

# NevilleThetaN

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## Notations

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### Traditional name

The Neville theta function  $\vartheta_n$

### Traditional notation

$\vartheta_n(z | m)$

### Mathematica StandardForm notation

NevilleThetaN[ $z$ ,  $m$ ]

## Primary definition

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09.11.02.0001.01

$$\vartheta_n(z | m) = \frac{\sqrt{\pi}}{\sqrt{2} \sqrt{1-m} \sqrt{K(m)}} \left( 1 + 2 \sum_{k=1}^{\infty} (-1)^k q(m)^{k^2} \cos\left(\frac{k\pi z}{K(m)}\right) \right)$$

## Specific values

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### Specialized values

For fixed  $z$

09.11.03.0001.01

$$\vartheta_n(z | 0) = 1$$

09.11.03.0002.01

$$\vartheta_n(z | 1) = 1$$

For fixed  $m$

09.11.03.0003.01

$$\vartheta_n(0 | m) = 1$$

## General characteristics

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### Domain and analyticity

$\vartheta_n(z | m)$  is a meromorphic function of  $z$  and  $m$  which is defined over  $\mathbb{C}^2$ .

09.11.04.0001.01

$$(z * m) \rightarrow \vartheta_n(z | m) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

## Symmetries and periodicities

### Parity

$\vartheta_n(z | m)$  is an even function with respect to  $z$ .

09.11.04.0002.01

$$\vartheta_n(-z | m) = \vartheta_n(z | m)$$

### Mirror symmetry

09.11.04.0003.01

$$\vartheta_n(\bar{z} | \bar{m}) = \overline{\vartheta_n(z | m)}$$

### Periodicity

$\vartheta_n(z | m)$  is a periodic function with respect to  $z$  with period  $2K(m)$ .

09.11.04.0004.01

$$\vartheta_n(z + 2K(m) | m) = \vartheta_n(z | m)$$

09.11.04.0006.01

$$\vartheta_n(z + 4K(m) | m) = \vartheta_n(z | m)$$

09.11.04.0005.01

$$\vartheta_n(z + 2rK(m) | m) = \vartheta_n(z | m) ; r \in \mathbb{Z}$$

## Branch points

Branch points locations: complicated

## Branch cuts

Branch cut locations: complicated

## Series representations

### Generalized power series

09.11.06.0001.01

$$\vartheta_n(z | m) = \frac{\sqrt{\pi}}{\sqrt{2} \sqrt[4]{1-m} \sqrt{K(m)}} \left( 1 + 2 \sum_{k=1}^{\infty} (-1)^k q(m)^{k^2} \cos\left(\frac{k\pi z}{K(m)}\right) \right)$$

## Product representations

09.11.08.0001.01

$$\vartheta_n(z | m) = \frac{\sqrt[12]{m}}{\sqrt[3]{2} \sqrt[6]{1-m} \sqrt[12]{q(m)}} \prod_{k=1}^{\infty} \left( q(m)^{4k-2} - 2 \cos\left(\frac{\pi z}{K(m)}\right) q(m)^{2k-1} + 1 \right)$$

## Differential equations

### Partial differential equations

09.11.13.0001.01

$$K(m) \frac{\partial^2 \vartheta_n(z | m)}{\partial z^2} + 2z (E(m) + (m-1)K(m)) \frac{\partial \vartheta_n(z | m)}{\partial z} - 4(m-1)mK(m) \frac{\partial \vartheta_n(z | m)}{\partial m} + (E(m) - K(m)) \vartheta_n(z | m) = 0$$

## Transformations

### Transformations and argument simplifications

#### Argument involving basic arithmetic operations

09.11.16.0001.01

$$\vartheta_n(z + K(m) | m) = \frac{1}{\sqrt[4]{1-m}} \vartheta_n(z | m)$$

09.11.16.0002.01

$$\vartheta_n(z + (2r+1)K(m) | m) = \frac{1}{\sqrt[4]{1-m}} \vartheta_n(z | m); r \in \mathbb{Z}$$

