Hypergeometric2F1Regularized

Notations

Traditional name

Regularized hypergeometric function ${}_2\tilde{F}_1$

Traditional notation

${}_2\tilde{F}_1(a, b; c; z)$

Mathematica StandardForm notation

`Hypergeometric2F1Regularized[a, b, c, z]`

Primary definition

Basic definition

07.24.02.0001.01

$${}_2\tilde{F}_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{\Gamma(c+k)\, k!} \;/;\; |z| < 1 \lor |z| = 1 \land \operatorname{Re}(c-a-b) > 0$$

For $|z| < 1$ and generic parameters a, b, c, the regularized hypergeometric function ${}_2\tilde{F}_1(a, b; c; z)$ is defined by the above infinite sum (that is convergent). Outside of the unit circle $|z| < 1$the function ${}_2\tilde{F}_1(a, b; c; z)$ is defined as the analytic continuation with respect to z of this sum, with the parameters a, b, c held fixed.

Complete definition

07.24.02.0004.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{\pi}{\sin(\pi(b-a))} \left(\frac{(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k (a-c+1)_k z^{-k}}{k!\,\Gamma(a-b+k+1)} - \frac{(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(b)_k (b-c+1)_k z^{-k}}{k!\,\Gamma(-a+b+k+1)} \right) /;$$
$$|z| > 1 \land a - b \notin \mathbb{Z}$$

Outside of the unit circle $|z| < 1$the function ${}_2\tilde{F}_1(a, b; c; z)$ can be defined by the above formula. Under the stated restrictions, all occurring sums are convergent.

07.24.02.0005.01

$${}_2\tilde{F}_1(a, b; c; z) = \lim_{\epsilon \to 0} {}_2\tilde{F}_1(a, b+\epsilon; c; z) \;/;\; |z| > 1 \land a - b \in \mathbb{Z}$$

07.24.02.0006.01

$${}_2\tilde{F}_1(a, b; c; z) = \lim_{r \to 1} {}_2\tilde{F}_1(a, b; c; r z) \;/;\; |z| = 1 \land r \in \mathbb{R}$$

For $a == -n$, $c == -m$ being nonpositive integers, the function ${}_2\tilde{F}_1(a, b; c; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a, c can approach nonpositive integers $-n$, $-m$ at different speeds. For nonpositive integers $a == -n$, $c == -m$ one defines:

07.24.02.0002.01

$${}_2\tilde{F}_1(-n, b; -m; z) = \sum_{k=0}^{n-m-1} \frac{(b)_{k+m+1}\,(-n)_{k+m+1}\,z^{k+m+1}}{k!\,(k+m+1)!} \;/;\; m \in \mathbb{N} \land n \in \mathbb{N} \land m < n$$

07.24.02.0007.01

$${}_2\tilde{F}_1(-n, b; -m; z) = 0 \;/;\; m \in \mathbb{N} \land n \in \mathbb{N} \land m \geq n$$

Using the symmetry ${}_2\tilde{F}_1(a, b; c; z) == {}_2\tilde{F}_1(b, a; c; z)$, we have analogeously for $b == -n$, $c == -m$ nonpositive integers:

07.24.02.0003.01

$${}_2\tilde{F}_1(a, -n; -m; z) = \sum_{k=0}^{n-m-1} \frac{(a)_{k+m+1}\,(-n)_{k+m+1}\,z^{k+m+1}}{k!\,(k+m+1)!} \;/;\; m \in \mathbb{N} \land n \in \mathbb{N} \land m < n$$

07.24.02.0008.01

$${}_2\tilde{F}_1(a, -n; -m; z) = 0 \;/;\; m \in \mathbb{N} \land n \in \mathbb{N} \land m \geq n$$

General characteristics

Some abbreviations

07.24.04.0001.01

$$\mathcal{NT}(\{a_1, a_2\}) == \neg\,(-a_1 \in \mathbb{N} \bigvee -a_2 \in \mathbb{N})$$

Domain and analyticity

${}_2\tilde{F}_1(a, b; c; z)$ is an analytical entire function of a, b, c and z which is defined in $\mathbb{C}^4$. For negative integer $a$or b, ${}_2\tilde{F}_1(a, b; c; z)$ degenerates to a polynomial in z of order $-a$or $-b$.

07.24.04.0002.01

$$(a * b * c * z) \longrightarrow {}_2\tilde{F}_1(a, b; c; z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.24.04.0003.02

$${}_2\tilde{F}_1(\overline{a}, \overline{b}; \overline{c}; \overline{z}) == \overline{{}_2\tilde{F}_1(a, b; c; z)} \;/;\; z \notin (1, \infty)$$

Permutation symmetry

07.24.04.0004.01

$${}_2\tilde{F}_1(a, b; c; z) == {}_2\tilde{F}_1(b, a; c; z)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed a, b, c in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \lor -b \in \mathbb{N})$), the function ${}_2\tilde{F}_1(a, b; c; z)$ does not have poles and essential singularities.

07.24.04.0005.01

$$\mathcal{S}ing_z\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\} \;/;\; \mathcal{NT}(\{a, b\})$$

For negative integer a or $b$and fixed c, the function ${}_2\tilde{F}_1(a, b; c; z)$ is a polynomial and has pole of order $-a$or $-b$at $z = \tilde{\infty}$.

07.24.04.0006.01

$$\mathcal{S}ing_z\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\{\tilde{\infty}, -\alpha\}\} \;/;\; \left(-a \in \mathbb{N}^+ \land \alpha == a\right) \lor \left(-b \in \mathbb{N}^+ \land \alpha == b\right) \lor \left(-a \in \mathbb{N}^+ \land -b \in \mathbb{N}^+ \land \alpha == \min(-a, -b)\right)$$

With respect to c

For fixed a, b, z, the function ${}_2\tilde{F}_1(a, b; c; z)$ has only one singular point at $c = \tilde{\infty}$. It is an essential singular point.

