DigitCount

Notations

Traditional name

Counts of the digits of an integer for a given base

Traditional notation

$\{s_b^{(1)}(n),\ s_b^{(2)}(n),\ \ldots,\ s_b^{(b-1)}(n),\ s_b^{(0)}(n)\}$

Mathematica StandardForm notation

```
DigitCount[n, b]
```

Primary definition

13.10.02.0001.01

$$s_2^{(1)}(n) = n - \sum_{k=1}^{\infty} \left\lfloor \frac{n}{2^k} \right\rfloor$$

13.10.02.0002.01

$$\{s_b^{(1)}(n),\ s_b^{(2)}(n),\ \ldots,\ s_b^{(b-1)}(n),\ s_b^{(0)}(n)\}$$

$\{s_b^{(1)}(n),\ s_b^{(2)}(n),\ \ldots,\ s_b^{(b-1)}(n),\ s_b^{(0)}(n)\}$ is the a list of the numbers of $1,\ 2,\ \ldots b-1,\ 0$ digits in the base b representation of the integer n. $s_b^{(k)}(n)$ is the number of times the digit k appears in the base b representation of the integer n.

For example:

$s_2^{(1)}(0) = 0$, because $0 = 0_2$.

$s_2^{(1)}(1) = 1$, because $1 = 1_2$.

$s_2^{(1)}(2) = 1$, because $2 = 10_2$.

$s_2^{(1)}(3) = 2$, because $3 = 11_2$.

$s_2^{(1)}(4) = 1$, because $4 = 100_2$.

$s_2^{(1)}(5) = 2$, because $5 = 101_2$.

$s_2^{(1)}(6) = 2$, because $6 = 110_2$.

$s_2^{(1)}(7) = 3$, because $7 = 111_2$.

$s_2^{(1)}(8) = 1$, because $8 = 1000_2$.

Specific values

Specialized values

13.10.03.0001.01

$$\left\{s_b^{(1)}(0),\, s_b^{(2)}(0),\, \ldots,\, s_b^{(b-1)}(0),\, s_b^{(0)}(0)\right\} = \{s_1,\, s_2,\, \ldots,\, s_{b-1},\, 1\} \;/;\; s_1 = s_2 = \ldots = s_{b-1} = 0$$

13.10.03.0002.01

$$\left\{s_b^{(1)}(0),\, s_b^{(2)}(0),\, \ldots,\, s_b^{(b-1)}(0),\, s_b^{(0)}(0)\right\} = \{1,\, s_1,\, \ldots,\, s_{b-2},\, s_{b-1}\} \;/;\; s_1 = \ldots = s_{b-2} = s_{b-1} = 0$$

13.10.03.0003.01

$$s_2^{(1)}(n) = 2\,n - \log_2 q \;/;\; \frac{p}{q} = \frac{1}{n!}\left(\frac{\partial^n}{\partial x^n}\frac{1}{\sqrt{1-x}}\right)\bigg|_{x=0} \land \gcd(p,\, q) = 1$$

