Sign

Notations

Traditional name

Sign function

Traditional notation

$\operatorname{sgn}(z)$

Mathematica StandardForm notation

Sign[z]

Primary definition

12.06.02.0001.01

$\operatorname{sgn}(x) = 1 \;/;\; x \in \mathbb{R} \land x > 0$

12.06.02.0002.01

$\operatorname{sgn}(x) = -1 \;/;\; x \in \mathbb{R} \land x < 0$

12.06.02.0003.01

$\operatorname{sgn}(0) = 0$

12.06.02.0004.01

$$\operatorname{sgn}(z) = \frac{z}{|z|} \;/;\; z \neq 0$$

Specific values

Specialized values

12.06.03.0001.01

$$\operatorname{sgn}(x) = \frac{x}{|x|} \;/;\; x \in \mathbb{R} \land x \neq 0$$

12.06.03.0002.01

$$\operatorname{sgn}(x + i\, y) = \frac{x + i\, y}{\sqrt{x^2 + y^2}} \;/;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

Values at fixed points

12.06.03.0003.01

$\operatorname{sgn}(0) = 0$

12.06.03.0004.01

$$\operatorname{sgn}(1) = 1$$

12.06.03.0005.01

$$\operatorname{sgn}(-1) = -1$$

12.06.03.0006.01

$$\operatorname{sgn}(i) = i$$

12.06.03.0007.01

$$\operatorname{sgn}(-i) = -i$$

12.06.03.0019.01

$$\operatorname{sgn}(1+i) = \frac{1+i}{\sqrt{2}}$$

12.06.03.0020.01

$$\operatorname{sgn}(-1+i) = \frac{-1+i}{\sqrt{2}}$$

12.06.03.0021.01

$$\operatorname{sgn}(-1-i) = -\frac{1+i}{\sqrt{2}}$$

12.06.03.0022.01

$$\operatorname{sgn}(1-i) = \frac{1-i}{\sqrt{2}}$$

12.06.03.0023.01

$$\operatorname{sgn}\left(\sqrt{3}+i\right) = \frac{\sqrt{3}+i}{2}$$

12.06.03.0024.01

$$\operatorname{sgn}\left(1+i\sqrt{3}\right) = \frac{1+i\sqrt{3}}{2}$$

12.06.03.0025.01

$$\operatorname{sgn}\left(-1+i\sqrt{3}\right) = \frac{-1+i\sqrt{3}}{2}$$

12.06.03.0026.01

$$\operatorname{sgn}\left(-\sqrt{3}+i\right) = \frac{-\sqrt{3}+i}{2}$$

12.06.03.0027.01

$$\operatorname{sgn}\left(-\sqrt{3}-i\right) = -\frac{\sqrt{3}+i}{2}$$

12.06.03.0028.01

$$\operatorname{sgn}\left(-1-i\sqrt{3}\right) = -\frac{1+i\sqrt{3}}{2}$$

12.06.03.0029.01

$$\operatorname{sgn}\left(1-i\sqrt{3}\right) = \frac{1-i\sqrt{3}}{2}$$

12.06.03.0030.01

$$\operatorname{sgn}\left(\sqrt{3} - i\right) = \frac{\sqrt{3} - i}{2}$$

12.06.03.0008.01

$$\operatorname{sgn}(2) = 1$$

12.06.03.0009.01

$$\operatorname{sgn}(-2) = -1$$

12.06.03.0010.01

$$\operatorname{sgn}(\pi) = 1$$

12.06.03.0011.01

$$\operatorname{sgn}(3\, i) = i$$

12.06.03.0012.01

$$\operatorname{sgn}(-2\, i) = -i$$

12.06.03.0013.01

$$\operatorname{sgn}(2 + i) = \frac{2 + i}{\sqrt{5}}$$

Values at infinities

12.06.03.0014.01

$$\operatorname{sgn}(\infty) = 1$$

12.06.03.0015.01

$$\operatorname{sgn}(-\infty) = -1$$

12.06.03.0016.01

$$\operatorname{sgn}(i\, \infty) = i$$

12.06.03.0017.01

$$\operatorname{sgn}(-i\, \infty) = -i$$

12.06.03.0018.01

$$\operatorname{sgn}(\tilde{\infty}) = ¿$$

General characteristics

Domain and analyticity

sgn(z) is a nonanalytical function. The real and the imaginary parts of sign(z) are real-analytic functions of the variable z.

