UnitStep2

Notations

Traditional name

Multivariate Heaviside step function ??

Traditional notation

$\theta(x_1, x_2, \ldots)$

Mathematica StandardForm notation

UnitStep[x_1, x_2, …]

Primary definition

14.02.02.0001.01

$\theta(x_1, x_2, \ldots) = 1 \;/;\; x_k \in \mathbb{R} \bigwedge x_k \geq 0$

14.02.02.0002.01

$\theta(x_1, x_2, \ldots) = 0 \;/;\; x_k \in \mathbb{R} \bigwedge \neg\, (x_1 \geq 0 \bigwedge x_2 \geq 0 \bigwedge \ldots)$

14.02.02.0003.01

$\theta(x_1, x_2, \ldots, x_n) = \{\; 1 \quad x_1 \geq 0 \bigwedge x_2 \geq 0 \bigwedge \ldots \bigwedge x_n \geq 0 \;/;\; x_k \in \mathbb{R} \bigwedge 1 \leq k \leq n$

Specific values

Specialized values

14.02.03.0001.01

$\theta(x_1, x_1, \ldots, x_1) = \theta(x_1)$

Values at fixed points

14.02.03.0002.01

$\theta(0, 0, \ldots, 0) = 1$

14.02.03.0003.01

$\theta(1, 1, \ldots, 1) = 1$

14.02.03.0004.01

$\theta(-1, 1, \ldots, 1) = 0$

Values at infinities

14.02.03.0005.01

$\theta(\infty, \infty, \ldots, \infty) = 1$

14.02.03.0006.01

$\theta(-\infty, \infty, \ldots, \infty) = 0$

General characteristics

Domain and analyticity

$\theta(x_1, x_2, \ldots)$ is a nonanalytical function; it is a piecewise constant function which is defined for all real x_k.

14.02.04.0001.01

$(x_1 * x_2 * \ldots * x_n) \longrightarrow \theta(x_1, x_2, \ldots, x_n) :: \mathbb{R}^n \longrightarrow \mathbb{Z}$

Symmetries and periodicities

Permutation symmetry

14.02.04.0002.01

$\theta(x_2, x_1) = \theta(x_1, x_2)$

14.02.04.0003.01

$\theta(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n) = \theta(x_1, x_2, \ldots, x_j, \ldots, x_k, \ldots, x_n) /; x_k \neq x_j \bigwedge k \neq j$

Periodicity

No periodicity

Sets of discontinuity

The function $\theta(x_1, x_2, \ldots)$ has discontinuity at the point $x_1 = x_2 = \ldots = 0$.

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

14.02.16.0001.01

$\theta(-x_1, x_2, \ldots) = \theta(x_2, \ldots) - \theta(x_1, x_2, \ldots) /; x_1 \neq 0$

Products, sums, and powers of the direct function

Powers of the direct function

14.02.16.0002.01

$\theta(x_1, x_2, \ldots)^a = \theta(x_1, x_2, \ldots) /; a > 0$

Identities

Functional identities

14.02.17.0001.01

$$\theta(x_1, x_2, \ldots, x_n) = \theta(x_1, x_2, \ldots, x_{n-1})\,\theta(x_n)$$

Complex characteristics

Real part

14.02.19.0001.01

$$\operatorname{Re}(\theta(x_1, x_2, \ldots, x_n)) = \theta(x_1, x_2, \ldots, x_n)$$

Imaginary part

14.02.19.0002.01

$$\operatorname{Im}(\theta(x_1, x_2, \ldots, x_n)) = 0$$

Absolute value

14.02.19.0003.01

$$|\theta(x_1, x_2, \ldots, x_n)| = \theta(x_1, x_2, \ldots, x_n)$$

Argument

14.02.19.0004.01

$$\arg(\theta(x_1, x_2, \ldots, x_n)) = \tan^{-1}(\theta(x_1, x_2, \ldots, x_n), 0)$$

Conjugate value

14.02.19.0005.01

$$\overline{\theta(x_1, x_2, \ldots, x_n)} = \theta(x_1, x_2, \ldots, x_n)$$

Differentiation

Low-order differentiation

In a distributional sense.