07.24.04.0007.01

$$\mathcal{S}ing_c\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\{\tilde{\infty}, \infty\}\}$$

With respect to b

For fixed a, c, z, the function ${}_2\tilde{F}_1(a, b; c; z)$ has only one singular point at $b = \tilde{\infty}$. It is an essential singular point.

07.24.04.0008.01

$$\mathcal{S}ing_b\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\{\tilde{\infty}, \infty\}\}$$

With respect to a

For fixed b, c, z, the function ${}_2\tilde{F}_1(a, b; c; z)$ has only one singular point at $a = \tilde{\infty}$. It is an essential singular point.

07.24.04.0009.01

$$\mathcal{S}ing_a\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\{\tilde{\infty}, \infty\}\}$$

Branch points

With respect to z

For fixed a, b, c in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \lor -b \in \mathbb{N})$), the function ${}_2F_1(a, b; c; z)$ has two branch points: $z == 1$, $z == \tilde{\infty}$.

07.24.04.0010.01

$$\mathcal{BP}_z\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{1, \tilde{\infty}\} \;/;\; \mathcal{NT}(\{a, b\})$$

07.24.04.0011.01

$$\mathcal{R}_z\left({}_2\tilde{F}_1(a, b; c; z), 1\right) == \log \;/;\; c - a - b \in \mathbb{Z} \lor c - a - b \notin \mathbb{Q} \land \mathcal{NT}(\{a, b\})$$

07.24.04.0012.01

$$\mathcal{R}_z\left({}_2\tilde{F}_1(a, b; c; z), 1\right) == s \;/;\; c - a - b == \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) == 1 \bigwedge \mathcal{NT}(\{a, b\})$$

07.24.04.0013.01

$$\mathcal{R}_z\left({}_2\tilde{F}_1(a,b;c;z),\tilde{\infty}\right)=\log\ /;\ a-b\in\mathbb{Z}\bigvee\neg(a\in\mathbb{Q}\bigwedge b\in\mathbb{Q})$$

07.24.04.0014.01

$$\mathcal{R}_z\left({}_2\tilde{F}_1(a,b;c;z),\tilde{\infty}\right)=\operatorname{lcm}(s,u)\ /;$$
$$a=\frac{r}{s}\bigwedge b=\frac{t}{u}\bigwedge\{r,s,t,u\}\in\mathbb{Z}\bigwedge s>1\bigwedge u>1\bigwedge\gcd(r,s)=1\bigwedge\gcd(t,u)=1\bigwedge\mathcal{NT}(\{a,b\})$$

With respect to *c*

For fixed a, b, z, the function ${}_2\tilde{F}_1(a,b;c;z)$ does not have branch points.

07.24.04.0015.01

$$\mathcal{BP}_c\left({}_2\tilde{F}_1(a,b;c;z)\right)=\{\}$$

With respect to *b*

For fixed a, c, z, the function ${}_2\tilde{F}_1(a,b;c;z)$ does not have branch points.

07.24.04.0016.01

$$\mathcal{BP}_b\left({}_2\tilde{F}_1(a,b;c;z)\right)=\{\}$$

With respect to *a*

For fixed b, c, z, the function ${}_2\tilde{F}_1(a,b;c;z)$ does not have branch points.

07.24.04.0017.01

$$\mathcal{BP}_a\left({}_2\tilde{F}_1(a,b;c;z)\right)=\{\}$$

Branch cuts

With respect to *z*

For fixed a, b, c in nonpolynomial cases (when $\neg(-a\in\mathbb{N}\bigvee -b\in\mathbb{N})$), the function ${}_2\tilde{F}_1(a,b;c;z)$ is a single-valued function on the z-plane cut along the interval $(1,\infty)$, where it is continuous from below.

07.24.04.0018.01

$$\mathcal{BC}_z\left({}_2\tilde{F}_1(a,b;c;z)\right)=\{\{(1,\infty),i\}\}\ /;\ \mathcal{NT}(\{a,b\})$$

07.24.04.0019.01

$$\lim_{\epsilon\to+0}{}_2\tilde{F}_1(a,b;c;x-i\epsilon)={}_2\tilde{F}_1(a,b;c;x)\ /;\ x>1$$

07.24.04.0020.01

$$\lim_{\epsilon\to+0}{}_2\tilde{F}_1(a,b;c;x+i\epsilon)=\frac{2i\pi e^{i(a+b-c)\pi}}{\Gamma(c-a)\Gamma(c-b)}{}_2\tilde{F}_1(a,b;a+b-c+1;1-x)+e^{2i(a+b-c)\pi}{}_2\tilde{F}_1(a,b;c;x)\ /;\ x>1$$

With respect to *c*

For fixed a, b, z, the function ${}_2\tilde{F}_1(a,b;c;z)$ does not have branch cuts.

07.24.04.0021.01

$$\mathcal{BC}_c\left({}_2\tilde{F}_1(a,b;c;z)\right)=\{\}$$

With respect to *b*

For fixed a, c, z, the function ${}_2\tilde{F}_1(a, b; c; z)$ does not have branch cuts.

07.24.04.0022.01

$$\mathcal{BC}_b\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\}$$

With respect to a

For fixed b, c, z, the function ${}_2\tilde{F}_1(a, b; c; z)$ does not have branch cuts.

