

Hypergeometric1F0

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Notations

Traditional name

Generalized hypergeometric function ${}_1F_0$

Traditional notation

${}_1F_0(a; ; z)$

Mathematica StandardForm notation

HypergeometricPFQ[{a}, {}, z]

Primary definition

07.19.02.0001.01

$${}_1F_0(a; ; z) = \sum_{k=0}^{\infty} \frac{(a)_k z^k}{k!} ; |z| < 1$$

07.19.02.0002.01

$${}_1F_0(a; ; z) = (1 - z)^{-a}$$

Integral representations

Contour integral representations

07.19.07.0001.01

$${}_1F_0(a; ; z) = \frac{1}{2\pi i \Gamma(a)} \int_{\gamma-i\infty}^{\gamma+i\infty} \Gamma(s) \Gamma(a-s) (-z)^{-s} ds ; |\arg(-z)| < \pi \wedge 0 < \gamma < \operatorname{Re}(a)$$

Identities

Functional identities

07.19.17.0001.01

$${}_1F_0(a; ; z) = (-z)^{-a} {}_1F_0\left(a; ; \frac{1}{z}\right) ; z \notin (0, 1)$$

Complex characteristics

Theorems

Gelfond-Schneider theorem

The number α^β , where α, β are algebraic numbers, $\alpha \neq 0, 1, \beta \notin \mathbb{Q}$, is transcendental.

Mellin transformation and Parseval relation

$$\hat{f}(s) = \int_0^{\infty} x^{s-1} f(x) dx \Leftrightarrow f(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \hat{f}(s) x^{-s} ds; \int_0^{\infty} f_1\left(\frac{x}{t}\right) f_2(t) \frac{dt}{t} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \hat{f}_1(s) \hat{f}_2(s) x^{-s} ds.$$

Hilbert transformation

$$\hat{f}(y) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{x-y} dx \Leftrightarrow f(x) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{f}(y)}{y-x} dy.$$

The eigenfunctions of the Hilbert transformation are given by the following:

$$\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{f(x, n)}{x-y} dx = \operatorname{sgn}(n) f(y, n) /; f(x, n) = \frac{(1+ix)^n}{(1-ix)^{n+1}} \bigwedge n \in \mathbb{Z}$$

Zipf's law

The frequency f versus the rank r in a text has the form $f = r^{-\beta}$, where $\beta \approx 0.6$ for natural languages.

The quantum mechanical density of states of a d -dimensional lattice

The quantum mechanical density of states $\mathcal{D}(\varepsilon)$ of a d -dimensional lattice exhibits van Hove singularities of the form $\mathcal{D}(\varepsilon) \propto \varepsilon^{\frac{d-1}{2}}$, where ε is the energy.

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