

Min

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Notations

Traditional name

Minimum

Traditional notation

$\min(x_1, x_2, \dots)$

Mathematica StandardForm notation

`Min[x1, x2, ...]`

Primary definition

01.35.02.0001.01

$$\min(x_1, x_2) = \frac{1}{2} \left(x_1 + x_2 - \sqrt{(x_1 - x_2)^2} \right) /; x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R}$$

01.35.02.0002.01

$$\min(x_1, x_2, \dots, x_n) = \min(\min(x_1, x_2), x_3, \dots, x_n)$$

$\min(x_1, x_2, \dots)$ is the numerically smallest of the real numbers x_k .

$\min(z_1, z_2, \dots)$ is not defined for complex numbers z_k .

Specific values

Specialized values

01.35.03.0001.01

$$\min(x) = x$$

01.35.03.0002.01

$$\min(x_1, x_1, \dots, x_1) = x_1$$

01.35.03.0003.01

$$\min(x_1, x_2, x_3) = x_1 /; x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R} \wedge x_3 \in \mathbb{R} \wedge x_1 \leq x_2 \wedge x_1 \leq x_3$$

Values at fixed points

01.35.03.0004.01

$$\min() = \infty$$

01.35.03.0005.01

$$\min(-1, 3) = -1$$

01.35.03.0006.01

$$\min(2, 6, 1, \pi, 8, 12, 8, 2, 6, 4, 6, 9) = 1$$

Values at infinities

01.35.03.0007.01

$$\min(\infty, x_2, \dots, x_n) = \min(x_2, \dots, x_n)$$

01.35.03.0008.01

$$\min(-\infty, x_2, \dots, x_n) = -\infty$$

01.35.03.0009.01

$$\min(-\infty, \infty) = -\infty$$

General characteristics

Domain and analyticity

\min is real valued function of an arbitrary number of real variables. In \mathbb{R}^n it is a piecewise linear function. The derivative of $\min(x_1, x_2, \dots, x_k, \dots, x_j, \dots, x_n)$ is discontinuous at $x_j = x_k$ for all j, k .

01.35.04.0001.01

$$(x_1 * x_2 * \dots * x_n) \rightarrow \min(x_1, x_2, \dots, x_n) :: \mathbb{R}^n \rightarrow \mathbb{R}$$

Symmetries and periodicities

Permutation symmetry

01.35.04.0002.01

$$\min(x_1, x_2) = \min(x_2, x_1)$$

01.35.04.0003.01

$$\min(x_1, x_2, \dots, x_k, \dots, x_j, \dots, x_n) = \min(x_1, x_2, \dots, x_j, \dots, x_k, \dots, x_n)$$

Periodicity

No periodicity

Sets of discontinuity

The function $\min(x_1, x_2, \dots, x_n)$ is continuous function in \mathbb{R}^n .

01.35.04.0004.01

$$\mathcal{DS}_{x_k}(\min(x_1, x_2, \dots, x_n)) = \{ \} /; 1 \leq k \leq n$$

Limit representations

01.35.09.0001.01

$$\min(x_1, x_2) = -\left(\lim_{\varepsilon \rightarrow 0} \varepsilon \log \left(e^{-\frac{x_1}{\varepsilon}} + e^{-\frac{x_2}{\varepsilon}} \right) \right)$$

01.35.09.0002.01

$$\min(x_1, x_2, \dots, x_n) = \lim_{z \rightarrow -\infty} \left(\frac{1}{n} \sum_{k=1}^n x_k^z \right)^{1/z}$$

