Floor

Notations

Traditional name

Floor function

Traditional notation

$\lfloor z \rfloor$

Mathematica StandardForm notation

`Floor[z]`

Primary definition

04.01.02.0001.01

$\lfloor x \rfloor == n \;/;\; x \in \mathbb{R} \land n \in \mathbb{Z} \land n \leq x < n+1$

04.01.02.0002.01

$\lfloor z \rfloor == \lfloor \mathrm{Re}(z) \rfloor + i \lfloor \mathrm{Im}(z) \rfloor$

For real z, the function $\lfloor z \rfloor$ is the greatest integer less than or equal to z.

Examples: $\lfloor 3.2 \rfloor == 3$, $\lfloor 3 \rfloor == 3$, $\lfloor -0.2 \rfloor == -1$, $\lfloor -2.3 \rfloor == -3$, $\left\lfloor \frac{2}{3} \right\rfloor == 0$, $\lfloor -\pi \rfloor == -4$, $\left\lfloor -4 - \frac{5}{3} i \right\rfloor == -4 - 2 i$, $\left\lfloor \frac{5}{2} \right\rfloor == 2$, $\left\lfloor \frac{7}{2} \right\rfloor == 3$.

Specific values

Specialized values

04.01.03.0001.01

$\lfloor x \rfloor == x \;/;\; x \in \mathbb{Z}$

04.01.03.0002.01

$\lfloor i x \rfloor == i x \;/;\; x \in \mathbb{Z}$

04.01.03.0003.01

$\lfloor x + i y \rfloor == \lfloor x \rfloor + i \lfloor y \rfloor \;/;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$

Values at fixed points

04.01.03.0004.01

$\lfloor 0 \rfloor == 0$

04.01.03.0005.01

$\lfloor 1 \rfloor = 1$

04.01.03.0006.01

$\lfloor -1 \rfloor = -1$

04.01.03.0007.01

$\lfloor i \rfloor = i$

04.01.03.0008.01

$\lfloor -i \rfloor = -i$

04.01.03.0009.01

$\left\lfloor \frac{23}{10} \right\rfloor = 2$

04.01.03.0010.01

$\lfloor -3 \rfloor = -3$

04.01.03.0011.01

$\lfloor -\pi \rfloor = -4$

04.01.03.0012.01

$\left\lfloor -\frac{27}{10} \right\rfloor = -3$

04.01.03.0013.01

$\lfloor -3.4 \rfloor = -4$

04.01.03.0014.01

$\left\lfloor \frac{23}{10} - i\, e \right\rfloor = 2 - 3\, i$

Values at infinities

04.01.03.0015.01

$\lfloor \infty \rfloor = \infty$

04.01.03.0016.01

$\lfloor -\infty \rfloor = -\infty$

04.01.03.0017.01

$\lfloor i\, \infty \rfloor = i\, \infty$

04.01.03.0018.01

$\lfloor -i\, \infty \rfloor = -i\, \infty$

04.01.03.0019.01

$\lfloor \tilde{\infty} \rfloor = \tilde{\infty}$

General characteristics

Domain and analyticity

$\lfloor z \rfloor$ is a non-analytical function; it is a piecewise constant function which is defined in the whole complex z-plane.

04.01.04.0001.01

$z \longrightarrow \lfloor z \rfloor :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Mirror symmetry

04.01.04.0002.01

$$\lfloor \bar{z} \rfloor = \overline{\lfloor z \rfloor} - i\,(1 - \chi_{\mathbb{Z}}(\operatorname{Im}(z)))$$

Periodicity

No periodicity

Sets of discontinuity

The function $\lfloor z \rfloor$ is a piecewise constant function with unit jumps on the lines $\operatorname{Re}(z) = k \bigvee \operatorname{Im}(z) = l$ /; $k, l \in \mathbb{Z}$.
The function $\lfloor z \rfloor$ is continuous from the right on the intervals $(k - i\,\infty, k + i\,\infty)$, $k \in \mathbb{Z}$, and from above on the intervals $(i\,k - \infty, i\,k + \infty)$, $k \in \mathbb{Z}$.