07.24.04.0023.01

$$\mathcal{BC}_a\left({}_2\tilde{F}_1(a, b; c; z)\right) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

07.24.06.0037.01

$$\begin{aligned}
{}_2\tilde{F}_1(a, b; c; z) \propto & \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \Bigg(G_{2,2}^{2,2}\left(1-z_0 \,\middle|\, \begin{matrix} 1-a, 1-b \\ 0, -a-b+c \end{matrix}\right) \left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} - \\
& 2\, e^{i(c-a-b)\pi\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor}\, i\,\pi \left\lfloor \frac{\arg(1-z_0)+\pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor \Gamma(a)\,\Gamma(b)\, {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z_0) + \\
& \Bigg(G_{2,2}^{2,2}\left(1-z_0 \,\middle|\, \begin{matrix} -a, -b \\ 0, -a-b+c-1 \end{matrix}\right) \left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} + \\
& 2\, e^{i(c-a-b)\pi\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor}\, i\,\pi \left\lfloor \frac{\arg(1-z_0)+\pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor \Gamma(a+1)\,\Gamma(b+1)\, {}_2\tilde{F}_1(a+1, b+1; a+b-c+2; 1-z_0) \Bigg) \\
& (z-z_0) + \frac{1}{2} \Bigg(G_{2,2}^{2,2}\left(1-z_0 \,\middle|\, \begin{matrix} -a-1, -b-1 \\ 0, -a-b+c-2 \end{matrix}\right) \left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} - \\
& 2\, i\, e^{i(c-a-b)\pi\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor}\, \pi \left\lfloor \frac{\arg(1-z_0)+\pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor \Gamma(a+2)\,\Gamma(b+2) \\
& {}_2\tilde{F}_1(a+2, b+2; a+b-c+3; 1-z_0) \Bigg) (z-z_0)^2 + \ldots \Bigg) /; (z \to z_0)
\end{aligned}$$

07.24.06.0038.01

$$
{}_2\tilde{F}_1(a,b;c;z)\propto\frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\Bigg(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}1-a,1-b\\0,-a-b+c\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}-2\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z_0)+\Bigg(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}-a,-b\\0,-a-b+c-1\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}+2\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a+1)\,\Gamma(b+1)\,{}_2\tilde{F}_1(a+1,b+1;a+b-c+2;1-z_0)\Bigg)(z-z_0)+\frac{1}{2}\Bigg(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}-a-1,-b-1\\0,-a-b+c-2\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}-2\,i\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a+2)\,\Gamma(b+2)\,{}_2\tilde{F}_1(a+2,b+2;a+b-c+3;1-z_0)\Bigg)(z-z_0)^2+O\big((z-z_0)^3\big)\Bigg)
$$

07.24.06.0039.01

$$
{}_2\tilde{F}_1(a,b;c;z)=\frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{1}{k!}\Bigg(\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}-a-k+1,-b-k+1\\0,-a-b+c-k\end{matrix}\right)-2\,\pi\,i\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor(-1)^k\,\Gamma(a+k)\,\Gamma(b+k)\,{}_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-z_0)\Bigg)(z-z_0)^k
$$

07.24.06.0040.01

$$ {}_2\tilde{F}_1(a, b; c; z) == \frac{\pi}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Bigg(\Gamma(a+k)\,\Gamma(b+k) \Bigg(\csc((c-a-b)\,\pi) \left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} - 2\,i\,e^{i\,(c-a-b)\,\pi \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(1-z_0)+\pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor \Bigg) {}_2\tilde{F}_1(a+k, b+k; a+b-c+k+1; 1-z_0) - \Gamma(c-a)\,\Gamma(c-b)\,(1-z_0)^{c-a-b-k} \csc((c-a-b)\,\pi) \left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} {}_2\tilde{F}_1(c-a, c-b; -a-b+c-k+1; 1-z_0)\Bigg) (z-z_0)^k \;/;\; c-a-b \notin \mathbb{Z} $$

07.24.06.0041.01

$$ {}_2\tilde{F}_1(a, b; c; z) \propto \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \Bigg(G_{2,2}^{2,2}\!\left(1-z_0 \,\middle|\, \begin{matrix} 1-a,\, 1-b \\ 0,\, -a-b+c \end{matrix}\right) \left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{(c-a-b)\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} - 2\,e^{i\,(c-a-b)\,\pi \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor}\, i\,\pi \left\lfloor \frac{\arg(1-z_0)+\pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor \Gamma(a)\,\Gamma(b)\, {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z_0) + O(z-z_0) \Bigg) $$

Expansions on branch cuts

For the function itself

07.24.06.0042.01

$$ {}_2\tilde{F}_1(a, b; c; z) \propto \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \Bigg(e^{i\,(-a-b+c)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(e^{i\,(-a-b+c)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} 1-a,\, 1-b \\ 0,\, -a-b+c \end{matrix}\right) - 2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a)\,\Gamma(b)\, {}_2\tilde{F}_1(a, b; a+b-c+1; 1-x) \right) + e^{i\,(-a-b+c)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a+1)\,\Gamma(b+1)\, {}_2\tilde{F}_1(a+1, b+1; a+b-c+2; 1-x) + e^{i\,(-a-b+c)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} -a,\, -b \\ 0,\, -a-b+c-1 \end{matrix}\right) \right) (z-x) + \frac{1}{2}\, e^{i\,(-a-b+c)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(e^{i\,(-a-b+c)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} -a-1,\, -b-1 \\ 0,\, -a-b+c-2 \end{matrix}\right) - 2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a+2)\, \Gamma(b+2)\, {}_2\tilde{F}_1(a+2, b+2; a+b-c+3; 1-x) \right) (z-x)^2 + \ldots \Bigg) \;/;\; (z \to x) \land x \in \mathbb{R} \land x > 1 $$