Transformations

Transformations and argument simplifications

01.35.16.0001.01

$$\min(-x_1, -x_2, \dots, -x_n) = -\max(x_1, x_2, \dots, x_n)$$

01.35.16.0002.01

$$\min(|x|, -|x|) = -|x|$$

Identities

Functional identities

01.35.17.0001.01

$$\min(x_1, x_2) = \min(x_2, x_1)$$

01.35.17.0002.01

$$\min(x_1, x_2, x_3, \dots) = \min(x_1, \min(x_2, x_3, \dots))$$

Complex characteristics

Real part

01.35.19.0001.01

$$\operatorname{Re}(\min(x_1, x_2, \dots, x_n)) = \min(x_1, x_2, \dots, x_n)$$

Imaginary part

01.35.19.0002.01

$$\operatorname{Im}(\min(x_1, x_2, \dots, x_n)) = 0$$

Absolute value

01.35.19.0003.01

$$|\min(x_1, x_2, \dots, x_n)| = \sqrt{\min(x_1, x_2, \dots, x_n)^2}$$

Argument

01.35.19.0004.01

$$\arg(\min(x_1, x_2, \dots, x_n)) = \tan^{-1}(\min(x_1, x_2, \dots, x_n), 0)$$

Conjugate value

01.35.19.0005.01

$$\overline{\min(x_1, x_2, \dots, x_n)} = \min(x_1, x_2, \dots, x_n)$$

Summation

01.35.23.0001.01

$$\sum_{k=0}^{m-1} \sum_{l=0}^{n-1} \min\left(\frac{k}{m}, \frac{l}{n}\right) = \frac{mn}{3} - \frac{m+n-1}{4} - \frac{m^2+n^2-\text{gcd}(m,n)^2}{12mn} ; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+$$

01.35.23.0002.01

$$\sum_{j=0}^{m-1} \sum_{k=0}^{n-1} \sum_{l=0}^{o-1} \min\left(\frac{j}{m}, \frac{k}{n}, \frac{l}{o}\right) = \frac{mno}{4} + \frac{1}{8}(m+n+o-1) - \frac{1}{6}(mn+on+mo) + \frac{m+o-2om}{24n} + \frac{n+o-2on}{24m} + \frac{m+n-2nm}{24o} + \frac{(o-1)\text{gcd}(m,n)^2}{24mn} + \frac{(n-1)\text{gcd}(m,o)^2}{24mo} + \frac{(m-1)\text{gcd}(n,o)^2}{24no} ; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge o \in \mathbb{N}^+$$

Integral transforms

Fourier exp transforms

01.35.22.0001.01

$$\mathcal{F}_{[t_1, t_2]}[\min(t_1, t_2)](z_1, z_2) = \frac{\delta(z_1 + z_2)}{z_1^2} - i\pi(\delta(z_2)\delta'(z_1) + \delta(z_1)\delta'(z_2))$$

Laplace transforms

01.35.22.0002.01

$$\mathcal{L}_{(t_1, t_2)}[\min(t_1, t_2)](z_1, z_2) = \frac{1}{z_1 z_2 (z_1 + z_2)}$$

Representations through more general functions

Through other functions

01.35.26.0001.01

$$\min(x_1, x_2) = \frac{1}{2} \left(x_1 + x_2 - \sqrt{(x_1 - x_2)^2} \right) ; x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R}$$

Representations through equivalent functions

01.35.27.0001.01

$$\min(x_1, x_2) = x_2 + (x_1 - x_2)\theta(x_2 - x_1)$$

01.35.27.0002.01

$$\min(x_1, x_2, x_3) = x_2 + (x_1 - x_2)\theta(x_2 - x_1) + (x_3 - x_2 - (x_1 - x_2)\theta(x_2 - x_1))(\theta(x_1 - x_2) + \theta(x_2 - x_1)\theta(x_1 - x_3))\theta(x_2 - x_3)$$

Inequalities

01.35.29.0001.01

$$\min(x_1, x_2, \dots, x_n) \leq \left(\frac{1}{n} \sum_{k=1}^n x_k^z \right)^{1/z} \leq \max(x_1, x_2, \dots, x_n) ; x_k > 0 \wedge 1 \leq k \leq n$$

01.35.29.0002.01

$$\min(x_1, x_2, \dots, x_n) \leq \left(\prod_{k=1}^n x_k \right)^{1/n} \leq \max(x_1, x_2, \dots, x_n) /; x_k > 0 \wedge 1 \leq k \leq n$$

Theorems

Operations in Boolean algebra

The identification negation(x) $\rightarrow -x$, conjunction(x, y) $\rightarrow \min(x, y)$ and disjunction(x, y) $\rightarrow \max(x, y)$ is a complete system of R -operations representing the Boolean algebra.

History

The function min is encountered often in mathematics and the natural sciences.

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