04.01.04.0003.01

$$\mathcal{DS}_z(\lfloor z \rfloor) = \{\{\{(k - i\,\infty, k + i\,\infty), -1\} /; k \in \mathbb{Z}\}, \{\{(i\,k - \infty, i\,k + \infty), -i\} /; k \in \mathbb{Z}\}\}$$

04.01.04.0004.01

$$\lim_{\epsilon \to +0} \lfloor z + \epsilon \rfloor = \lfloor z \rfloor \;/;\; \operatorname{Re}(z) \in \mathbb{Z}$$

04.01.04.0005.01

$$\lim_{\epsilon \to +0} \lfloor z - \epsilon \rfloor = \lfloor z \rfloor - 1 \;/;\; \operatorname{Re}(z) \in \mathbb{Z}$$

04.01.04.0006.01

$$\lim_{\epsilon \to +0} \lfloor z + i\,\epsilon \rfloor = \lfloor z \rfloor \;/;\; \operatorname{Im}(z) \in \mathbb{Z}$$

04.01.04.0007.01

$$\lim_{\epsilon \to +0} \lfloor z - i\,\epsilon \rfloor = \lfloor z \rfloor - i \;/;\; \operatorname{Im}(z) \in \mathbb{Z}$$

Series representations

Exponential Fourier series

04.01.06.0001.01

$$\lfloor x \rfloor = x - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2\,\pi\,k\,x)}{k} \;/;\; x \in \mathbb{R} \bigwedge x \notin \mathbb{Z}$$

Other series representations

04.01.06.0002.01

$$\left\lfloor \frac{m}{n} \right\rfloor = \frac{m}{n} + \frac{1}{2\,n} \sum_{k=1}^{n-1} \sin\!\left(\frac{2\,\pi\,k\,m}{n}\right) \cot\!\left(\frac{\pi\,k}{n}\right) - \frac{1}{2} \;/;\; m \in \mathbb{Z} \bigwedge n \in \mathbb{Z} \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigwedge n > 1$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

04.01.16.0001.01

$\lfloor -z \rfloor = -\lfloor z \rfloor \;/;\; \mathrm{Re}(z) \in \mathbb{Z} \land \mathrm{Im}(z) \in \mathbb{Z}$

04.01.16.0002.01

$\lfloor -z \rfloor = -\lfloor z \rfloor - \mathrm{sgn}(|\mathrm{Re}(z)|) - i\, \mathrm{sgn}(|\mathrm{Im}(z)|) \;/;\; \mathrm{Re}(z) \notin \mathbb{Z} \bigwedge \mathrm{Im}(z) \notin \mathbb{Z}$

04.01.16.0003.01

$\lfloor -z \rfloor = \chi_{\mathbb{Z}}(z) - \lfloor z \rfloor - 1 \;/;\; z \in \mathbb{R}$

04.01.16.0004.01

$\lfloor -z \rfloor = -\lfloor z \rfloor - i\,(1 - \chi_{\mathbb{Z}}(\mathrm{Im}(z)))\, \mathrm{sgn}(|\mathrm{Im}(z)|) - (1 - \chi_{\mathbb{Z}}(\mathrm{Re}(z)))\, \mathrm{sgn}(|\mathrm{Re}(z)|)$

04.01.16.0005.01

$\lfloor i\, z \rfloor = i\, \lfloor z \rfloor + \chi_{\mathbb{Z}}(\mathrm{Im}(z)) - 1$

04.01.16.0006.01

$\lfloor i\, z \rfloor = \lfloor -\mathrm{Im}(z) \rfloor + i\, \lfloor \mathrm{Re}(z) \rfloor$

04.01.16.0007.01

$\lfloor -i\, z \rfloor = -i\, \lfloor z \rfloor - i\,(1 - \chi_{\mathbb{Z}}(\mathrm{Re}(z)))$

