# CoshIntegral



## Notations

### Traditional name

Hyperbolic cosine integral

### Traditional notation

$\mathrm{Chi}(z)$

### *Mathematica* StandardForm notation

`CoshIntegral[z]`

## Primary definition

06.40.02.0001.01

$$\mathrm{Chi}(z) = \int_0^z \frac{\cosh(t)-1}{t}\,dt + \log(z) + \gamma$$

## Specific values

### Values at fixed points

06.40.03.0001.01

$$\mathrm{Chi}(0) = -\infty$$

### Values at infinities

06.40.03.0002.01

$$\mathrm{Chi}(\infty) = \infty$$

06.40.03.0003.01

$$\mathrm{Chi}(-\infty) = \infty$$

06.40.03.0004.01

$$\mathrm{Chi}(i\,\infty) = \frac{\pi\,i}{2}$$

06.40.03.0005.01

$$\mathrm{Chi}(-i\,\infty) = -\frac{\pi\,i}{2}$$

06.40.03.0006.01

$$\mathrm{Chi}(\tilde{\infty}) = ¿$$

# General characteristics

## Domain and analyticity

Chi($z$) is an analytical function of $z$ which is defined over the whole complex $z$-plane. It has one infinitely long branch cut.

06.40.04.0001.01

$$z \longrightarrow \text{Chi}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

## Symmetries and periodicities

### Mirror symmetry

06.40.04.0002.01

$$\text{Chi}(\bar{z}) = \overline{\text{Chi}(z)} \;/;\; z \notin (-\infty, 0)$$

### Periodicity

No periodicity

## Poles and essential singularities

The function Chi($z$) has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point.

06.40.04.0003.01

$$\mathcal{Sing}_z(\text{Chi}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

## Branch points

The function Chi($z$) has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

06.40.04.0004.01

$$\mathcal{BP}_z(\text{Chi}(z)) = \{0, \tilde{\infty}\}$$

06.40.04.0005.01

$$\mathcal{R}_z(\text{Chi}(z), 0) = \log$$

06.40.04.0006.01

$$\mathcal{R}_z(\text{Chi}(z), \tilde{\infty}) = \log$$

## Branch cuts

The function Chi($z$) is a single-valued function on the $z$-plane cut along the interval $(-\infty, 0)$ where it is continuous from above.

06.40.04.0007.01

$$\mathcal{BC}_z(\text{Chi}(z)) = \{\{(-\infty, 0), -i\}\}$$

06.40.04.0008.01

$$\lim_{\epsilon \to +0} \text{Chi}(x + i\,\epsilon) = \text{Chi}(x) \;/;\; x < 0$$

06.40.04.0009.01

$$\lim_{\epsilon \to +0} \text{Chi}(x - i\,\epsilon) = \text{Chi}(x) - 2\,\pi\,i \;/;\; x < 0$$

# Series representations

## Generalized power series

### Expansions at generic point $z = z_0$

## For the function itself

06.40.06.0009.01

$$\text{Chi}(z) \propto \text{Chi}(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left( \log\!\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \frac{\sinh(z_0)\,z_0 - \cosh(z_0)}{2\,z_0^2}\,(z - z_0)^2 + \frac{\cosh(z_0)}{z_0}\,(z - z_0) + \ldots \;/;\; (z \to z_0)$$

06.40.06.0010.01

$$\text{Chi}(z) \propto \text{Chi}(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left( \log\!\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \frac{\cosh(z_0)}{z_0}\,(z - z_0) + \frac{\sinh(z_0)\,z_0 - \cosh(z_0)}{2\,z_0^2}\,(z - z_0)^2 + O\!\left((z - z_0)^3\right)$$

06.40.06.0011.01

$$\text{Chi}(z) = \text{Chi}(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left( \log\!\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \sum_{k=1}^{\infty} \left( \frac{(-1)^{k-1}\,z_0^{-k}}{k} + \frac{\sqrt{\pi}\;z_0^{2-k}\,2^{k-3}}{k!}\,{}_2\tilde{F}_3\!\left(1, 1; 2, \frac{3-k}{2}, 2 - \frac{k}{2}; \frac{z_0^2}{4}\right) \right) (z - z_0)^k$$

06.40.06.0012.01

$$\text{Chi}(z) \propto \text{Chi}(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left( \log\!\left(\frac{1}{z_0}\right) + \log(z_0) \right) + O(z - z_0)$$

### Expansions on branch cuts

## For the function itself

06.40.06.0013.01

$$\text{Chi}(z) \propto \text{Chi}(x) + 2\,i\,\pi \left\lfloor \frac{\arg(z - x)}{2\,\pi} \right\rfloor + \frac{\cosh(x)}{x}\,(z - x) + \frac{\sinh(x)\,x - \cosh(x)}{2\,x^2}\,(z - x)^2 + \ldots \;/;\; (z \to x) \land x \in \mathbb{R} \land x < 0$$

06.40.06.0014.01

$$\text{Chi}(z) \propto \text{Chi}(x) + 2\,i\,\pi \left\lfloor \frac{\arg(z - x)}{2\,\pi} \right\rfloor + \frac{\cosh(x)}{x}\,(z - x) + \frac{\sinh(x)\,x - \cosh(x)}{2\,x^2}\,(z - x)^2 + O\!\left((z - x)^3\right) \;/;\; x \in \mathbb{R} \land x < 0$$

06.40.06.0015.01

$$\text{Chi}(z) = \text{Chi}(x) + 2\,i\,\pi \left\lfloor \frac{\arg(z - x)}{2\,\pi} \right\rfloor + \sum_{k=1}^{\infty} \left( \frac{(-1)^{k-1}\,x^{-k}}{k} + \frac{\sqrt{\pi}\;x^{2-k}\,2^{k-3}}{k!}\,{}_2\tilde{F}_3\!\left(1, 1; 2, \frac{3-k}{2}, 2 - \frac{k}{2}; \frac{x^2}{4}\right) \right) (z - x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

06.40.06.0016.01

$$\text{Chi}(z) \propto \text{Chi}(x) + 2\,i\,\pi \left\lfloor \frac{\arg(z - x)}{2\,\pi} \right\rfloor + O(z - x) \;/;\; x \in \mathbb{R} \land x < 0$$

### Expansions at $z = 0$

## For the function itself

06.40.06.0001.02

$$\text{Chi}(z) \propto \log(z) + \gamma + \frac{z^2}{4}\left(1 + \frac{z^2}{24} + \frac{z^4}{1080} + \ldots\right) /; (z \to 0)$$

06.40.06.0017.01

$$\text{Chi}(z) \propto \log(z) + \gamma + \frac{z^2}{4}\left(1 + \frac{z^2}{24} + \frac{z^4}{1080} + O\left(z^6\right)\right)$$

06.40.06.0002.01

$$\text{Chi}(z) = \log(z) + \gamma + \frac{1}{2}\sum_{k=1}^{\infty} \frac{z^{2k}}{k\,(2k)!}$$

06.40.06.0003.01

$$\text{Chi}(z) = \log(z) + \gamma + \frac{z^2}{4}\, {}_2F_3\left(1, 1; 2, 2, \frac{3}{2}; \frac{z^2}{4}\right)$$

06.40.06.0004.02

$$\text{Chi}(z) \propto \log(z) + \gamma + \frac{z^2}{4}\left(1 + O\left(z^2\right)\right)$$

06.40.06.0018.01

$$\text{Chi}(z) = F_\infty(z) /;$$
$$\left(\left(F_n(z) = \log(z) + \gamma + \frac{1}{2}\sum_{k=1}^{n} \frac{z^{2k}}{k\,(2k)!} = \text{Chi}(z) - \frac{z^{2n+2}}{4\,(n+1)^2\,(2n+1)!}\, {}_2F_3\left(1, n+1; n+\frac{3}{2}, n+2, n+2; \frac{z^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

### Asymptotic series expansions

06.40.06.0005.01

$$\text{Ci}(z) \propto \log(z) - \frac{1}{2}\log\left(-z^2\right) + \frac{\cosh(z)}{z^2}\, {}_3F_0\left(1, 1, \frac{3}{2};; \frac{4}{z^2}\right) + \frac{\sinh(z)}{z}\, {}_3F_0\left(\frac{1}{2}, 1, 1;; \frac{4}{z^2}\right) /; (|z| \to \infty)$$

06.40.06.0006.01

$$\text{Chi}(z) \propto \log(z) - \frac{\log\left(-z^2\right)}{2} + \frac{\sinh(z)}{z}\left(1 + O\left(\frac{1}{z^2}\right)\right) + \frac{\cosh(z)}{z^2}\left(1 + O\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

06.40.06.0019.01

$$\text{Chi}(z) \propto \begin{cases} -\frac{i\pi}{2} & \arg(z) = -\frac{\pi}{2} \\ \frac{i\pi}{2} & \arg(z) = \frac{\pi}{2} \\ \frac{\sinh(z)}{z} & \text{True} \end{cases} /; (|z| \to \infty)$$

### Residue representations

06.40.06.0007.02

$$\text{Chi}(z) = -\frac{1}{2}\left(\log\left(-z^2\right) - 2\log(z)\right) - \frac{\sqrt{\pi}}{2}\,\text{res}_s\left(\frac{\left(-\frac{z^2}{4}\right)^{-s}}{\Gamma\left(\frac{1}{2}-s\right)}\,\frac{\Gamma(s)}{s}\right)(0) - \frac{\sqrt{\pi}}{2}\sum_{j=1}^{\infty}\text{res}_s\left(\frac{\left(-\frac{z^2}{4}\right)^{-s}}{s\,\Gamma\left(\frac{1}{2}-s\right)}\,\Gamma(s)\right)(-j)$$

06.40.06.0008.02

$$\mathrm{Chi}(z) = -\frac{\sqrt{\pi}}{2}\,\mathrm{res}_s\left(\frac{\left(\frac{i z}{2}\right)^{-2 s}}{\Gamma\left(\frac{1}{2}-s\right)}\,\frac{\Gamma(s)}{s}\right)(0) - \frac{\sqrt{\pi}}{2}\sum_{j=1}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{i z}{2}\right)^{-2 s}}{s\,\Gamma\left(\frac{1}{2}-s\right)}\,\Gamma(s)\right)(-j) + \log(z) - \log(i z)$$

# Integral representations

## On the real axis

### Of the direct function

06.40.07.0001.01

$$\mathrm{Chi}(z) = \int_0^z \frac{\cosh(t)-1}{t}\,dt + \log(z) + \gamma$$

## Contour integral representations

06.40.07.0002.01

$$\mathrm{Chi}(z) = -\frac{1}{2}\left(\log\left(-z^2\right) - 2\log(z)\right) - \frac{\sqrt{\pi}}{4\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)^2}{\Gamma(s+1)\,\Gamma\left(\frac{1}{2}-s\right)}\left(-\frac{z^2}{4}\right)^{-s} ds$$

06.40.07.0003.01

$$\mathrm{Chi}(z) = -\frac{\sqrt{\pi}}{4\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)^2}{\Gamma(s+1)\,\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{i z}{2}\right)^{-2 s} ds + \log(z) - \log(i z)$$

# Differential equations

## Ordinary linear differential equations and wronskians

### For the direct function itself

06.40.13.0001.01

$$z\,w^{(3)}(z) + 2\,w''(z) - z\,w'(z) = 0 \;/;\; w(z) = c_1\,\mathrm{Chi}(z) + c_2\,\mathrm{Shi}(z) + c_3$$

06.40.13.0004.01

$$W_z(1, \mathrm{Chi}(z), \mathrm{Shi}(z)) = \frac{1}{z^2}$$

06.40.13.0002.01

$$z\,w^{(3)}(z) + 2\,w''(z) - z\,w'(z) = 0 \;/;\; w(z) = c_1\,\mathrm{Chi}(z) + c_2\,\mathrm{Ei}(z) + c_3$$

06.40.13.0005.01

$$W_z(1, \mathrm{Chi}(z), \mathrm{Ei}(z)) = \frac{1}{z^2}$$

06.40.13.0003.01

$$z\,w^{(3)}(z) + 2\,w''(z) - z\,w'(z) = 0 \;/;\; w(z) = c_1\,\mathrm{Chi}(z) + c_2\,\mathrm{Ei}(-z) + c_3$$

06.40.13.0006.01

$$W_z(1, \mathrm{Chi}(z), \mathrm{Ei}(-z)) = -\frac{1}{z^2}$$

06.40.13.0007.01

$$w^{(3)}(z)+\left(\frac{2\,g'(z)}{g(z)}-\frac{3\,g''(z)}{g'(z)}\right)w''(z)+\left(-g'(z)^2+\frac{3\,g''(z)^2}{g'(z)^2}-\frac{2\,g''(z)}{g(z)}-\frac{g^{(3)}(z)}{g'(z)}\right)w'(z)=0\;/;\;w(z)=c_1\,\mathrm{Chi}(g(z))+c_2\,\mathrm{Shi}(g(z))\;+c_3$$

