

# Mod

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## Notations

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### Traditional name

Congruence function

### Traditional notation

$m \bmod n$

### Mathematica StandardForm notation

Mod[ $m$ ,  $n$ ]

## Primary definition

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04.06.02.0001.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

$m \bmod n$  is the remainder on division of  $m$  by  $n$ . The sign of  $m \bmod n$  for real  $m, n$  is always the same as the sign of  $n$ .

Examples:  $5 \bmod 2 = 1$ ,  $8 \bmod 3 = 2$ ,  $-5 \bmod 3 = 1$ ,  $(7\pi) \bmod 3 = -21 + 7\pi$ ,  $(27 - 3i) \bmod 4 = 3 + i$ ,  
 $\text{frac}(-\pi) = 3 - \pi$ ,  $(2.7 - 3i) \bmod 5 = 2.7 + 2i$ .

## Specific values

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### Specialized values

04.06.03.0001.01

$$0 \bmod n = 0 ; n \neq 0$$

04.06.03.0002.01

$$m \bmod 1 = 0 ; m \in \mathbb{Z}$$

04.06.03.0003.01

$$1 \bmod n = n + 1 ; -n \in \mathbb{N}^+$$

04.06.03.0004.01

$$1 \bmod n = 1 ; n \in \mathbb{Z} \wedge n > 1$$

04.06.03.0005.01

$$m \bmod n = m ; m \in \mathbb{N} \wedge n \in \mathbb{Z} \wedge m < n$$

04.06.03.0006.01

$$m \bmod n = m - n \lfloor m/n \rfloor; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n \leq m < 2n$$

04.06.03.0007.01

$$m \bmod n = m - kn \lfloor m/(kn) \rfloor; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge k \in \mathbb{N}^+ \wedge kn \leq m < (k+1)n$$

04.06.03.0008.01

$$n \bmod n = 0$$

04.06.03.0009.01

$$(2n) \bmod n = 0$$

04.06.03.0010.01

$$(p-1)! \bmod p = p-1 \lfloor p \in \mathbb{P} \rfloor$$

04.06.03.0011.01

$$\binom{2p-1}{p-1} \bmod p^3 = 1 \lfloor p \in \mathbb{P} \wedge p > 3 \rfloor$$

04.06.03.0012.01

$$|B_{2n}| \bmod 1 = \delta_{\frac{n+1}{2} \bmod 1, 0} + (-1)^n \left( \sum_{k=3}^{2n+1} \frac{1}{k} \chi_{\mathbb{Z}} \left( \frac{2n}{k-1} \right) \chi_{\mathbb{P}} \left( k + \frac{1}{2} \right) \right) \bmod 1$$

## Values at fixed points

04.06.03.0013.01

$$0 \bmod 1 = 0$$

04.06.03.0014.01

$$1 \bmod 2 = 1$$

04.06.03.0015.01

$$1 \bmod 3 = 1$$

04.06.03.0016.01

$$2 \bmod 3 = 2$$

04.06.03.0017.01

$$3 \bmod 3 = 0$$

04.06.03.0018.01

$$4 \bmod 3 = 1$$

04.06.03.0019.01

$$5 \bmod 3 = 2$$

04.06.03.0020.01

$$12 \bmod 8 = 4$$

04.06.03.0021.01

$$-3 \bmod -2 = -1$$

04.06.03.0022.01

$$-\frac{27}{10} \bmod \frac{23}{5} = \frac{19}{10}$$

04.06.03.0023.01

$$(2\pi) \bmod e = 2\pi - 2e$$

04.06.03.0024.01

$$-\pi \bmod 2 = 4 - \pi$$

04.06.03.0025.01

$$\pi \bmod e = \pi - e$$

04.06.03.0026.01

$$(-3 + \pi i) \bmod (-2 - 3 i e) = -3 + (1 + i)(-2 - 3 i e) + i \pi$$

04.06.03.0027.01

$$5.2 \bmod 3.1 = 2.1$$

## General characteristics

### Domain and analyticity

$m \bmod n$  is a nonanalytical function; it is a piecewise continuous function which is defined over  $\mathbb{C}^2$ .