Values at fixed points

13.10.03.0004.01

$$\left\{s_2^{(1)}(0),\, s_2^{(0)}(0)\right\} = \{0,\, 1\}$$

13.10.03.0005.01

$$\left\{s_2^{(1)}(1),\, s_2^{(0)}(1)\right\} = \{1,\, 0\}$$

13.10.03.0006.01

$$\left\{s_2^{(1)}(2),\, s_2^{(0)}(2)\right\} = \{1,\, 1\}$$

13.10.03.0007.01

$$\left\{s_2^{(1)}(3),\, s_2^{(0)}(3)\right\} = \{2,\, 0\}$$

13.10.03.0008.01

$$\left\{s_2^{(1)}(4),\, s_2^{(0)}(4)\right\} = \{1,\, 2\}$$

13.10.03.0009.01

$$\left\{s_2^{(1)}(5),\, s_2^{(0)}(5)\right\} = \{2,\, 1\}$$

13.10.03.0010.01

$$\left\{s_2^{(1)}(6),\, s_2^{(0)}(6)\right\} = \{2,\, 1\}$$

13.10.03.0011.01

$$\left\{s_2^{(1)}(7),\, s_2^{(0)}(7)\right\} = \{3,\, 0\}$$

13.10.03.0012.01

$$\left\{s_2^{(1)}(8),\, s_2^{(0)}(8)\right\} = \{1,\, 3\}$$

13.10.03.0013.01

$$\left\{s_2^{(1)}(9),\, s_2^{(0)}(9)\right\} = \{2,\, 2\}$$

13.10.03.0014.01

$$\left\{s_2^{(1)}(10),\, s_2^{(0)}(10)\right\} = \{2,\, 2\}$$

13.10.03.0015.01

$$\left\{s_3^{(1)}(0),\, s_3^{(2)}(0),\, s_3^{(0)}(0)\right\} = \{0,\, 0,\, 1\}$$

13.10.03.0016.01

$$\left\{s_3^{(1)}(1),\, s_3^{(2)}(1),\, s_3^{(0)}(1)\right\} = \{1,\, 0,\, 0\}$$

13.10.03.0017.01

$$\left\{s_3^{(1)}(2),\, s_3^{(2)}(2),\, s_3^{(0)}(2)\right\} = \{0,\, 1,\, 0\}$$

13.10.03.0018.01

$$\left\{s_3^{(1)}(3),\, s_3^{(2)}(3),\, s_3^{(0)}(3)\right\} = \{1, 0, 1\}$$

13.10.03.0019.01

$$\left\{s_3^{(1)}(4),\, s_3^{(2)}(4),\, s_3^{(0)}(4)\right\} = \{2, 0, 0\}$$

13.10.03.0020.01

$$\left\{s_3^{(1)}(5),\, s_3^{(2)}(5),\, s_3^{(0)}(5)\right\} = \{1, 1, 0\}$$

13.10.03.0021.01

$$\left\{s_3^{(1)}(6),\, s_3^{(2)}(6),\, s_3^{(0)}(6)\right\} = \{0, 1, 1\}$$

13.10.03.0022.01

$$\left\{s_3^{(1)}(7),\, s_3^{(2)}(7),\, s_3^{(0)}(7)\right\} = \{1, 1, 0\}$$

13.10.03.0023.01

$$\left\{s_3^{(1)}(8),\, s_3^{(2)}(8),\, s_3^{(0)}(8)\right\} = \{0, 2, 0\}$$

13.10.03.0024.01

$$\left\{s_3^{(1)}(9),\, s_3^{(2)}(9),\, s_3^{(0)}(9)\right\} = \{1, 0, 2\}$$

13.10.03.0025.01

$$\left\{s_3^{(1)}(10),\, s_3^{(2)}(10),\, s_3^{(0)}(10)\right\} = \{2, 0, 1\}$$

13.10.03.0026.01

$$\left\{s_3^{(1)}(0),\, s_3^{(2)}(0),\, s_3^{(3)}(0),\, s_3^{(0)}(0)\right\} = \{0, 0, 0, 1\}$$

13.10.03.0027.01

$$\left\{s_3^{(1)}(1),\, s_3^{(2)}(1),\, s_3^{(3)}(1),\, s_3^{(0)}(1)\right\} = \{1, 0, 0, 0\}$$

13.10.03.0028.01

$$\left\{s_3^{(1)}(2),\, s_3^{(2)}(2),\, s_3^{(3)}(2),\, s_3^{(0)}(2)\right\} = \{0, 1, 0, 0\}$$

13.10.03.0029.01

$$\left\{s_3^{(1)}(3),\, s_3^{(2)}(3),\, s_3^{(3)}(3),\, s_3^{(0)}(3)\right\} = \{0, 0, 1, 0\}$$

13.10.03.0030.01

$$\left\{s_3^{(1)}(4),\, s_3^{(2)}(4),\, s_3^{(3)}(4),\, s_3^{(0)}(4)\right\} = \{1, 0, 0, 1\}$$