06.40.13.0008.01

$$W_z(\mathrm{Chi}(g(z)),\,\mathrm{Shi}(g(z)),\,1)=\frac{g'(z)^3}{g(z)^2}$$

06.40.13.0009.01

$$w^{(3)}(z)+\left(\frac{2\,g'(z)}{g(z)}-\frac{3\,h'(z)}{h(z)}-\frac{3\,g''(z)}{g'(z)}\right)w''(z)+$$
$$\left(-g'(z)^2-\frac{4\,h'(z)\,g'(z)}{g(z)\,h(z)}+\frac{6\,h'(z)^2}{h(z)^2}+\frac{3\,g''(z)^2}{g'(z)^2}+\frac{6\,h'(z)\,g''(z)}{h(z)\,g'(z)}-\frac{2\,g''(z)}{g(z)}-\frac{3\,h''(z)}{h(z)}-\frac{g^{(3)}(z)}{g'(z)}\right)w'(z)+$$
$$\left(-\frac{6\,h'(z)^3}{h(z)^3}+\frac{4\,g'(z)\,h'(z)^2}{g(z)\,h(z)^2}-\frac{6\,g''(z)\,h'(z)^2}{h(z)^2\,g'(z)}+\frac{6\,h''(z)\,h'(z)}{h(z)^2}-\frac{3\,g'(z)^2\,h'(z)}{h(z)\,g'(z)^2}+\frac{2\,h'(z)\,g''(z)-2\,g'(z)\,h''(z)}{g(z)\,h(z)}+\right.$$
$$\left.\frac{3\,g''(z)\,h''(z)+h'(z)\,g^{(3)}(z)}{h(z)\,g'(z)}+\frac{g'(z)^2\,h'(z)-h^{(3)}(z)}{h(z)}\right)w(z)=0\;/;\;w(z)=c_1\,h(z)\,\mathrm{Chi}(g(z))\;+c_2\,\mathrm{Shi}(g(z))\,h(z)\;+c_3\,h(z)$$

06.40.13.0010.01

$$W_z(h(z)\,\mathrm{Chi}(g(z)),\,h(z)\,\mathrm{Shi}(g(z))\,,\,h(z))=\frac{h(z)^3\,g'(z)^3}{g(z)^2}$$

06.40.13.0011.01

$$z^3\,w^{(3)}(z)-(r+3\,s-3)\,z^2\,w''(z)\;-\left(a^2\,r^2\,z^{2\,r}-3\,s^2+r-2\,r\,s+3\,s-1\right)z\,w'(z)-s\left(-a^2\,r^2\,z^{2\,r}+s^2+r\,s\right)w(z)=0\;/;$$
$$w(z)=c_1\,z^s\,\mathrm{Chi}(a\,z^r)\;+c_2\,z^s\,\mathrm{Shi}(a\,z^r)\;+c_3\,z^s$$

06.40.13.0012.01

$$W_z(z^s\,\mathrm{Chi}(a\,z^r),\,z^s\,\mathrm{Shi}(a\,z^r),\,z^s)=a\,r^3\,z^{r+3\,s-3}$$

06.40.13.0013.01

$$w^{(3)}(z)-(\log(r)+3\log(s))\,w''(z)+\left(-a^2\log^2(r)\,r^{2\,z}+3\log^2(s)+2\log(r)\log(s)\right)w'(z)-$$
$$\log(s)\left(-a^2\log^2(r)\,r^{2\,z}+\log^2(s)+\log(r)\log(s)\right)w(z)=0\;/;\;w(z)=c_1\,s^z\,\mathrm{Chi}(a\,r^z)\;+c_2\,s^z\,\mathrm{Shi}(a\,r^z)\;+c_3\,s^z$$

06.40.13.0014.01

$$W_z(s^z\,\mathrm{Chi}(a\,r^z),\,s^z\,\mathrm{Shi}(a\,r^z),\,s^z)=a\,r^z\,s^{3\,z}\log^3(r)$$

# Transformations

## Transformations and argument simplifications

### Argument involving basic arithmetic operations

06.40.16.0001.01

$$\mathrm{Chi}(-z)=\mathrm{Chi}(z)+\log(-z)-\log(z)$$

06.40.16.0002.01

$$\mathrm{Chi}(i\,z)=\mathrm{Ci}(z)-\log(z)+\log(i\,z)$$

06.40.16.0003.01

$$\mathrm{Chi}(-i\,z)=\mathrm{Ci}(z)-\log(z)+\log(-i\,z)$$

06.40.16.0004.01

$$\mathrm{Chi}\left(\sqrt{z^2}\right) = \mathrm{Chi}(z) - \log(z) + \log\left(\sqrt{z^2}\right)$$

## Complex characteristics

### Real part

06.40.19.0001.01

$$\mathrm{Re}(\mathrm{Chi}(x + i\, y)) = \mathrm{Ci}(y) - \log(y) + \frac{1}{2}\log\left(x^2 + y^2\right) + \frac{1}{2}\sum_{k=1}^{\infty} \frac{x^{2k}}{k\,(2k)!}\, {}_1F_2\left(k; \frac{1}{2}, k+1; -\frac{y^2}{4}\right)$$

06.40.19.0002.01

$$\mathrm{Re}(\mathrm{Chi}(x + i\, y)) = \frac{1}{2}\log\left(x^2 + y^2\right) + \frac{1}{2}\sum_{k=1}^{\infty} \frac{(-1)^k \left(x^2 + y^2\right)^k}{k\,(2k)!} \cos\left(2\,k \tan^{-1}\left(\frac{x}{y}\right)\right) + \gamma$$

06.40.19.0003.01

$$\mathrm{Re}(\mathrm{Ci}(x + i\, y)) = \frac{1}{2}\log\left(x^2 + y^2\right) + \frac{1}{2}\sum_{k=1}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{j+k}\, x^{2j}\, y^{2k-2j}}{k\,(2j)!\,(2k-2j)!} + \gamma$$

06.40.19.0004.01

$$\mathrm{Re}(\mathrm{Chi}(x + i\, y)) = \frac{1}{2}\left(\mathrm{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

### Imaginary part

06.40.19.0005.01

$$\mathrm{Im}(\mathrm{Chi}(x + i\, y)) = \tan^{-1}(x, y) + y\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+2)!}\, {}_1F_2\left(k+1; \frac{3}{2}, k+2; -\frac{y^2}{4}\right)$$

06.40.19.0006.01

$$\mathrm{Im}(\mathrm{Chi}(x + i\, y)) = \tan^{-1}(x, y) + \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k \left(x^2 + y^2\right)^{k+1}}{(k+1)\,(2k+2)!} \sin\left(2\,(k+1) \tan^{-1}\left(\frac{x}{y}\right)\right)$$

06.40.19.0007.01

$$\mathrm{Im}(\mathrm{Chi}(x + i\, y)) = \tan^{-1}(x, y) + \frac{1}{2}\sum_{k=0}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{j+k}\, y^{2k-2j+1}\, x^{2j+1}}{(k+1)\,(2j+1)!\,(2k-2j+1)!}$$

06.40.19.0008.01

$$\mathrm{Im}(\mathrm{Chi}(x + i\, y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\mathrm{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Chi}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

### Absolute value

06.40.19.0009.01

$$|\text{Chi}(x+i\,y)| = \sqrt{\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\text{Chi}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)}$$

### Argument

06.40.19.0010.01

$$\arg(\text{Chi}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\text{Chi}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Chi}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

### Conjugate value

06.40.19.0011.01

$$\overline{\text{Chi}(x+i\,y)} = \frac{1}{2}\left(\text{Chi}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right) - \frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Chi}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

### Signum value

06.40.19.0012.01

$$\text{sgn}(\text{Chi}(x+i\,y)) = \left(\frac{1}{y}\left(i\sqrt{-\frac{y^2}{x^2}}\;x\left(\text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Chi}\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)\right)\right) + \text{Chi}\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right) + \text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right) \Big/ \left(2\sqrt{\text{Chi}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\text{Chi}\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)}\right)$$

## Differentiation

### Low-order differentiation

06.40.20.0001.01

$$\frac{\partial\,\text{Chi}(z)}{\partial z} = \frac{\cosh(z)}{z}$$

06.40.20.0002.01

$$\frac{\partial^2\,\text{Chi}(z)}{\partial z^2} = \frac{\sinh(z)}{z} - \frac{\cosh(z)}{z^2}$$

### Symbolic differentiation

06.40.20.0006.01

$$\frac{\partial^n\,\text{Chi}(z)}{\partial z^n} = \delta_n\,\text{Chi}(z) - \sum_{k=0}^{n-1}\frac{i^k\,(-1)^n\,(n-1)!\,z^{k-n}}{k!}\cosh\left(\frac{i\,\pi\,k}{2} + z\right) /;\, n \in \mathbb{N}$$

06.40.20.0003.01

$$\frac{\partial^n \mathrm{Ci}(z)}{\partial z^n} = \delta_n \mathrm{Ci}(z) + \mathrm{Boole}\left(n \neq 0,\, (n-1)! \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1} z^{k-n}}{k!} \cos\left(z + \frac{\pi k}{2}\right)\right) /;\, n \in \mathbb{N}$$

06.40.20.0004.01

$$\frac{\partial^n \mathrm{Chi}(z)}{\partial z^n} = (-1)^{n-1} z^{-n} (n-1)! + 2^{n-3} \sqrt{\pi}\, z^{2-n}\, {}_2\tilde{F}_3\left(1, 1; 2, \frac{3-n}{2}, 2 - \frac{n}{2}; \frac{z^2}{4}\right) /;\, n \in \mathbb{N}^+$$

### Fractional integro-differentiation

06.40.20.0005.01

$$\frac{\partial^\alpha \mathrm{Chi}(z)}{\partial z^\alpha} = \left(\mathcal{F}C_{\log}^{(\alpha)}(z) + \frac{\gamma}{\Gamma(1-\alpha)}\right) z^{-\alpha} + 2^{\alpha-3} \sqrt{\pi}\, z^{2-\alpha}\, {}_2\tilde{F}_3\left(1, 1; 2, \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; \frac{z^2}{4}\right)$$

## Integration

### Indefinite integration

#### Involving only one direct function

06.40.21.0001.01

$$\int \mathrm{Chi}(b + a z)\, dz = \frac{(b + a z)\, \mathrm{Chi}(b + a z) - \sinh(b + a z)}{a}$$

06.40.21.0002.01

$$\int \mathrm{Chi}(a z)\, dz = z\, \mathrm{Chi}(a z) - \frac{\sinh(a z)}{a}$$

06.40.21.0003.01

$$\int \mathrm{Chi}(z)\, dz = z\, \mathrm{Chi}(z) - \sinh(z)$$

#### Involving one direct function and elementary functions

### Involving power function

Involving power

### Linear argument

06.40.21.0004.01

$$\int z^{\alpha-1} \mathrm{Chi}(a z)\, dz = \frac{z^\alpha}{2 \alpha} \left(\Gamma(\alpha, -a z) (-a z)^{-\alpha} + 2\, \mathrm{Chi}(a z) + (a z)^{-\alpha}\, \Gamma(\alpha, a z)\right)$$

06.40.21.0005.01

$$\int z^{\alpha-1} \mathrm{Chi}(z)\, dz = \frac{z^\alpha}{\alpha} \mathrm{Chi}(z) + \frac{1}{2 \alpha} \left((-z)^{-\alpha} z^\alpha\, \Gamma(\alpha, -z) + \Gamma(\alpha, z)\right)$$

06.40.21.0006.01

$$\int z\, \mathrm{Chi}(a z)\, dz = \frac{a^2\, \mathrm{Chi}(a z)\, z^2 - a \sinh(a z)\, z + \cosh(a z)}{2 a^2}$$

06.40.21.0007.01

$$\int \frac{\text{Chi}(a z)}{z} \, dz = \frac{1}{2} \left(a z \left({}_3F_3(1, 1, 1; 2, 2, 2; a z) - {}_3F_3(1, 1, 1; 2, 2, 2; -a z)\right) + \log(z) \left(2 \left(\log(a z) + \gamma\right) - \log(z)\right)\right)$$

06.40.21.0008.01

$$\int \frac{\text{Chi}(a z)}{z^2} \, dz = -\frac{\cosh(a z) + \text{Chi}(a z) - a z \, \text{Shi}(a z)}{z}$$

06.40.21.0009.01

$$\int \frac{\text{Chi}(b + a z)}{z^2} \, dz = \frac{1}{b z} \left(a z \cosh(b) \, \text{Chi}(a z) - (b + a z) \, \text{Chi}(b + a z) + a z \sinh(b) \, \text{Shi}(a z)\right)$$

