Min

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Notations

Traditional name

Minimum

Traditional notation

$\min(x_1, x_2, \ldots)$

Mathematica StandardForm notation

`Min[`x_1 `,` x_2`, …]`

Primary definition

01.35.02.0001.01

$$\min(x_1, x_2) = \frac{1}{2}\left(x_1 + x_2 - \sqrt{(x_1 - x_2)^2}\right) /; x_1 \in \mathbb{R} \bigwedge x_2 \in \mathbb{R}$$

01.35.02.0002.01

$$\min(x_1, x_2, \ldots, x_n) = \min(\min(x_1, x_2), x_3, \ldots, x_n)$$

$\min(x_1, x_2, \ldots)$ is the numerically smallest of the real numbers x_k.

$\min(z_1, z_2, \ldots)$ is not defined for complex numbers z_k.

Specific values

Specialized values

01.35.03.0001.01

$$\min(x) = x$$

01.35.03.0002.01

$$\min(x_1, x_1, \ldots, x_1) = x_1$$

01.35.03.0003.01

$$\min(x_1, x_2, x_3) = x_1 /; x_1 \in \mathbb{R} \bigwedge x_2 \in \mathbb{R} \bigwedge x_3 \in \mathbb{R} \bigwedge x_1 \leq x_2 \bigwedge x_1 \leq x_3$$

Values at fixed points

01.35.03.0004.01

$$\min() = \infty$$

01.35.03.0005.01

$\min(-1, 3) == -1$

01.35.03.0006.01

$\min(2, 6, 1, \pi, 8, 12, 8, 2, 6, 4, 6, 9) == 1$

Values at infinities

01.35.03.0007.01

$\min(\infty, x_2, \ldots, x_n) == \min(x_2, \ldots, x_n)$

01.35.03.0008.01

$\min(-\infty, x_2, \ldots, x_n) == -\infty$

01.35.03.0009.01

$\min(-\infty, \infty) == -\infty$

General characteristics

Domain and analyticity

min is real valued function of an arbitrary number of real variables. In $\mathbb{R}^n$ it is a piecewise linear function. The derivative of $\min(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n)$ is discontinuous at $x_j == x_k$ for all $j,\ k$.

01.35.04.0001.01

$(x_1 * x_2 * \ldots * x_n) \longrightarrow \min(x_1, x_2, \ldots, x_n) :: \mathbb{R}^n \longrightarrow \mathbb{R}$

Symmetries and periodicities

Permutation symmetry

01.35.04.0002.01

$\min(x_1, x_2) == \min(x_2, x_1)$

01.35.04.0003.01

$\min(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n) == \min(x_1, x_2, \ldots, x_j, \ldots, x_k, \ldots, x_n)$

Periodicity

No periodicity

Sets of discontinuity

The function $\min(x_1, x_2, \ldots, x_n)$ is continuous function in $\mathbb{R}^n$.

01.35.04.0004.01

$\mathcal{DS}_{x_k}(\min(x_1, x_2, \ldots, x_n)) == \{\} /; 1 \leq k \leq n$

Limit representations

01.35.09.0001.01

$\min(x_1, x_2) == -\left(\lim_{\varepsilon \to 0} \varepsilon \log\left(e^{-\frac{x_1}{\varepsilon}} + e^{-\frac{x_2}{\varepsilon}}\right)\right)$

01.35.09.0002.01

$$\min(x_1, x_2, \ldots, x_n) = \lim_{z \to -\infty} \left(\frac{1}{n} \sum_{k=1}^{n} x_k^z \right)^{1/z}$$

Transformations

Transformations and argument simplifications

01.35.16.0001.01

$$\min(-x_1, -x_2, \ldots, -x_n) = -\max(x_1, x_2, \ldots, x_n)$$

01.35.16.0002.01

$$\min(|x|, -|x|) = -|x|$$

Identities

Functional identities

01.35.17.0001.01

$$\min(x_1, x_2) = \min(x_2, x_1)$$

01.35.17.0002.01

$$\min(x_1, x_2, x_3, \ldots) = \min(x_1, \min(x_2, x_3, \ldots))$$

Complex characteristics

Real part

01.35.19.0001.01

$$\operatorname{Re}(\min(x_1, x_2, \ldots, x_n)) = \min(x_1, x_2, \ldots, x_n)$$

Imaginary part

01.35.19.0002.01

$$\operatorname{Im}(\min(x_1, x_2, \ldots, x_n)) = 0$$

Absolute value

01.35.19.0003.01

$$|\min(x_1, x_2, \ldots, x_n)| = \sqrt{\min(x_1, x_2, \ldots, x_n)^2}$$

Argument

01.35.19.0004.01

$$\arg(\min(x_1, x_2, \ldots, x_n)) = \tan^{-1}(\min(x_1, x_2, \ldots, x_n), 0)$$

