

Quotient

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Notations

Traditional name

Integer part of the quotient

Traditional notation

$\text{quotient}(m, n)$

Mathematica StandardForm notation

`Quotient[m, n]`

Primary definition

04.07.02.0001.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} \right\rfloor$$

$\text{quotient}(m, n)$ is the integer quotient of m and n .

Examples: $\text{quotient}(5, 2) = 2$, $\text{quotient}(13, 3) = 4$, $\text{quotient}(-4, 3) = -2$, $\text{quotient}(\pi, 2) = 1$,
 $\text{quotient}(27 - 3i, 5) = 5 - i$, $\text{quotient}(-\pi, 2) = -2$, $\text{quotient}(2.7 - 3i, 5) = -i$.

Specific values

Specialized values

For fixed m

04.07.03.0001.01

$$\text{quotient}(m, 1) = m \text{ ; } m \in \mathbb{Z}$$

For fixed n

04.07.03.0002.01

$$\text{quotient}(0, n) = 0 \text{ ; } n \neq 0$$

04.07.03.0003.01

$$\text{quotient}(1, n) = -1 \text{ ; } n \in \mathbb{Z} \wedge n < 0$$

04.07.03.0004.01

$$\text{quotient}(1, n) = 0 \text{ ; } n \in \mathbb{Z} \wedge n > 1$$

04.07.03.0005.01

$$\text{quotient}(m, n) = 0 \text{ ; } m \in \mathbb{N} \wedge n \in \mathbb{N}^+ \wedge m < n$$

04.07.03.0006.01

$$\text{quotient}(m, n) = 1 \text{ ; } m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n \leq m < 2n$$

04.07.03.0007.01

$$\text{quotient}(m, n) = k \text{ ; } m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge k \in \mathbb{N}^+ \wedge kn \leq m < (k+1)n$$

04.07.03.0008.01

$$\text{quotient}(n, n) = 1$$

04.07.03.0009.01

$$\text{quotient}(2n, n) = 2$$

04.07.03.0010.01

$$\text{quotient}((p-1)!, p) = \frac{(p-1)! + 1 - p}{p} \text{ ; } p \in \mathbb{P}$$

04.07.03.0011.01

$$\text{quotient}\left(\binom{2p-1}{p-1}, p^3\right) = \frac{1}{p^3} \left(\binom{2p-1}{p-1} - 1\right) \text{ ; } p \in \mathbb{P} \wedge p > 3$$

Values at fixed points

04.07.03.0012.01

$$\text{quotient}(0, 1) = 0$$

04.07.03.0013.01

$$\text{quotient}(1, 2) = 0$$

04.07.03.0014.01

$$\text{quotient}(1, 3) = 0$$

04.07.03.0015.01

$$\text{quotient}(2, 3) = 0$$

04.07.03.0016.01

$$\text{quotient}(3, 3) = 1$$

04.07.03.0017.01

$$\text{quotient}(4, 3) = 1$$

04.07.03.0018.01

$$\text{quotient}(5, 3) = 1$$

04.07.03.0019.01

$$\text{quotient}(12, 8) = 1$$

04.07.03.0020.01

$$\text{quotient}(-3, -2) = 1$$

04.07.03.0021.01

$$\text{quotient}\left(-\frac{27}{10}, \frac{23}{5}\right) = -1$$

04.07.03.0022.01

$$\text{quotient}(2\pi, e) = 2$$

04.07.03.0023.01

$$\text{quotient}(-\pi, 2) = -2$$

04.07.03.0024.01

$$\text{quotient}(\pi, e) = 1$$

04.07.03.0025.01

$$\text{quotient}(-3 + \pi i, -2 - 3 i e) = -1 - i$$

04.07.03.0026.01

$$\text{quotient}(5.2, 3.1) = 1$$

Values at infinities

04.07.03.0027.01

$$\text{quotient}(n, \infty) = 0$$

04.07.03.0028.01

$$\text{quotient}(n, -\infty) = 0$$

04.07.03.0029.01

$$\text{quotient}(n, i \infty) = 0$$

04.07.03.0030.01

$$\text{quotient}(n, -i \infty) = 0$$

04.07.03.0031.01

$$\text{quotient}(n, \tilde{\infty}) = 0$$

General characteristics

Domain and analyticity

$\text{quotient}(m, n)$ is a nonanalytical function; it is a piecewise continuous function which is defined over \mathbb{C}^2 .