## Power arguments

06.40.21.0010.01

$$\int z^{\alpha-1} \, \text{Chi}(a z^r) \, dz = \frac{z^\alpha}{2 \alpha} \left(\Gamma\left(\frac{\alpha}{r}, -a z^r\right) (-a z^r)^{-\frac{\alpha}{r}} + 2 \, \text{Chi}(a z^r) + (a z^r)^{-\frac{\alpha}{r}} \, \Gamma\left(\frac{\alpha}{r}, a z^r\right)\right)$$

## Involving exponential function

### Involving exp

06.40.21.0011.01

$$\int e^{b z} \, \text{Chi}(a z) \, dz = -\frac{-2 \, e^{b z} \, \text{Chi}(a z) + \text{Ei}((b - a) z) + \text{Ei}((a + b) z)}{2 b}$$

06.40.21.0012.01

$$\int e^{a z} \, \text{Chi}(a z) \, dz = -\frac{-2 \, e^{a z} \, \text{Chi}(a z) + \text{Ei}(2 a z) + \log(a z)}{2 a}$$

06.40.21.0013.01

$$\int e^{-a z} \, \text{Chi}(a z) \, dz = -\frac{2 \, e^{-a z} \, \text{Chi}(a z) - \text{Ei}(-2 a z) - \log(a z)}{2 a}$$

## Involving exponential function and a power function

### Involving exp and power

06.40.21.0014.01

$$\int z^n \, e^{b z} \, \text{Chi}(a z) \, dz = \frac{1}{2} (-b)^{-n-1} \, n! \left(\text{Ei}((b - a) z) + \text{Ei}((a + b) z) - 2 \, e^{b z} \, \text{Chi}(a z) \sum_{k=0}^{n} \frac{(-b z)^k}{k!} - \right.$$
$$\left. e^{(a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a + b}\right)^m \sum_{k=0}^{m-1} \frac{(-a - b)^k \, z^k}{k!} - e^{(b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b - a}\right)^m \sum_{k=0}^{m-1} \frac{(a - b)^k \, z^k}{k!}\right) /; n \in \mathbb{N}$$

06.40.21.0015.01

$$\int z^n e^{a z} \operatorname{Chi}(a z) \, dz = \frac{(-a)^{-n}}{2 a} \left( 2 \operatorname{Chi}(a z) \Gamma(n+1, -a z) - n! \left( \operatorname{Ei}(2 a z) + \log(z) + 2 \sum_{k=1}^{n} \frac{1}{k!} \left( \frac{(-a z)^k}{2 k} - 2^{-k-1} \Gamma(k, -2 a z) \right) \right) \right) /; n \in \mathbb{N}$$

06.40.21.0016.01

$$\int z^n e^{-a z} \operatorname{Chi}(a z) \, dz = \frac{1}{2} a^{-n-1} \left( n! \left( \operatorname{Ei}(-2 a z) + \log(z) + 2 \sum_{k=1}^{n} \frac{1}{k!} \left( \frac{(a z)^k}{2 k} - 2^{-k-1} \Gamma(k, 2 a z) \right) \right) - 2 \operatorname{Chi}(a z) \Gamma(n+1, a z) \right) /; n \in \mathbb{N}$$

06.40.21.0017.01

$$\int z e^{b z} \operatorname{Chi}(a z) \, dz = \frac{1}{2 b^2} \left( 2 e^{b z} (b z - 1) \operatorname{Chi}(a z) + \operatorname{Ei}((b-a) z) + \operatorname{Ei}((a+b) z) - \frac{2 b e^{b z} (b \cosh(a z) - a \sinh(a z))}{(b-a)(a+b)} \right)$$

06.40.21.0018.01

$$\int z^2 e^{b z} \operatorname{Chi}(a z) \, dz = -\frac{1}{b^3} \Bigg( -e^{b z} \left(b^2 z^2 - 2 b z + 2\right) \operatorname{Chi}(a z) + \operatorname{Ei}((b-a) z) + \operatorname{Ei}((a+b) z) + \frac{1}{(a-b)^2 (a+b)^2} \left( b e^{b z} \left( b \left( (1 - b z) a^2 + b^2 (b z - 3) \right) \cosh(a z) + a \left( (b z - 2) a^2 + b^2 (4 - b z) \right) \sinh(a z) \right) \right) \Bigg)$$

06.40.21.0019.01

$$\int z^3 e^{b z} \operatorname{Chi}(a z) \, dz = \frac{1}{b^4} \Bigg( e^{b z} \left(b^3 z^3 - 3 b^2 z^2 + 6 b z - 6\right) \operatorname{Chi}(a z) + 3 (\operatorname{Ei}((b-a) z) + \operatorname{Ei}((a+b) z)) - \frac{1}{(b-a)^3 (a+b)^3} \Big( b e^{b z} \Big( b \left( \left(b^2 z^2 - b z + 3\right) a^4 - 2 b^2 \left(b^2 z^2 - 3 b z + 3\right) a^2 + b^4 \left(b^2 z^2 - 5 b z + 11\right) \right) \cosh(a z) - a \left( \left(b^2 z^2 - 3 b z + 6\right) a^4 - 2 b^2 \left(b^2 z^2 - 5 b z + 8\right) a^2 + b^4 \left(b^2 z^2 - 7 b z + 18\right) \right) \sinh(a z) \Big) \Big) \Bigg)$$

## Involving trigonometric functions

### Involving sin

06.40.21.0020.01

$$\int \sin(b z) \operatorname{Chi}(a z) \, dz = \frac{-2 \cos(b z) \operatorname{Chi}(a z) + \operatorname{Chi}((a + b i) z) + \operatorname{Chi}((a - i b) z)}{2 b}$$

### Involving cos

06.40.21.0021.01

$$\int \cos(b z) \operatorname{Chi}(a z) \, dz = \frac{2 \operatorname{Chi}(a z) \sin(b z) + i \operatorname{Shi}((a + b i) z) - i \operatorname{Shi}((a - i b) z)}{2 b}$$

## Involving trigonometric functions and a power function

### Involving sin and power

06.40.21.0022.01

$$\int z^n \sin(b z) \operatorname{Chi}(a z) \, dz = -\frac{i}{4} (i b)^{-n-1} n! \Bigg( -\operatorname{Ei}(-(a + b i) z) - (-1)^n \operatorname{Ei}((a + b i) z) - \operatorname{Ei}((a - i b) z) - (-1)^n \operatorname{Ei}(i b z - a z) + 4 \operatorname{Chi}(a z) \left( \cos(b z) \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{((i b) z)^{2k+n-2\lfloor \frac{n}{2} \rfloor}}{\left(2k + n - 2\lfloor \frac{n}{2} \rfloor\right)!} - i \sin(b z) \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{((i b) z)^{2k+n-2\lfloor \frac{n-1}{2} \rfloor - 1}}{\left(2k + n - 2\lfloor \frac{n-1}{2} \rfloor - 1\right)!} \right) + e^{(a - i b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{i b}{i b - a}\right)^m \left((i b - a)^k z^k\right)}{m \, k!} + (-1)^n e^{i b z - a z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{i b}{i b - a}\right)^m \left((-1)^k (i b - a)^k z^k\right)}{m \, k!} + e^{-(a + b i) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{i b}{a + b i}\right)^m \left((a + b i)^k z^k\right)}{m \, k!} + (-1)^n e^{(a + b i) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{i b}{a + b i}\right)^m \left((-1)^k (a + b i)^k z^k\right)}{m \, k!} \Bigg) /; n \in \mathbb{N}$$

06.40.21.0023.01

$$\int z \sin(b z) \operatorname{Chi}(a z) \, dz = -\frac{1}{4 b^2 (a^2 + b^2)} \left( e^{-i b z} \left( 2 \left(-1 + e^{2 i b z}\right) i \cosh(a z) b^2 - 2 a \left(1 + e^{2 i b z}\right) \sinh(a z) b + \left(a^2 + b^2\right) \left( 2 \left(-i + b z + e^{2 i b z} (i + b z)\right) \operatorname{Chi}(a z) + e^{i b z} i \left(\operatorname{Ei}(-(a + b i) z) - \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((a - i b) z) - \operatorname{Ei}(i b z - a z)\right) \right) \right) \right)$$

06.40.21.0024.01

$$\int z^2 \sin(b z) \operatorname{Chi}(a z) \, dz = -\frac{1}{2 b^3} \Bigg( 2 \operatorname{Chi}(a z) \left( \left(b^2 z^2 - 2\right) \cos(b z) - 2 b z \sin(b z) \right) + \frac{1}{\left(a^2 + b^2\right)^2} \Big( -2 \cosh(a z) \left( \left(a^2 + 3 b^2\right) \cos(b z) + b \left(a^2 + b^2\right) z \sin(b z) \right) b^2 - 2 a \left( b \left(a^2 + b^2\right) z \cos(b z) - 2 \left(a^2 + 2 b^2\right) \sin(b z) \right) \sinh(a z) b + \left(a^2 + b^2\right)^2 \left(\operatorname{Ei}(-(a + b i) z) + \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z)\right) \Big) \Bigg)$$

06.40.21.0025.01

$$\int z^3 \sin(b z) \operatorname{Chi}(a z) \, dz = -\frac{1}{2 b^4} \Bigg( 2 \operatorname{Chi}(a z) \left( b z \left(b^2 z^2 - 6\right) \cos(b z) - 3 \left(b^2 z^2 - 2\right) \sin(b z) \right) - \frac{1}{\left(a^2 + b^2\right)^3} \Big( 3 i \left(\operatorname{Ei}(-(a + b i) z) - \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((a - i b) z) - \operatorname{Ei}(i b z - a z)\right) \left(a^2 + b^2\right)^3 + 2 b^2 \cosh(a z) \left( b \left(a^2 + b^2\right) \left(a^2 + 5 b^2\right) z \cos(b z) + \left(-3 a^4 - 6 b^2 a^2 - 11 b^4 + b^2 \left(a^2 + b^2\right)^2 z^2\right) \sin(b z) \right) + 2 a b \left( \left( b^2 \left(a^2 + b^2\right)^2 z^2 - 2 \left(3 a^4 + 8 b^2 a^2 + 9 b^4\right) \right) \cos(b z) - b \left(a^2 + b^2\right) \left(3 a^2 + 7 b^2\right) z \sin(b z) \right) \sinh(a z) \Big) \Bigg)$$

## Involving cos and power

06.40.21.0026.01

$$\int z^n \cos(b\,z)\,\text{Chi}(a\,z)\,dz = \frac{1}{4}(i\,b)^{-n-1}\,n!\left(\text{Ei}(-(a+b\,i)\,z) - (-1)^n\,\text{Ei}((a+b\,i)\,z) + \text{Ei}((a-i\,b)\,z) - (-1)^n\,\text{Ei}(i\,b\,z-a\,z) + \text{Chi}(a\,z)\left(4\,i\sin(b\,z)\sum_{k=0}^{\lfloor \frac{n}{2}\rfloor}\frac{((i\,b)\,z)^{2k+n-2\lfloor \frac{n}{2}\rfloor}}{\left(2k+n-2\lfloor \frac{n}{2}\rfloor\right)!} - 4\cos(b\,z)\sum_{k=0}^{\lfloor \frac{n-1}{2}\rfloor}\frac{((i\,b)\,z)^{2k+n-2\lfloor \frac{n-1}{2}\rfloor-1}}{\left(2k+n-2\lfloor \frac{n-1}{2}\rfloor-1\right)!}\right) - e^{(a-i\,b)\,z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{i\,b}{i\,b-a}\right)^m\left((i\,b-a)^k\,z^k\right)}{m\,k!} + (-1)^n\,e^{i\,b\,z-a\,z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{i\,b}{i\,b-a}\right)^m\left((-1)^k\,(i\,b-a)^k\,z^k\right)}{m\,k!} - e^{-(a+b\,i)\,z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{i\,b}{a+b\,i}\right)^m\left((a+b\,i)^k\,z^k\right)}{m\,k!} + (-1)^n\,e^{(a+b\,i)\,z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{i\,b}{a+b\,i}\right)^m\left((-1)^k\,(a+b\,i)^k\,z^k\right)}{m\,k!}\right) /;\, n\in\mathbb{N}$$

06.40.21.0027.01

$$\int z\cos(b\,z)\,\text{Chi}(a\,z)\,dz = -\frac{1}{4\,b^2\left(a^2+b^2\right)}\left(i\left(i\left(4\cos(b\,z)\cosh(a\,z)\,b^2 - 4\,a\sin(b\,z)\sinh(a\,z)\,b - \left(a^2+b^2\right)(\text{Ei}(-(a+b\,i)\,z) + \text{Ei}((a+b\,i)\,z) + \text{Ei}((a-i\,b)\,z) + \text{Ei}(i\,b\,z-a\,z)) + 4\left(a^2+b^2\right)\text{Chi}(a\,z)\,(\cos(b\,z)+b\,z\sin(b\,z))\right)\right)\right)$$