07.24.06.0043.01

$$ {}_2\tilde{F}_1(a,b;c;z) \propto \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \Bigg(e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \Bigg(e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} 1-a,\,1-b \\ 0,\,-a-b+c \end{matrix}\right) - 2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-x)\Bigg) + e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor}\Bigg(2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a+1)\,\Gamma(b+1)\,{}_2\tilde{F}_1(a+1,b+1;a+b-c+2;1-x) + e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} -a,\,-b \\ 0,\,-a-b+c-1 \end{matrix}\right)\Bigg)(z-x) + \frac{1}{2} e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor}\Bigg(e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} -a-1,\,-b-1 \\ 0,\,-a-b+c-2 \end{matrix}\right) - 2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a+2)\,\Gamma(b+2)\,{}_2\tilde{F}_1(a+2,b+2;a+b-c+3;1-x)\Bigg)(z-x)^2 + O\big((z-x)^3\big)\Bigg) \;/;\; x\in\mathbb{R} \land x>1 $$

07.24.06.0044.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} e^{(c-a-b)\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{1}{k!} \Bigg(e^{i(c-a-b)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} -a-k+1,\,-b-k+1 \\ 0,\,-a-b+c-k \end{matrix}\right) - 2\,\pi\,i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor (-1)^k\,\Gamma(a+k)\,\Gamma(b+k)\,{}_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-x)\Bigg)(z-x)^k \;/;\; x\in\mathbb{R} \land x>1 $$

07.24.06.0045.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\pi}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} e^{2\,i\,(c-a-b)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Bigg((1-x)^{c-a-b-k} \csc((a+b-c)\pi)\,\Gamma(c-a)\,\Gamma(c-b)\,{}_2\tilde{F}_1(c-a,c-b;c-a-b-k+1;1-x) - \left(\csc((a+b-c)\pi) + 2\,e^{i(a+b-c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor}\, i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor\right) \Gamma(a+k)\,\Gamma(b+k)\,{}_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-x)\Bigg)(z-x)^k \;/;\; c-a-b\notin\mathbb{Z} \land x\in\mathbb{R} \land x>1 $$

07.24.06.0046.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\pi^2 \csc(c\,\pi)\,(1-x)^{c-a-b+c}}{\Gamma(a)\,\Gamma(b)} e^{i(c-a-b)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{(1-x)^{-k}}{k!} \Bigg(\frac{2\,i\,x^{-c-k+1}(1-x)^{a+b-c+k}}{\Gamma(c-a)\,\Gamma(c-b)} \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor {}_2\tilde{F}_1(a-c+1,b-c+1;-c-k+2;x) + \left(\frac{\csc((a+b-c)\pi)}{\Gamma(1-a-k)\,\Gamma(1-b-k)} e^{i(-a-b+c)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} - \frac{\sin((c-a)\pi)\,\sin((c-b)\pi)\,\Gamma(a+k)\,\Gamma(b+k)}{\pi^2} \left(e^{i(c-a-b)\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \csc((a+b-c)\pi) + 2\,i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \right)\right) {}_2\tilde{F}_1(c-a,c-b;c+k;x)\Bigg)(z-x)^k \;/;\; c-a-b\notin\mathbb{Z} \land c\notin\mathbb{Z} \land x\in\mathbb{R} \land x>1 $$

07.24.06.0047.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) \propto \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, e^{i\,(c-a-b)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(e^{i\,(c-a-b)\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} G_{2,2}^{2,2}\!\left(1-x \,\middle|\, \begin{matrix} 1-a,\,1-b \\ 0,\,-a-b+c \end{matrix}\right) - 2\,i\,\pi \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor \Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,\,b;\,a+b-c+1;\,1-x) \right) + O(z-x) \,/;\, x \in \mathbb{R} \land x > 1 $$

Expansions at $z == 0$

For the function itself

General case

07.24.06.0001.02

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) \propto \frac{1}{\Gamma(c)} \left(1 + \frac{a\,b}{c}\,z + \frac{a\,(1+a)\,b\,(1+b)}{2\,c\,(1+c)}\,z^2 + \ldots\right) /;\, (z \to 0) $$

07.24.06.0048.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) \propto \frac{1}{\Gamma(c)} \left(1 + \frac{a\,b}{c}\,z + \frac{a\,(1+a)\,b\,(1+b)}{2\,c\,(1+c)}\,z^2 + O\!\left(z^3\right)\right) $$

07.24.06.0002.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) == \sum_{k=0}^{\infty} \frac{(a)_k\,(b)_k\,z^k}{\Gamma(c+k)\,k!} \,/;\, |z| < 1 $$

07.24.06.0003.02

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) \propto \frac{1}{\Gamma(c)}\,(1 + O(z)) $$

07.24.06.0049.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) == F_\infty(z,\,a,\,b,\,c) \,/; \left(\left(F_n(z,\,a,\,b,\,c) == \sum_{k=0}^{n} \frac{(a)_k\,(b)_k\,z^k}{\Gamma(c+k)\,k!} == {}_2\tilde{F}_1(a,\,b;\,c;\,z) - z^{n+1}\,(a)_{n+1}\,(b)_{n+1}\,{}_3\tilde{F}_2(1,\,a+n+1,\,b+n+1;\,n+2,\,c+n+1;\,z)\right) \bigwedge n \in \mathbb{N}\right) $$

Summed form of the truncated series expansion.