13.10.03.0031.01

$$\left\{s_3^{(1)}(5),\, s_3^{(2)}(5),\, s_3^{(3)}(5),\, s_3^{(0)}(5)\right\} = \{2, 0, 0, 0\}$$

13.10.03.0032.01

$$\left\{s_3^{(1)}(6),\, s_3^{(2)}(6),\, s_3^{(3)}(6),\, s_3^{(0)}(6)\right\} = \{1, 1, 0, 0\}$$

13.10.03.0033.01

$$\left\{s_3^{(1)}(7),\, s_3^{(2)}(7),\, s_3^{(3)}(7),\, s_3^{(0)}(7)\right\} = \{1, 0, 1, 0\}$$

13.10.03.0034.01

$$\left\{s_3^{(1)}(8),\, s_3^{(2)}(8),\, s_3^{(3)}(8),\, s_3^{(0)}(8)\right\} = \{0, 1, 0, 1\}$$

13.10.03.0035.01

$$\left\{s_3^{(1)}(9),\, s_3^{(2)}(9),\, s_3^{(3)}(9),\, s_3^{(0)}(9)\right\} = \{1, 1, 0, 0\}$$

13.10.03.0036.01

$$\left\{s_3^{(1)}(10),\, s_3^{(2)}(10),\, s_3^{(3)}(10),\, s_3^{(0)}(10)\right\} = \{0, 2, 0, 0\}$$

13.10.03.0037.01

$$\left\{s_3^{(1)}(0),\, s_3^{(2)}(0),\, s_3^{(3)}(0),\, s_3^{(4)}(0),\, s_3^{(0)}(0)\right\} = \{0,\, 0,\, 0,\, 0,\, 1\}$$

13.10.03.0038.01

$$\left\{s_3^{(1)}(1),\, s_3^{(2)}(1),\, s_3^{(3)}(1),\, s_3^{(4)}(1),\, s_3^{(0)}(1)\right\} = \{1,\, 0,\, 0,\, 0,\, 0\}$$

13.10.03.0039.01

$$\left\{s_3^{(1)}(2),\, s_3^{(2)}(2),\, s_3^{(3)}(2),\, s_3^{(4)}(2),\, s_3^{(0)}(2)\right\} = \{0,\, 1,\, 0,\, 0,\, 0\}$$

13.10.03.0040.01

$$\left\{s_3^{(1)}(3),\, s_3^{(2)}(3),\, s_3^{(3)}(3),\, s_3^{(4)}(3),\, s_3^{(0)}(3)\right\} = \{0,\, 0,\, 1,\, 0,\, 0\}$$

13.10.03.0041.01

$$\left\{s_3^{(1)}(4),\, s_3^{(2)}(4),\, s_3^{(3)}(4),\, s_3^{(4)}(4),\, s_3^{(0)}(4)\right\} = \{0,\, 0,\, 0,\, 1,\, 0\}$$

13.10.03.0042.01

$$\left\{s_3^{(1)}(5),\, s_3^{(2)}(5),\, s_3^{(3)}(5),\, s_3^{(4)}(5),\, s_3^{(0)}(5)\right\} = \{1,\, 0,\, 0,\, 0,\, 1\}$$

13.10.03.0043.01

$$\left\{s_3^{(1)}(6),\, s_3^{(2)}(6),\, s_3^{(3)}(6),\, s_3^{(4)}(6),\, s_3^{(0)}(6)\right\} = \{2,\, 0,\, 0,\, 0,\, 0\}$$

13.10.03.0044.01

$$\left\{s_3^{(1)}(7),\, s_3^{(2)}(7),\, s_3^{(3)}(7),\, s_3^{(4)}(7),\, s_3^{(0)}(7)\right\} = \{1,\, 1,\, 0,\, 0,\, 0\}$$

13.10.03.0045.01

$$\left\{s_3^{(1)}(8),\, s_3^{(2)}(8),\, s_3^{(3)}(8),\, s_3^{(4)}(8),\, s_3^{(0)}(8)\right\} = \{1,\, 0,\, 1,\, 0,\, 0\}$$