04.06.04.0001.01

$$(m * n) \rightarrow m \bmod n :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Parity

$m \bmod n$  is an odd function.

04.06.04.0002.01

$$-m \bmod -n = -(m \bmod n)$$

#### Mirror symmetry

04.06.04.0005.01

$$[\bar{z}] = \overline{[z]} - i(1 - \chi_{\mathbb{Z}}(\text{Im}(z)))$$

#### Periodicity

$m \bmod n$  is a periodic function with respect to  $m$  with period  $n$ .

04.06.04.0006.01

$$(m + n) \bmod n = m \bmod n$$

04.06.04.0007.01

$$(m + k n) \bmod n = m \bmod n ; k \in \mathbb{Z}$$

### Sets of discontinuity

The function  $m \bmod n$  is a piecewise continuous function with jumps on the curves

$\text{Re}\left(\frac{m}{n}\right) = k \vee \text{Im}\left(\frac{m}{n}\right) = l ; k, l \in \mathbb{Z}$ . The functional property  $m \bmod n = n\left(\frac{m}{n} \bmod 1\right) = n\left(\frac{m}{n} - \left\lfloor \frac{m}{n} \right\rfloor\right)$  makes the behaviour of the  $m \bmod n$  similar to the behaviour of  $\left\lfloor \frac{m}{n} \right\rfloor$ .

04.06.04.0003.01

$$\mathcal{DS}_m(m \bmod n) = \{ \{(n k - i \infty, n k + i \infty), -1\} ; k \in \mathbb{Z} \}, \{ \{(i n k - \infty, i n k + \infty), -i\} ; k \in \mathbb{Z} \} \}$$

04.06.04.0004.01

$$\mathcal{DS}_n(m \bmod n) = \left\{ \left\{ \left\{ \frac{m}{(k-i\infty, k+i\infty)}, -1 \right\}; k \in \mathbb{Z} \right\}, \left\{ \left\{ \frac{m}{(ik-\infty, ik+\infty)}, -i \right\}; k \in \mathbb{Z} \right\} \right\}$$

04.06.04.0008.01

$$\lim_{\epsilon \rightarrow +0} (m + \epsilon) \bmod n = m \bmod n; \operatorname{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

04.06.04.0009.01

$$\lim_{\epsilon \rightarrow +0} (m - \epsilon) \bmod n = m \bmod n + n; \operatorname{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

04.06.04.0010.01

$$\lim_{\epsilon \rightarrow +0} (m + i\epsilon) \bmod n = m \bmod n; \operatorname{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

04.06.04.0011.01

$$\lim_{\epsilon \rightarrow +0} (m - i\epsilon) \bmod n = i n + m \bmod n; \operatorname{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

## Series representations

### Exponential Fourier series

04.06.06.0001.01

$$m \bmod n = \frac{n}{2} - \frac{n}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{2\pi k m}{n}\right); \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

### Other series representations

04.06.06.0002.01

$$m \bmod n = \frac{n}{2} - \frac{1}{2} \sum_{k=1}^{n-1} \sin\left(\frac{2\pi k m}{n}\right) \cot\left(\frac{\pi k}{n}\right); m \in \mathbb{Z} \wedge n-1 \in \mathbb{N}^+ \wedge \frac{m}{n} \notin \mathbb{Z}$$

## Transformations

### Transformations and argument simplifications

#### Argument involving basic arithmetic operations

04.06.16.0001.01

$$(-m) \bmod (-n) = -(m \bmod n)$$

04.06.16.0002.01

$$m \bmod -n = m \bmod n + \chi_{\mathbb{Z}}\left(\frac{m}{n}\right) n - n; m \in \mathbb{R} \wedge n \in \mathbb{R}$$