04.01.16.0008.01

$\lfloor -i\, z \rfloor = \lfloor \mathrm{Im}(z) \rfloor + i\, \lfloor -\mathrm{Re}(z) \rfloor$

04.01.16.0009.01

$\lfloor z + n \rfloor = \lfloor z \rfloor + n \;/;\; n \in \mathbb{Z}$

04.01.16.0021.01

$\left\lfloor \frac{\lfloor n\, x \rfloor}{n} \right\rfloor = \lfloor x \rfloor \;/;\; x \in \mathbb{R} \land n \in \mathbb{Z}$

Argument involving related functions

04.01.16.0010.01

$\lfloor \lfloor z \rfloor \rfloor = \lfloor z \rfloor$

04.01.16.0011.01

$\lfloor z - \lfloor z \rfloor \rfloor = 0$

04.01.16.0012.01

$\lfloor \lfloor z \rceil \rfloor = \lfloor z \rceil$

04.01.16.0013.01

$\lfloor \lceil z \rceil \rfloor = \lceil z \rceil$

04.01.16.0014.01

$\lfloor \mathrm{int}(z) \rfloor = \mathrm{int}(z)$

04.01.16.0022.01

$\lfloor \mathrm{frac}(z) \rfloor = (\theta(\mathrm{Im}(z)) - 1)\, i\,(1 - \chi_{\mathbb{Z}}(\mathrm{Im}(z))) + (\theta(\mathrm{Re}(z)) - 1)\,(1 - \chi_{\mathbb{Z}}(\mathrm{Re}(z)))$

04.01.16.0023.01

$\lfloor m \bmod n \rfloor = \left\lfloor m - n \left\lfloor \frac{m}{n} \right\rfloor \right\rfloor$

04.01.16.0024.01

$\lfloor m \bmod 1 \rfloor = 0$

04.01.16.0025.01

$$\lfloor \text{quotient}(m,\, n) \rfloor = \left\lfloor \frac{m}{n} \right\rfloor$$

Addition formulas

04.01.16.0015.01

$$\lfloor z + n \rfloor = \lfloor z \rfloor + n \;/;\; n \in \mathbb{Z}$$

04.01.16.0016.01

$$\lfloor z_1 + z_2 \rfloor = \lfloor z_1 + z_2 - \lfloor z_1 \rfloor - \lfloor z_2 \rfloor \rfloor + \lfloor z_1 \rfloor + \lfloor z_2 \rfloor$$

Multiple arguments

04.01.16.0017.01

$$\lfloor n\, z \rfloor = n \lfloor z \rfloor + \sum_{k=0}^{n-1} k\, \theta\!\left(z \bmod 1 - \frac{k}{n}\right) \left(1 - \theta\!\left(z \bmod 1 - \frac{k+1}{n}\right)\right) /;\; n \in \mathbb{N} \land z \in \mathbb{R}$$

Products, sums, and powers of the direct function

Sums of the direct function

04.01.16.0018.01

$$\lfloor z_1 \rfloor + \lfloor z_2 \rfloor = \lfloor z_1 + z_2 \rfloor - \lfloor z_1 + z_2 - \lfloor z_1 \rfloor - \lfloor z_2 \rfloor \rfloor$$

04.01.16.0019.01

$$\sum_{k=0}^{n-1} \left\lfloor \frac{k\, m + x}{n} \right\rfloor = \sum_{k=0}^{m-1} \left\lfloor \frac{k\, n + x}{m} \right\rfloor /;\; x \in \mathbb{R} \bigwedge n \in \mathbb{N}^+ \bigwedge m \in \mathbb{N}^+$$

Related transformations

04.01.16.0020.01

$$z_1\, z_2 = (z_1 - \lfloor z_1 \rfloor)\, (z_2 - \lfloor z_2 \rfloor) - \lfloor z_1 \rfloor \lfloor z_2 \rfloor + \lfloor z_2 \rfloor\, z_1 + \lfloor z_1 \rfloor\, z_2$$

