FactorInteger

Notations

Traditional name

Prime decomposition

Traditional notation

$\text{factors}(n)$

Mathematica StandardForm notation

`FactorInteger[n]`

Primary definition

13.01.02.0001.01

$$\text{factors}(0) == \{\{0, 1\}\}$$

13.01.02.0002.01

$$\text{factors}(n) == \{\{p_1, n_1\}, \{p_2, n_2\}, \ldots, \{p_m, n_m\}\} /; n == \prod_{k=1}^{m} p_k^{n_k} \bigwedge p_k \in \mathbb{P} \bigwedge n_k \in \mathbb{N}^+ \bigwedge p_k < p_{k+1} \bigwedge 1 \leq k \leq m-1 \bigwedge m == \sigma_0(n)$$

13.01.02.0003.01

$$\text{factors}(n) == \{\{-1, 1\}, \{p_1, n_1\}, \{p_2, n_2\}, \ldots, \{p_m, n_m\}\} /;$$
$$n == -\prod_{k=1}^{m} p_k^{n_k} \bigwedge p_k \in \mathbb{P} \bigwedge n_k \in \mathbb{N}^+ \bigwedge p_k < p_{k+1} \bigwedge 1 \leq k \leq m-1 \bigwedge m == \sigma_0(n)$$

For integer n, the function prime factors $\text{factors}(n)$ is the list of the prime factors of the integer n, together with their exponents.

Examples: The prime factor of 2 is 2, so $\text{factors}(2) == \{\{2, 1\}\}$; the prime factor of 8 is 2 but it is taken with exponent 3, so $\text{factors}(8) == \{\{2, 3\}\}$; the prime factors of 6 are 2 and 3, so $\text{factors}(6) == \{\{2, 1\}, \{3, 1\}\}$; the prime factors of -6 are -1, 2 and 3, so $\text{factors}(-6) == \{\{-1, 1\}, \{2, 1\}, \{3, 1\}\}$.

Specific values

Specialized values

13.01.03.0001.01

$$\text{factors}(p^n) == \{\{p, n\}\} /; p \in \mathbb{P} \land n \in \mathbb{N}^+$$

13.01.03.0002.01

$\text{factors}\left(p_1^{n_1}\, p_2^{n_2}\right) = \{\{p_1, n_1\}, \{p_2, n_2\}\} \;/;\; p_k \in \mathbb{P} \bigwedge n_k \in \mathbb{N}^+ \bigwedge k \in \{1, 2\} \bigwedge p_1 < p_2$

