RamanujanTauL

Notations

Traditional name

Ramanujan`s tau L function

Traditional notation

$\tau\text{L}(z)$

Mathematica StandardForm notation

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RamanujanTauL[z]
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Primary definition

10.10.02.0001.01

$$\tau\text{L}(z) = \sum_{n=1}^{\infty} \frac{\tau(n)}{n^z} \;/;\; \text{Re}(z) > 6$$

10.10.02.0002.01

$$\tau\text{L}(z) = (2\,\pi)^{2\,z-12} \frac{\Gamma(12-z)}{\Gamma(z)} \sum_{n=1}^{\infty} \frac{\tau(n)}{n^{12-z}} \;/;\; \text{Re}(z) \leq 6$$

Specific values

Specialized values

10.10.03.0001.01

$$\tau\text{L}(-n) = 0 \;/;\; n \in \mathbb{N}$$

Values at fixed points

10.10.03.0002.01

$$\tau\text{L}(0) = 0$$

Values at infinities

10.10.03.0003.01

$$\tau\text{L}(\infty) = 1$$

General characteristics

Domain and analyticity

$\tau L(z)$ is an analytical function of z which is defined over the complex z-plane.

10.10.04.0001.01

$$z \longrightarrow \tau L(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

10.10.04.0002.01

$$\tau L(\bar{z}) = \overline{\tau L(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function $\tau L(z)$ does not have poles and essential singularities.

10.10.04.0003.01

$$\mathcal{Sing}_z(\tau L(z)) = \{\}$$

Branch points

The function $\tau L(z)$ does not have branch points.

10.10.04.0004.01

$$\mathcal{BP}_z(\tau L(z)) = \{\}$$

Branch cuts

The function $\tau L(z)$ does not have branch cuts.

10.10.04.0005.01

$$\mathcal{BC}_z(\tau L(z)) = \{\}$$

Product representations

10.10.08.0001.01

$$\tau L(z) = \prod_{j=1}^{\infty} \frac{1}{1 - \tau(p_j)\, p_j^{-z} + p_j^{11-2z}} \;/;\; p_j \in \mathbb{P} \bigwedge \operatorname{Re}(z) > \frac{13}{2}$$

Identities

Functional identities

10.10.17.0001.01

$$\frac{\tau L(z)\, \Gamma(z)}{(2\pi)^z} = \frac{\tau L(12 - z)\, \Gamma(12 - z)}{(2\pi)^{12-z}}$$

Differentiation

Low-order differentiation

10.10.20.0001.01

$$\frac{\partial \tau\mathrm{L}(z)}{\partial z} = -\sum_{n=1}^{\infty} \frac{\log(n)\,\tau(n)}{n^z}$$

10.10.20.0002.01

$$\frac{\partial^2 \tau\mathrm{L}(z)}{\partial z^2} = \sum_{n=1}^{\infty} \frac{\log^2(n)\,\tau(n)}{n^z}$$

Representations through equivalent functions

With related functions

10.10.27.0001.01

$$\tau\mathrm{L}(z) = \frac{(2\pi)^{z-6}\,\tau\mathrm{Z}(i\,(6-z))}{\sqrt{\Gamma(z)}}\, e^{\frac{1}{2}(-\log(\Gamma(12-z))+\log(\Gamma(z))+\log\Gamma(12-z)-\log\Gamma(z))} \sqrt{\Gamma(1-z)\left(\left(z^2-12\,z+11\right)\left(z^2-12\,z+20\right)\left(z^2-12\,z+27\right)\left(z^2-12\,z+32\right)\left(z^2-12\,z+35\right)(6-z)\right)}$$

10.10.27.0002.01

$$\tau\mathrm{L}(z) = e^{-i\,\tau\theta(-i\,(z-6))}\,\tau\mathrm{Z}(-i\,(z-6))$$

Zeros

There exists the hypothethis that all nontrivial zeroes of $\tau\mathrm{L}(z)$ lie on the line $\mathrm{Re}(z) == 6$.

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