Identities

Functional identities

04.01.17.0001.01

$$\left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n+2}{6} \right\rfloor + \left\lfloor \frac{n+4}{6} \right\rfloor = \left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n+3}{6} \right\rfloor /;\; n \in \mathbb{Z}$$

04.01.17.0002.01

$$\left\lfloor \sqrt{n + \frac{1}{2}} + \frac{1}{2} \right\rfloor = \left\lfloor \sqrt{n + \frac{1}{4}} + \frac{1}{2} \right\rfloor /;\; n \in \mathbb{Z}$$

04.01.17.0003.01

$$\left\lfloor \sqrt{n} + \sqrt{n+1} \right\rfloor = \left\lfloor \sqrt{4\, n + 2} \right\rfloor /;\; n \in \mathbb{Z}$$

Complex characteristics

Real part

04.01.19.0001.01

$$\operatorname{Re}(\lfloor x + i\, y \rfloor) = \lfloor x \rfloor$$

04.01.19.0006.01

$$\operatorname{Re}(\lfloor z \rfloor) = \lfloor \operatorname{Re}(z) \rfloor$$

Imaginary part

04.01.19.0002.01

$$\operatorname{Im}(\lfloor x + i\, y \rfloor) = \lfloor y \rfloor$$

04.01.19.0007.01

$$\operatorname{Im}(\lfloor z \rfloor) = \lfloor \operatorname{Im}(z) \rfloor$$

Absolute value

04.01.19.0003.01

$$|\lfloor x + i\, y \rfloor| = \sqrt{\lfloor x \rfloor^2 + \lfloor y \rfloor^2}$$

04.01.19.0008.01

$$|\lfloor z \rfloor| = \sqrt{\lfloor \operatorname{Im}(z) \rfloor^2 + \lfloor \operatorname{Re}(z) \rfloor^2}$$

Argument

04.01.19.0004.01

$$\arg(\lfloor x + i\, y \rfloor) = \tan^{-1}(\lfloor x \rfloor, \lfloor y \rfloor)$$

04.01.19.0009.01

$$\arg(\lfloor z \rfloor) = \tan^{-1}(\lfloor \operatorname{Re}(z) \rfloor, \lfloor \operatorname{Im}(z) \rfloor)$$

Conjugate value

04.01.19.0005.01

$$\overline{\lfloor x + i\, y \rfloor} = \lfloor x \rfloor - i\, \lfloor y \rfloor$$

04.01.19.0010.01

$$\overline{\lfloor z \rfloor} = \lfloor \operatorname{Re}(z) \rfloor - i\, \lfloor \operatorname{Im}(z) \rfloor$$

Signum value

04.01.19.0011.01

$$\operatorname{sgn}(\lfloor x + i\, y \rfloor) = \frac{\lfloor x + i\, y \rfloor}{|\lfloor x + i\, y \rfloor|}$$

04.01.19.0012.01

$$\operatorname{sgn}(\lfloor z \rfloor) = \frac{\lfloor z \rfloor}{|\lfloor z \rfloor|}$$

Differentiation

Low-order differentiation

04.01.20.0001.01

$$\frac{\partial \lfloor z \rfloor}{\partial z} == 0$$

In a distributional sense, for $x \in \mathbb{R}$.

04.01.20.0002.01

$$\frac{\partial \lfloor x \rfloor}{\partial x} == \sum_{k=-\infty}^{\infty} \delta(x-k)$$

Fractional integro-differentiation

04.01.20.0003.01

$$\frac{\partial^{\alpha} \lfloor z \rfloor}{\partial z^{\alpha}} == \frac{\lfloor z \rfloor z^{-\alpha}}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

Involving only one direct function

04.01.21.0001.01

$$\int \lfloor z \rfloor \, dz == z \lfloor z \rfloor$$

Involving one direct function and elementary functions

Involving power function

04.01.21.0002.01

$$\int z^{\alpha-1} \lfloor z \rfloor \, dz == \frac{z^{\alpha} \lfloor z \rfloor}{\alpha}$$

04.01.21.0003.01

$$\int \frac{\lfloor z \rfloor}{z} \, dz == \log(z) \lfloor z \rfloor$$

Definite integration

For the direct function itself

In the following formulas $a \in \mathbb{R}$.