Special cases

07.24.06.0050.01

$$ {}_2\tilde{F}_1(a,\,b;\,-n;\,z) \propto \frac{(a)_{n+1}\,(b)_{n+1}}{(n+1)!}\,z^{n+1} \left(1 + \frac{(a+n+1)\,(b+n+1)\,z}{n+2} + \frac{(a+n+1)\,(a+n+2)\,(b+n+1)\,(b+n+2)\,z^2}{2\,(n+2)\,(n+3)} + \ldots\right) /;\, (z \to 0) \land n \in \mathbb{N} $$

07.24.06.0051.01

$$ {}_2\tilde{F}_1(a,\,b;\,-n;\,z) \propto \frac{(a)_{n+1}\,(b)_{n+1}}{(n+1)!}\,z^{n+1} \left(1 + \frac{(a+n+1)\,(b+n+1)\,z}{n+2} + \frac{(a+n+1)\,(a+n+2)\,(b+n+1)\,(b+n+2)\,z^2}{2\,(n+2)\,(n+3)} + O\!\left(z^3\right)\right) /;\, n \in \mathbb{N} $$

07.24.06.0052.01

$${}_2\tilde{F}_1(a,\,b;\,-n;\,z) = \frac{(a)_{n+1}\,(b)_{n+1}}{(n+1)!}\,z^{n+1}\sum_{k=0}^{\infty}\frac{(a+n+1)_k\,(b+n+1)_k\,z^k}{(n+2)_k\,k!}\;/;\;|z|<1\land n\in\mathbb{N}$$

07.24.06.0053.01

$${}_2\tilde{F}_1(a,\,b;\,-n;\,z) = (a)_{n+1}\,(b)_{n+1}\,z^{n+1}\,{}_2\tilde{F}_1(a+n+1,\,b+n+1;\,n+2;\,z)\;/;\;n\in\mathbb{N}$$

07.24.06.0054.01

$${}_2\tilde{F}_1(a,\,b;\,-n;\,z) = \frac{(a)_{n+1}\,(b)_{n+1}\,z^{n+1}}{(n+1)!}\,{}_2F_1(a+n+1,\,b+n+1;\,n+2;\,z)\;/;\;n\in\mathbb{N}$$

07.24.06.0055.01

$${}_2\tilde{F}_1(a,\,b;\,-n;\,z) \propto \frac{(a)_{n+1}\,(b)_{n+1}}{(n+1)!}\,z^{n+1}(1+O(z))\;/;\;n\in\mathbb{N}$$

07.24.06.0056.01

$${}_2\tilde{F}_1(a,\,b;\,-n;\,z) = F_\infty(z,\,a,\,b,\,n)\;/;$$
$$\left(\left(F_m(z,\,a,\,b,\,n) = \frac{(a)_{n+1}\,(b)_{n+1}\,z^{n+1}}{(n+1)!}\sum_{k=0}^{m}\frac{(a+n+1)_k\,(b+n+1)_k\,z^k}{(n+2)_k\,k!} = z^{n+1}\,(a)_{n+1}\,(b)_{n+1}\,{}_2\tilde{F}_1(a+n+1,\,b+n+1;\,n+2;\,z) - z^{m+n+2}\,(a)_{m+n+2}\,(b)_{m+n+2}\,{}_3\tilde{F}_2(1,\,a+m+n+2,\,b+m+n+2;\,m+2,\,m+n+3;\,z)\right)\bigwedge m\in\mathbb{N}\right)\bigwedge n\in\mathbb{N}$$

Summed form of the truncated series expansion.

Generic formulas for main term

07.24.06.0057.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) \propto \begin{cases} 0 & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a\le 0)\bigwedge(-b\in\mathbb{N}\bigwedge c-b\le 0)) \\ \frac{(a)_{1-c}\,(b)_{1-c}\,z^{1-c}}{(1-c)!} & -c\in\mathbb{N} \\ \frac{1}{\Gamma(c)} & \text{True} \end{cases}\;/;\;(|z|\to 0)$$

Expansions at $z = 1$

For the function itself

General case

07.24.06.0004.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \mathcal{A}_{\tilde{F}}\left(\begin{matrix} a,\,b; \\ c; \end{matrix}\;\{z,\,1,\,\infty\}\right)$$

07.24.06.0005.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \mathcal{A}_{\tilde{F}}^{(\text{power})}\left(\begin{matrix} a,\,b; \\ c; \end{matrix}\;\{z,\,1,\,\infty\}\right)$$

07.24.06.0006.02

$$ {}_2\tilde{F}_1(a,b;c;z)\propto \frac{\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\left(1+\frac{(a-c)(-b+c)(z-1)}{1-a-b+c}+\frac{(-a+c)(1-a+c)(-b+c)(1-b+c)(z-1)^2}{2(1-a-b+c)(2-a-b+c)}+\ldots\right)+ \frac{\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\left(1-\frac{a\,b\,(z-1)}{1+a+b-c}+\frac{a(1+a)\,b\,(1+b)(z-1)^2}{2(1+a+b-c)(2+a+b-c)}+\ldots\right)/;\ (z\to 1)\bigwedge c-a-b\notin\mathbb{Z} $$

07.24.06.0058.01

$$ {}_2F_1(a,b;c;z)\propto \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\left(1+\frac{(a-c)(-b+c)(z-1)}{1-a-b+c}+\frac{(-a+c)(1-a+c)(-b+c)(1-b+c)(z-1)^2}{2(1-a-b+c)(2-a-b+c)}+O\big((z-1)^3\big)\right)+ \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\left(1-\frac{a\,b\,(z-1)}{1+a+b-c}+\frac{a(1+a)\,b\,(1+b)(z-1)^2}{2(1+a+b-c)(2+a+b-c)}+O\big((z-1)^3\big)\right)/;\ (z\to 1)\bigwedge c-a-b\notin\mathbb{Z} $$

07.24.06.0007.01

$$ {}_2\tilde{F}_1(a,b;c;z)=\frac{\pi}{\sin(\pi(c-a-b))}\left(\frac{1}{\Gamma(c-a)\,\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k\,(1-z)^k}{\Gamma(a+b-c+k+1)\,k!}-\frac{(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)}\sum_{k=0}^{\infty}\frac{(c-a)_k\,(c-b)_k\,(1-z)^k}{\Gamma(-a-b+c+k+1)\,k!}\right)/;\ |z-1|<1\bigwedge c-a-b\notin\mathbb{Z} $$

