{r}, \ldots, \frac{b_q}{r}, \ldots, \frac{b_q+r-1}{r} \end{matrix}\right) /;$$

$$\xi = \left(\frac{p+q}{2} - m - n\right)(r-1) \bigwedge \theta = \frac{p-q}{2} - \sum_{k=1}^{p} a_k + \sum_{k=1}^{q} b_k + 1 \bigwedge r \in \mathbb{N}^+$$

History

J. Keiper and O.I. Marichev (1993) extended `MeijerG` function as part of the *Mathematica* system development effort.

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