04.01.21.0004.01

$$\int_0^n \lfloor t \rfloor \, dt == \frac{n(n-1)}{2} \;/;\; n \in \mathbb{N}$$

04.01.21.0005.01

$$\int_0^a \lfloor t \rfloor \, dt == \frac{1}{2} (2a - \lfloor a \rfloor - 1) \lfloor a \rfloor$$

04.01.21.0006.01

$$\int_0^a t^{\alpha-1} \lfloor t \rfloor \, dt = \frac{1}{\alpha} \left(\lfloor a \rfloor a^\alpha - \zeta(-\alpha) + \zeta(-\alpha, \lfloor a \rfloor + 1)\right)$$

04.01.21.0007.01

$$\int_a^\infty t^{\alpha-1} \lfloor t \rfloor \, dt = -\frac{1}{\alpha} \left(\lfloor a \rfloor a^\alpha + \zeta(-\alpha, \lfloor a \rfloor + 1)\right) /; \operatorname{Re}(\alpha) < -1$$

04.01.21.0008.01

$$\int_0^\infty t^{\alpha-1} \lfloor t \rfloor \, dt = -\frac{\zeta(-\alpha)}{\alpha} /; \operatorname{Re}(\alpha) < -1$$

04.01.21.0009.01

$$\int_{-a}^a \lfloor t \rfloor \, dt = -a$$

04.01.21.0010.01

$$\int_0^{a b} \left(\frac{x}{a} - \left\lfloor \frac{x}{a} \right\rfloor - \frac{1}{2}\right) \left(\frac{x}{b} - \left\lfloor \frac{x}{b} \right\rfloor - \frac{1}{2}\right) dx = \frac{a b}{12} \frac{\gcd(a, b)}{\operatorname{lcm}(a, b)}$$

Integral transforms

Fourier exp transforms

04.01.22.0001.01

$$\mathcal{F}_t[\lfloor t \rfloor](z) = -\sqrt{\frac{\pi}{2}} \delta(z) + \frac{i}{\sqrt{2\pi}} \sum_{k=1}^\infty \frac{\delta(2 k \pi - z) - \delta(2 \pi k + z)}{k} - i \sqrt{2\pi} \, \delta'(z)$$

Fourier cos transforms

04.01.22.0002.01

$$\mathcal{F}c_t[\lfloor t \rfloor](z) = -\frac{1}{\sqrt{2\pi} z} \cot\left(\frac{z}{2}\right) - \sqrt{\frac{\pi}{2}} \delta(z)$$

Fourier sin transforms

04.01.22.0003.01

$$\mathcal{F}s_t[\lfloor t \rfloor](z) = -\frac{1}{\sqrt{2\pi} z} - \sqrt{2\pi} \, \delta'(z) + \frac{1}{\sqrt{2\pi}} \sum_{k=1}^\infty \frac{\delta(2 k \pi - z) - \delta(2 \pi k + z)}{k}$$

Laplace transforms

04.01.22.0004.01

$$\mathcal{L}_t[\lfloor t \rfloor](z) = \frac{1}{(e^z - 1) z} /; \operatorname{Re}(z) > 0$$

Mellin transforms

04.01.22.0005.01

$$\mathcal{M}_t[\lfloor t \rfloor](z) = -\frac{\zeta(-z)}{z} /; \operatorname{Re}(z) < -1$$