04.06.16.0003.01

$$m \bmod -n = m \bmod n - n \left( 1 - \chi_{\mathbb{Z}}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) \operatorname{sgn}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) - i n \left( 1 - \chi_{\mathbb{Z}}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) \operatorname{sgn}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) \right) \right)$$

04.06.16.0004.01

$$-m \bmod n = n - m \bmod n; m \in \mathbb{R} \wedge n \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.16.0005.01

$$-m \bmod n = -\chi_Z\left(\frac{m}{n}\right)n + n - m \bmod n /; m \in \mathbb{R} \wedge n \in \mathbb{R}$$

04.06.16.0006.01

$$-m \bmod n = -(m \bmod n) + n \left(1 - \chi_Z\left(\operatorname{Re}\left(\frac{m}{n}\right)\right)\right) \operatorname{sgn}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) + i n \left(1 - \chi_Z\left(\operatorname{Im}\left(\frac{m}{n}\right)\right)\right) \operatorname{sgn}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right)$$

04.06.16.0007.01

$$(i m) \bmod (i n) = i (m \bmod n)$$

04.06.16.0008.01

$$(i m) \bmod n = n - n \chi_Z\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) + i (m \bmod n)$$

04.06.16.0009.01

$$(-i m) \bmod n = -i n \left(\chi_Z\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) - 1\right) - i (m \bmod n)$$

04.06.16.0010.01

$$m \bmod (i n) = m \bmod n + \left(\chi_Z\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) - 1\right) n$$

04.06.16.0011.01

$$m \bmod (-i n) = m \bmod n + \left(\chi_Z\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) - 1\right) i n$$

04.06.16.0012.01

$$\frac{m}{n} \bmod 1 = \frac{m \bmod n}{n}$$

### Argument involving related functions

04.06.16.0016.01

$$\lfloor m \rfloor \bmod n = \lfloor m \rfloor - n \left\lfloor \frac{\lfloor m \rfloor}{n} \right\rfloor$$

04.06.16.0017.01

$$\lfloor m \rfloor \bmod 1 = 0$$

04.06.16.0018.01

$$\lceil m \rceil \bmod n = \lceil m \rceil - n \left\lceil \frac{\lceil m \rceil}{n} \right\rceil$$

04.06.16.0019.01

$$\lceil m \rceil \bmod 1 = 0$$

04.06.16.0020.01

$$\lceil m \rceil \bmod n = \lceil m \rceil - n \left\lceil \frac{\lceil m \rceil}{n} \right\rceil$$

04.06.16.0021.01

$$\lceil m \rceil \bmod 1 = 0$$

04.06.16.0022.01

$$\operatorname{int}(m) \bmod n = \operatorname{int}(m) - n \left\lfloor \frac{\operatorname{int}(m)}{n} \right\rfloor$$

04.06.16.0023.01

$$\text{int}(m) \bmod 1 = 0$$

04.06.16.0024.01

$$\text{frac}(m) \bmod n = \text{frac}(m) - n \left\lfloor \frac{\text{frac}(m)}{n} \right\rfloor$$

04.06.16.0025.01

$$(m \bmod n) \bmod n = m \bmod n$$

04.06.16.0026.01

$$(m \bmod n) \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

04.06.16.0027.01

$$\text{quotient}(m, n) \bmod n = \left\lfloor \frac{m}{n} \right\rfloor - n \left\lfloor \frac{1}{n} \left\lfloor \frac{m}{n} \right\rfloor \right\rfloor$$

04.06.16.0028.01

$$\text{quotient}(m, 1) \bmod 1 = 0$$

## Addition formulas

04.06.16.0029.01

$$(m + kn) \bmod n = m \bmod n ; k \in \mathbb{Z}$$

## Multiple arguments

04.06.16.0013.01

$$(km) \bmod n = k(m \bmod n) - n \sum_{j=0}^{k-1} j \theta \left( \frac{m}{n} - \frac{j}{k} - \text{quotient}(m, n) \right) \left( 1 - \theta \left( \frac{m}{n} - \frac{j+1}{k} - \text{quotient}(m, n) \right) \right) ; k \in \mathbb{N} \wedge \frac{m}{n} \in \mathbb{R}$$