12.06.04.0001.01

$$z \longrightarrow \operatorname{sgn}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

sgn(z) is an odd function.

12.06.04.0002.01

$\operatorname{sgn}(-z) = -\operatorname{sgn}(z)$

Mirror symmetry

12.06.04.0003.01

$\operatorname{sgn}(\bar{z}) = \overline{\operatorname{sgn}(z)}$

Periodicity

No periodicity

Homogeneity

12.06.04.0005.01

$\operatorname{sgn}(a\, z) = \operatorname{sgn}(a) \operatorname{sgn}(z)$

Scale symmetry

12.06.04.0006.01

$\operatorname{sgn}(z^a) = \operatorname{sgn}(z)^a \,/;\, a \in \mathbb{R}$

Sets of discontinuity

The function sgn(z) has discontinuity at point $z = 0$.

12.06.04.0004.01

$\mathcal{DS}_z(\operatorname{sgn}(z)) = \{0\}$

Series representations

Residue representations

12.06.06.0002.02

$$\operatorname{sgn}(x) = 2 \operatorname{res}_s\left((x+1)^{-s} \frac{1}{s}\right)(0) - 1 \,/;\, x \in \mathbb{R} \land x > 0$$

Other series representations

12.06.06.0003.01

$$\operatorname{sgn}(x) = \frac{1}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k}\,(2k+1)\,k!} H_{2k+1}(x) \,/;\, x \in \mathbb{R} \land -1 < x < 1$$

12.06.06.0004.01

$$\operatorname{sgn}(x) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1} T_{2k-1}(x) \,/;\, x \in \mathbb{R} \land -1 < x < 1$$

12.06.06.0005.01

$$\operatorname{sgn}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k\,(4k+3)\,(2k)!}{2^{2k+1}\,(k+1)!\,k!} P_{2k+1}(x) \,/;\, x \in \mathbb{R} \land -1 < x < 1$$

Limit representations

12.06.09.0001.01

$$\operatorname{sgn}(x) = \lim_{m+n\to\infty} \frac{4\,n!\,\Gamma\left(m+\frac{3}{2}\right)\Gamma(m+n+2)}{\sqrt{\pi}\;m!\,\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(m+n+\frac{3}{2}\right)}\,x\,\frac{{}_3F_2\left(-m,\,\frac{1}{2}-n,\,m+n+2;\,\frac{3}{2},\,\frac{3}{2};\,x^2\right)}{{}_3F_2\left(-n,\,-m-\frac{1}{2},\,m+n+\frac{3}{2};\,\frac{1}{2},\,1;\,x^2\right)} \;/;\; -1<x<1 \land n\in\mathbb{N} \land m\in\mathbb{N}$$

(generalized Padé approximation)

12.06.09.0002.01

$$\operatorname{sgn}(x) = \lim_{m+n\to\infty} \frac{4\,(m+1)\,(2\,n+1)}{\pi}\,x\,\frac{{}_4F_3\left(-m,\,m+2,\,\frac{1}{2}-n,\,n+\frac{3}{2};\,\frac{3}{2},\,\frac{3}{2},\,\frac{3}{2};\,x^2\right)}{{}_4F_3\left(-n,\,n+1,\,-m-\frac{1}{2},\,m+\frac{3}{2};\,\frac{1}{2},\,1,\,1;\,x^2\right)} \;/;\; -1<x<1 \land n\in\mathbb{N} \land m\in\mathbb{N}$$

(generalized Padé approximation)

Integral representations

Contour integral representations

12.06.07.0001.01

$$\operatorname{sgn}(x) = \frac{1}{\pi\,i}\int_{\gamma-i\,\infty}^{\gamma+i\,\infty}\frac{\Gamma(-s)\,(x+1)^{-s}}{\Gamma(1-s)}\,ds \;/;\; 0<\gamma \bigwedge x>-2$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

12.06.16.0001.01

$$\operatorname{sgn}(-z) = -\operatorname{sgn}(z)$$

12.06.16.0002.01

$$\operatorname{sgn}(x) = \frac{x}{|x|} \;/;\; x\in\mathbb{R} \land x\neq 0$$

12.06.16.0003.01

$$\operatorname{sgn}(x+i\,y) = \frac{x+i\,y}{\sqrt{x^2+y^2}} \;/;\; x\in\mathbb{R} \bigwedge y\in\mathbb{R} \bigwedge \{x,\,y\}\neq\{0,\,0\}$$