04.07.04.0001.01

$$(m * n) \rightarrow \text{quotient}(m, n) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\text{quotient}(m, n)$ is an even function.

04.07.04.0002.01

$$\text{quotient}(-m, -n) = \text{quotient}(m, n)$$

Periodicity

No periodicity

Sets of discontinuity

The function $\text{quotient}(m, n)$ is a piecewise continuous function with jumps on the curves $\text{Re}\left(\frac{m}{n}\right) = k \vee \text{Im}\left(\frac{m}{n}\right) = l$; $k, l \in \mathbb{Z}$. The functional property $\text{quotient}(m, n) = \text{quotient}\left(\frac{m}{n}, 1\right) = \left\lfloor \frac{m}{n} \right\rfloor$ makes the behaviour of the $\text{quotient}(m, n)$ similar to behavior of $\left\lfloor \frac{m}{n} \right\rfloor$.

04.07.04.0003.01

$$\mathcal{DS}_m(\text{quotient}(m, n)) = \{ \{(nk - i\infty, nk + i\infty), -1\}; k \in \mathbb{Z} \}, \{ \{(in k - \infty, in k + \infty), -i\}; k \in \mathbb{Z} \}$$

04.07.04.0004.01

$$\mathcal{DS}_n(\text{quotient}(m, n)) = \left\{ \left\{ \left(\frac{m}{k - i\infty}, k + i\infty \right), -1 \right\}; k \in \mathbb{Z} \right\}, \left\{ \left\{ \left(\frac{m}{i k - \infty}, i k + \infty \right), -i \right\}; k \in \mathbb{Z} \right\}$$

04.07.04.0005.01

$$\lim_{\epsilon \rightarrow +0} \text{quotient}(m + \epsilon, n) = \text{quotient}(m, n); \text{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

04.07.04.0006.01

$$\lim_{\epsilon \rightarrow +0} \text{quotient}(m - \epsilon, n) = \text{quotient}(m, n) - 1; \text{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

04.07.04.0007.01

$$\lim_{\epsilon \rightarrow +0} \text{quotient}(m + i\epsilon, n) = \text{quotient}(m, n); \text{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

04.07.04.0008.01

$$\lim_{\epsilon \rightarrow +0} \text{quotient}(m - i\epsilon, n) = \text{quotient}(m, n) - i; \text{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge n \in \mathbb{R} \wedge n > 0$$

Series representations

Exponential Fourier series

04.07.06.0001.01

$$\text{quotient}(m, n) = \frac{m}{n} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{2\pi k m}{n}\right) - \frac{1}{2}; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

Other series representations

04.07.06.0002.01

$$\text{quotient}(m, n) = \frac{m}{n} + \frac{1}{2n} \sum_{k=1}^{n-1} \sin\left(\frac{2\pi k m}{n}\right) \cot\left(\frac{\pi k}{n}\right) - \frac{1}{2}; m \in \mathbb{Z} \wedge n - 1 \in \mathbb{N}^+ \wedge \frac{m}{n} \notin \mathbb{Z}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

04.07.16.0001.01

$$\text{quotient}(-m, -n) = \text{quotient}(m, n)$$

04.07.16.0002.01

$$\text{quotient}(m, -n) = -\text{quotient}(m, n) + \chi_{\mathbb{Z}}\left(\frac{m}{n}\right) - 1 \quad ; \quad m \in \mathbb{R} \wedge n \in \mathbb{R}$$

04.07.16.0003.01

$$\text{quotient}(m, -n) = -\text{quotient}(m, n) - \left(1 - \chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Re}\left(\frac{m}{n}\right)\right|\right) - i \left(1 - \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Im}\left(\frac{m}{n}\right)\right|\right)$$

04.07.16.0004.01

$$\text{quotient}(-m, n) = \chi_{\mathbb{Z}}\left(\frac{m}{n}\right) - 1 - \text{quotient}(m, n) \quad ; \quad m \in \mathbb{R} \wedge n \in \mathbb{R}$$

04.07.16.0005.01

$$\text{quotient}(-m, n) = -\text{quotient}(m, n) - \left(1 - \chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Re}\left(\frac{m}{n}\right)\right|\right) - i \left(1 - \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Im}\left(\frac{m}{n}\right)\right|\right)$$

04.07.16.0006.01

$$\text{quotient}(i m, i n) = \text{quotient}(m, n)$$

04.07.16.0007.01

$$\text{quotient}(i m, n) = i \text{quotient}(m, n) + \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) - 1$$