## Related transformations

04.06.16.0015.01

$$a = b \bmod \text{lcm}(n, m) ; a = b \bmod n \wedge a = b \bmod m \wedge a \in \mathbb{N}^+ \wedge b \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge m \in \mathbb{N}^+$$

## Identities

### Functional identities

04.06.17.0001.01

$$\frac{m}{n} \bmod 1 = \frac{m \bmod n}{n}$$

04.06.17.0002.01

$$(a + c) \bmod n = (b + d) \bmod n ; a \bmod n = b \bmod n \wedge c \bmod n = d \bmod n \wedge a \in \mathbb{R} \wedge b \in \mathbb{R} \wedge c \in \mathbb{R} \wedge d \in \mathbb{R} \wedge n \in \mathbb{R}$$

## Complex characteristics

### Real part



04.06.19.0007.01

$$\operatorname{sgn}(m \bmod n) = \operatorname{sgn}(n) \quad ; \quad m \in \mathbb{R} \wedge n \in \mathbb{R}$$

## Differentiation

### Low-order differentiation

With respect to  $m$

04.06.20.0001.01

$$\frac{\partial(m \bmod n)}{\partial m} = 1$$

04.06.20.0002.01

$$\frac{\partial^2(m \bmod n)}{\partial m^2} = 0$$

In a distributional sense for  $x \in \mathbb{R}$ .

04.06.20.0003.01

$$\frac{\partial(x \bmod n)}{\partial x} = x + \sum_{k=-\infty}^{\infty} \delta(x - kn)$$

With respect to  $n$

04.06.20.0004.01

$$\frac{\partial(m \bmod n)}{\partial n} = -\left\lfloor \frac{m}{n} \right\rfloor$$

In a distributional sense for  $x \in \mathbb{R}$ .

04.06.20.0005.01

$$\frac{\partial(m \bmod x)}{\partial x} = \operatorname{sgn}(x) \left( \operatorname{int}\left(\frac{m}{x}\right) - \frac{m}{x^2} \sum_{k=-\infty}^{\infty} \delta_{k,0} \delta\left(\frac{m}{x} - k\right) \right)$$

### Fractional integro-differentiation

With respect to  $m$

04.06.20.0006.01

$$\frac{\partial^\alpha(m \bmod n)}{\partial m^\alpha} = \frac{\alpha m^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{(m \bmod n) m^{-\alpha}}{\Gamma(1-\alpha)}$$

With respect to  $n$

04.06.20.0007.01

$$\frac{\partial^\alpha(m \bmod n)}{\partial n^\alpha} = \frac{n^{-\alpha}(m \bmod n)}{\Gamma(2-\alpha)} - \frac{m \alpha n^{-\alpha}}{\Gamma(2-\alpha)}$$

## Integration

### Indefinite integration



**Involving only one direct function with respect to  $m$**

04.06.21.0001.01

$$\int m \bmod n \, dm = m(m \bmod n) - \frac{m^2}{2}$$

**Involving one direct function and elementary functions with respect to  $m$**

**Involving power function**

04.06.21.0002.01

$$\int m^{\alpha-1} (m \bmod n) \, dm = \frac{m^\alpha}{\alpha(\alpha+1)} ((\alpha+1)(m \bmod n) - m)$$

04.06.21.0003.01

$$\int \frac{m \bmod n}{m} \, dm = m(1 - \log(m)) + \log(m)(m \bmod n)$$

**Involving only one direct function with respect to  $n$**

04.06.21.0004.01

$$\int m \bmod n \, dn = \frac{1}{2} n(m + m \bmod n)$$

**Involving one direct function and elementary functions with respect to  $n$**

**Involving power function**

04.06.21.0005.01

$$\int n^{\alpha-1} (m \bmod n) \, dn = \frac{n^\alpha}{\alpha(\alpha+1)} (m + \alpha(m \bmod n))$$

04.06.21.0006.01

$$\int \frac{m \bmod n}{n} \, dn = (\log(n) - 1)m + m \bmod n$$

**Definite integration**

**For the direct function with respect to  $m$**

In the following formulas  $a \in \mathbb{R}$ .