13.10.03.0046.01

$$\left\{s_3^{(1)}(9),\, s_3^{(2)}(9),\, s_3^{(3)}(9),\, s_3^{(4)}(9),\, s_3^{(0)}(9)\right\} = \{1,\, 0,\, 0,\, 1,\, 0\}$$

13.10.03.0047.01

$$\left\{s_3^{(1)}(10),\, s_3^{(2)}(10),\, s_3^{(3)}(10),\, s_3^{(4)}(10),\, s_3^{(0)}(10)\right\} = \{0,\, 1,\, 0,\, 0,\, 1\}$$

General characteristics

Domain and analyticity

$\left\{s_b^{(1)}(n),\, s_b^{(2)}(n),\, \ldots,\, s_b^{(b-1)}(n),\, s_b^{(0)}(n)\right\}$ is a nonanalytical vector-valued function of n and b which is defined for integer n and positive integers b $(b > 1)$.

13.10.04.0001.01

$$(n * b) \longrightarrow \left\{s_b^{(1)}(n),\, s_b^{(2)}(n),\, \ldots,\, s_b^{(b-1)}(n),\, s_b^{(0)}(n)\right\} :: (\mathbb{Z} \otimes \mathbb{Z}) \longrightarrow \mathbb{Z}^b$$

Symmetries and periodicities

Parity

$\left\{s_b^{(1)}(n),\, s_b^{(2)}(n),\, \ldots,\, s_b^{(b-1)}(n),\, s_b^{(0)}(n)\right\}$ is an even function with respect to n.

13.10.04.0002.01

$$\left\{s_b^{(1)}(-n),\, s_b^{(2)}(-n),\, \ldots,\, s_b^{(b-1)}(-n),\, s_b^{(0)}(-n)\right\} = \left\{s_b^{(1)}(n),\, s_b^{(2)}(n),\, \ldots,\, s_b^{(b-1)}(n),\, s_b^{(0)}(n)\right\}$$

Periodicity

No periodicity

Series representations

Other series representations

13.10.06.0001.01

$$s_2^{(1)}(n) = n - \sum_{k=1}^{\infty} \left\lfloor \frac{n}{2^k} \right\rfloor$$

13.10.06.0002.01

$$s_2^{(1)}(n) = \sum_{k=1}^{m} a_k \;/;\; n = \sum_{k=1}^{m} a_k \, 2^{k-1} \bigwedge m = \lceil \log_2(n) \rceil \bigwedge (a_k = 0 \lor a_k = 1)$$

Identities

Functional identities

13.10.17.0001.01

$$t(n) = n - 2^{\lfloor \log_2(n) \rfloor} + t\left(2^{\lfloor \log_2(n) \rfloor}\right) + t\left(n - 2^{\lfloor \log_2(n) \rfloor}\right) \;/;\; t(n) = \sum_{k=0}^{n-1} s_2^{(1)}(k)$$

Summation

Finite summation

13.10.23.0001.01

$$\sum_{k=0}^{n-1} s_2^{(1)}(k) = n\, F(\log_2(n)) + \frac{1}{2}\, n \log_2(n) \;/;$$
$$F(n) = F(n+1) \bigwedge F(n) = 2^{\lfloor \log_2(n) \rfloor - 1} \left(-2 \left(\frac{n}{2^{\lfloor \log_2(n) \rfloor}} - 1 \right) + 2\, f\left(\frac{n}{2^{\lfloor \log_2(n) \rfloor}} - 1 \right) + \frac{n}{2^{\lfloor \log_2(n) \rfloor}} \log_2\left(\frac{n}{2^{\lfloor \log_2(n) \rfloor}} \right) \right) \bigwedge$$
$$\left(f(x) = \sum_{k=0}^{\infty} 2^{-k}\, g\left(2^k x\right) \;/;\; g(x) = \frac{1}{2}\,(1 - x \bmod 1)\, \theta\left(x \bmod 1 - \frac{1}{2} \right) + \frac{1}{2}\,(x \bmod 1)\, \theta\left(\frac{1}{2} - x \bmod 1 \right) \right) \bigvee$$
$$\left(F(x) = \sum_{k=0}^{\infty} c_k\, e^{2 \pi i k x} \;/;\; c_0 = \frac{\log_2(\pi)}{2} - \frac{1}{4} - \frac{1}{2 \log(2)} \bigwedge c_k = -\frac{\log(2)}{2\, i\, k\, \pi \log(2) - 4\, k^2\, \pi^2}\, \zeta\left(\frac{2\, i\, k\, \pi}{\log(2)} \right) \right)$$