04.07.16.0008.01

$$\text{quotient}(-i m, n) = -i \text{quotient}(m, n) + i \left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) - 1\right)$$

04.07.16.0009.01

$$\text{quotient}(m, i n) = -i \text{quotient}(m, n) + i \left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) - 1\right)$$

04.07.16.0010.01

$$\text{quotient}(m, -i n) = \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) + i \text{quotient}(m, n) - 1$$

04.07.16.0011.01

$$\text{quotient}\left(\frac{m}{n}, 1\right) = \text{quotient}(m, n)$$

Argument involving related functions

04.07.16.0014.01

$$\text{quotient}(\lfloor m \rfloor, n) = \left\lfloor \frac{\lfloor m \rfloor}{n} \right\rfloor$$

04.07.16.0015.01

$$\text{quotient}(\lfloor m \rfloor, 1) = \lfloor m \rfloor$$

04.07.16.0016.01

$$\text{quotient}(\lfloor m \rfloor, n) = \left\lfloor \frac{\lfloor m \rfloor}{n} \right\rfloor$$

04.07.16.0017.01

$$\text{quotient}(\lfloor m \rfloor, 1) = \lfloor m \rfloor$$

04.07.16.0018.01

$$\text{quotient}(\lceil m \rceil, n) = \left\lceil \frac{\lceil m \rceil}{n} \right\rceil$$

04.07.16.0019.01

$$\text{quotient}(\lceil m \rceil, 1) = \lceil m \rceil$$

04.07.16.0020.01

$$\text{quotient}(\text{int}(m), n) = \left\lfloor \frac{\text{int}(m)}{n} \right\rfloor$$

04.07.16.0021.01

$$\text{quotient}(\text{int}(m), 1) = \text{int}(m)$$

04.07.16.0022.01

$$\text{quotient}(\text{frac}(m), n) = \left\lfloor \frac{\text{frac}(m)}{n} \right\rfloor$$

04.07.16.0023.01

$$\text{quotient}(m \bmod n, n) = \left\lfloor \frac{m \bmod n}{n} \right\rfloor$$

04.07.16.0024.01

$$\text{quotient}(m \bmod 1, 1) = 0$$

04.07.16.0025.01

$$\text{quotient}(\text{quotient}(m, n), n) = \left\lfloor \frac{1}{n} \left\lfloor \frac{m}{n} \right\rfloor \right\rfloor$$

04.07.16.0026.01

$$\text{quotient}(\text{quotient}(m, 1), 1) = \lfloor m \rfloor$$

Addition formulas

04.07.16.0027.01

$$\text{quotient}(m + k n, n) = \text{quotient}(m, n) + k \quad ; k \in \mathbb{Z}$$

Multiple arguments

04.07.16.0012.01

$$\text{quotient}(k m, n) = k \text{quotient}(m, n) + \sum_{j=0}^{k-1} j \left(\theta \left(\frac{m}{n} \bmod 1 - \frac{j}{k} \right) \left(1 - \theta \left(\frac{m}{n} \bmod 1 - \frac{j+1}{k} \right) \right) \right) \quad ; k \in \mathbb{N} \wedge \frac{m}{n} \in \mathbb{R}$$

Related transformations

04.07.16.0013.01

$$\text{quotient}(m, n) = \left\lfloor \text{Re} \left(\frac{m}{n} \right) \right\rfloor + i \left\lfloor \text{Im} \left(\frac{m}{n} \right) \right\rfloor$$

Identities

Functional identities

04.07.17.0001.01

$$\text{quotient} \left(\frac{m}{n}, 1 \right) = \text{quotient}(m, n)$$

Complex characteristics

Real part

04.07.19.0001.01

$$\operatorname{Re}(\operatorname{quotient}(m, n)) = \left[\frac{\operatorname{Re}(m) \operatorname{Re}(n) + \operatorname{Im}(m) \operatorname{Im}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]$$

Imaginary part

04.07.19.0002.01

$$\operatorname{Im}(\operatorname{quotient}(m, n)) = \left[\frac{\operatorname{Im}(m) \operatorname{Re}(n) - \operatorname{Im}(n) \operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]$$

Absolute value

04.07.19.0003.01

$$|\operatorname{quotient}(m, n)| = \sqrt{\left[\frac{\operatorname{Re}(m) \operatorname{Re}(n) + \operatorname{Im}(m) \operatorname{Im}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]^2 + \left[\frac{\operatorname{Im}(m) \operatorname{Re}(n) - \operatorname{Im}(n) \operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]^2}$$