04.06.21.0007.01

$$\int_0^a t \bmod n \, dt = \frac{1}{2} ((a \bmod n)^2 - n(a \bmod n) + an)$$

04.06.21.0008.01

$$\int_0^a t^{\alpha-1} (t \bmod n) \, dt = \frac{a^{\alpha+1}}{\alpha+1} - \frac{1}{\alpha} \left( -(a \bmod n) a^\alpha + a^{\alpha+1} - n^{\alpha+1} \zeta(-\alpha) + n^{\alpha+1} \zeta\left(-\alpha, \frac{a+n-a \bmod n}{n}\right) \right); \operatorname{Re}(\alpha) > -1$$

04.06.21.0009.01

$$\int_a^\infty t^{\alpha-1} (t \bmod n) \, dt = \frac{1}{\alpha(\alpha+1)} \left( -(\alpha+1)(a \bmod n) a^\alpha + a^{\alpha+1} + n^{\alpha+1} (\alpha+1) \zeta\left(-\alpha, \frac{a+n-a \bmod n}{n}\right) \right); \operatorname{Re}(\alpha) < 0$$

04.06.21.0010.01

$$\int_0^\infty t^{\alpha-1} (t \bmod n) dt = \frac{n^{\alpha+1} \zeta(-\alpha)}{\alpha} \quad ; -1 < \operatorname{Re}(\alpha) < 0$$

04.06.21.0011.01

$$\int_{-a}^a t \bmod n dt = a n$$

**For the direct function with respect to n**

In the following formulas  $a \in \mathbb{R}$  .

04.06.21.0012.01

$$\int_0^a m \bmod t dt = \frac{1}{2} \left( -\psi^{(1)} \left( \frac{a+m-m \bmod a}{a} \right) m^2 + a m + a (m \bmod a) \right)$$

04.06.21.0013.01

$$\int_0^a t^{\alpha-1} (m \bmod t) dt = \frac{1}{\alpha^2 + \alpha} \left( m a^\alpha + \alpha (m \bmod a) a^\alpha - m^{\alpha+1} \alpha \zeta \left( \alpha + 1, \frac{a+m-m \bmod a}{a} \right) \right) \quad ; \operatorname{Re}(\alpha) > -1$$

04.06.21.0014.01

$$\int_a^\infty t^{\alpha-1} (m \bmod t) dt = -\frac{1}{\alpha(\alpha+1)} \left( \alpha (m \bmod a) a^\alpha + m \left( a^\alpha + m^\alpha \alpha \zeta(\alpha+1) - m^\alpha \alpha \zeta \left( \alpha + 1, \frac{a+m-m \bmod a}{a} \right) \right) \right) \quad ; \operatorname{Re}(\alpha) < 0$$

04.06.21.0015.01

$$\int_0^\infty t^{\alpha-1} (m \bmod t) dt = -\frac{m^{\alpha+1} \zeta(\alpha+1)}{\alpha+1} \quad ; -1 < \operatorname{Re}(\alpha) < 0$$

## Integral transforms

### Fourier exp transforms

04.06.22.0001.01

$$\mathcal{F}_i[t \bmod n](z) = n \sqrt{\frac{\pi}{2}} \delta(z) - \frac{i n}{\sqrt{2 \pi}} \sum_{k=1}^\infty \frac{1}{k} \left( \delta \left( \frac{2 k \pi}{n} - z \right) - \delta \left( \frac{2 \pi k}{n} + z \right) \right)$$