## Differentiation

### Low-order differentiation

#### With respect to $z$

09.11.20.0001.01

$$\frac{\partial \vartheta_n(z | m)}{\partial z} = \frac{\sqrt{2} \pi^{3/2}}{\sqrt[4]{1-m} K(m)^{3/2}} \sum_{k=1}^{\infty} (-1)^{k-1} k q(m)^{k^2} \sin\left(\frac{k\pi z}{K(m)}\right)$$

09.11.20.0002.01

$$\frac{\partial^2 \vartheta_n(z | m)}{\partial z^2} = \frac{\sqrt{2}}{\sqrt[4]{1-m}} \left(\frac{\pi}{K(m)}\right)^{5/2} \sum_{k=1}^{\infty} (-1)^{k-1} k^2 q(m)^{k^2} \cos\left(\frac{k\pi z}{K(m)}\right)$$

### Symbolic differentiation

#### With respect to $z$

09.11.20.0003.01

$$\frac{\partial^n \vartheta_n(z | m)}{\partial z^n} = \frac{\sqrt{2}}{\sqrt[4]{1-m}} \left(\frac{\pi}{K(m)}\right)^{n+\frac{1}{2}} \sum_{k=1}^{\infty} (-1)^k k^n q(m)^{k^2} \cos\left(\frac{\pi n}{2} + \frac{k\pi z}{K(m)}\right); n \in \mathbb{N}^+$$

### Fractional integro-differentiation

#### With respect to $z$

09.11.20.0004.01

$$\frac{\partial^\alpha \vartheta_n(z | m)}{\partial z^\alpha} = \frac{\sqrt{\pi} z^{-\alpha}}{\sqrt{2} \sqrt[4]{1-m} \sqrt{K(m)}} \left( 2^{\alpha+1} \sqrt{\pi} \sum_{k=1}^{\infty} (-1)^k q(m)^{k^2} {}_1\tilde{F}_2 \left( 1; \frac{1-\alpha}{2}, 1 - \frac{\alpha}{2}; -\frac{k^2 \pi^2 z^2}{4 K(m)^2} \right) + \frac{1}{\Gamma(1-\alpha)} \right)$$

## Integration

### Indefinite integration

#### Involving only one direct function

09.11.21.0001.01

$$\int \vartheta_n(z | m) dz = \frac{\sqrt{\pi}}{\sqrt{2} \sqrt[4]{1-m} \sqrt{K(m)}} \left( z + \frac{2 K(m)}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k q(m)^{k^2}}{k} \sin \left( \frac{k \pi z}{K(m)} \right) \right)$$

## Representations through equivalent functions

### With related functions

#### Involving Jacobi and other Neville functions

09.11.27.0001.01

$$\vartheta_n(z | m) = \operatorname{nc}(z | m) \vartheta_c(z | m)$$

09.11.27.0002.01

$$\vartheta_n(z | m) = \frac{\vartheta_c(z | m)}{\operatorname{cn}(z | m)}$$

09.11.27.0003.01

$$\vartheta_n(z | m) = \operatorname{nd}(z | m) \vartheta_d(z | m)$$

09.11.27.0004.01

$$\vartheta_n(z | m) = \frac{\vartheta_d(z | m)}{\operatorname{dn}(z | m)}$$

09.11.27.0005.01

$$\vartheta_n(z | m) = \frac{1}{\sqrt[4]{1-m}} \vartheta_d(z + K(m) | m)$$

09.11.27.0006.01

$$\vartheta_n(z | m) = \operatorname{ns}(z | m) \vartheta_s(z | m)$$

09.11.27.0007.01

$$\vartheta_n(z | m) = \frac{\vartheta_s(z | m)}{\operatorname{sn}(z | m)}$$

#### Involving theta functions

09.11.27.0008.01

$$\vartheta_n(z | m) = \frac{\sqrt{\pi}}{\sqrt{2} \sqrt[4]{1-m} \sqrt{K(m)}} \vartheta_4 \left( \frac{\pi z}{2 K(m)}, q(m) \right)$$

09.11.27.0009.01

$$\vartheta_n(z | m) = \frac{1}{\vartheta_4(0, q(m))} \vartheta_4\left(\frac{\pi z}{2K(m)}, q(m)\right)$$

## History

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- K. Weierstrass (1894)
- E. N. Neville (1944)

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