Argument

04.07.19.0004.01

$$\arg(\operatorname{quotient}(m, n)) = \tan^{-1} \left(\left[\frac{\operatorname{Im}(m) \operatorname{Im}(n) + \operatorname{Re}(m) \operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right], \left[\frac{\operatorname{Im}(m) \operatorname{Re}(n) - \operatorname{Im}(n) \operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right] \right)$$

Conjugate value

04.07.19.0005.01

$$\overline{\operatorname{quotient}(m, n)} = \left[\frac{\operatorname{Im}(m) \operatorname{Im}(n) + \operatorname{Re}(m) \operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right] - i \left[\frac{\operatorname{Im}(m) \operatorname{Re}(n) - \operatorname{Im}(n) \operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]$$

Signum value

04.07.19.0006.01

$$\operatorname{sgn}(\operatorname{quotient}(m, n)) = \frac{i \left[\frac{\operatorname{Im}(m) \operatorname{Re}(n) - \operatorname{Im}(n) \operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right] + \left[\frac{\operatorname{Im}(m) \operatorname{Im}(n) + \operatorname{Re}(m) \operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]}{\sqrt{\left[\frac{\operatorname{Im}(m) \operatorname{Re}(n) - \operatorname{Im}(n) \operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]^2 + \left[\frac{\operatorname{Im}(m) \operatorname{Im}(n) + \operatorname{Re}(m) \operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right]^2}}$$

04.07.19.0007.01

$$\operatorname{sgn}(\operatorname{quotient}(m, n)) = \operatorname{sgn} \left(\frac{m}{n} \right); m \in \mathbb{R} \wedge n \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to m

$$\frac{\partial \text{quotient}(m, n)}{\partial m} = 0$$

In a distributional sense for $x \in \mathbb{R}$.

$$\frac{\partial \text{quotient}(x, n)}{\partial x} = \text{sgn}(n) \sum_{k=-\infty}^{\infty} \delta(x - kn)$$

With respect to n

$$\frac{\partial \text{quotient}(m, n)}{\partial n} = 0$$

In a distributional sense for $x \in \mathbb{R}$.

$$\frac{\partial \text{quotient}(m, x)}{\partial x} = -\frac{\text{sgn}(m) m}{x^2} \sum_{k=-\infty}^{\infty} \delta_{k,0} \delta\left(\frac{m}{x} - k\right)$$

Fractional integro-differentiation

With respect to m

$$\frac{\partial^\alpha \text{quotient}(m, n)}{\partial m^\alpha} = \frac{\text{quotient}(m, n) m^{-\alpha}}{\Gamma(1 - \alpha)}$$

With respect to n

$$\frac{\partial^\alpha \text{quotient}(m, n)}{\partial n^\alpha} = \frac{\text{quotient}(m, n) n^{-\alpha}}{\Gamma(1 - \alpha)}$$

Integration

Indefinite integration

Involving only one direct function with respect to m

$$\int \text{quotient}(m, n) dm = m \text{quotient}(m, n)$$

Involving one direct function and elementary functions with respect to m

Involving power function

$$\int m^{\alpha-1} \text{quotient}(m, n) dm = \frac{m^\alpha \text{quotient}(m, n)}{\alpha}$$

$$\int \frac{\text{quotient}(m, n)}{m} dm = \log(m) \text{quotient}(m, n)$$

Involving only one direct function with respect to n

$$\int \text{quotient}(m, n) dn = n \text{quotient}(m, n)$$

Involving one direct function and elementary functions with respect to n

Involving power function

$$\int n^{\alpha-1} \text{quotient}(m, n) dn = \frac{n^\alpha \text{quotient}(m, n)}{\alpha}$$

$$\int \frac{\text{quotient}(m, n)}{n} dn = \log(n) \text{quotient}(m, n)$$

Definite integration

For the direct function with respect to m

$$\int_0^a \text{quotient}(t, n) dt = \frac{1}{2} \text{quotient}(a, n) (2a - n - n \text{quotient}(a, n))$$

$$\int_0^a t^{\alpha-1} \text{quotient}(t, n) dt = \frac{1}{\alpha} (\text{quotient}(a, n) a^\alpha + n^\alpha (\zeta(-\alpha, \text{quotient}(a, n) + 1) - \zeta(-\alpha)))$$

$$\int_a^\infty t^{\alpha-1} \text{quotient}(t, n) dt = -\frac{\text{quotient}(a, n) a^\alpha + n^\alpha \zeta(-\alpha, \text{quotient}(a, n) + 1)}{\alpha} ; \text{Re}(\alpha) < -1$$

$$\int_0^\infty t^{\alpha-1} \text{quotient}(t, n) dt = -\frac{n^\alpha}{\alpha} \zeta(-\alpha) ; \text{Re}(\alpha) < -1$$

$$\int_{-a}^a \text{Quotient}(t, n) dt = -a$$

For the direct function with respect to n

In the following formulas $a \in \mathbb{R}$.

