Hypergeometric1F0

View the online version at
functions.wolfram.com
Download the
PDF File

Notations

Traditional name

Generalized hypergeometric function ${}_1F_0$

Traditional notation

${}_1F_0(a;\,;\,z)$

Mathematica StandardForm notation

```
HypergeometricPFQ[{a}, {}, z]
```

Primary definition

07.19.02.0001.01

$${}_1F_0(a;\,;\,z) = \sum_{k=0}^{\infty} \frac{(a)_k\, z^k}{k!} \;/;\; |z| < 1$$

07.19.02.0002.01

$${}_1F_0(a;\,;\,z) = (1-z)^{-a}$$

Integral representations

Contour integral representations

07.19.07.0001.01

$${}_1F_0(a;\,;\,z) = \frac{1}{2\,\pi\,i\,\Gamma(a)} \int_{\gamma-i\,\infty}^{\gamma+i\,\infty} \Gamma(s)\,\Gamma(a-s)\,(-z)^{-s}\,ds \;/;\; |\arg(-z)| < \pi \bigwedge 0 < \gamma < \mathrm{Re}(a)$$

Identities

Functional identities

07.19.17.0001.01

$${}_1F_0(a;\,;\,z) = (-z)^{-a}\,{}_1F_0\!\left(a;\,;\,\frac{1}{z}\right) /;\; z \notin (0,\,1)$$

Complex characteristics

Theorems

Gelfond-Schneider theorem

The number α^{β}, where α, β are algebraic numbers, $\alpha \neq 0, 1, \beta \notin \mathbb{Q}$, is transcendental.

Mellin transformation and Parseval relation

$$\hat{f}(s) = \int_0^{\infty} x^{s-1} f(x)\, dx \Leftrightarrow f(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \hat{f}(s)\, x^{-s}\, ds; \int_0^{\infty} f_1\left(\frac{x}{t}\right) f_2(t) \frac{dt}{t} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \hat{f}_1(s)\, \hat{f}_2(s)\, x^{-s}\, ds.$$

Hilbert transformation

$$\hat{f}(y) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{x-y}\, dx \Leftrightarrow f(x) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{f}(y)}{y-x}\, dy.$$

The eigenfunctions of the Hilbert transformation are given by the following:

$$\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{f(x, n)}{x-y}\, dx = \operatorname{sgn}(n)\, f(y, n) \;/;\; f(x, n) = \frac{(1 + i x)^n}{(1 - i x)^{n+1}} \bigwedge n \in \mathbb{Z}$$

Zipf's law

The frequency f versus the rank r in a text has the form $f = r^{-\beta}$, where $\beta \approx 0.6$ for natural languages.

The quantum mechanical density of states of a d-dimensional lattice

The quantum mechanical density of states $\mathcal{D}(\varepsilon)$ of a d-dimensional lattice exhibits van Hove singularities of the form $\mathcal{D}(\varepsilon) \propto \varepsilon^{\frac{d-1}{2}}$, where ε is the energy.

Copyright