### Fourier cos transforms

04.06.22.0002.01

$$\mathcal{F}_c[t \bmod n](z) = \frac{1}{\sqrt{2 \pi} z^2} \left( n z \cot \left( \frac{n z}{2} \right) - 2 \right) + \sqrt{\frac{\pi}{2}} n \delta(z)$$

### Fourier sin transforms

04.06.22.0003.01

$$\mathcal{F}_s[t \bmod n](z) = \frac{n}{\sqrt{2 \pi} z} - \frac{n}{\sqrt{2 \pi}} \sum_{k=1}^\infty \frac{1}{k} \left( \delta \left( \frac{2 k \pi}{n} - z \right) - \delta \left( \frac{2 \pi k}{n} + z \right) \right)$$

### Laplace transforms

04.06.22.0004.01

$$\mathcal{L}_t[t \bmod n](z) = \frac{1}{z^2} \left( 1 - \frac{nz}{e^{nz} - 1} \right); \operatorname{Re}(nz) > 0$$

## Mellin transforms

04.06.22.0005.01

$$\mathcal{M}_t[t \bmod n](z) = \frac{n^{z+1} \zeta(-z)}{z}; -1 < \operatorname{Re}(z) < 0$$

04.06.22.0006.01

$$\mathcal{M}_t[m \bmod t](z) = -\frac{m^{z+1} \zeta(z+1)}{z+1}; -1 < \operatorname{Re}(z) < 0$$

## Representations through equivalent functions

### With related functions

#### With Floor

04.06.27.0001.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

#### With Round

### For real arguments

04.06.27.0008.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rfloor; \frac{m}{n} \in \mathbb{R} \wedge \frac{m+n}{2n} \notin \mathbb{Z}$$

04.06.27.0009.01

$$m \bmod n = m - n - n \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rfloor; \frac{m+n}{2n} \in \mathbb{Z}$$

04.06.27.0010.01

$$m \bmod n = m - n \left( \chi_{\mathbb{Z}} \left( \frac{m+n}{2n} \right) + \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rfloor \right); \frac{m}{n} \in \mathbb{R}$$

### For complex arguments

04.06.27.0003.01

$$m \bmod n = m + n \left( \left\lfloor \frac{1+i}{2} - \frac{m}{n} \right\rfloor - \chi_{\mathbb{Z}} \left( \frac{1}{2} \left( \operatorname{Re} \left( \frac{m}{n} \right) + 1 \right) \right) - i \chi_{\mathbb{Z}} \left( \frac{1}{2} \left( \operatorname{Im} \left( \frac{m}{n} \right) + 1 \right) \right) \right)$$

#### With Ceiling

### For real arguments

04.06.27.0011.01

$$m \bmod n = m + n - n \left\lceil \frac{m}{n} \right\rceil /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.27.0012.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor /; \frac{m}{n} \in \mathbb{Z}$$

04.06.27.0013.01

$$m \bmod n = m + n - n \left\lceil \frac{m}{n} \right\rceil - n \theta \left( \chi_{\mathbb{Z}} \left( \frac{m}{n} \right) - 1 \right) /; \frac{m}{n} \in \mathbb{R}$$

### For complex arguments

04.06.27.0014.01

$$m \bmod n = m + n - n \left\lceil \frac{m}{n} \right\rceil + i n /; \operatorname{Re} \left( \frac{m}{n} \right) \notin \mathbb{Z} \wedge \operatorname{Im} \left( \frac{m}{n} \right) \notin \mathbb{Z}$$

04.06.27.0015.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor + n /; \operatorname{Re} \left( \frac{m}{n} \right) \notin \mathbb{Z} \wedge \operatorname{Im} \left( \frac{m}{n} \right) \in \mathbb{Z}$$

04.06.27.0016.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor + i n /; \operatorname{Re} \left( \frac{m}{n} \right) \in \mathbb{Z} \wedge \operatorname{Im} \left( \frac{m}{n} \right) \notin \mathbb{Z}$$