12.06.16.0004.01

$$\operatorname{sgn}(a\,z) = \operatorname{sgn}(z) \;/;\; a\in\mathbb{R} \land a>0$$

12.06.16.0005.01

$$\operatorname{sgn}(a\,z) = -\operatorname{sgn}(z) \;/;\; a\in\mathbb{R} \land a<0$$

12.06.16.0006.01

$$\operatorname{sgn}(i\,z) = i\operatorname{sgn}(z)$$

12.06.16.0007.01

$$\operatorname{sgn}(-i\,z) = -i\operatorname{sgn}(z)$$

12.06.16.0024.01

$$\operatorname{sgn}\left(\frac{1}{z}\right) == \frac{|z|}{z}$$

Addition formulas

12.06.16.0009.01

$$\operatorname{sgn}(x + i\, y) == \frac{x + i\, y}{\sqrt{x^2 + y^2}} \;/;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

Multiple arguments

12.06.16.0010.01

$$\operatorname{sgn}(a\, z) == \operatorname{sgn}(z) \;/;\; a \in \mathbb{R} \land a > 0$$

12.06.16.0011.01

$$\operatorname{sgn}(a\, z) == -\operatorname{sgn}(z) \;/;\; a \in \mathbb{R} \land a < 0$$

12.06.16.0012.01

$$\operatorname{sgn}(i\, z) == i \operatorname{sgn}(z)$$

12.06.16.0013.01

$$\operatorname{sgn}(-i\, z) == -i \operatorname{sgn}(z)$$

12.06.16.0014.01

$$\operatorname{sgn}\left(\prod_{k=1}^{n} z_k\right) == \prod_{k=1}^{n} \operatorname{sgn}(z_k)$$

12.06.16.0015.01

$$\operatorname{sgn}(z_1\, z_2) == \operatorname{sgn}(z_1) \operatorname{sgn}(z_2)$$

Ratio of arguments

12.06.16.0025.01

$$\operatorname{sgn}\left(\frac{z_1}{z_2}\right) == \frac{\operatorname{sgn}(z_1)}{\operatorname{sgn}(z_2)}$$

Power of arguments

12.06.16.0016.01

$$\operatorname{sgn}(x^a) == x^{i \operatorname{Im}(a)} \;/;\; x \in \mathbb{R} \land x > 0$$

12.06.16.0017.01

$$\operatorname{sgn}(z^a) == \operatorname{sgn}(z)^a \;/;\; a \in \mathbb{R}$$

12.06.16.0018.01

$$\operatorname{sgn}(z^a) == \exp(a \operatorname{Re}(\log(z))) \;/;\; i\, a \in \mathbb{R}$$

12.06.16.0019.01

$$\operatorname{sgn}(z^a) == |z|^a \;/;\; i\, a \in \mathbb{R}$$

12.06.16.0020.01

$$\operatorname{sgn}(z^a) == z^a \exp(-\operatorname{Re}(a \log(z)))$$

12.06.16.0021.01

$$\operatorname{sgn}(z^a) == |z|^{i \operatorname{Im}(a)} \exp(i \operatorname{Re}(a) \arg(z))$$

12.06.16.0022.01

$$\operatorname{sgn}(z^a) = |z|^{i \operatorname{Im}(a)} \exp\left(i \operatorname{Re}(a) \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right)$$

12.06.16.0023.01

$$\operatorname{sgn}(z^a) = \exp(i (\operatorname{Im}(a) \log(|z|) + \arg(z) \operatorname{Re}(a)))$$

