NevilleThetaD

Notations

Traditional name

Neville theta function ϑ_d

Traditional notation

$\vartheta_d(z \mid m)$

Mathematica StandardForm notation

NevilleThetaD[z, m]

Primary definition

09.10.02.0001.02

$$\vartheta_d(z \mid m) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{K(m)}} \left(1 + 2 \sum_{k=1}^{\infty} q(m)^{k^2} \cos\left(\frac{k \pi z}{K(m)}\right)\right)$$

Specific values

Specialized values

For fixed *z*

09.10.03.0001.01

$$\vartheta_d(z \mid 0) = 1$$

09.10.03.0002.01

$$\vartheta_d(z \mid 1) = 1$$

For fixed *m*

09.10.03.0003.01

$$\vartheta_d(0 \mid m) = 1$$

General characteristics

Domain and analyticity

$\vartheta_d(z \mid m)$ is an analytical meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.10.04.0001.01

$(z * m) \longrightarrow \vartheta_d(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\vartheta_d(z \mid m)$ is an even function with respect to z.

09.10.04.0002.01

$\vartheta_d(-z \mid m) == \vartheta_d(z \mid m)$

Mirror symmetry

09.10.04.0003.01

$\vartheta_d(\bar{z} \mid \bar{m}) == \overline{\vartheta_d(z \mid m)}$

Periodicity

$\vartheta_d(z \mid m)$ is a periodic function with respect to z with period $2 K(m)$.

09.10.04.0004.01

$\vartheta_d(z + 2 K(m) \mid m) == \vartheta_d(z \mid m)$

09.10.04.0006.01

$\vartheta_d(z + 4 K(m) \mid m) = \vartheta_d(z \mid m)$

09.10.04.0005.01

$\vartheta_d(z + 2 r K(m) \mid m) == \vartheta_d(z \mid m) /; r \in \mathbb{Z}$

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

09.10.06.0001.02

$$\vartheta_d(z \mid m) == \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{K(m)}} \left(1 + 2 \sum_{k=1}^{\infty} q(m)^{k^2} \cos\left(\frac{k \pi z}{K(m)}\right)\right)$$

Product representations

09.10.08.0001.01

$$\vartheta_d(z \mid m) == \frac{\sqrt[12]{1-m}\ \sqrt[12]{m}}{\sqrt[3]{2}\ \sqrt[12]{q(m)}} \prod_{k=1}^{\infty} \left(q(m)^{4k-2} + 2 \cos\left(\frac{\pi z}{K(m)}\right) q(m)^{2k-1} + 1\right)$$

Differential equations

Partial differential equations

09.10.13.0001.01

$$K(m)\frac{\partial^2 \vartheta_d(z\mid m)}{\partial z^2} + 2\,z\,(E(m)+(m-1)\,K(m))\frac{\partial \vartheta_d(z\mid m)}{\partial z} - 4\,(m-1)\,m\,K(m)\frac{\partial \vartheta_d(z\mid m)}{\partial m} + ((K(m)-1)\,m+E(m))\,\vartheta_d(z\mid m) == 0$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.10.16.0001.01

$$\vartheta_d(z+K(m)\mid m) == \sqrt[4]{1-m}\;\vartheta_n(z\mid m)$$

09.10.16.0002.01

$$\vartheta_d(z+(2\,r+1)\,K(m)\mid m) == \sqrt[4]{1-m}\;\vartheta_n(z\mid m)\;/;\; r\in\mathbb{Z}$$

Differentiation

Low-order differentiation

With respect to *z*

09.10.20.0001.02

$$\frac{\partial \vartheta_d(z\mid m)}{\partial z} == -\frac{\sqrt{2}\,\pi^{3/2}}{K(m)^{3/2}}\sum_{k=1}^{\infty} k\,q(m)^{k^2}\sin\left(\frac{k\,\pi\,z}{K(m)}\right)$$

09.10.20.0002.02

$$\frac{\partial^2 \vartheta_d(z\mid m)}{\partial z^2} == -\frac{\sqrt{2}\,\pi^{5/2}}{K(m)^{5/2}}\sum_{k=1}^{\infty} k^2\,q(m)^{k^2}\cos\left(\frac{k\,\pi\,z}{K(m)}\right)$$

Symbolic differentiation

With respect to *z*

09.10.20.0003.02

$$\frac{\partial^n \vartheta_d(z\mid m)}{\partial z^n} == \sqrt{2}\,\pi^{n+\frac{1}{2}}\,K(m)^{-n-\frac{1}{2}}\sum_{k=1}^{\infty} k^n\,q(m)^{k^2}\cos\left(\frac{\pi\,n}{2}+\frac{k\,\pi\,z}{K(m)}\right)/;\; n\in\mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.10.20.0004.02

$$\frac{\partial^\alpha \vartheta_d(z\mid m)}{\partial z^\alpha} == \sqrt{\frac{\pi}{2}}\,\frac{1}{\sqrt{K(m)}}\,z^{-\alpha}\left(2^{\alpha+1}\sqrt{\pi}\sum_{k=1}^{\infty} q(m)^{k^2}\,{}_1\tilde{F}_2\left(1;\frac{1}{2}-\frac{\alpha}{2},1-\frac{\alpha}{2};-\frac{k^2\,\pi^2\,z^2}{4\,K(m)^2}\right)+\frac{1}{\Gamma(1-\alpha)}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.10.21.0001.02

$$\int \vartheta_d(z \mid m)\, dz = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{K(m)}} \left(z + \frac{2\,K(m)}{\pi} \sum_{k=1}^{\infty} \frac{q(m)^{k^2}}{k} \sin\left(\frac{k\,\pi\,z}{K(m)}\right) \right)$$

Representations through equivalent functions

With related functions

Involving Jacobi and other Neville functions

09.10.27.0001.01

$$\vartheta_d(z \mid m) = \mathrm{dc}(z \mid m)\, \vartheta_c(z \mid m)$$

09.10.27.0002.01

$$\vartheta_d(z \mid m) = \frac{\vartheta_c(z \mid m)}{\mathrm{cd}(z \mid m)}$$

09.10.27.0003.01

$$\vartheta_d(z \mid m) = \mathrm{dn}(z \mid m)\, \vartheta_n(z \mid m)$$

09.10.27.0004.01

$$\vartheta_d(z \mid m) = \frac{\vartheta_n(z \mid m)}{\mathrm{nd}(z \mid m)}$$

09.10.27.0005.01

$$\vartheta_d(z \mid m) = \sqrt[4]{1-m}\; \vartheta_n(K(m) - z \mid m)$$

09.10.27.0006.01

$$\vartheta_d(z \mid m) = \mathrm{ds}(z \mid m)\, \vartheta_s(z \mid m)$$

09.10.27.0007.01

$$\vartheta_d(z \mid m) = \frac{\vartheta_s(z \mid m)}{\mathrm{sd}(z \mid m)}$$

Involving theta functions

09.10.27.0008.02

$$\vartheta_d(z \mid m) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{K(m)}} \vartheta_3\left(\frac{\pi\,z}{2\,K(m)}, q(m)\right)$$

09.10.27.0009.01

$$\vartheta_d(z \mid m) = \frac{1}{\vartheta_3(0, q(m))} \vartheta_3\left(\frac{\pi\,z}{2\,K(m)}, q(m)\right)$$

History

– K. Weierstrass (1894)
– E. N. Neville (1944)

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