07.24.06.0008.01

$$ {}_2\tilde{F}_1(a,b;c;z)=\frac{\pi}{\sin(\pi(c-a-b))}\left(\frac{1}{\Gamma(c-a)\,\Gamma(c-b)}\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z)-\frac{1}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\,{}_2\tilde{F}_1(c-a,c-b;c-a-b+1;1-z)\right)/;\ c-a-b\notin\mathbb{Z} $$

07.24.06.0009.01

$$ {}_2\tilde{F}_1(a,b;c;z)\propto \frac{\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}(1+O(z-1))+\frac{\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}(1+O(z-1))/;\ (z\to 1)\bigwedge c-a-b\notin\mathbb{Z} $$

07.24.06.0059.01

$$ {}_2\tilde{F}_1(a,b;c;z)=F_\infty(z,a,b,c)/; \left(\left(F_n(z,a,b,c)=\frac{\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\sum_{k=0}^{n}\frac{(c-a)_k\,(c-b)_k}{(c-a-b+1)_k\,k!}(1-z)^k+\frac{\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\sum_{k=0}^{n}\frac{(a)_k\,(b)_k}{(a+b-c+1)_k\,k!}(1-z)^k = {}_2\tilde{F}_1(a,b;c;z)-\frac{\Gamma(c-a-b)\,(a)_{n+1}\,(b)_{n+1}}{(n+1)!\,\Gamma(c-a)\,\Gamma(c-b)\,(a+b-c+1)_{n+1}}(1-z)^{n+1}\,{}_3F_2(1,a+n+1,b+n+1;n+2,a+b-c+n+2;1-z)-\frac{\Gamma(a+b-c)\,(c-a)_{n+1}\,(c-b)_{n+1}}{(n+1)!\,\Gamma(a)\,\Gamma(b)\,(c-a-b+1)_{n+1}}(1-z)^{c-a-b+n+1}\,{}_3F_2(1,-a+c+n+1,-b+c+n+1;n+2,c-a-b+n+2;1-z)\right)\bigwedge n\in\mathbb{N}\right)\bigwedge c-a-b\notin\mathbb{Z} $$

Summed form of the truncated series expansion.

Logarithmic cases

07.24.06.0010.01

$${}_2\tilde{F}_1(a,\,b;\,a+b-n;\,z) = \frac{(n-1)!}{\Gamma(a)\,\Gamma(b)}\,(1-z)^{-n}\sum_{k=0}^{n-1}\frac{(a-n)_k\,(b-n)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{(-1)^n}{\Gamma(a-n)\,\Gamma(b-n)}\sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k}{k!\,(k+n)!}\,(-\log(1-z)+\psi(k+1)+\psi(k+n+1)-\psi(a+k)-\psi(b+k))\,(1-z)^k\;/;\;n\in\mathbb{N}^+\bigwedge|1-z|<1$$

07.24.06.0011.01

$${}_2\tilde{F}_1(a,\,b;\,a+b-n;\,z) = \frac{(n-1)!}{\Gamma(a)\,\Gamma(b)}\,(1-z)^{-n}\sum_{k=0}^{n-1}\frac{(a-n)_k\,(b-n)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{(-1)^{n-1}}{\Gamma(a-n)\,\Gamma(b-n)\,n!}\log(1-z)\,{}_2F_1(a,\,b;\,n+1;\,1-z) + \frac{(-1)^n}{\Gamma(a-n)\,\Gamma(b-n)}\sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k}{k!\,(k+n)!}\,(\psi(k+1)+\psi(k+n+1)-\psi(a+k)-\psi(b+k))\,(1-z)^k\;/;\;n\in\mathbb{N}^+\bigwedge|1-z|<1$$

07.24.06.0012.01

$${}_2\tilde{F}_1(a,\,b;\,a+b-n;\,z) \propto \frac{(n-1)!\,(1-z)^{-n}}{\Gamma(a)\,\Gamma(b)}\,(1+O(z-1)) + \frac{(-1)^{n-1}}{n!\,\Gamma(a-n)\,\Gamma(b-n)}\,(\log(1-z)+\psi(a)+\psi(b)-\psi(n+1)+\gamma)\,(1+O(z-1))\;/;\;(z\to 1)\bigwedge n\in\mathbb{N}^+$$

07.24.06.0013.01

$${}_2\tilde{F}_1(a,\,b;\,a+b;\,z) = \frac{1}{\Gamma(a)\,\Gamma(b)}\left(\sum_{k=0}^{\infty}\frac{1}{k!^2}\left((a)_k\,(b)_k\,(-\log(1-z)+2\,\psi(k+1)-\psi(a+k)-\psi(b+k))\,(1-z)^k\right)\right)\;/;\;|1-z|<1$$

07.24.06.0014.01

$${}_2\tilde{F}_1(a,\,b;\,a+b;\,z) = \frac{1}{\Gamma(a)\,\Gamma(b)}\left(\sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k\,(2\,\psi(k+1)-\psi(a+k)-\psi(b+k))\,(1-z)^k}{k!^2} - \log(1-z)\,{}_2F_1(a,\,b;\,1;\,1-z)\right)\;/;\;|1-z|<1$$

07.24.06.0015.01

$${}_2\tilde{F}_1(a,\,b;\,a+b;\,z) \propto -\frac{\Gamma(a+b)\,(\log(1-z)+\psi(a)+\psi(b)+2\,\gamma)}{\Gamma(a)\,\Gamma(b)}\,(1+O(z-1))\;/;\;(z\to 1)$$