Summation

Finite summation

04.01.23.0001.01

$$\sum_{k=0}^{y}\left\lfloor \frac{k}{y}+x\right\rfloor = \lfloor x\,y+(\lceil y\rceil - y)\lfloor x+1\rfloor\rfloor \;/;\; x\in\mathbb{R}\land y\in\mathbb{R}\land y>0$$

04.01.23.0002.01

$$\sum_{k=1}^{p-1}\left\lfloor \frac{k\,q}{p}\right\rfloor = \frac{1}{2}(p-1)(q-1) \;/;\; p\in\mathbb{N}^+\land q\in\mathbb{N}^+\land \gcd(p,q)=1$$

04.01.23.0003.01

$$\sum_{k=1}^{n-1}\left(\frac{k}{n}-\left\lfloor\frac{k}{n}\right\rfloor-\frac{1}{2}\right)\left(\frac{k\,m}{n}-\left\lfloor\frac{m\,k}{n}\right\rfloor-\frac{1}{2}\right) = \frac{1}{4\,n}\sum_{k=1}^{n-1}\cot\left(\frac{k\,\pi}{n}\right)\cot\left(\frac{m\,k\,\pi}{n}\right) \;/;\; m\in\mathbb{N}^+\land n\in\mathbb{N}^+\land \gcd(m,n)=1$$

04.01.23.0004.01

$$\sum_{l=1}^{p-1}\sum_{k=1}^{p-1}\left\lfloor\frac{k\,l}{p}\right\rfloor = \left(\frac{p-1}{2}\right)^2(p-2) \;/;\; p\in\mathbb{P}$$

04.01.23.0006.01

$$\sum_{k=1}^{\frac{p-1}{2}}\left\lfloor\frac{q\,k}{p}\right\rfloor+\sum_{k=1}^{\frac{q-1}{2}}\left\lfloor\frac{p\,k}{q}\right\rfloor = \frac{1}{4}(p-1)(q-1) \;/;\; p\in\mathbb{N}^+\land q\in\mathbb{N}^+\land \gcd(p,q)=1$$

04.01.23.0007.01

$$(x-1)\sum_{k=1}^{p-1}x^{k-1}\,y^{\left\lfloor\frac{k\,q}{p}\right\rfloor}+(y-1)\sum_{k=1}^{q-1}y^{k-1}\,x^{\left\lfloor\frac{k\,p}{q}\right\rfloor} = y^{q-1}\,x^{p-1}-1 \;/;\; p\in\mathbb{N}^+\land q\in\mathbb{N}^+\land \gcd(p,q)=1$$

04.01.23.0008.01

$$(x-1)\sum_{k=1}^{\frac{p-1}{2}}x^{k-1}\,y^{\left\lfloor\frac{k\,q}{p}\right\rfloor}+(y-1)\sum_{k=1}^{\frac{q-1}{2}}y^{k-1}\,x^{\left\lfloor\frac{k\,p}{q}\right\rfloor} = y^{\frac{q-1}{2}}\,x^{\frac{p-1}{2}}-1 \;/;\; p\in\mathbb{N}^+\land q\in\mathbb{N}^+\land \gcd(p,q)=1$$

Infinite summation

04.01.23.0005.01

$$\sum_{n=1}^{\infty}\frac{\left\lfloor\frac{n}{m}\right\rfloor}{k^n} = \frac{k}{(k-1)(k^m-1)} \;/;\; k\in\mathbb{N}^+\land m\in\mathbb{N}^+$$

Representations through equivalent functions

With related functions

With Round

For real arguments

04.01.27.0011.01

$$\lfloor x \rfloor = \left\lceil x - \frac{1}{2} \right\rceil /; x \in \mathbb{R} \bigwedge \frac{x+1}{2} \notin \mathbb{Z}$$

04.01.27.0012.01

$$\lfloor x \rfloor = \left\lceil x - \frac{1}{2} \right\rceil + 1 /; \frac{x+1}{2} \in \mathbb{Z}$$