$$\int_0^a t^{\alpha-1} \text{quotient}(m, t) dt = \frac{\text{quotient}(m, a) a^\alpha + m^\alpha \zeta(\alpha, \text{quotient}(m, a) + 1)}{\alpha} ; \text{Re}(\alpha) > 1$$

04.07.21.0013.01

$$\int_a^{\infty} t^{\alpha-1} \text{quotient}(m, t) dt = \frac{1}{\alpha} (m^{\alpha} (\zeta(\alpha) - \zeta(\alpha, \text{quotient}(m, a) + 1)) - a^{\alpha} \text{quotient}(m, a))$$

04.07.21.0014.01

$$\int_0^{\infty} t^{\alpha-1} \text{quotient}(m, t) dt = \frac{m^{\alpha} \zeta(\alpha)}{\alpha} \quad ; \quad \text{Re}(\alpha) > 1$$

Integral transforms

Fourier exp transforms

04.07.22.0001.01

$$\mathcal{F}_i[\text{quotient}(t, n)](z) = -\sqrt{\frac{\pi}{2}} \delta(z) + \frac{i}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{1}{k} \left(\delta\left(\frac{2k\pi}{n} - z\right) - \delta\left(\frac{2\pi k}{n} + z\right) \right) - \frac{i\sqrt{2\pi}}{n} \delta'(z)$$

Fourier cos transforms

04.07.22.0002.01

$$\mathcal{F}_{c_i}[\text{quotient}(t, n)](z) = -\frac{1}{\sqrt{2\pi} z} \cot\left(\frac{nz}{2}\right) - \sqrt{\frac{\pi}{2}} \delta(z)$$

Fourier sin transforms

04.07.22.0003.01

$$\mathcal{F}_{s_i}[\text{quotient}(t, n)](z) = \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{1}{k} \left(\delta\left(\frac{2k\pi}{n} - z\right) - \delta\left(\frac{2\pi k}{n} + z\right) \right) - \frac{\sqrt{2\pi} \delta'(z)}{n} - \frac{1}{\sqrt{2\pi} z}$$

Laplace transforms

04.07.22.0004.01

$$\mathcal{L}_i[\text{quotient}(t, n)](z) = \frac{1}{(e^{nz} - 1)z} \quad ; \quad \text{Re}(nz) > 0$$

Mellin transforms

04.07.22.0005.01

$$\mathcal{M}_i[\text{quotient}(t, n)](z) = -\frac{n^z \zeta(-z)}{z} \quad ; \quad \text{Re}(z) < -1$$

04.07.22.0006.01

$$\mathcal{M}_i[\text{quotient}(m, t)](z) = \frac{m^z \zeta(z)}{z} \quad ; \quad \text{Re}(z) > 1$$

Representations through equivalent functions

With related functions

With Floor

04.07.27.0001.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} \right\rfloor$$

With Round**For real arguments**

04.07.27.0008.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rfloor /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m+n}{2n} \notin \mathbb{Z}$$

04.07.27.0009.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rfloor + 1 /; \frac{m+n}{2n} \in \mathbb{Z}$$

04.07.27.0010.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rfloor + \chi_{\mathbb{Z}}\left(\frac{m+n}{2n}\right) /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0003.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} - \frac{1+i}{2} \right\rfloor + \chi_{\mathbb{Z}}\left(\frac{1}{2}\left(\text{Re}\left(\frac{m}{n}\right) + 1\right)\right) + i \chi_{\mathbb{Z}}\left(\frac{1}{2}\left(\text{Im}\left(\frac{m}{n}\right) + 1\right)\right)$$

With Ceiling**For real arguments**

04.07.27.0011.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil - 1 /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.07.27.0012.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil /; \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0013.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil + \theta\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) - 1\right) - 1 /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0014.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil /; \text{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge \text{Im}\left(\frac{m}{n}\right) \in \mathbb{Z}$$