Conjugate value

01.35.19.0005.01

$$\overline{\min(x_1, x_2, \ldots, x_n)} = \min(x_1, x_2, \ldots, x_n)$$

Summation

01.35.23.0001.01

$$\sum_{k=0}^{m-1}\sum_{l=0}^{n-1}\min\left(\frac{k}{m},\frac{l}{n}\right)=\frac{m\,n}{3}-\frac{m+n-1}{4}-\frac{m^2+n^2-\gcd(m,n)^2}{12\,m\,n}\;/;\;m\in\mathbb{N}^+\bigwedge n\in\mathbb{N}^+$$

01.35.23.0002.01

$$\sum_{j=0}^{m-1}\sum_{k=0}^{n-1}\sum_{l=0}^{o-1}\min\left(\frac{j}{m},\frac{k}{n},\frac{l}{o}\right)=\frac{m\,n\,o}{4}+\frac{1}{8}(m+n+o-1)-\frac{1}{6}(m\,n+o\,n+m\,o)+\frac{m+o-2\,o\,m}{24\,n}+\frac{n+o-2\,o\,n}{24\,m}+\frac{m+n-2\,n\,m}{24\,o}+\frac{(o-1)\gcd(m,n)^2}{24\,m\,n}+\frac{(n-1)\gcd(m,o)^2}{24\,m\,o}+\frac{(m-1)\gcd(n,o)^2}{24\,n\,o}\;/;\;m\in\mathbb{N}^+\bigwedge n\in\mathbb{N}^+\bigwedge o\in\mathbb{N}^+$$

Integral transforms

Fourier exp transforms

01.35.22.0001.01

$$\mathcal{F}_{\{t_1,t_2\}}[\min(t_1,t_2)]\,(z_1,z_2)=\frac{\delta(z_1+z_2)}{z_1^2}-i\,\pi\,(\delta(z_2)\,\delta'(z_1)+\delta(z_1)\,\delta'(z_2))$$

Laplace transforms

01.35.22.0002.01

$$\mathcal{L}_{(t_1,t_2)}[\min(t_1,t_2)]\,(z_1,z_2)=\frac{1}{z_1\,z_2\,(z_1+z_2)}$$

Representations through more general functions

Through other functions

01.35.26.0001.01

$$\min(x_1,x_2)=\frac{1}{2}\left(x_1+x_2-\sqrt{(x_1-x_2)^2}\right)\;/;\;x_1\in\mathbb{R}\bigwedge x_2\in\mathbb{R}$$

Representations through equivalent functions

01.35.27.0001.01

$$\min(x_1,x_2)=x_2+(x_1-x_2)\,\theta(x_2-x_1)$$

01.35.27.0002.01

$$\min(x_1,x_2,x_3)=x_2+(x_1-x_2)\,\theta(x_2-x_1)+(x_3-x_2-(x_1-x_2)\,\theta(x_2-x_1))\,(\theta(x_1-x_2)+\theta(x_2-x_1)\,\theta(x_1-x_3))\,\theta(x_2-x_3)$$

Inequalities

01.35.29.0001.01

$$\min(x_1,x_2,\ldots,x_n)\le\left(\frac{1}{n}\sum_{k=1}^{n}x_k^z\right)^{1/z}\le\max(x_1,x_2,\ldots,x_n)\;/;\;x_k>0\bigwedge 1\le k\le n$$

01.35.29.0002.01

$$\min(x_1, x_2, \ldots, x_n) \leq \left(\prod_{k=1}^{n} x_k\right)^{1/n} \leq \max(x_1, x_2, \ldots, x_n) \;/;\; x_k > 0 \bigwedge 1 \leq k \leq n$$

Theorems

Operations in Boolean algebra

The identification negation$(x) \to -x$, conjunction$(x, y) \to \min(x, y)$ and disjunction$(x, y) \to \max(x, y)$ is a complete system of R-operations representing the Boolean algebra.

History

The function min is encountered often in mathematics and the natural sciences.

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