06.40.21.0028.01

$$\int z^2\cos(b\,z)\,\text{Chi}(a\,z)\,dz = -\frac{1}{2\,b^3}\left(i\left(2\,i\,\text{Chi}(a\,z)\left(2\,b\,z\cos(b\,z) + \left(b^2 z^2-2\right)\sin(b\,z)\right) + \frac{1}{\left(a^2+b^2\right)^2}\left(2\,i\cosh(a\,z)\left(b\left(a^2+b^2\right)z\cos(b\,z) - \left(a^2+3\,b^2\right)\sin(b\,z)\right)b^2 - 2\,i\,a\left(2\left(a^2+2\,b^2\right)\cos(b\,z) + b\left(a^2+b^2\right)z\sin(b\,z)\right)\sinh(a\,z)\,b - \left(a^2+b^2\right)^2(\text{Ei}(-(a+b\,i)\,z) - \text{Ei}((a+b\,i)\,z) + \text{Ei}((a-i\,b)\,z) - \text{Ei}(i\,b\,z-a\,z))\right)\right)\right)$$

06.40.21.0029.01

$$\int z^3\cos(b\,z)\,\text{Chi}(a\,z)\,dz = -\frac{1}{2\,b^4}\left(i\left(i\left(2\,\text{Chi}(a\,z)\left(3\left(b^2 z^2-2\right)\cos(b\,z) + b\,z\left(b^2 z^2-6\right)\sin(b\,z)\right) + \frac{1}{\left(a^2+b^2\right)^3}\left(3\,(\text{Ei}(-(a+b\,i)\,z) + \text{Ei}((a+b\,i)\,z) + \text{Ei}((a-i\,b)\,z) + \text{Ei}(i\,b\,z-a\,z))\left(a^2+b^2\right)^3 + 2\,b^2\cosh(a\,z)\left(\left(-3\,a^4 - 6\,b^2 a^2 - 11\,b^4 + b^2\left(a^2+b^2\right)^2 z^2\right)\cos(b\,z) - b\left(a^2+b^2\right)\left(a^2+5\,b^2\right)z\sin(b\,z)\right) - 2\,a\,b\left(b\left(a^2+b^2\right)\left(3\,a^2+7\,b^2\right)z\cos(b\,z) + \left(b^2\left(a^2+b^2\right)^2 z^2 - 2\left(3\,a^4+8\,b^2 a^2+9\,b^4\right)\right)\sin(b\,z)\right)\sinh(a\,z)\right)\right)\right)\right)$$

## Involving hyperbolic functions

## Involving sinh

06.40.21.0030.01

$$\int \sinh(b\,z)\,\text{Chi}(a\,z)\,dz = -\frac{-2\cosh(b\,z)\,\text{Chi}(a\,z)+\text{Chi}((a-b)\,z)+\text{Chi}((a+b)\,z)}{2\,b}$$

06.40.21.0031.01

$$\int \sinh(a\,z)\,\text{Chi}(a\,z)\,dz = -\frac{-2\cosh(a\,z)\,\text{Chi}(a\,z)+\text{Chi}(2\,a\,z)+\log(a\,z)}{2\,a}$$

## Involving cosh

06.40.21.0032.01

$$\int \cosh(b\,z)\,\text{Chi}(a\,z)\,dz = \frac{2\,\text{Chi}(a\,z)\sinh(b\,z)+\text{Shi}((a-b)\,z)-\text{Shi}((a+b)\,z)}{2\,b}$$

# Involving hyperbolic functions and a power function

## Involving sinh and power

06.40.21.0033.01

$$\int z^n \sinh(b\,z)\,\text{Chi}(a\,z)\,dz = \frac{1}{4}\left(b^{-n-1}\,n!\right)\Bigg(-\text{Ei}((-a-b)\,z)-\text{Ei}((a-b)\,z)-(-1)^n\,\text{Ei}((b-a)\,z)-(-1)^n\,\text{Ei}((a+b)\,z)-$$

$$4\sinh(b\,z)\,\text{Chi}(a\,z)\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{(b\,z)^{2k+n-2\left\lfloor\frac{n-1}{2}\right\rfloor-1}}{\left(2\,k+n-2\left\lfloor\frac{n-1}{2}\right\rfloor-1\right)!}+4\cosh(b\,z)\,\text{Chi}(a\,z)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(b\,z)^{2k+n-2\left\lfloor\frac{n}{2}\right\rfloor}}{\left(2\,k+n-2\left\lfloor\frac{n}{2}\right\rfloor\right)!}+$$

$$e^{(a-b)z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{b}{b-a}\right)^m\left((b-a)^k\,z^k\right)}{m\,k!}+(-1)^n\,e^{(b-a)z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{b}{b-a}\right)^m\left((-1)^k\,(b-a)^k\,z^k\right)}{m\,k!}+$$

$$e^{(-a-b)z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{b}{a+b}\right)^m\left((a+b)^k\,z^k\right)}{m\,k!}+(-1)^n\,e^{(a+b)z}\sum_{k=0}^{n}\sum_{m=k+1}^{n}\frac{\left(\frac{b}{a+b}\right)^m\left((-1)^k\,(a+b)^k\,z^k\right)}{m\,k!}\Bigg)\,/;\,n\in\mathbb{N}$$

06.40.21.0034.01

$$\int z^n \sinh(a\,z)\,\text{Chi}(a\,z)\,dz =$$

$$\frac{(-1)^n\,a^{-n-1}}{4}\Bigg(2\,\text{Chi}(a\,z)\left(\Gamma(n+1,-a\,z)+(-1)^n\,\Gamma(n+1,a\,z)\right)-n!\Bigg((-1)^n\,\text{Ei}(-2\,a\,z)+\text{Ei}(2\,a\,z)+((-1)^n+1)\log(z)+$$

$$2\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(-a\,z)^k}{2\,k}-2^{-k-1}\,\Gamma(k,-2\,a\,z)\right)+2\,(-1)^n\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(a\,z)^k}{2\,k}-2^{-k-1}\,\Gamma(k,2\,a\,z)\right)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}$$

06.40.21.0035.01

$$\int z \sinh(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{4 b^2 (a^2 - b^2)} \left( i \left( i e^{b z} \left( 2 \left( -1 + e^{-2 b z} \right) \cosh(a z) b^2 + 2 a \left( 1 + e^{-2 b z} \right) \sinh(a z) b - \left( a^2 - b^2 \right) e^{-2 b z} \left( 2 \left( b z + e^{2 b z} (b z - 1) + 1 \right) \operatorname{Chi}(a z) + e^{b z} \left( -\operatorname{Ei}((a - b) z) + \operatorname{Ei}((b - a) z) - \operatorname{Ei}(-(a + b) z) + \operatorname{Ei}((a + b) z) \right) \right) \right) \right) \right)$$

06.40.21.0036.01

$$\int z^2 \sinh(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{2 b^3} \left( i \left( i \left( 2 \operatorname{Chi}(a z) \left( 2 b z \sinh(b z) - \left( b^2 z^2 + 2 \right) \cosh(b z) \right) + \frac{1}{\left( a^2 - b^2 \right)^2} \left( 2 \cosh(a z) \left( \left( a^2 - 3 b^2 \right) \cosh(b z) + b \left( b^2 - a^2 \right) z \sinh(b z) \right) b^2 + 2 a \sinh(a z) \left( b \left( a^2 - b^2 \right) z \cosh(b z) - 2 \left( a^2 - 2 b^2 \right) \sinh(b z) \right) b + \left( a^2 - b^2 \right)^2 \left( \operatorname{Ei}((a - b) z) + \operatorname{Ei}((b - a) z) + \operatorname{Ei}(-(a + b) z) + \operatorname{Ei}((a + b) z) \right) \right) \right) \right) \right)$$

06.40.21.0037.01

$$\int z^3 \sinh(b z) \operatorname{Chi}(a z) \, dz = -\frac{1}{2 b^4} \left( i \left( i \left( 2 \operatorname{Chi}(a z) \left( b z \left( b^2 z^2 + 6 \right) \cosh(b z) - 3 \left( b^2 z^2 + 2 \right) \sinh(b z) \right) + \frac{1}{\left( a^2 - b^2 \right)^3} \left( 3 \left( -\operatorname{Ei}((a - b) z) + \operatorname{Ei}((b - a) z) - \operatorname{Ei}(-(a + b) z) + \operatorname{Ei}((a + b) z) \right) \left( a^2 - b^2 \right)^3 + 2 a b \sinh(a z) \left( \left( -b^2 \left( a^2 - b^2 \right)^2 z^2 - 2 \left( 3 a^4 - 8 b^2 a^2 + 9 b^4 \right) \right) \cosh(b z) + b \left( b^2 - a^2 \right) \left( 7 b^2 - 3 a^2 \right) z \sinh(b z) \right) - 2 b^2 \cosh(a z) \left( b \left( a^2 - b^2 \right) \left( a^2 - 5 b^2 \right) z \cosh(b z) + \left( -3 a^4 + 6 b^2 a^2 - 11 b^4 - b^2 \left( a^2 - b^2 \right)^2 z^2 \right) \sinh(b z) \right) \right) \right) \right) \right)$$

## Involving cosh and power

06.40.21.0038.01

$$\int z^n \cosh(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{4} b^{-n-1} n! \left( \operatorname{Ei}((-a - b) z) + \operatorname{Ei}((a - b) z) - (-1)^n \operatorname{Ei}((b - a) z) - (-1)^n \operatorname{Ei}((a + b) z) - 4 \cosh(b z) \operatorname{Chi}(a z) \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(b z)^{2 k + n - 2 \left\lfloor \frac{n-1}{2} \right\rfloor - 1}}{\left( 2 k + n - 2 \left\lfloor \frac{n-1}{2} \right\rfloor - 1 \right)!} + 4 \operatorname{Chi}(a z) \sinh(b z) \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(b z)^{2 k + n - 2 \left\lfloor \frac{n}{2} \right\rfloor}}{\left( 2 k + n - 2 \left\lfloor \frac{n}{2} \right\rfloor \right)!} - e^{(a-b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left( \frac{b}{b-a} \right)^m \left( (b - a)^k z^k \right)}{m \, k!} + (-1)^n e^{(b-a) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left( \frac{b}{b-a} \right)^m \left( (-1)^k (b - a)^k z^k \right)}{m \, k!} - e^{(-a-b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left( \frac{b}{a+b} \right)^m \left( (a + b)^k z^k \right)}{m \, k!} + (-1)^n e^{(a+b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left( \frac{b}{a+b} \right)^m \left( (-1)^k (a + b)^k z^k \right)}{m \, k!} \right) /; n \in \mathbb{N}$$

06.40.21.0039.01

$$\int z^n \cosh(a z) \operatorname{Chi}(a z) \, dz = \frac{(-1)^n}{4} a^{-n-1} \left( 2 \operatorname{Chi}(a z) \left( \Gamma(n+1, -a z) - (-1)^n \Gamma(n+1, a z) \right) + n! \left( (-1)^n \operatorname{Ei}(-2 a z) - \operatorname{Ei}(2 a z) + ((-1)^n - 1) \log(z) + 2 (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left( \frac{(a z)^k}{2 k} - 2^{-k-1} \Gamma(k, 2 a z) \right) - 2 \sum_{k=1}^{n} \frac{1}{k!} \left( \frac{(-a z)^k}{2 k} - 2^{-k-1} \Gamma(k, -2 a z) \right) \right) \right) /; n \in \mathbb{N}$$

06.40.21.0040.01

$$\int z \cosh(b z) \operatorname{Chi}(a z) \, dz = -\frac{1}{4 b^2 (a^2 - b^2)} \left( -4 \cosh(a z) \cosh(b z) b^2 + 4 a \sinh(a z) \sinh(b z) b - (a^2 - b^2) (\operatorname{Ei}((a-b) z) + \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) + \operatorname{Ei}((a+b) z)) + 4 (a^2 - b^2) \operatorname{Chi}(a z) (\cosh(b z) - b z \sinh(b z)) \right)$$

06.40.21.0041.01

$$\int z^2 \cosh(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{2 b^3} \left( 2 \operatorname{Chi}(a z) \left( (b^2 z^2 + 2) \sinh(b z) - 2 b z \cosh(b z) \right) + \frac{1}{(a^2 - b^2)^2} \left( -2 \cosh(a z) \left( b (b^2 - a^2) z \cosh(b z) + (a^2 - 3 b^2) \sinh(b z) \right) b^2 + 2 a \sinh(a z) \left( 2 (a^2 - 2 b^2) \cosh(b z) + b (b^2 - a^2) z \sinh(b z) \right) b + (a^2 - b^2)^2 (\operatorname{Ei}((a-b) z) - \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) - \operatorname{Ei}((a+b) z)) \right) \right)$$