13.10.23.0002.01

$$4 \sum_{k=0}^{2^n - 1} s_2^{(1)}(k) - \sum_{k=0}^{2^{n+1}-1} s_2^{(1)}(k) = 2^n\, (n-1)$$

13.10.23.0003.01

$$\sum_{k=0}^{n} \sum_{j=1}^{10} s_{10}^{(j)}(k) = (n+1) \left(\lfloor \log_{10}(n) \rfloor + 1 \right) - \sum_{k=0}^{\lfloor \log_{10}(n) \rfloor} 10^k + 1$$

13.10.23.0004.01

$$\sum_{d_j|n} \mu(d_j) \sum_{k=1}^{p-1} k\, s_p^{(k)}\left(\frac{n}{d_j}\right) == 0 \;/;\; d_j \in \text{divisors}(n) \bigwedge p \in \mathbb{P} \bigwedge \frac{n}{p} \in \mathbb{Z} \bigwedge n > 0$$

Infinite summation

13.10.23.0005.01

$$\sum_{k=1}^{\infty} \frac{1}{k\,(k+1)}\, s_2^{(1)}(k) == 2\log(2)$$

13.10.23.0006.01

$$\sum_{k=1}^{\infty} \frac{2\,k+1}{k^2\,(k+1)^2}\, s_2^{(1)}(k) == \frac{\pi^2}{9}$$

13.10.23.0007.01

$$\sum_{k=1}^{\infty} \frac{1}{k\,(k+1)} \sum_{j=1}^{b-1} j\, s_b^{(j)}(k) == \frac{b\log(b)}{b-1} \;/;\; b-1 \in \mathbb{N}^+$$

Products

Infinite products

13.10.24.0001.01

$$\prod_{k=0}^{\infty} \left(\frac{2\,k+1}{2\,k+2}\right)^{s_2^{(1)}(k)\,(-1)^{s_2^{(1)}(k)}} == \sqrt[4]{2}$$

Inequalities

13.10.29.0001.01

$$\sum_{k=0}^{n} (-1)^{s_2^{(1)}(3\,k)} > 0$$

13.10.29.0002.01

$$\sum_{k=0}^{n} (-1)^{s_2^{(1)}(5\,k)} > 0$$

Other identities

Congruence properties

13.10.32.0001.01

$$\sum_{k=0}^{n} \binom{n}{k} \bmod 2 == 2^{s_2^{(1)}(n)}$$

13.10.32.0002.01

$$\underbrace{s(s(\ldots(s(n))))}_{\text{n-times}} == 15 \;/;\; \left(s(n) == \sum_{d|n} \sum_{j=1}^{9} j\, s_{10}^{(j)}(d) \;/;\; n \in \mathbb{Z} \land n > 1\right)$$

Theorems

Numbers in Pascal's triangle

The number of odd numbers in the n th row of Pascal's triangle is given by $2^{s_2^{(1)}(n)}$.

The classical Thue-Morse sequence

The sequence $(-1)^{s_2^{(1)}(n)}$ is the classical Thue–Morse sequence, which does not contain any triple repetition of a subsequence.

History

– E. Waring (1770)
– E. Kummer (1852)
– E. Lucas (1878)
– M.D. Ocagne (1886)
– L.E. Dickson (1896)
– J. W. L. Glaisher (1899)
– S. Ramanujan
– J. R. Trollope (1968)
– D. R. Woods (1978)
– D. Robbins (1979)

Copyright