Exponent of arguments

12.06.16.0026.01

$$\operatorname{sgn}\left(e^{x+i y}\right) = e^{i y}$$

12.06.16.0027.01

$$\operatorname{sgn}(e^z) = e^{i \operatorname{Im}(z)}$$

12.06.16.0028.01

$$\operatorname{sgn}\left(e^{i z}\right) = e^{i \operatorname{Re}(z)}$$

Complex characteristics

Real part

12.06.19.0001.01

$$\operatorname{Re}(\operatorname{sgn}(x + i y)) = \frac{x}{\sqrt{x^2 + y^2}}$$

12.06.19.0008.01

$$\operatorname{Re}(\operatorname{sgn}(z)) = \frac{\operatorname{Re}(z)}{|z|}$$

Imaginary part

12.06.19.0002.01

$$\operatorname{Im}(\operatorname{sgn}(x + i y)) = \frac{y}{\sqrt{x^2 + y^2}}$$

12.06.19.0009.01

$$\operatorname{Im}(\operatorname{sgn}(z)) = \frac{\operatorname{Im}(z)}{|z|}$$

Absolute value

12.06.19.0003.01

$$|\operatorname{sgn}(x + i y)| = 1 \;/;\; x + i y \neq 0$$

12.06.19.0004.01

$$|\operatorname{sgn}(z)| = 1 \;/;\; z \neq 0$$

Argument

12.06.19.0005.01

$$\arg(\operatorname{sgn}(x + i y)) = \tan^{-1}(x, y)$$

12.06.19.0006.01

$$\arg(\operatorname{sgn}(z)) = \arg(z)$$

Conjugate value

12.06.19.0007.01

$$\overline{\operatorname{sgn}(x+i\,y)} = \frac{x-i\,y}{\sqrt{x^2+y^2}}$$

12.06.19.0010.01

$$\overline{\operatorname{sgn}(z)} = \frac{\bar{z}}{|z|}$$

Signum value

12.06.19.0011.01

$$\operatorname{sign}(\operatorname{sgn}(x+i\,y)) = \frac{x+i\,y}{\sqrt{x^2+y^2}}$$

12.06.19.0012.01

$$\operatorname{sgn}(\operatorname{sgn}(z)) = \operatorname{sgn}(z)$$

Differentiation

Low-order differentiation

In a distributional sense for $x \in \mathbb{R}$.

12.06.20.0001.01

$$\frac{\partial \operatorname{sgn}(x)}{\partial x} = 2\,\delta(x)$$

Integration

Indefinite integration

Involving only one direct function

In a distributional sense for $x \in \mathbb{R}$.

12.06.21.0001.01

$$\int \operatorname{sgn}(x)\,dx = |x|$$

Definite integration

For the direct function itself

12.06.21.0002.01

$$\int_{-a}^{a} \operatorname{sgn}(t)\,dt = 0$$

12.06.21.0003.01

$$\int_{-a}^{a} t^k \operatorname{sgn}(t)\, dt = \frac{\left(1-(-1)^k\right) a^{k+1}}{k+1} /;\ a > 0 \bigwedge \operatorname{Re}(k) > -1$$

Integral transforms

Fourier exp transforms

12.06.22.0004.01

$$\mathcal{F}_t[\operatorname{sgn}(t)](x) = \sqrt{\frac{2}{\pi}}\, \frac{i}{x}$$

12.06.22.0005.01

$$\mathcal{F}_t[t^n (\operatorname{sgn}(t)+1)](x) = (-i)^n \sqrt{2\pi}\, \frac{\partial^n \delta(x)}{\partial x^n} - i^{n-1} n! \sqrt{\frac{2}{\pi}}\, x^{-n-1} /;\ n \in \mathbb{N}$$

12.06.22.0006.01

$$\mathcal{F}_t[|t|^\alpha \operatorname{sgn}(t)](x) = i \sqrt{\frac{2}{\pi}}\, |x|^{-\alpha-1} \cos\left(\frac{\pi \alpha}{2}\right) \Gamma(\alpha+1) \operatorname{sgn}(x) /;\ \operatorname{Re}(\alpha) > -1$$

Fourier cos transforms

12.06.22.0001.01

$$\mathcal{F}c_t[\operatorname{sgn}(t)](z) = \sqrt{\frac{\pi}{2}}\, \delta(z)$$

Fourier sin transforms

12.06.22.0002.01

$$\mathcal{F}s_t[\operatorname{sgn}(t)](z) = \sqrt{\frac{2}{\pi}}\, \frac{1}{z}$$

Laplace transforms

12.06.22.0003.01

$$\mathcal{L}_t[\operatorname{sgn}(t)](z) = \frac{1}{z}$$

Representations through more general functions

Through Meijer *G*

Classical cases involving the direct function

12.06.26.0001.01

$$((1-z) \operatorname{sgn}(1-|z|))^\nu = \frac{\pi}{\Gamma(-\nu)} \sec\left(\frac{\pi \nu}{2}\right) G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+1, \frac{\nu+1}{2} \\ 0, \frac{\nu+1}{2} \end{matrix}\right)$$