04.01.27.0013.01

$$\lfloor x \rfloor = \left\lceil x - \frac{1}{2} \right\rceil + \chi_{\mathbb{Z}}\left(\frac{x+1}{2}\right) /; x \in \mathbb{R}$$

For complex arguments

04.01.27.0001.01

$$\lfloor z \rfloor = \left\lceil z - \frac{1+i}{2} \right\rceil + \chi_{\mathbb{Z}}\left(\frac{\mathrm{Re}(z)+1}{2}\right) + i\,\chi_{\mathbb{Z}}\left(\frac{\mathrm{Im}(z)+1}{2}\right)$$

With Ceiling

For real arguments

04.01.27.0014.01

$$\lfloor x \rfloor = \lceil x \rceil - 1 /; x \in \mathbb{R} \bigwedge x \notin \mathbb{Z}$$

04.01.27.0015.01

$$\lfloor x \rfloor = \lceil x \rceil /; x \in \mathbb{Z}$$

04.01.27.0016.01

$$\lfloor x \rfloor = \lceil x \rceil + \theta(\chi_{\mathbb{Z}}(x) - 1) - 1$$

For complex arguments

04.01.27.0017.01

$$\lfloor z \rfloor = \lceil z \rceil - 1 - i /; \mathrm{Re}(z) \notin \mathbb{Z} \bigwedge \mathrm{Im}(z) \notin \mathbb{Z}$$

04.01.27.0018.01

$$\lfloor z \rfloor = \lceil z \rceil - 1 /; \mathrm{Re}(z) \notin \mathbb{Z} \bigwedge \mathrm{Im}(z) \in \mathbb{Z}$$

04.01.27.0019.01

$$\lfloor z \rfloor = \lceil z \rceil - i /; \mathrm{Re}(z) \in \mathbb{Z} \bigwedge \mathrm{Im}(z) \notin \mathbb{Z}$$

04.01.27.0020.01

$$\lfloor z \rfloor = \lceil z \rceil /; \mathrm{Re}(z) \in \mathbb{Z} \land \mathrm{Im}(z) \in \mathbb{Z}$$

04.01.27.0003.01

$$\lfloor z \rfloor = \lceil z \rceil + \theta(\chi_{\mathbb{Z}}(\mathrm{Re}(z)) - 1) - i\,\theta(-\chi_{\mathbb{Z}}(\mathrm{Im}(z))) - 1$$

04.01.27.0002.01

$$\lfloor z \rfloor = -\lceil -z \rceil$$

With IntegerPart

For real arguments

04.01.27.0021.01

$\lfloor x \rfloor = \text{int}(x) \;/;\; x \in \mathbb{R} \land x > 0 \lor x \in \mathbb{Z}$

04.01.27.0022.01

$\lfloor x \rfloor = \text{int}(x) - 1 \;/;\; x \in \mathbb{R} \bigwedge x < 0 \bigwedge x \notin \mathbb{Z}$

04.01.27.0023.01

$\lfloor x \rfloor = \text{int}(x) - 1 + \text{sgn}(\chi_{\mathbb{Z}}(x) + \theta(x)) \;/;\; x \in \mathbb{R}$

For complex arguments

04.01.27.0004.01

$\lfloor z \rfloor = \text{int}(z) \;/;\; \text{Re}(z) \geq 0 \land \text{Im}(z) \geq 0 \lor z \in \mathbb{Z} \lor i\, z \in \mathbb{Z}$

04.01.27.0024.01

$\lfloor z \rfloor = \text{int}(z) - 1 \;/;\; z \in \mathbb{R} \bigwedge z < 0 \bigwedge z \notin \mathbb{Z} \bigvee \text{Re}(z) < 0 \land \text{Im}(z) > 0$

04.01.27.0025.01

$\lfloor z \rfloor = \text{int}(z) - i \;/;\; i\, z \in \mathbb{R} \bigwedge i\, z > 0 \bigwedge i\, z \notin \mathbb{Z} \bigvee \text{Re}(z) > 0 \land \text{Im}(z) < 0$