04.06.27.0017.01

$$m \bmod n = m - n \left\lceil \frac{m}{n} \right\rceil /; \operatorname{Re} \left( \frac{m}{n} \right) \in \mathbb{Z} \wedge \operatorname{Im} \left( \frac{m}{n} \right) \in \mathbb{Z}$$

04.06.27.0018.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor - n \theta \left( \chi_{\mathbb{Z}} \left( \operatorname{Re} \left( \frac{m}{n} \right) \right) - 1 \right) + n i \theta \left( -\chi_{\mathbb{Z}} \left( \operatorname{Im} \left( \frac{m}{n} \right) \right) \right) + n$$

04.06.27.0002.01

$$m \bmod n = m + n \left\lfloor -\frac{m}{n} \right\rfloor$$

### With IntegerPart

### For real arguments

04.06.27.0019.01

$$m \bmod n = m - n \operatorname{int} \left( \frac{m}{n} \right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} > 0 \vee \frac{m}{n} \in \mathbb{Z}$$

04.06.27.0020.01

$$m \bmod n = m - n \left( \operatorname{int} \left( \frac{m}{n} \right) - 1 \right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.27.0021.01

$$m \bmod n = m - n \left( \operatorname{int} \left( \frac{m}{n} \right) + \operatorname{sgn} \left( \chi_{\mathbb{Z}} \left( \frac{m}{n} \right) + \theta \left( \frac{m}{n} \right) - 1 \right) \right) /; \frac{m}{n} \in \mathbb{R}$$

### For complex arguments

04.06.27.0022.01

$$m \bmod n = m - n \operatorname{int}\left(\frac{m}{n}\right) /; \operatorname{Re}\left(\frac{m}{n}\right) \geq 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) \geq 0 \vee \frac{m}{n} \in \mathbb{Z} \vee \frac{im}{n} \in \mathbb{Z}$$

04.06.27.0023.01

$$m \bmod n = m - n \left(\operatorname{int}\left(\frac{m}{n}\right) - 1\right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z} \vee \operatorname{Re}\left(\frac{m}{n}\right) < 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) > 0$$

04.06.27.0024.01

$$m \bmod n = m - n \left(\operatorname{int}\left(\frac{m}{n}\right) - i\right) /; \frac{im}{n} \in \mathbb{R} \wedge \frac{im}{n} > 0 \wedge \frac{im}{n} \notin \mathbb{Z} \vee \operatorname{Re}\left(\frac{m}{n}\right) > 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0025.01

$$m \bmod n = m - n \left(\operatorname{int}\left(\frac{m}{n}\right) - 1 - i\right) /; \operatorname{Re}\left(\frac{m}{n}\right) < 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0004.01

$$m \bmod n = m + n \left(-\operatorname{int}\left(\frac{m}{n}\right) + 1 + i - i \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\operatorname{Im}\left(\frac{m}{n}\right)\right)\right) - \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\operatorname{Re}\left(\frac{m}{n}\right)\right)\right)\right)$$

### With FractionalPart

### For real arguments

04.06.27.0026.01

$$m \bmod n = n \operatorname{frac}\left(\frac{m}{n}\right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} > 0 \vee \frac{m}{n} \in \mathbb{Z}$$

04.06.27.0027.01

$$m \bmod n = n \left(\operatorname{frac}\left(\frac{m}{n}\right) + 1\right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.27.0028.01

$$m \bmod n = n \left(\operatorname{frac}\left(\frac{m}{n}\right) - \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) + \theta\left(\frac{m}{n}\right)\right) + 1\right) /; \frac{m}{n} \in \mathbb{R}$$

### For complex arguments

04.06.27.0029.01

$$m \bmod n = n \operatorname{frac}\left(\frac{m}{n}\right) /; \operatorname{Re}\left(\frac{m}{n}\right) \geq 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) \geq 0 \vee \frac{m}{n} \in \mathbb{Z} \vee \frac{im}{n} \in \mathbb{Z}$$