06.40.21.0042.01

$$\int z^3 \cosh(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{2 b^4} \left( 2 \operatorname{Chi}(a z) \left( b z (b^2 z^2 + 6) \sinh(b z) - 3 (b^2 z^2 + 2) \cosh(b z) \right) + \frac{1}{(a^2 - b^2)^3} \left( 3 (\operatorname{Ei}((a-b) z) + \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) + \operatorname{Ei}((a+b) z)) (a^2 - b^2)^3 - 2 b^2 \cosh(a z) \left( \left( -3 a^4 + 6 b^2 a^2 - 11 b^4 - b^2 (a^2 - b^2)^2 z^2 \right) \cosh(b z) + b (a^2 - b^2) (a^2 - 5 b^2) z \sinh(b z) \right) + 2 a b \sinh(a z) \left( b (b^2 - a^2) (7 b^2 - 3 a^2) z \cosh(b z) + \left( -b^2 (a^2 - b^2)^2 z^2 - 2 (3 a^4 - 8 b^2 a^2 + 9 b^4) \right) \sinh(b z) \right) \right) \right)$$

## Involving logarithm

### Involving log

06.40.21.0043.01

$$\int \log(b z) \operatorname{Chi}(a z) \, dz = \frac{a z \operatorname{Chi}(a z) (\log(b z) - 1) - \sinh(a z) (\log(b z) - 1) + \operatorname{Shi}(a z)}{a}$$

## Involving logarithm and a power function

## Involving log and power

06.40.21.0044.01

$$\int z^{\alpha-1}\log(b\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{2\,\alpha^3}\left(z^\alpha\left(-a^2 z^2\right)^{-\alpha}\left(-\alpha\,\Gamma(\alpha,a\,z)\,(-a\,z)^\alpha - \alpha\,\Gamma(\alpha+1)\log(z)\,(-a\,z)^\alpha + \alpha^2\,\Gamma(\alpha,a\,z)\log(b\,z)\,(-a\,z)^\alpha - (a\,z)^\alpha\,\alpha\,\Gamma(\alpha,-a\,z) + \left(-a^2 z^2\right)^\alpha\,{}_2F_2(\alpha,\alpha;\alpha+1,\alpha+1;-a\,z) + \left(-a^2 z^2\right)^\alpha\,{}_2F_2(\alpha,\alpha;\alpha+1,\alpha+1;a\,z) - (a\,z)^\alpha\,\alpha\,\Gamma(\alpha+1)\log(z) + (a\,z)^\alpha\,\alpha^2\,\Gamma(\alpha,-a\,z)\log(b\,z) + 2\left(-a^2 z^2\right)^\alpha\,\alpha\,\mathrm{Chi}(a\,z)\,(\alpha\log(b\,z)-1)\right)\right)$$

06.40.21.0045.01

$$\int z\log(b\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{4\,a^2}\left(\cosh(a\,z)\,(2\log(b\,z)+1) + \mathrm{Chi}(a\,z)\left(-a^2 z^2 + 2\,a^2\log(b\,z)\,z^2 - 2\right) + a\,z\,(1-2\log(b\,z))\sinh(a\,z)\right)$$

06.40.21.0046.01

$$\int z^2\log(b\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{9\,a^3}\left(a^3\,\mathrm{Chi}(a\,z)\,(3\log(b\,z)-1)\,z^3 + a^2\sinh(a\,z)\,z^2 - 3\,a^2\log(b\,z)\sinh(a\,z)\,z^2 + a\cosh(a\,z)\,z + 6\,a\cosh(a\,z)\log(b\,z)\,z - 6\log(b\,z)\sinh(a\,z) - 7\sinh(a\,z) + 6\,\mathrm{Shi}(a\,z)\right)$$

06.40.21.0047.01

$$\int z^3\log(b\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{16\,a^4}\left(\mathrm{Chi}(a\,z)\left(-a^4 z^4 + 4\,a^4\log(b\,z)\,z^4 - 24\right) + \cosh(a\,z)\left(a^2 z^2 + 12\left(a^2 z^2+2\right)\log(b\,z) + 38\right) + a\,z\left(a^2 z^2 - 4\left(a^2 z^2+6\right)\log(b\,z) - 14\right)\sinh(a\,z)\right)$$

**Involving functions of the direct function**

# Involving elementary functions of the direct function

## Involving powers of the direct function

06.40.21.0048.01

$$\int \mathrm{Chi}(a\,z)^2\,dz = \frac{a\,z\,\mathrm{Chi}(a\,z)^2 - 2\sinh(a\,z)\,\mathrm{Chi}(a\,z) + \mathrm{Shi}(2\,a\,z)}{a}$$

## Involving products of the direct function

06.40.21.0049.01

$$\int \mathrm{Chi}(a\,z)\,\mathrm{Chi}(b\,z)\,dz = \frac{1}{2\,a\,b}\,(2\,a\,b\,z\,\mathrm{Chi}(a\,z)\,\mathrm{Chi}(b\,z) - 2\,b\sinh(a\,z)\,\mathrm{Chi}(b\,z) - 2\,a\,\mathrm{Chi}(a\,z)\sinh(b\,z) - a\,\mathrm{Shi}((a-b)\,z) + b\,\mathrm{Shi}((a-b)\,z) + a\,\mathrm{Shi}((a+b)\,z) + b\,\mathrm{Shi}((a+b)\,z))$$

**Involving functions of the direct function and elementary functions**

# Involving elementary functions of the direct function and elementary functions

## Involving powers of the direct function and a power function

06.40.21.0050.01

$$\int z^n \operatorname{Chi}(a z)^2 \, dz = \frac{z^{n+1} \operatorname{Chi}(a z)^2}{n+1} - \frac{(-1)^n a^{-n-1}}{2(n+1)} \left(2 \operatorname{Chi}(a z) (\Gamma(n+1, -a z) - (-1)^n \Gamma(n+1, a z)) + n! \left((-1)^n \operatorname{Ei}(-2 a z) - \operatorname{Ei}(2 a z) + ((-1)^n - 1) \log(z) - 2 \sum_{k=1}^{n} \frac{(-a)^k}{k!} \left(\frac{z^k}{2k} - 2^{-k-1} (-a)^{-k} \Gamma(k, -2 a z)\right) + 2 (-1)^n \sum_{k=1}^{n} \frac{a^k}{k!} \left(\frac{z^k}{2k} - 2^{-k-1} a^{-k} \Gamma(k, 2 a z)\right)\right)\right) /; n \in \mathbb{N}$$

06.40.21.0051.01

$$\int z \operatorname{Chi}(a z)^2 \, dz = \frac{1}{4 a^2} \left(2 a^2 z^2 \operatorname{Chi}(a z)^2 + 4 (\cosh(a z) - a z \sinh(a z)) \operatorname{Chi}(a z) + \cosh(2 a z) - 2 \operatorname{Chi}(2 a z) - 2 \log(z)\right)$$

06.40.21.0052.01

$$\int z^2 \operatorname{Chi}(a z)^2 \, dz = \frac{1}{12 a^3} \left(4 a^3 \operatorname{Chi}(a z)^2 z^3 - 8 a z + 2 a \cosh(2 a z) z - 8 \operatorname{Chi}(a z) \left(\left(a^2 z^2 + 2\right) \sinh(a z) - 2 a z \cosh(a z)\right) - 5 \sinh(2 a z) + 8 \operatorname{Shi}(2 a z)\right)$$

06.40.21.0053.01

$$\int z^3 \operatorname{Chi}(a z)^2 \, dz = \frac{1}{8 a^4} \left(2 a^4 \operatorname{Chi}(a z)^2 z^4 - 3 a^2 z^2 + a^2 \cosh(2 a z) z^2 - 4 a \sinh(2 a z) z + 8 \cosh(2 a z) - 12 \operatorname{Chi}(2 a z) - 12 \log(z) - 4 \operatorname{Chi}(a z) \left(a z \left(a^2 z^2 + 6\right) \sinh(a z) - 3 \left(a^2 z^2 + 2\right) \cosh(a z)\right)\right)$$

## Involving products of the direct function and a power function

06.40.21.0054.01

$$\int z^n \operatorname{Chi}(a z) \operatorname{Chi}(b z) \, dz = \frac{\operatorname{Chi}(b z)}{n+1} \left(\operatorname{Chi}(a z) z^{n+1} + \frac{a^{-n} \Gamma(n+1, a z) - (-a)^{-n} \Gamma(n+1, -a z)}{2 a}\right) - \frac{1}{4(n+1)} \left(n! \sum_{k=0}^{n} \left(\frac{1}{k!} \left((-a)^{k-n-1} \left(-\Gamma(k, (-a-b) z) (-a-b)^{-k} - (b-a)^{-k} \Gamma(k, (b-a) z)\right)\right) + \frac{1}{k!} \left(a^{k-n-1} \left(-\Gamma(k, (a-b) z) (a-b)^{-k} - (a+b)^{-k} \Gamma(k, (a+b) z)\right)\right)\right)\right) - \frac{1}{4(n+1)} \left(\left(b^{-n-1} n!\right) \left(\operatorname{Ei}((a-b) z) - (-1)^n \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) - (-1)^n \operatorname{Ei}((a+b) z) + 4 \operatorname{Chi}(a z) \left(\sinh(b z) \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(b z)^{2k+n-2\left\lfloor \frac{n}{2} \right\rfloor}}{\left(2k+n-2\left\lfloor \frac{n}{2} \right\rfloor\right)!} - \cosh(b z) \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(b z)^{2k+n-2\left\lfloor \frac{n-1}{2} \right\rfloor - 1}}{\left(2k+n-2\left\lfloor \frac{n-1}{2} \right\rfloor - 1\right)!}\right) - e^{(a-b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{b}{b-a}\right)^m (b-a)^k z^k}{m\, k!} + (-1)^n e^{(a+b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{b}{a+b}\right)^m (-a-b)^k z^k}{m\, k!} + (-1)^n e^{(b-a) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{b}{b-a}\right)^m (a-b)^k z^k}{m\, k!} - e^{-(a+b) z} \sum_{k=0}^{n} \sum_{m=k+1}^{n} \frac{\left(\frac{b}{a+b}\right)^m (a+b)^k z^k}{m\, k!}\right)\right) /; n \in \mathbb{N}$$

06.40.21.0055.01

$$\int z\,\mathrm{Chi}(a\,z)\,\mathrm{Chi}(b\,z)\,dz = \frac{1}{8\,a^2\,b^2}\left(4\,\mathrm{Chi}(a\,z)\,(\cosh(b\,z) + b\,z\,(b\,z\,\mathrm{Chi}(b\,z) - \sinh(b\,z)))\,a^2 + 4\,b\sinh(a\,z)\sinh(b\,z)\,a - \left(a^2 + b^2\right)(\mathrm{Ei}((a-b)\,z) + \mathrm{Ei}((b-a)\,z) + \mathrm{Ei}(-(a+b)\,z) + \mathrm{Ei}((a+b)\,z)) + 4\,b^2\,\mathrm{Chi}(b\,z)\,(\cosh(a\,z) - a\,z\sinh(a\,z))\right)$$

06.40.21.0056.01

$$\int z^2\,\mathrm{Chi}(a\,z)\,\mathrm{Chi}(b\,z)\,dz = \frac{1}{3\,a^3\,(a-b)\,b^3\,(a+b)}\left(\left(\left(a^2 + 2\,b^2\right)\cosh(a\,z)\,b^2 + a\,(a-b)\,(a+b)\left(b^2\,z\sinh(a\,z) - a\left(b^2\,z^2 + 2\right)\mathrm{Chi}(a\,z)\right)\right)\sinh(b\,z)\,a - a^2\,b\cosh(b\,z)\left(\left(2\,a^2 + b^2\right)\sinh(a\,z) - 2\,a\,(a-b)\,(a+b)\,z\,\mathrm{Chi}(a\,z)\right) + (a-b)\,(a+b)\left(\mathrm{Chi}(b\,z)\left(a^3\,\mathrm{Chi}(a\,z)\,z^3 + 2\,a\cosh(a\,z)\,z - \left(a^2\,z^2 + 2\right)\sinh(a\,z)\right)b^3 + \left(b^3 - a^3\right)\mathrm{Shi}((a-b)\,z) + \left(a^3 + b^3\right)\mathrm{Shi}((a+b)\,z)\right)\right)$$

