

RamanujanTauL

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Notations

Traditional name

Ramanujan's tau L function

Traditional notation

$\tau L(z)$

Mathematica StandardForm notation

RamanujanTauL[z]

Primary definition

10.10.02.0001.01

$$\tau L(z) = \sum_{n=1}^{\infty} \frac{\tau(n)}{n^z} \quad /; \operatorname{Re}(z) > 6$$

10.10.02.0002.01

$$\tau L(z) = (2\pi)^{2z-12} \frac{\Gamma(12-z)}{\Gamma(z)} \sum_{n=1}^{\infty} \frac{\tau(n)}{n^{12-z}} \quad /; \operatorname{Re}(z) \leq 6$$

Specific values

Specialized values

10.10.03.0001.01

$$\tau L(-n) = 0 \quad /; n \in \mathbb{N}$$

Values at fixed points

10.10.03.0002.01

$$\tau L(0) = 0$$

Values at infinities

10.10.03.0003.01

$$\tau L(\infty) = 1$$

General characteristics

Domain and analyticity

$\tau L(z)$ is an analytical function of z which is defined over the complex z -plane.

10.10.04.0001.01

$$z \rightarrow \tau L(z) :: \mathbb{C} \rightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

10.10.04.0002.01

$$\tau L(\bar{z}) = \overline{\tau L(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function $\tau L(z)$ does not have poles and essential singularities.

10.10.04.0003.01

$$\text{Sing}_z(\tau L(z)) = \{\}$$

Branch points

The function $\tau L(z)$ does not have branch points.

10.10.04.0004.01

$$\mathcal{BP}_z(\tau L(z)) = \{\}$$

Branch cuts

The function $\tau L(z)$ does not have branch cuts.

10.10.04.0005.01

$$\mathcal{BC}_z(\tau L(z)) = \{\}$$

Product representations

10.10.08.0001.01

$$\tau L(z) = \prod_{j=1}^{\infty} \frac{1}{1 - \tau(p_j) p_j^{-z} + p_j^{11-2z}} ; p_j \in \mathbb{P} \bigwedge \text{Re}(z) > \frac{13}{2}$$

Identities

Functional identities

10.10.17.0001.01

$$\frac{\tau L(z) \Gamma(z)}{(2\pi)^z} = \frac{\tau L(12-z) \Gamma(12-z)}{(2\pi)^{12-z}}$$

Differentiation

Low-order differentiation

$$\frac{\partial \tau L(z)}{\partial z} = - \sum_{n=1}^{\infty} \frac{\log(n) \tau(n)}{n^z}$$

$$\frac{\partial^2 \tau L(z)}{\partial z^2} = \sum_{n=1}^{\infty} \frac{\log^2(n) \tau(n)}{n^z}$$

Representations through equivalent functions

With related functions

$$\tau L(z) = \frac{(2\pi)^{z-6} \tau Z(i(6-z))}{\sqrt{\Gamma(z)}} e^{\frac{1}{2}(-\log(\Gamma(12-z)) + \log(\Gamma(z)) + \log\Gamma(12-z) - \log\Gamma(z))}$$

$$\sqrt{\Gamma(1-z)((z^2 - 12z + 11)(z^2 - 12z + 20)(z^2 - 12z + 27)(z^2 - 12z + 32)(z^2 - 12z + 35)(6-z))}$$

$$\tau L(z) = e^{-i\tau\theta(-i(z-6))} \tau Z(-i(z-6))$$

Zeros

There exists the hypothesis that all nontrivial zeroes of $\tau L(z)$ lie on the line $\text{Re}(z) = 6$.

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