04.06.27.0030.01

$$m \bmod n = n \left(\operatorname{frac}\left(\frac{m}{n}\right) + 1\right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z} \vee \operatorname{Re}\left(\frac{m}{n}\right) < 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) > 0$$

04.06.27.0031.01

$$m \bmod n = n \left(\operatorname{frac}\left(\frac{m}{n}\right) + i\right) /; \frac{im}{n} \in \mathbb{R} \wedge \frac{im}{n} > 0 \wedge \frac{im}{n} \notin \mathbb{Z} \vee \operatorname{Re}\left(\frac{m}{n}\right) > 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0032.01

$$m \bmod n = n \left(\operatorname{frac}\left(\frac{m}{n}\right) + 1 + i\right) /; \operatorname{Re}\left(\frac{m}{n}\right) < 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0005.01

$$m \bmod n = n \left(\operatorname{frac}\left(\frac{m}{n}\right) + 1 + i - i \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\operatorname{Im}\left(\frac{m}{n}\right)\right)\right) - \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\operatorname{Re}\left(\frac{m}{n}\right)\right)\right)\right)$$

**With Quotient**

04.06.27.0006.01

$$m \bmod n = m - n \operatorname{quotient}(m, n)$$

**With elementary functions**

04.06.27.0007.01

$$m \bmod n = \frac{n}{2} - \frac{n}{\pi} \tan^{-1} \left( \cot \left( \frac{\pi m}{n} \right) \right); \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

**Zeros**

04.06.30.0001.01

$$m \bmod n = 0 /; m = 0 \wedge n \neq 0$$

**Theorems****Linear congruential random number generator**

A sequence of pseudorandom numbers  $r_k$  is generated by  $r_{k+1} = (a r_k + c) \bmod m$ , with  $a, c, m \in \mathbb{N}$ ,  $m > 0$ ,  $0 \leq a < m$ ,  $0 \leq c < m$ ,  $0 \leq r_0 < m$ .

**Chinese remainder theorem**

Let  $m_1, m_2, \dots, m_n$  be pairwise relatively prime integers ( $\gcd(m_i, m_k) = 1 /; i \neq k$ ). Then for given integers  $z_1, z_2, \dots, z_n$  there exists a unique  $(\bmod m_k)$  integer  $z$  such that  $z \bmod m_k = z_k$ .

**Legendre theorem**

If  $a, b, c \in \mathbb{N}^+ \wedge \gcd(a, b) = \gcd(a, c) = \gcd(b, c) = 0 \wedge \sqrt{a}, \sqrt{b}, \sqrt{c} \notin \mathbb{N}$ , then the equation  $a x^2 + b y^2 + c z^2 = 0$  has nontrivial solutions for  $x, y, z$  if and only if the equations  $x^2 - b c \bmod a = 0 \wedge y^2 - a c \bmod b = 0 \wedge z^2 - a b \bmod c = 0$  are solvable.

**Gauss' Easter formula**

Easter Sunday is the  $i + j + 1$  th day after the 21st of March, where

$$i = 28 \delta_{h,29} + 27 \delta_{h,28} \theta(a - 11) + 11 (1 - \delta_{h,29}) (1 - \delta_{h,28} \theta(a - 11)), j = (2b + 4c + 6i + g) \bmod 7,$$

$$h = (f + 19a) \bmod 30, a = \text{year} \bmod 19, b = \text{year} \bmod 4, c = \text{year} \bmod 7,$$

$$d = \left\lfloor \frac{(8 \lfloor \text{year} / 100 \rfloor + 13)}{25} \right\rfloor - 2, e = \left\lfloor \frac{\text{year}}{100} \right\rfloor - \left\lfloor \frac{\text{year}}{400} \right\rfloor - 2, f = (15 + e - d) \bmod 30, g = (6 + e) \bmod 7.$$

**History**

–C. F. Gauss (1801) introduced the symbol mod

Applications include pseudo-random number generation.

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