06.40.21.0057.01

$$\int z^3\,\mathrm{Chi}(a\,z)\,\mathrm{Chi}(b\,z)\,dz = \left((a-b)^2\,(a+b)^2\,\mathrm{Chi}(a\,z)\left(b^4\,\mathrm{Chi}(b\,z)\,z^4 - b\left(b^2\,z^2 + 6\right)\sinh(b\,z)\,z + 3\left(b^2\,z^2 + 2\right)\cosh(b\,z)\right)a^4 + b^2\cosh(b\,z)\left(\left(-3\,a^4 + 14\,b^2\,a^2 - 3\,b^4\right)\cosh(a\,z) + a\left(-3\,a^4 + 2\,b^2\,a^2 + b^4\right)z\sinh(a\,z)\right)a^2 + b\left(\left(a^5 + 2\,b^2\,a^3 - 3\,b^4\,a\right)z\cosh(a\,z)\,b^2 + \left(a^2\,b^2\left(a^2 - b^2\right)^2 z^2 + 2\left(a^2 + b^2\right)\left(3\,a^4 - 8\,b^2\,a^2 + 3\,b^4\right)\right)\sinh(a\,z)\right)\sinh(b\,z)\,a + (a-b)^2\,(a+b)^2\left(3\left(a^2\,z^2 + 2\right)\cosh(a\,z)\,\mathrm{Chi}(b\,z)\,b^4 - a\,z\left(a^2\,z^2 + 6\right)\mathrm{Chi}(b\,z)\sinh(a\,z)\,b^4 - 3\left(a^4 + b^4\right)(\mathrm{Chi}((a-b)\,z) + \mathrm{Chi}((a+b)\,z))\right)\right) \Big/ \left(4\,a^4\,(a-b)^2\,b^4\,(a+b)^2\right)$$

**Involving direct function and Gamma-, Beta-, Erf-type functions**

## Involving exponential integral-type functions

### Involving Ei

06.40.21.0058.01

$$\int \mathrm{Ei}(b\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{2\,a\,b}\left(-2\,a\,\mathrm{Chi}(a\,z)\left(e^{b\,z} - b\,z\,\mathrm{Ei}(b\,z)\right) + (a-b)\,\mathrm{Ei}((b-a)\,z) + a\,\mathrm{Ei}((a+b)\,z) + b\,\mathrm{Ei}((a+b)\,z) - 2\,b\,\mathrm{Ei}(b\,z)\sinh(a\,z)\right)$$

06.40.21.0059.01

$$\int \mathrm{Ei}(a\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{a}(\mathrm{Chi}(2\,a\,z) - \mathrm{Ei}(a\,z)\sinh(a\,z) - \mathrm{Chi}(a\,z)\,(\cosh(a\,z) - a\,z\,\mathrm{Ei}(a\,z) + \sinh(a\,z)) + \mathrm{Shi}(2\,a\,z))$$

06.40.21.0060.01

$$\int \mathrm{Ei}(-a\,z)\,\mathrm{Chi}(a\,z)\,dz = \frac{1}{a}(-\mathrm{Chi}(2\,a\,z) + \mathrm{Chi}(a\,z)\,(\cosh(a\,z) + a\,z\,\mathrm{Ei}(-a\,z) - \sinh(a\,z)) - \mathrm{Ei}(-a\,z)\sinh(a\,z) + \mathrm{Shi}(2\,a\,z))$$

### Involving Si

06.40.21.0061.01

$$\int \mathrm{Si}(b\,z)\,\mathrm{Chi}(a\,z)\,dz = -\frac{1}{2\,a\,b}(a\,\mathrm{Chi}((a + b\,i)\,z) + b\,i\,\mathrm{Chi}((a + b\,i)\,z) + (a - i\,b)\,\mathrm{Chi}((a - i\,b)\,z) + 2\,b\sinh(a\,z)\,\mathrm{Si}(b\,z) - 2\,a\,\mathrm{Chi}(a\,z)\,(\cos(b\,z) + b\,z\,\mathrm{Si}(b\,z)))$$

### Involving Ci

06.40.21.0062.01

$$\int \text{Ci}(b z) \text{Chi}(a z) \, dz = \frac{1}{2 a b} (2 a b z \text{Chi}(a z) \text{Ci}(b z) - 2 b \sinh(a z) \text{Ci}(b z) - 2 a \text{Chi}(a z) \sin(b z) - i a \text{Shi}((a + b i) z) + b \text{Shi}((a + b i) z) + b \text{Shi}((a - i b) z) + a i \text{Shi}((a - i b) z))$$

## Involving Shi

06.40.21.0063.01

$$\int \text{Shi}(b z) \text{Chi}(a z) \, dz = \frac{1}{2 a b} ((a - b) \text{Chi}((a - b) z) + a \text{Chi}((a + b) z) + b \text{Chi}((a + b) z) - 2 b \sinh(a z) \text{Shi}(b z) - 2 a \text{Chi}(a z) (\cosh(b z) - b z \text{Shi}(b z)))$$

# Involving exponential integral-type functions and a power function

## Involving Ei and power

06.40.21.0064.01

$$\int z^n \text{Ei}(b z) \text{Chi}(a z) \, dz = \frac{\text{Chi}(a z)}{n + 1} \left(\Gamma(n + 1, -b z) (-b)^{-n-1} + z^{n+1} \text{Ei}(b z)\right) + \frac{1}{2 a (n + 1)} \left((-a)^{-n} \left(-(-1)^n \text{Ei}((b - a) z) n! + \text{Ei}((a + b) z) n! + (-1)^n \left(\sum_{k=1}^{n} \frac{a^k (a - b)^{-k} \Gamma(k, (a - b) z)}{k!}\right) n! - \left(\sum_{k=1}^{n} \frac{a^k (a + b)^{-k} \Gamma(k, -(a + b) z)}{k!}\right) n! - \text{Ei}(b z) \Gamma(n + 1, -a z) + (-1)^n \text{Ei}(b z) \Gamma(n + 1, a z)\right)\right) + \frac{(-b)^{-n} n!}{2 b (n + 1)} \sum_{k=0}^{n} \frac{1}{k!} \left(b^k \left(-(b - a)^{-k} \Gamma(k, (a - b) z) - (a + b)^{-k} \Gamma(k, -(a + b) z)\right)\right) /; n \in \mathbb{N}$$

06.40.21.0065.01

$$\int z^n \text{Ei}(a z) \text{Chi}(a z) \, dz = \frac{1}{2 (n + 1)} \left(-a^{-n-1} \left(\text{Ei}(a z) ((-1)^n \Gamma(n + 1, -a z) - \Gamma(n + 1, a z)) + n! (\log(z) - (-1)^n \text{Ei}(2 a z)) - \frac{1}{(n + 1)^2} \left(a z \, {}_2F_2(1, n + 1; n + 2, n + 2; a z) (a z)^n + (n + 1)^2 n! (\Gamma(0, -a z) + \log(-a z) + \gamma)\right) + (-1)^n n! \sum_{k=1}^{n} \frac{2^{-k} \Gamma(k, -2 a z)}{k!}\right) + 2 \text{Chi}(a z) \left(\Gamma(n + 1, -a z) (-a)^{-n-1} + z^{n+1} \text{Ei}(a z)\right) + \frac{2 (-a)^{-n} n!}{a} \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-a z)^k}{2 k} - 2^{-k-1} \Gamma(k, -2 a z)\right) + \frac{(-a)^{-n} n! (\text{Ei}(2 a z) + \log(z))}{a}\right) /; n \in \mathbb{N}$$

06.40.21.0066.01

$$\int z^n \operatorname{Ei}(-a z) \operatorname{Chi}(a z) \, dz = \frac{1}{2(n+1)} \left( (-1)^n \left( \frac{(-1)^n a^n z^{n+1}}{(n+1)^2} \, {}_2F_2(1, n+1; n+2, n+2; -a z) - \frac{1}{a} \left( 2(-1)^n \operatorname{Ei}(-2 a z) n! + \Gamma(0, a z) n! + ((-1)^n - 1) \log(z) n! + \log(a z) n! - (-1)^n n! \sum_{k=1}^{n} \frac{2^{-k} \Gamma(k, 2 a z)}{k!} + 2(-1)^n n! \sum_{k=1}^{n} \frac{1}{k!} \left( \frac{(a z)^k}{2k} - 2^{-k-1} \Gamma(k, 2 a z) \right) + \gamma n! + \operatorname{Ei}(-a z) \Gamma(n+1, -a z) - (-1)^n \operatorname{Ei}(-a z) \Gamma(n+1, a z) \right) \right) a^{-n} + \operatorname{Chi}(a z) \left( 2 \Gamma(n+1, a z) a^{-n-1} + 2 z^{n+1} \operatorname{Ei}(-a z) \right) \right) /; n \in \mathbb{N}$$

06.40.21.0067.01

$$\int z \operatorname{Ei}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{4} \left( -\frac{2 e^{b z} (b z - 1) \operatorname{Chi}(a z)}{b^2} + \frac{2 e^{b z} \sinh(a z)}{a b} + \frac{2 \operatorname{Ei}(b z) \left( a^2 \operatorname{Chi}(a z) z^2 - a \sinh(a z) z + \cosh(a z) \right)}{a^2} - \frac{\left( a^2 + b^2 \right) (\operatorname{Ei}((b-a) z) + \operatorname{Ei}((a+b) z))}{a^2 b^2} \right)$$

06.40.21.0068.01

$$\int z^2 \operatorname{Ei}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{3} \left( \frac{\left( a^2 + 2 b^2 \right) e^{b z} \cosh(a z)}{a^2 (a-b) b (a+b)} + \frac{\left( a^3 - b^3 \right) \operatorname{Ei}((b-a) z) + \left( a^3 + b^3 \right) \operatorname{Ei}((a+b) z)}{a^3 b^3} + \frac{1}{a^3} \left( \operatorname{Ei}(b z) \left( a^3 \operatorname{Chi}(a z) z^3 + 2 a \cosh(a z) z - \left( a^2 z^2 + 2 \right) \sinh(a z) \right) \right) - \frac{e^{b z} \left( b^2 z^2 - 2 b z + 2 \right) \operatorname{Chi}(a z)}{b^3} - \frac{e^{b z} \left( (2 - b z) a^2 + b^2 (b z + 1) \right) \sinh(a z)}{a (a-b) b^2 (a+b)} \right)$$

06.40.21.0069.01

$$\int z^3 \operatorname{Ei}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{4} \left( \left( e^{b z} \left( (b z - 3) a^4 + 2 b^2 (b z + 7) a^2 - 3 b^4 (b z + 1) \right) \cosh(a z) \right) \Big/ \left( a^2 (a-b)^2 b^2 (a+b)^2 \right) + \left( e^{b z} \left( \left( b^2 z^2 - 3 b z + 6 \right) a^6 - 2 b^2 \left( b^2 z^2 - b z + 5 \right) a^4 + b^4 \left( b^2 z^2 + b z - 10 \right) a^2 + 6 b^6 \right) \sinh(a z) \right) \Big/ \left( a^3 (a-b)^2 b^3 (a+b)^2 \right) + \frac{1}{a^4} \left( \operatorname{Ei}(b z) \left( a^4 \operatorname{Chi}(a z) z^4 - a \left( a^2 z^2 + 6 \right) \sinh(a z) z + 3 \left( a^2 z^2 + 2 \right) \cosh(a z) \right) \right) - \frac{e^{b z} \left( b^3 z^3 - 3 b^2 z^2 + 6 b z - 6 \right) \operatorname{Chi}(a z)}{b^4} - \frac{3 \left( a^4 + b^4 \right) (\operatorname{Ei}((b-a) z) + \operatorname{Ei}((a+b) z))}{a^4 b^4} \right)$$

## Involving Si and power

06.40.21.0070.01

$$\int z^n \operatorname{Si}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{4(n+1)} \Bigg( i\, n!\, a^{-n-1} \Bigg( -\operatorname{Ei}(-(a+b i) z) - (-1)^n \operatorname{Ei}((a+b i) z) + (-1)^n \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z) + \frac{1}{(n+1)!} (2 i (n+1) ((-1)^n \Gamma(n+1, -a z) - \Gamma(n+1, a z)) \operatorname{Si}(b z)) - (-1)^n e^{(a - i b) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a - i b}\right)^m \sum_{k=0}^{m-1} \frac{(i b - a)^k z^k}{k!} + (-1)^n e^{(a+b i) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a+b i}\right)^m \sum_{k=0}^{m-1} \frac{(-a - i b)^k z^k}{k!} + e^{-(a+b i) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a+b i}\right)^m \sum_{k=0}^{m-1} \frac{(a+b i)^k z^k}{k!} - e^{i b z - a z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a - i b}\right)^m \sum_{k=0}^{m-1} \frac{(a - i b)^k z^k}{k!} \Bigg) - \frac{(i b)^{-n} n!}{b} (\operatorname{Ei}(-(a+b i) z) + \operatorname{Ei}((a - i b) z) + (-1)^n (\operatorname{Ei}((a+b i) z) + \operatorname{Ei}(i b z - a z))) + \frac{2 (i b)^{-n}}{b} \operatorname{Chi}(a z) ((-1)^n \Gamma(n+1, -i b z) + \Gamma(n+1, i b z)) + 4 z^{n+1} \operatorname{Chi}(a z) \operatorname{Si}(b z) - \frac{2 (i b)^{-n} n!}{b} \Bigg( \sum_{k=1}^n \frac{1}{2 k!} \left(b^k \left(-(b + a i)^{-k} \Gamma(k, (i b - a) z) - (b - i a)^{-k} \Gamma(k, (a+b i) z)\right)\right) + (-1)^n \sum_{k=1}^n \frac{1}{2 k!} \left(b^k \left(-\Gamma(k, -(a+b i) z) (b - i a)^{-k} - (b + a i)^{-k} \Gamma(k, (a - i b) z)\right)\right) \Bigg) \Bigg) /; n \in \mathbb{N}$$