07.24.06.0016.01

$${}_2\tilde{F}_1(a,\,b;\,a+b+n;\,z) = \frac{(n-1)!}{\Gamma(a+n)\,\Gamma(b+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,(b)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{1}{\Gamma(a)\,\Gamma(b)}\,(z-1)^n\sum_{k=0}^{\infty}\frac{(a+n)_k\,(b+n)_k}{k!\,(k+n)!}\,(-\log(1-z)+\psi\,(k+1)+\psi(k+n+1)-\psi(a+k+n)-\psi(b+k+n))\,(1-z)^k\;/;\;n\in\mathbb{N}^+\bigwedge|1-z|<1$$

07.24.06.0017.01

$${}_2\tilde{F}_1(a,\,b;\,a+b+n;\,z) = \frac{(n-1)!}{\Gamma(a+n)\,\Gamma(b+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,(b)_k\,(1-z)^k}{k!\,(1-n)_k} - \frac{1}{\Gamma(a)\,\Gamma(b)\,n!}\log(1-z)\,(z-1)^n\,{}_2F_1(a+n,\,b+n;\,n+1;\,1-z) + \frac{1}{\Gamma(a)\,\Gamma(b)}\,(z-1)^n\sum_{k=0}^{\infty}\frac{(a+n)_k\,(b+n)_k}{k!\,(k+n)!}\,(\psi(k+1)+\psi(k+n+1)-\psi(a+k+n)-\psi(b+k+n))\,(1-z)^k\;/;\;n\in\mathbb{N}^+\bigwedge|1-z|<1$$

07.24.06.0018.01

$$ {}_2\tilde{F}_1(a,\,b;\,a+b+n;\,z)\propto\frac{(n-1)!}{\Gamma(a+n)\,\Gamma(b+n)}\,(1+O(z-1))-\frac{1}{n!\,\Gamma(a)\,\Gamma(b)}\,(z-1)^n\,(\log(1-z)-\psi(n+1)+\psi(a+n)+\psi(b+n)+\gamma)\,(1+O(z-1))\,/;\,(z\to1)\bigwedge n\in\mathbb{N}^+ $$

07.24.06.0060.01

$$ {}_2\tilde{F}_1(a,\,b;\,a+b-n;\,z)=F_\infty(z,\,a,\,b,\,n)\,/;\,\Bigg(\Bigg(F_m(z,\,a,\,b,\,n)=\frac{(n-1)!\,(1-z)^{-n}}{\Gamma(a)\,\Gamma(b)}\sum_{k=0}^{n-1}\frac{(a-n)_k\,(b-n)_k\,(1-z)^k}{k!\,(1-n)_k}+\frac{(-1)^n}{\Gamma(a-n)\,\Gamma(b-n)}\sum_{k=0}^{m}\frac{(a)_k\,(b)_k}{k!\,(k+n)!}\,(-\log(1-z)+\psi(k+1)+\psi(k+n+1)-\psi(a+k)-\psi(b+k))\,(1-z)^k= {}_2\tilde{F}_1(a,\,b;\,a+b-n;\,z)-\frac{(-1)^n}{\Gamma(a)\,\Gamma(b)\,\Gamma(a-n)\,\Gamma(b-n)}\,G_{4,4}^{2,4}\!\left(1-z\,\middle|\,\begin{matrix}m+1,\,m+1,\,1-a,\,1-b\\ m+1,\,m+1,\,0,\,-n\end{matrix}\right)\Bigg)\bigwedge m\in\mathbb{N}\Bigg)\bigwedge n\in\mathbb{N} $$

Summed form of the truncated series expansion.

07.24.06.0061.01

$$ {}_2\tilde{F}_1(a,\,b;\,a+b+n;\,z)=F_\infty(z,\,a,\,b,\,n)\,/;\,\Bigg(\Bigg(F_m(z,\,a,\,b,\,n)=\frac{1}{\Gamma(a+n)\,\Gamma(b+n)}\sum_{k=0}^{n-1}\frac{\Gamma(n-k)\,(a)_k\,(b)_k}{k!}\,(z-1)^k+\frac{(z-1)^n}{\Gamma(a)\,\Gamma(b)}\sum_{k=0}^{m}\frac{(a+n)_k\,(b+n)_k}{k!\,(k+n)!}\,(-\log(1-z)+\psi(k+1)+\psi(k+n+1)-\psi(a+k+n)-\psi(b+k+n))\,(1-z)^k= {}_2\tilde{F}_1(a,\,b;\,a+b+n;\,z)-\frac{(-1)^n}{\Gamma(a)\,\Gamma(b)\,\Gamma(a+n)\,\Gamma(b+n)}\,G_{4,4}^{2,4}\!\left(1-z\,\middle|\,\begin{matrix}m+n+1,\,m+n+1,\,1-a,\,1-b\\ m+n+1,\,m+n+1,\,0,\,n\end{matrix}\right)\Bigg)\bigwedge m\in\mathbb{N}\Bigg)\bigwedge n\in\mathbb{N} $$

Summed form of the truncated series expansion.