04.07.27.0015.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil - 1 /; \text{Re}\left(\frac{m}{n}\right) \notin \mathbb{Z} \wedge \text{Im}\left(\frac{m}{n}\right) \in \mathbb{Z}$$

04.07.27.0016.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil - i /; \text{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \wedge \text{Im}\left(\frac{m}{n}\right) \notin \mathbb{Z}$$

04.07.27.0017.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} \right\rfloor - 1 - i \text{ ; } \operatorname{Re}\left(\frac{m}{n}\right) \notin \mathbb{Z} \wedge \operatorname{Im}\left(\frac{m}{n}\right) \notin \mathbb{Z}$$

04.07.27.0018.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} \right\rfloor + \theta\left(\chi_{\mathbb{Z}}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) - 1\right) - i \theta\left(-\chi_{\mathbb{Z}}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right)\right) - 1$$

04.07.27.0002.01

$$\text{quotient}(m, n) = -\left\lfloor -\frac{m}{n} \right\rfloor$$

With IntegerPart

For real arguments

04.07.27.0019.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) \text{ ; } \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} > 0 \vee \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0020.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) - 1 \text{ ; } \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.07.27.0021.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) - 1 + \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) + \theta\left(\frac{m}{n}\right)\right) \text{ ; } \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0022.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) \text{ ; } \operatorname{Re}\left(\frac{m}{n}\right) \geq 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) \geq 0 \vee \frac{m}{n} \in \mathbb{Z} \vee i \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0023.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) - 1 \text{ ; } \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z} \vee \operatorname{Re}\left(\frac{m}{n}\right) < 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) > 0$$

04.07.27.0024.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) - i \text{ ; } i \frac{m}{n} \in \mathbb{R} \wedge i \frac{m}{n} > 0 \wedge i \frac{m}{n} \notin \mathbb{Z} \vee \operatorname{Re}\left(\frac{m}{n}\right) > 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0025.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) - 1 - i \text{ ; } \operatorname{Re}\left(\frac{m}{n}\right) < 0 \wedge \operatorname{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0004.01

$$\text{quotient}(m, n) = \operatorname{int}\left(\frac{m}{n}\right) - 1 - i + i \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\operatorname{Im}\left(\frac{m}{n}\right)\right)\right) + \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\operatorname{Re}\left(\frac{m}{n}\right)\right)\right)$$

With FractionalPart

For real arguments

04.07.27.0026.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} > 0 \vee \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0027.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z}$$

04.07.27.0028.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 + \text{sgn}\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) + \theta\left(\frac{m}{n}\right)\right) /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0029.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) /; \text{Re}\left(\frac{m}{n}\right) \geq 0 \wedge \text{Im}\left(\frac{m}{n}\right) \geq 0 \vee \frac{m}{n} \in \mathbb{Z} \vee i \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0030.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} < 0 \wedge \frac{m}{n} \notin \mathbb{Z} \vee \text{Re}\left(\frac{m}{n}\right) < 0 \wedge \text{Im}\left(\frac{m}{n}\right) > 0$$

04.07.27.0031.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - i /; i \frac{m}{n} \in \mathbb{R} \wedge i \frac{m}{n} > 0 \wedge i \frac{m}{n} \notin \mathbb{Z} \vee \text{Re}\left(\frac{m}{n}\right) > 0 \wedge \text{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0032.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 - i /; \text{Re}\left(\frac{m}{n}\right) < 0 \wedge \text{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0005.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 - i + i \text{sgn}\left(\chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) + \text{sgn}\left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\text{Re}\left(\frac{m}{n}\right)\right)\right)$$

With Mod

04.07.27.0006.01

$$\text{quotient}(m, n) = \frac{m - m \bmod n}{n}$$

With elementary functions

04.07.27.0007.01

$$\text{quotient}(m, n) = \frac{m}{n} + \frac{1}{\pi} \tan^{-1}\left(\cot\left(\frac{\pi m}{n}\right)\right) - \frac{1}{2} /; \frac{m}{n} \in \mathbb{R} \wedge \frac{m}{n} \notin \mathbb{Z}$$

Zeros

04.07.30.0001.01

$$\text{quotient}(m, n) = 0 /; 0 \leq \text{Re}\left(\frac{m}{n}\right) < 1 \wedge 0 \leq \text{Im}\left(\frac{m}{n}\right) < 1$$

History

- J. Nemorarius (1237)
- the word "quotient" appears for the first time around 1250 in the writings of Meister Gernadus.

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