12.06.26.0002.01

$$\operatorname{sgn}(1-|z|)\,((1-z)\operatorname{sgn}(1-|z|))^{\nu} = -\frac{\pi}{\Gamma(-\nu)}\csc\left(\frac{\nu\pi}{2}\right) G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+1, \frac{\nu}{2}+1 \\ 0, \frac{\nu}{2}+1 \end{matrix}\right)$$

Classical cases involving cosh

12.06.26.0003.01

$$((1-z)\operatorname{sgn}(1-|z|))^{\nu}\cosh\left(\nu\tanh^{-1}\left(\frac{2\sqrt{z}}{z+1}\right)\right) = \frac{\sqrt{\pi}}{\Gamma(-\nu)}\Gamma\left(\frac{1}{2}+\nu\right) G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1+\nu, \frac{1}{2}+\nu \\ 0, \frac{1}{2} \end{matrix}\right)$$

12.06.26.0004.01

$$((1-z)\operatorname{sgn}(1-|z|))^{\nu}\cosh\left(\nu\coth^{-1}\left(\frac{1+z}{2\sqrt{z}}\right)\right) = \frac{\sqrt{\pi}}{\Gamma(-\nu)}\Gamma\left(\nu+\frac{1}{2}\right) G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+1, \nu+\frac{1}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving sinh

12.06.26.0005.01

$$((1-z)\operatorname{sgn}(1-|z|))^{\nu}\sinh\left(\nu\tanh^{-1}\left(\frac{2\sqrt{z}}{z+1}\right)\right) = -\frac{\sqrt{\pi}}{\Gamma(-\nu)}\Gamma\left(\nu+\frac{1}{2}\right) G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+\frac{1}{2}, \nu+1 \\ \frac{1}{2}, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

12.06.26.0006.01

$$((1-z)\operatorname{sgn}(1-|z|))^{\nu}\sinh\left(\nu\coth^{-1}\left(\frac{1+z}{2\sqrt{z}}\right)\right) = -\frac{\sqrt{\pi}}{\Gamma(-\nu)}\Gamma\left(\nu+\frac{1}{2}\right) G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+\frac{1}{2}, \nu+1 \\ \frac{1}{2}, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

Representations through equivalent functions

With related functions

With Re

12.06.27.0007.01

$$\operatorname{sgn}(z) = \frac{z}{\sqrt{2\,z\operatorname{Re}(z)-z^2}}$$

With Im

12.06.27.0008.01

$$\operatorname{sgn}(z) = \frac{z}{\sqrt{z^2-2\,i\,z\operatorname{Im}(z)}}$$

12.06.27.0009.01

$$\operatorname{sgn}(z) = \frac{i\operatorname{Im}(z)+\operatorname{Re}(z)}{\sqrt{\operatorname{Im}(z)^2+\operatorname{Re}(z)^2}}$$

12.06.27.0004.01

$$\operatorname{sgn}(z) = \frac{z}{\sqrt{\operatorname{Im}(z)^2+\operatorname{Re}(z)^2}} /; z \neq 0$$

With Abs

12.06.27.0001.01

$$\operatorname{sgn}(z) = \frac{z}{|z|} \;/;\; z \neq 0$$

With Arg

12.06.27.0002.01

$$\operatorname{sgn}(z) = e^{i \arg(z)}$$

With Conjugate

12.06.27.0003.01

$$\operatorname{sgn}(z) = \frac{z}{\sqrt{z \bar{z}}} \;/;\; z \neq 0$$

With UnitStep

12.06.27.0005.01

$$\operatorname{sgn}(x) = \theta(x) - \theta(-x) \;/;\; x \in \mathbb{R}$$

12.06.27.0006.01

$$\operatorname{sgn}(x) = 2\,\theta(x) - 1 \;/;\; x \in \mathbb{R} \land x \neq 0$$

Inequalities

12.06.29.0001.01

$$|\operatorname{sgn}(z)| \leq 1$$

12.06.29.0002.01

$$\operatorname{Re}(\operatorname{sgn}(z)) \leq 1$$

12.06.29.0003.01

$$\operatorname{Im}(\operatorname{sgn}(z)) \leq 1$$

Zeros

12.06.30.0001.01

$$\operatorname{sgn}(z) = 0 \;/;\; z = 0$$

Theorems

Rademacher functions

The functions $r_n(x) = \operatorname{sgn}(\sin(2^n \pi x))$ form an orthogonal sequence over (0,1).

History

The function sgn is encountered often in mathematics and the natural sciences.

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