04.01.27.0026.01

$\lfloor z \rfloor = \text{int}(z) - 1 - i \;/;\; \text{Re}(z) < 0 \land \text{Im}(z) < 0$

04.01.27.0005.01

$\lfloor z \rfloor = \text{int}(z) - 1 - i + \text{sgn}(\chi_{\mathbb{Z}}(\text{Re}(z)) + \theta(\text{Re}(z))) + i\, \text{sgn}(\chi_{\mathbb{Z}}(\text{Im}(z)) + \theta(\text{Im}(z)))$

With FractionalPart

For real arguments

04.01.27.0027.01

$\lfloor x \rfloor = x - \text{frac}(x) \;/;\; x \in \mathbb{R} \land x > 0 \lor x \in \mathbb{Z}$

04.01.27.0028.01

$\lfloor x \rfloor = x - \text{frac}(x) - 1 \;/;\; x \in \mathbb{R} \bigwedge x < 0 \bigwedge x \notin \mathbb{Z}$

04.01.27.0029.01

$\lfloor x \rfloor = x - \text{frac}(x) - 1 + \text{sgn}(\chi_{\mathbb{Z}}(x) + \theta(x)) \;/;\; x \in \mathbb{R}$

For complex arguments

04.01.27.0006.01

$\lfloor z \rfloor = z - \text{frac}(z) \;/;\; \text{Re}(z) \geq 0 \land \text{Im}(z) \geq 0 \lor z \in \mathbb{Z} \lor i\, z \in \mathbb{Z}$

04.01.27.0030.01

$\lfloor z \rfloor = z - \text{frac}(z) - 1 \;/;\; z \in \mathbb{R} \bigwedge z < 0 \bigwedge z \notin \mathbb{Z} \bigvee \text{Re}(z) < 0 \land \text{Im}(z) > 0$

04.01.27.0031.01

$\lfloor z \rfloor = z - \text{frac}(z) - i \;/;\; i\, z \in \mathbb{R} \bigwedge i\, z > 0 \bigwedge i\, z \notin \mathbb{Z} \bigvee \text{Re}(z) > 0 \land \text{Im}(z) < 0$

04.01.27.0032.01

$\lfloor z \rfloor = z - \text{frac}(z) - 1 - i \;/;\; \text{Re}(z) < 0 \land \text{Im}(z) < 0$

04.01.27.0007.01

$\lfloor z \rfloor = z - \text{frac}(z) - 1 - i + \text{sgn}(\chi_{\mathbb{Z}}(\text{Re}(z)) + \theta(\text{Re}(z))) + i\, \text{sgn}(\chi_{\mathbb{Z}}(\text{Im}(z)) + \theta(\text{Im}(z)))$

With Mod

04.01.27.0008.01

$$\lfloor z \rfloor = z - z \bmod 1$$

With Quotient

04.01.27.0009.01

$$\lfloor z \rfloor = \text{quotient}(z, 1)$$

With elementary functions

04.01.27.0010.01

$$\lfloor z \rfloor = z + \frac{\tan^{-1}(\cot(\pi z))}{\pi} - \frac{1}{2} \;/;\; z \in \mathbb{R} \bigwedge z \notin \mathbb{Z}$$

Zeros

04.01.30.0001.01

$$\lfloor z \rfloor = 0 \;/;\; 0 \leq \text{Re}(z) < 1 \wedge 0 \leq \text{Im}(z) < 1$$

Theorems

The Arnold cat map

$\{x, y\} \Longrightarrow \{x + y - \lfloor x + y \rfloor, x + 2y - \lfloor x + 2y \rfloor\}$.

History

– C.F. Gauss (1808)
– J. Liouville (1838)
– K.E. Iverson (1962) suggested the notation $\lfloor z \rfloor$

The function $\lfloor z \rfloor$ is encountered often in mathematics and the natural sciences.

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