06.40.21.0071.01

$$\int z \operatorname{Si}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{8 a^2 b^2} \Big( e^{-(a+b i) z} \Big( -a b \left(-1 + e^{2 a z}\right) \left(1 + e^{2 i b z}\right) + (a - b)(a + b) e^{(a+b i) z} i (\operatorname{Ei}(-(a+b i) z) - \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a - i b) z) - \operatorname{Ei}(i b z - a z)) + 2 e^{(a+b i) z} \left( i \operatorname{Chi}(a z) (\Gamma(2, -i b z) - \Gamma(2, i b z)) a^2 + b^2 \left(2 a^2 \operatorname{Chi}(a z) z^2 + \Gamma(2, -a z) + \Gamma(2, a z)\right) \operatorname{Si}(b z) \right) \Big) \Big)$$

06.40.21.0072.01

$$\int z^2 \operatorname{Si}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{12} \Bigg( 4 \operatorname{Chi}(a z) \operatorname{Si}(b z) z^3 + \frac{1}{b^3} (2 (\operatorname{Ei}(-(a+b i) z) + \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z))) + \frac{1}{b^2 \left(a^2 + b^2\right)} \Big( b \left(a^2 + b^2\right) (E_{-1}(-(a+b i) z) + E_{-1}((a+b i) z) + E_{-1}((a - i b) z) + E_{-1}(i b z - a z)) z^2 - 8 b \cos(b z) \cosh(a z) + 8 a \sin(b z) \sinh(a z) \Big) + \frac{1}{a^3} \Bigg( i \Bigg( \frac{1}{\left(a^2 + b^2\right)^2} \Big( -4 i \sin(b z) \left(a \left(a^2 + b^2\right) z \cosh(a z) - \left(3 a^2 + b^2\right) \sinh(a z)\right) a^2 + 4 b i \cos(b z) \left(a \left(a^2 + b^2\right) z \sinh(a z) - 2 \left(2 a^2 + b^2\right) \cosh(a z)\right) a + 2 \left(a^2 + b^2\right)^2 (-\operatorname{Ei}(-(a+b i) z) - \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z)) \Big) + 2 i (\Gamma(3, -a z) - \Gamma(3, a z)) \operatorname{Si}(b z) \Bigg) \Bigg) - \frac{2 \operatorname{Chi}(a z) (\Gamma(3, -i b z) + \Gamma(3, i b z))}{b^3} \Bigg)$$

06.40.21.0073.01

$$\int z^3 \operatorname{Si}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{16}\left(4 \operatorname{Chi}(a z) \operatorname{Si}(b z) z^4 - \frac{1}{b^3}\left(i\left(\frac{\Gamma(3, -(a+b i) z) b^2}{(b-i a)^3} + \frac{\Gamma(3, (a-i b) z) b^2}{(b+a i)^3} - \frac{\Gamma(3, (a+b i) z) b^2}{(b-i a)^3} - \frac{\Gamma(3, i b z - a z) b^2}{(b+a i)^3} + \frac{3 \Gamma(2, -(a+b i) z) b}{(b-i a)^2} + \frac{3 \Gamma(2, (a-i b) z) b}{(b+a i)^2} - \frac{3 \Gamma(2, (a+b i) z) b}{(b-i a)^2} - \frac{3 \Gamma(2, i b z - a z) b}{(b+a i)^2} + \frac{6 e^{-(a+b i) z}}{i a - b} + \frac{6 e^{i b z - a z}}{b + a i} + \frac{6 e^{(a+b i) z} i}{a + b i} - \frac{6 e^{(a-i b) z}}{b + a i}\right)\right) - \frac{1}{b^4}(6 i (\operatorname{Ei}(-(a+b i) z) - \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a-i b) z) - \operatorname{Ei}(i b z - a z))) + \frac{2 i \operatorname{Chi}(a z) (\Gamma(4, i b z) - \Gamma(4, -i b z))}{b^4} + \frac{1}{a^4}\left(i\left(\frac{1}{(a-i b)^3}\left(a e^{i b z - a z}\left(-z^2 a^4 + z(2 i b z - 5) a^3 + \left(b^2 z^2 + 8 b i z - 11\right) a^2 + 3 b (5 i + b z) a + 6 b^2\right)\right) - \frac{1}{(a+b i)^3}\left(a e^{(a+b i) z}\left(z^2 a^4 + z(2 i b z - 5) a^3 - \left(b^2 z^2 + 8 b i z - 11\right) a^2 + 3 b (5 i + b z) a - 6 b^2\right)\right) + \frac{1}{(a-i b)^3}\left(a e^{(a-i b) z}\left(z^2 a^4 + z(-2 i b z - 5) a^3 + \left(-b^2 z^2 + 8 b i z + 11\right) a^2 + 3 b (-5 i + b z) a - 6 b^2\right)\right) + \frac{1}{(a+b i)^3}\left(a e^{-(a+b i) z}\left(z^2 a^4 + z(2 b i z + 5) a^3 + \left(-b^2 z^2 + 8 b i z + 11\right) a^2 - 3 b (-5 i + b z) a - 6 b^2\right)\right) - 6 \operatorname{Ei}(-(a+b i) z) + 6 \operatorname{Ei}((a+b i) z) - 6 \operatorname{Ei}((a-i b) z) + 6 \operatorname{Ei}(i b z - a z) - 2 i (\Gamma(4, -a z) + \Gamma(4, a z)) \operatorname{Si}(b z)\right)\right)\right)$$

## Involving Ci and power

06.40.21.0074.01

$$\int z^n \operatorname{Ci}(b z) \operatorname{Chi}(a z) \, dz = \frac{z^{n+1} \operatorname{Ci}(b z) \operatorname{Chi}(a z)}{n+1} - \frac{1}{2(n+1)} \left( (i b)^{-n-1} ((-1)^n \Gamma(n+1, -i b z) - \Gamma(n+1, i b z)) \operatorname{Chi}(a z) \right) + \frac{(i b)^{-n-1} n!}{4(n+1)} ((-1)^n (\operatorname{Ei}((i b - a) z) + \operatorname{Ei}((a + b i) z)) - \operatorname{Ei}(-a z - i b z) - \operatorname{Ei}(a z - i b z)) - \frac{a^{-n-1} n!}{4(n+1)} \left( \operatorname{Ei}((i b - a) z) - (-1)^n \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((-a - i b) z) - (-1)^n \operatorname{Ei}((a - i b) z) + \frac{(2 \operatorname{Ci}(b z)) ((-1)^n \Gamma(n+1, -a z) - \Gamma(n+1, a z))}{n!} + (-1)^n e^{(a+b i) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a + b i} \right)^m \sum_{k=0}^{m-1} \frac{(-a + b(-i))^k z^k}{k!} + (-1)^n e^{(a - i b) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a - i b} \right)^m \sum_{k=0}^{m-1} \frac{(i b - a)^k z^k}{k!} - e^{(i b - a) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a - i b} \right)^m \sum_{k=0}^{m-1} \frac{(a - i b)^k z^k}{k!} - e^{(-a - i b) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a + b i} \right)^m \sum_{k=0}^{m-1} \frac{(a + b i)^k z^k}{k!} \right) + \frac{1}{2(n+1)} \left( \left( (i b)^{-n-1} n! \right) \left( (-1)^n \sum_{k=1}^{n} \frac{1}{2 k!} \left( b^k \left( -\Gamma(k, -(a + b i) z) (b - i a)^{-k} - (b + a i)^{-k} \Gamma(k, (a - i b) z) \right) \right) - \sum_{k=1}^{n} \frac{1}{2 k!} \left( b^k \left( -(b + a i)^{-k} \Gamma(k, (i b - a) z) - (b - i a)^{-k} \Gamma(k, (a + b i) z) \right) \right) \right) \right) /; n \in \mathbb{N}$$

06.40.21.0075.01

$$\int z \operatorname{Ci}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{8 a^2 b^2} \left( e^{-(a+b i) z} \left( -i a b \left( -1 + e^{2 a z} \right) \left( -1 + e^{2 i b z} \right) + e^{(a+b i) z} \left( -4 \operatorname{Chi}(a z) (\cos(b z) + b z (\sin(b z) - b z \operatorname{Ci}(b z))) a^2 + (a - b)(a + b) (\operatorname{Ei}(-(a + b i) z) + \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z)) + 4 b^2 \operatorname{Ci}(b z) (\cosh(a z) - a z \sinh(a z)) \right) \right) \right)$$

06.40.21.0076.01

$$\int z^2 \operatorname{Ci}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{6} \left( 2 \operatorname{Chi}(a z) \operatorname{Ci}(b z) z^3 + \frac{1}{b^3} (i (-\operatorname{Ei}(-(a + b i) z) + \operatorname{Ei}((a + b i) z) - \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z))) - \frac{1}{a^3} \left( \operatorname{Ei}(-(a + b i) z) - \operatorname{Ei}((a + b i) z) - \operatorname{Ei}((a - i b) z) + \operatorname{Ei}(i b z - a z) + \operatorname{Ci}(b z) (\Gamma(3, -a z) - \Gamma(3, a z)) + \frac{2 a}{\left(a^2 + b^2\right)^2} \left( a \cos(b z) \left( \left( 3 a^2 + b^2 \right) \sinh(a z) - a \left( a^2 + b^2 \right) z \cosh(a z) \right) + b \sin(b z) \left( 2 \left( 2 a^2 + b^2 \right) \cosh(a z) - a \left( a^2 + b^2 \right) z \sinh(a z) \right) \right) \right) + \frac{i}{2 b^2 \left(a^2 + b^2\right)} \left( b \left( a^2 + b^2 \right) (E_{-1}(-(a + b i) z) - E_{-1}((a + b i) z) + E_{-1}((a - i b) z) - E_{-1}(i b z - a z)) z^2 - 8 i b \cosh(a z) \sin(b z) - 8 i a \cos(b z) \sinh(a z) \right) - \frac{i \operatorname{Chi}(a z) (\Gamma(3, -i b z) - \Gamma(3, i b z))}{b^3} \right)$$

06.40.21.0077.01

$$\int z^3 \operatorname{Ci}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{8} \left( 2 \operatorname{Chi}(a z) \operatorname{Ci}(b z) z^4 + \frac{1}{2 b^3} \left( b \, i \, (b z (-E_{-2}((i b - a) z) + E_{-2}(-(a + b i) z) - E_{-2}((a + b i) z) + E_{-2}((a - i b) z)) + 3 i (E_{-1}((i b - a) z) + E_{-1}(-(a + b i) z) + E_{-1}((a + b i) z) + E_{-1}((a - i b) z))) z^2 + \frac{24 (b \cos(b z) \cosh(a z) - a \sin(b z) \sinh(a z))}{a^2 + b^2} \right) - \frac{1}{b^4} (3 (\operatorname{Ei}((i b - a) z) + \operatorname{Ei}(-(a + b i) z) + \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((a - i b) z))) - \frac{3}{a^4} \left( \operatorname{Ei}((i b - a) z) + \operatorname{Ei}(-(a + b i) z) + \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((a - i b) z) - \frac{1}{3} \operatorname{Ci}(b z) (\Gamma(4, -a z) + \Gamma(4, a z)) + \frac{2 a}{3 (a^2 + b^2)^3} \left( a \cos(b z) \left( a (a^2 + b^2) (5 a^2 + b^2) z \sinh(a z) - \left( 11 a^4 + 6 b^2 a^2 + (a^2 + b^2)^2 z^2 a^2 + 3 b^4 \right) \cosh(a z) \right) + \sin(b z) \left( a b (a^2 + b^2) (7 a^2 + 3 b^2) z \cosh(a z) - b \left( a^2 (a^2 + b^2)^2 z^2 + 2 (9 a^4 + 8 b^2 a^2 + 3 b^4) \right) \sinh(a z) \right) \right) \right) + \frac{\operatorname{Chi}(a z) (\Gamma(4, -i b z) + \Gamma(4, i b z))}{b^4} \right)$$

