ComplexInfinity

Notations

Traditional name

The complex quantity with infinite magnitude but indeterminate phase

Traditional notation

∞

Mathematica StandardForm notation

ComplexInfinity

Primary definition

∞ represents an infinite numerical quantity whose direction in the complex plane is unknown (undetermined).

General characteristics

∞ is a special symbol. On the Riemann sphere it is the north pole. In the projective complex plane it is the line at infinity.

Limit representations

\[ \lim_{z \to \infty} \frac{1}{z} = 02.12.09.0001.01 \]

Transformations

Products, sums, and powers of the direct function

Products involving the direct function

\[ 0 \cdot \infty = 02.12.16.0001.01 \]
\[ a \cdot \infty = \infty /; a \neq 0 \]
Sums of the direct function

02.12.16.0003.01
\[ \infty + \infty = i \]

02.12.16.0004.01
\[ \infty - \infty = i \]

Related transformations

02.12.16.0006.01
\[ \infty^0 = i \]

02.12.16.0007.01
\[ 1^{\infty} = i \]

02.12.16.0008.01
\[ (\infty)^0 = \infty \]

Identities

Functional identities

02.12.17.0001.01
\[ \infty = \infty \infty \]

Complex characteristics

Real part

02.12.19.0001.01
\[ \text{Re}(\infty) = i \]

Imaginary part

02.12.19.0002.01
\[ \text{Im}(\infty) = i \]

Absolute value

02.12.19.0003.01
\[ |\infty| = \infty \]

Argument

02.12.19.0004.01
\[ \arg(\infty) \in (-\pi, \pi] \]

Conjugate value
Differentiation

Low-order differentiation

\[
\frac{d\infty}{dz} = 0
\]

Integration

Indefinite integration

\[
\int \infty \, dz = z \infty
\]

Summation

Finite summation

\[
\sum_{k=0}^{m} \infty = \infty
\]

Integral transforms

Fourier exp transforms

\[
\mathcal{F}_1[\infty](z) = \infty
\]

Inverse Fourier exp transforms

\[
\mathcal{F}_1^{-1}[\infty](z) = \infty
\]

Fourier cos transforms

\[
\mathcal{F}_{\cos}[\infty](z) = \infty
\]

Fourier sin transforms
Laplace transforms

\[ \mathcal{F}[\infty](s) = \infty \]

Inverse Laplace transforms

\[ \mathcal{L}^{-1}[\infty](\sigma) = \infty \]

Representations through more general functions

Through other functions

\[ \infty = \iota \infty \]

\[ \infty = (0 \infty) \]

Representations through equivalent functions

\[ \infty = \frac{1}{0} \]

History

– John Wallis (1655) introduced the sign \( \infty \) to signify infinite number
– K. Weierstrass (1876) used symbol \( \infty \) to represent an actual infinity, which is prototype of symbol ComplexInfinity \( \infty \) in Mathematica

The symbol \( \infty \) is encountered often in mathematics and the natural sciences.
Copyright

This document was downloaded from functions.wolfram.com, a comprehensive online compendium of formulas involving the special functions of mathematics. For a key to the notations used here, see http://functions.wolfram.com/Notations/.

Please cite this document by referring to the functions.wolfram.com page from which it was downloaded, for example:

http://functions.wolfram.com/Constants/E/

To refer to a particular formula, cite functions.wolfram.com followed by the citation number.

e.g.:  http://functions.wolfram.com/01.03.03.0001.01

This document is currently in a preliminary form. If you have comments or suggestions, please email comments@functions.wolfram.com.