Values at fixed points

13.01.03.0003.01

$\text{factors}(-1) = \{\{-1, 1\}\}$

13.01.03.0004.01

$\text{factors}(0) = \{\{0, 1\}\}$

13.01.03.0005.01

$\text{factors}(1) = \{\}$

13.01.03.0006.01

$\text{factors}(2) = \{\{2, 1\}\}$

13.01.03.0007.01

$\text{factors}(3) = \{\{3, 1\}\}$

13.01.03.0008.01

$\text{factors}(4) = \{\{2, 2\}\}$

13.01.03.0009.01

$\text{factors}(5) = \{\{5, 1\}\}$

13.01.03.0010.01

$\text{factors}(6) = \{\{2, 1\}, \{3, 1\}\}$

13.01.03.0011.01

$\text{factors}(7) = \{\{7, 1\}\}$

13.01.03.0012.01

$\text{factors}(8) = \{\{2, 3\}\}$

13.01.03.0013.01

$\text{factors}(9) = \{\{3, 2\}\}$

13.01.03.0014.01

$\text{factors}(10) = \{\{2, 1\}, \{5, 1\}\}$

13.01.03.0015.01

$\text{factors}(11) = \{\{11, 1\}\}$

13.01.03.0016.01

$\text{factors}(12) = \{\{2, 2\}, \{3, 1\}\}$

13.01.03.0017.01

$\text{factors}(13) = \{\{13, 1\}\}$

13.01.03.0018.01

$\text{factors}(14) = \{\{2, 1\}, \{7, 1\}\}$

13.01.03.0019.01

$\text{factors}(15) = \{\{3, 1\}, \{5, 1\}\}$

13.01.03.0020.01

$\text{factors}(16) = \{\{2, 4\}\}$

13.01.03.0021.01
factors(17) == {{17, 1}}

13.01.03.0022.01
factors(18) == {{2, 1}, {3, 2}}

13.01.03.0023.01
factors(19) == {{19, 1}}

13.01.03.0024.01
factors(20) == {{2, 2}, {5, 1}}

13.01.03.0025.01
factors(21) == {{3, 1}, {7, 1}}

13.01.03.0026.01
factors(22) == {{2, 1}, {11, 1}}

13.01.03.0027.01
factors(23) == {{23, 1}}

13.01.03.0028.01
factors(24) == {{2, 3}, {3, 1}}

13.01.03.0029.01
factors(25) == {{5, 2}}

13.01.03.0030.01
factors(26) == {{2, 1}, {13, 1}}

13.01.03.0031.01
factors(27) == {{3, 3}}

13.01.03.0032.01
factors(28) == {{2, 2}, {7, 1}}

13.01.03.0033.01
factors(29) == {{29, 1}}

13.01.03.0034.01
factors(30) == {{2, 1}, {3, 1}, {5, 1}}

13.01.03.0035.01
factors(31) == {{31, 1}}

13.01.03.0036.01
factors(32) == {{2, 5}}

13.01.03.0037.01
factors(33) == {{3, 1}, {11, 1}}

13.01.03.0038.01
factors(34) == {{2, 1}, {17, 1}}

13.01.03.0039.01
factors(35) == {{5, 1}, {7, 1}}

13.01.03.0040.01
factors(36) == {{2, 2}, {3, 2}}

13.01.03.0041.01

$\text{factors}(37) = \{\{37, 1\}\}$

13.01.03.0042.01

$\text{factors}(38) = \{\{2, 1\}, \{19, 1\}\}$

13.01.03.0043.01

$\text{factors}(39) = \{\{3, 1\}, \{13, 1\}\}$

13.01.03.0044.01

$\text{factors}(40) = \{\{2, 3\}, \{5, 1\}\}$

13.01.03.0045.01

$\text{factors}(41) = \{\{41, 1\}\}$

13.01.03.0046.01

$\text{factors}(42) = \{\{2, 1\}, \{3, 1\}, \{7, 1\}\}$

13.01.03.0047.01

$\text{factors}(43) = \{\{43, 1\}\}$

13.01.03.0048.01

$\text{factors}(44) = \{\{2, 2\}, \{11, 1\}\}$

13.01.03.0049.01

$\text{factors}(45) = \{\{3, 2\}, \{5, 1\}\}$

13.01.03.0050.01

$\text{factors}(46) = \{\{2, 1\}, \{23, 1\}\}$

13.01.03.0051.01

$\text{factors}(47) = \{\{47, 1\}\}$

13.01.03.0052.01

$\text{factors}(48) = \{\{2, 4\}, \{3, 1\}\}$

13.01.03.0053.01

$\text{factors}(49) = \{\{7, 2\}\}$

13.01.03.0054.01

$\text{factors}(50) = \{\{2, 1\}, \{5, 2\}\}$

13.01.03.0055.01

$\text{factors}(100) = \{\{2, 2\}, \{5, 2\}\}$

13.01.03.0058.01

$\text{factors}(1000) = \{\{2, 3\}, \{5, 3\}\}$

13.01.03.0059.01

$\text{factors}(10\,000) = \{\{2, 4\}, \{5, 4\}\}$

13.01.03.0057.01

$\text{factors}(5 + 20\,i) = \{\{-i, 1\}, \{1 + 2\,i, 1\}, \{1 + 4\,i, 1\}, \{2 + i, 1\}\}$

13.01.03.0060.01

$\text{factors}(-100) = \{\{-1, 1\}, \{2, 2\}, \{5, 2\}\}$

13.01.03.0056.01

$\text{factors}(-5\,!) = \{\{-1, 1\}, \{2, 3\}, \{3, 1\}, \{5, 1\}\}$

General characteristics

Domain and analyticity

factors(n) is a nonanalytical function which is defined only for integer n.

13.01.04.0001.01

$$n \longrightarrow \text{factors}(n) :: \mathbb{N} \longrightarrow (\mathbb{N} \otimes \mathbb{N})^m$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Identities

Functional identities

13.01.17.0001.01

$$x^{\omega(n\,m)-\omega(m)} = \sum_{d|\gcd(n,m)} \left(x^{\omega\left(\frac{n}{d}\right)} (1-t)\right)^{\Omega(d)} /;\ x \in \mathbb{R} \bigwedge \left(\omega(k) = r /;\ k = \prod_{j=1}^{r} \text{prime}(j)^{n_j}\right) \bigwedge \left(\Omega(k) = \sum_{j=1}^{r} n_j /;\ k = \prod_{j=1}^{r} \text{prime}(j)^{n_j}\right)$$