## Involving Shi and power

06.40.21.0078.01

$$\int z^n \operatorname{Shi}(b z) \operatorname{Chi}(a z) \, dz = \frac{z^{n+1} \operatorname{Shi}(b z) \operatorname{Chi}(a z)}{n+1} + \frac{b^{-n-1} n!}{4 (n+1)} ((-1)^n (\operatorname{Ei}(b z - a z) + \operatorname{Ei}(a z + b z)) + \operatorname{Ei}(-a z - b z) + \operatorname{Ei}(a z - b z)) - \frac{b^{-n-1} \operatorname{Chi}(a z) ((-1)^n \Gamma(n+1, -b z) + \Gamma(n+1, b z))}{2 (n+1)} - \frac{a^{-n-1} n!}{4 (n+1)} \left( -\operatorname{Ei}((-a - b) z) + (-1)^n \operatorname{Ei}((a - b) z) + \operatorname{Ei}((b - a) z) + (-1)^{-n-1} \operatorname{Ei}((a + b) z) + \frac{2 (-1)^n \Gamma(n+1, -a z) \operatorname{Shi}(b z)}{n!} + (-1)^n e^{(a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a+b} \right)^m \sum_{k=0}^{m-1} \frac{(-a-b)^k z^k}{k!} - (-1)^n e^{(a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a-b} \right)^m \sum_{k=0}^{m-1} \frac{(b-a)^k z^k}{k!} - e^{(b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a-b} \right)^m \sum_{k=0}^{m-1} \frac{(a-b)^k z^k}{k!} + e^{(-a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left( \frac{a}{a+b} \right)^m \sum_{k=0}^{m-1} \frac{(a+b)^k z^k}{k!} - \frac{2 \Gamma(n+1, a z) \operatorname{Shi}(b z)}{n!} \right) + \frac{b^{-n-1} n!}{2 (n+1)} \left( \sum_{k=1}^{n} \frac{1}{2 k!} \left( b^k \left( -\Gamma(k, (b - a) z) (b - a)^{-k} - (a + b)^{-k} \Gamma(k, (a + b) z) \right) \right) + (-1)^n \sum_{k=1}^{n} \frac{1}{2 k!} \left( b^k \left( -(b - a)^{-k} \Gamma(k, (a - b) z) - (a + b)^{-k} \Gamma(k, -(a + b) z) \right) \right) \right) /; n \in \mathbb{N}$$

06.40.21.0079.01

$$\int z^n \operatorname{Shi}(a z) \operatorname{Chi}(a z) \, dz = \frac{z^{n+1} \operatorname{Shi}(a z) \operatorname{Chi}(a z)}{n+1} - \frac{a^{-n-1} \operatorname{Chi}(a z) \left((-1)^n \Gamma(n+1, -a z) + \Gamma(n+1, a z)\right)}{2(n+1)} + \frac{a^{-n-1} n!}{4(n+1)} \left(\operatorname{Ei}(-2 a z) + \log(z) + (-1)^n (\operatorname{Ei}(2 a z) + \log(z))\right) - \frac{(-a)^{-n}}{4 a (n+1)} \left(2 \left(\Gamma(n+1, -a z) - (-1)^n \Gamma(n+1, a z)\right) \operatorname{Shi}(a z) - n! \left((-1)^n \operatorname{Ei}(-2 a z) + \operatorname{Ei}(2 a z) - (1 + (-1)^n) \log(z) - 2 \sum_{k=1}^n \frac{1}{k!} \left(\frac{(-a z)^k}{2 k} + 2^{-k-1} \Gamma(k, -2 a z)\right) - 2 (-1)^n \sum_{k=1}^n \frac{1}{k!} \left(\frac{(a z)^k}{2 k} + 2^{-k-1} \Gamma(k, 2 a z)\right)\right)\right) + \frac{a^{-n-1} n!}{2(n+1)} \left((-1)^n \sum_{k=1}^n \frac{1}{k!} \left(\frac{(-a z)^k}{2 k} - 2^{-k-1} \Gamma(k, -2 a z)\right) + \sum_{k=1}^n \frac{1}{k!} \left(\frac{(a z)^k}{2 k} - 2^{-k-1} \Gamma(k, 2 a z)\right)\right) /; n \in \mathbb{N}$$

06.40.21.0080.01

$$\int z \operatorname{Shi}(b z) \operatorname{Chi}(a z) \, dz = -\frac{1}{8 a^2 b^2} \left(-2 \operatorname{Chi}(a z) \left(2 b^2 \operatorname{Shi}(b z) z^2 + \Gamma(2, -b z) - \Gamma(2, b z)\right) a^2 + b e^{(b-a) z} a + b e^{-(a+b) z} a - b e^{(a+b) z} a - a b e^{(a-b) z} - \left(a^2 + b^2\right) \left(\operatorname{Ei}((a-b) z) - \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) - \operatorname{Ei}((a+b) z)\right) - 2 b^2 \left(\Gamma(2, -a z) + \Gamma(2, a z)\right) \operatorname{Shi}(b z)\right)$$

06.40.21.0081.01

$$\int z^2 \operatorname{Shi}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{12} \left(4 \operatorname{Chi}(a z) \operatorname{Shi}(b z) z^3 + \frac{1}{b^3} \left(2 \left(\operatorname{Ei}((a-b) z) + \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) + \operatorname{Ei}((a+b) z)\right)\right) - \frac{2}{a^3} \left(\operatorname{Ei}((a-b) z) + \operatorname{Ei}((b-a) z) - \operatorname{Ei}(-(a+b) z) - \operatorname{Ei}((a+b) z) + \frac{1}{\left(a^2 - b^2\right)^2} \left(2 a \left(\cosh(b z) \left(2 b \left(b^2 - 2 a^2\right) \cosh(a z) + a (a-b) b (a+b) z \sinh(a z)\right) - a \left(a (a-b) (a+b) z \cosh(a z) + \left(b^2 - 3 a^2\right) \sinh(a z)\right) \sinh(b z)\right)\right) + \Gamma(3, -a z) \operatorname{Shi}(b z) - \Gamma(3, a z) \operatorname{Shi}(b z)\right) - \frac{2 \operatorname{Chi}(a z) \left(\Gamma(3, -b z) + \Gamma(3, b z)\right)}{b^3} - \frac{1}{(a-b) b^2 (a+b)} \left((a-b) b (a+b) \left(E_{-1}((a-b) z) + E_{-1}((b-a) z) + E_{-1}(-(a+b) z) + E_{-1}((a+b) z)\right) z^2 - 8 b \cosh(a z) \cosh(b z) + 8 a \sinh(a z) \sinh(b z)\right)\right)$$

06.40.21.0082.01

$$\int z^3 \operatorname{Shi}(b z) \operatorname{Chi}(a z) \, dz = \frac{1}{16} \Bigg( 4 \operatorname{Chi}(a z) \operatorname{Shi}(b z) z^4 + \frac{1}{b^4} (8 (\operatorname{Ei}((a-b) z) - \operatorname{Ei}((b-a) z) + \operatorname{Ei}(-(a+b) z) - \operatorname{Ei}((a+b) z))) + \frac{2 \operatorname{Chi}(a z) (\Gamma(4, -b z) - \Gamma(4, b z))}{b^4} + \frac{1}{b^3 \left(a^2 - b^2\right)} \Big( b \left(b^2 - a^2\right) (b z (E_{-2}((a-b) z) + E_{-2}((b-a) z) + E_{-2}(-(a+b) z) + E_{-2}((a+b) z)) - 3 (E_{-1}((a-b) z) - E_{-1}((b-a) z) + E_{-1}(-(a+b) z) - E_{-1}((a+b) z))) z^2 + 12 (a+b) \sinh((a-b) z) + 12 (a-b) \sinh((a+b) z) \Big) - \frac{1}{a^4} \Bigg( -6 \operatorname{Ei}((a-b) z) + 6 \operatorname{Ei}((b-a) z) - 6 \operatorname{Ei}(-(a+b) z) + 6 \operatorname{Ei}((a+b) z) + \frac{1}{\left(a^2 - b^2\right)^3} \Big( \left(a e^{-2 b z}\right) \Big( \Big( e^{(b-a) z} \left((a+b)^2 z^2 a^2 + 11 a^2 + 15 b a + (a+b)(5 a + 3 b) z a + 6 b^2\right) - e^{(a+3 b) z} \left((a+b)^2 z^2 a^2 + 11 a^2 + 15 b a - (a+b)(5 a + 3 b) z a + 6 b^2\right) \Big) (a-b)^3 + 2 (a+b)^3 e^{2 b z} \left( \left((a-b)^2 z^2 a^2 + 11 a^2 - 15 b a + 6 b^2\right) \sinh((a-b) z) - a (5 a - 3 b) (a-b) z \cosh((a-b) z) \right) \Big) \Big) - 2 \Gamma(4, -a z) \operatorname{Shi}(b z) - 2 \Gamma(4, a z) \operatorname{Shi}(b z) \Bigg) \Bigg)$$

## Definite integration

### Involving the direct function

06.40.21.0083.01

$$\int_0^{\infty} e^{-t z} \operatorname{Chi}(t) \, dt = -\frac{\log\left(z^2 - 1\right)}{2 z} \; /; \; \operatorname{Re}(z) > 1$$

# Integral transforms

## Laplace transforms

06.40.22.0001.01

$$\mathcal{L}_t[\operatorname{Chi}(t)](z) = -\frac{\log\left(z^2 - 1\right)}{2 z} \; /; \; \operatorname{Re}(z) > 1$$

# Operations

## Limit operation

06.40.25.0001.01

$$\lim_{x \to \infty} \operatorname{Chi}(a + b x) = \begin{cases} \frac{\pi i}{2} & \arg(b) = \frac{\pi}{2} \\ -\frac{\pi i}{2} & \arg(b) = -\frac{\pi}{2} \\ \infty & \operatorname{Im}(b) = 0 \bigwedge \operatorname{Im}(b) = 0 \\ \tilde{\infty} & \text{True} \end{cases}$$

# Representations through more general functions

## Through hypergeometric functions

### Involving ${}_pF_q$

06.40.26.0001.01

$$\text{Chi}(z) = \frac{z^2}{4}\, {}_2F_3\left(1, 1; 2, 2, \frac{3}{2}; \frac{z^2}{4}\right) + \log(z) + \gamma$$

## Through Meijer G

### Classical cases for the direct function itself

06.40.26.0002.01

$$\text{Chi}(z) = -\frac{\sqrt{\pi}}{2}\, G_{1,3}^{2,0}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right) - \frac{1}{2}\left(\log\left(-z^2\right) - 2\log(z)\right)$$

06.40.26.0003.01

$$\text{Chi}(z) = \frac{\pi i}{2} - \frac{\sqrt{\pi}}{2}\, G_{1,3}^{2,0}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right) /;\ \text{Im}(z) > 0$$

06.40.26.0008.01

$$\text{Chi}\left(\sqrt{z}\right) = -\frac{1}{2}\pi^{3/2}\, G_{2,4}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

### Generalized cases for the direct function itself

06.40.26.0004.01

$$\text{Chi}(z) = -\frac{1}{2}\pi^{3/2}\, G_{2,4}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

06.40.26.0005.01

$$\text{Chi}(z) = \log(z) - \log(i z) - \frac{\sqrt{\pi}}{2}\, G_{1,3}^{2,0}\left(\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

## Through other functions

06.40.26.0006.01

$$\text{Chi}(z) = -\frac{1}{2}\left(\Gamma(0, -z) + \Gamma(0, z) + \log(-z) - \log(z)\right)$$

06.40.26.0007.01

$$\text{Chi}(z) = -\frac{1}{2}\left(E_1(-z) + E_1(z) + \log(-z) - \log(z)\right)$$

# Representations through equivalent functions

## With related functions

06.40.27.0001.02

$$\mathrm{Chi}(z) = \mathrm{Ci}(i\, z) + \log(z) - \log(i\, z)$$

06.40.27.0002.01

$$\mathrm{Chi}(z) = \frac{1}{4}\left(2\,(\mathrm{Ei}(-z) + \mathrm{Ei}(z)) + \log\left(\frac{1}{z}\right) + \log\left(-\frac{1}{z}\right) - \log(-z) + 3\log(z)\right)$$

06.40.27.0003.01

$$\mathrm{Chi}(z) = \frac{1}{2}\left(\log\left(\frac{1}{z}\right) + \log(z)\right) + \frac{1}{2}\left(\mathrm{li}(e^{-z}) + \mathrm{li}(e^{z})\right) + \frac{\pi\, i}{2}\,\mathrm{sgn}(\mathrm{Im}(z)) \;/;\; |\mathrm{Im}(z)| < \pi$$

## History

– C. A. Bretschneider (1843)

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