14.02.20.0001.01

$$\frac{\partial \theta(x_1, x_2, \ldots)}{\partial x_1} = \delta(x_1)\,\theta(x_2, \ldots)$$

Integration

Indefinite integration

Involving only one direct function

14.02.21.0001.01

$$\int \theta(x_1, x_2, \ldots)\, dx_1 = x_1\,\theta(x_1, x_2, \ldots)$$

Involving one direct function and elementary functions

Involving power function

14.02.21.0002.01

$$\int x_1^{\alpha-1}\,\theta(x_1, x_2, \ldots)\,dx_1 = \frac{x_1^{\alpha}}{\alpha}\,\theta(x_1, x_2, \ldots)$$

14.02.21.0003.01

$$\int \frac{\theta(x_1, x_2, \ldots)}{x_1}\,dx_1 = \log(x_1)\,\theta(x_1, x_2, \ldots)$$

Integral transforms

Fourier exp transforms

14.02.22.0001.01

$$\mathcal{F}_t[\theta(x_1, x_2, \ldots, x_{n-1}, t)](x) = \theta(x_1, x_2, \ldots, x_{n-1})\left(\frac{i}{\sqrt{2\pi}\,x} + \sqrt{\frac{\pi}{2}}\,\delta(x)\right)$$

Inverse Fourier exp transforms

14.02.22.0002.01

$$\mathcal{F}_t^{-1}[\theta(x_1, x_2, \ldots, x_{n-1}, t)](x) = \theta(x_1, x_2, \ldots, x_{n-1})\left(-\frac{i}{\sqrt{2\pi}\,x} + \sqrt{\frac{\pi}{2}}\,\delta(x)\right)$$

Fourier cos transforms

14.02.22.0003.01

$$\mathcal{F}c_t[\theta(x_1, x_2, \ldots, x_{n-1}, t)](x) = \sqrt{\frac{\pi}{2}}\,\theta(x_1, x_2, \ldots, x_{n-1})\,\delta(x)$$

Fourier sin transforms

14.02.22.0004.01

$$\mathcal{F}s_t[\theta(x_1, x_2, \ldots, x_{n-1}, t)](z) = \sqrt{\frac{2}{\pi}}\,\frac{\theta(x_1, x_2, \ldots, x_{n-1})}{z}$$

Laplace transforms

14.02.22.0005.01

$$\mathcal{L}_t[\theta(x_1, x_2, \ldots, x_{n-1}, t)](z) = \frac{\theta(x_1, x_2, \ldots, x_{n-1})}{z}$$

Representations through more general functions

Through Meijer *G*

Classical cases for the direct function itself

14.02.26.0001.01

$$\theta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} G_{1,1}^{1,0}\left(1 - x_k \,\middle|\, \begin{matrix} 1 \\ 0 \end{matrix}\right) /; x_k < 2$$

14.02.26.0002.01

$$\theta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} G_{1,1}^{0,1}\left(x_k + 1 \,\middle|\, \begin{matrix} 1 \\ 0 \end{matrix}\right) /; x_k > -2$$

14.02.26.0003.01

$$\theta(1 - |z_1|, 1 - |z_2|, \ldots, 1 - |z_n|) = \prod_{k=1}^{n} G_{1,1}^{1,0}\left(z_k \,\middle|\, \begin{matrix} 1 \\ 0 \end{matrix}\right)$$

14.02.26.0004.01

$$\theta(|z_1| - 1, |z_2| - 1, \ldots, |z_n| - 1) = \prod_{k=1}^{n} G_{1,1}^{0,1}\left(z_k \,\middle|\, \begin{matrix} 1 \\ 0 \end{matrix}\right)$$

Representations through equivalent functions

14.02.27.0001.01

$$\theta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} \theta(x_k)$$

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