Operations

Limit operation

13.01.25.0001.01

$$\lim_{n\to\infty} \left(\frac{1}{n} \sum_{k=1}^{n} \omega(k) - (b + \log(\log(n)))\right) = 0 /;\ \left(\omega(k) = r /;\ k = \prod_{j=1}^{r} \text{prime}(j)^{n_j}\right) \bigwedge b = \sum_{k=1}^{\infty} \left(\log\left(1 - \frac{1}{\text{prime}(k)}\right) - \frac{1}{\text{prime}(k)}\right) + \gamma$$

13.01.25.0002.01

$$\lim_{n\to\infty} \left(\frac{1}{n} \sum_{k=1}^{n} \Omega(k) - (c + \log(\log(n)))\right) = 0 /;$$

$$\left(\Omega(k) = \sum_{j=1}^{r} n_j /;\ k = \prod_{j=1}^{r} \text{prime}(j)^{n_j}\right) \bigwedge c = \sum_{k=1}^{\infty} \frac{1}{\text{prime}(k)\,(\text{prime}(k)-1)} + \sum_{k=1}^{\infty} \left(\log\left(1 - \frac{1}{\text{prime}(k)}\right) - \frac{1}{\text{prime}(k)}\right) + \gamma$$

Summation

Asymptotic finite summation

13.01.23.0001.01

$$\sum_{k=2}^{n} p_1(k) \propto \frac{n^2}{2\log(n)} + O\left(\frac{n^2}{\log^2(n)}\right) /; n == \prod_{j=1}^{m} p_j(k)^{n_j} \bigwedge p_j(k) \in \mathbb{P} \bigwedge n_j \in \mathbb{Z} \bigwedge n_j > 0 \bigwedge p_j(k) < p_{j+1}(k) \bigwedge 1 \leq j \leq m-1$$

13.01.23.0002.01

$$\sum_{k=2}^{n} p_m(k) \propto \frac{\pi^2 n^2}{12\log(n)} + O\left(\frac{n^2 \log(\log(x))}{\log^{\frac{3}{2}}(n)}\right) /;$$

$$n == \prod_{j=1}^{m} p_j(k)^{n_j} \bigwedge p_j(k) \in \mathbb{P} \bigwedge n_j \in \mathbb{Z} \bigwedge n_j > 0 \bigwedge p_j(k) < p_{j+1}(k) \bigwedge 1 \leq j \leq m-1$$

13.01.23.0003.01

$$\sum_{k=2}^{n} \frac{p_1(k)}{p_m(k)} \propto \pi(n)\,(1 + o(n)) /; n == \prod_{j=1}^{m} p_j(k)^{n_j} \bigwedge p_j(k) \in \mathbb{P} \bigwedge n_j \in \mathbb{Z} \bigwedge n_j > 0 \bigwedge p_j(k) < p_{j+1}(k) \bigwedge 1 \leq j \leq m-1$$

13.01.23.0004.01

$$\sum_{k=2}^{n} \frac{1}{p_m(k)} \propto n\, e^{-\left(\sqrt{2\log(n)\log(\log(n))}\right) + O\left(\sqrt{\log(n)\log(\log(n))}\right)} /;$$

$$n == \prod_{j=1}^{m} p_j(k)^{n_j} \bigwedge p_j(k) \in \mathbb{P} \bigwedge n_j \in \mathbb{Z} \bigwedge n_j > 0 \bigwedge p_j(k) < p_{j+1}(k) \bigwedge 1 \leq j \leq m-1$$

Representations through equivalent functions

With related functions

13.01.27.0001.01

$$\text{factors}(n) == \{\{p_1, n_1\}, \{p_2, n_2\}, \ldots, \{p_m, n_m\}\} /; n == \prod_{k=1}^{m} p_k^{n_k} \bigwedge p_k \in \mathbb{P} \bigwedge n_k \in \mathbb{N}^+ \bigwedge p_k < p_{k+1} \bigwedge 1 \leq k \leq m-1 \bigwedge m == \sigma_0(n)$$

Theorems

Fundamental theorem of arithmetic

Every possible integer n can be uniquely decomposed into the form

$$n == \prod_{k=1}^{r} {p_k}^{n_k},\ p_k \in \mathbb{P} \wedge n_k \in \mathbb{N}^+,\ p_k < p_{k+1}.$$

History

– G. H. Hardy, S. Ramanujan (1917)

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