Generic formulas for main term

07.24.06.0062.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z)\propto\begin{cases}0 & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a\le0)\bigwedge(-\mathbb{N}\bigwedge c-b\le0))\\ \frac{(a+b-c-1)!\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)}+\frac{(-1)^{a+b-c-1}\,(\log(1-z)+\psi(a)+\psi(b)-\psi(a+b-c+1)+\gamma)}{(a+b-c)!\,\Gamma(c-a)\,\Gamma(c-b)} & a+b-c\in\mathbb{N}^+\\ -\frac{\log(1-z)+\psi(a)+\psi(b)+2\gamma}{\Gamma(a)\,\Gamma(b)} & c=a+b\\ \frac{(c-a-b-1)!}{\Gamma(c-a)\,\Gamma(c-b)}-\frac{(\log(1-z)+\psi(c-a)-\psi(c-a-b+1)+\psi(c-b)+\gamma)\,(z-1)^{c-a-b}}{(c-a-b)!\,\Gamma(a)\,\Gamma(b)} & c-a-b\in\mathbb{N}^+\\ \frac{\Gamma(a+b-c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)}+\frac{\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)} & \text{True}\end{cases}\;/;\,(z\to1) $$

07.24.06.0063.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z)\propto\begin{cases}0 & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a\le0)\bigwedge(-b\in\mathbb{N}\bigwedge c-b\le0))\\ \frac{\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)} & \operatorname{Re}(c-a-b)>0\\ \frac{\Gamma(a+b-c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)} & \operatorname{Re}(c-a-b)<0\\ -\frac{\log(1-z)}{\Gamma(a)\,\Gamma(b)} & c=a+b\\ \frac{\Gamma(a+b-c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)}+\frac{\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)} & \text{True}\end{cases}\;/;\,(z\to1) $$

Expansions at $z == \infty$

For the function itself

The general formulas

07.24.06.0019.01

$$ {}_2\tilde{F}_1(a, b; c; z) == \mathcal{A}_{\tilde{F}}\left(\begin{matrix} a, b; \\ c; \end{matrix} \; \{z, \tilde{\infty}, \infty\}\right) /; z \notin (0, 1) $$

07.24.06.0020.01

$$ {}_2\tilde{F}_1(a, b; c; z) == \mathcal{A}_{\tilde{F}}^{(\text{power})}\left(\begin{matrix} a, b; \\ c; \end{matrix} \; \{z, \tilde{\infty}, \infty\}\right) /; z \notin (0, 1) $$

Case of simple poles

07.24.06.0021.02

$$ {}_2\tilde{F}_1(a, b; c; z) \propto \frac{\Gamma(b-a)}{\Gamma(b)\,\Gamma(c-a)} (-z)^{-a} \left(1 + \frac{a\,(1+a-c)}{(1+a-b)\,z} + \frac{a\,(1+a)\,(1+a-c)\,(2+a-c)}{2\,(1+a-b)\,(2+a-b)\,z^2} + \ldots\right) + \frac{\Gamma(a-b)}{\Gamma(a)\,\Gamma(c-b)} (-z)^{-b} \left(1 + \frac{b\,(1+b-c)}{(1-a+b)\,z} + \frac{b\,(1+b)\,(1+b-c)\,(2+b-c)}{2\,(1-a+b)\,(2-a+b)\,z^2} + \ldots\right) /; (|z| \to \infty) \bigwedge a-b \notin \mathbb{Z} $$

07.24.06.0022.01

$$ {}_2\tilde{F}_1(a, b; c; z) == \frac{\pi}{\sin(\pi\,(b-a))} \left(\frac{(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k\,(a-c+1)_k\,z^{-k}}{k!\,\Gamma(a-b+k+1)} - \frac{(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(b)_k\,(b-c+1)_k\,z^{-k}}{k!\,\Gamma(-a+b+k+1)}\right) /; |z| > 1 \bigwedge a-b \notin \mathbb{Z} $$

07.24.06.0023.01

$$ {}_2\tilde{F}_1(a, b; c; z) == \frac{\pi}{\sin(\pi\,(b-a))} \left(\frac{1}{\Gamma(b)\,\Gamma(c-a)} (-z)^{-a}\, {}_2\tilde{F}_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) - \frac{1}{\Gamma(a)\,\Gamma(c-b)} (-z)^{-b}\, {}_2\tilde{F}_1\left(b, b-c+1; -a+b+1; \frac{1}{z}\right)\right) /; a-b \notin \mathbb{Z} \bigwedge z \notin (0, 1) $$

07.24.06.0024.01

$$ {}_2\tilde{F}_1(a, b; c; z) \propto \frac{\Gamma(b-a)}{\Gamma(b)\,\Gamma(c-a)} (-z)^{-a} \left(1 + O\left(\frac{1}{z}\right)\right) + \frac{\Gamma(a-b)}{\Gamma(a)\,\Gamma(c-b)} (-z)^{-b} \left(1 + O\left(\frac{1}{z}\right)\right) /; (|z| \to \infty) \bigwedge a \neq b $$

07.24.06.0064.01

$$ {}_2\tilde{F}_1(a, b; c; z) == F_\infty(z, a, b, c) /; \left(\left(F_n(z, a, b, c) == \frac{\Gamma(b-a)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} \sum_{k=0}^{n} \frac{(a)_k\,(a-c+1)_k\,z^{-k}}{k!\,(a-b+1)_k} + \frac{\Gamma(a-b)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} \sum_{k=0}^{n} \frac{(b)_k\,(b-c+1)_k\,z^{-k}}{k!\,(-a+b+1)_k} == {}_2\tilde{F}_1(a, b; c; z) - \frac{\Gamma(b-a)\,(a)_{n+1}\,(a-c+1)_{n+1}\,z^{-n-1}}{(n+1)!\,\Gamma(b)\,\Gamma(c-a)\,(a-b+1)_{n+1}} (-z)^{-a}\, {}_3F_2\left(1, a+n+1, a-c+n+2; n+2, a-b+n+2; \frac{1}{z}\right) - \frac{\Gamma(a-b)\,(b)_{n+1}\,(b-c+1)_{n+1}\,z^{-n-1}}{(n+1)!\,\Gamma(a)\,\Gamma(c-b)\,(-a+b+1)_{n+1}} (-z)^{-b}\, {}_3F_2\left(1, b+n+1, b-c+n+2; n+2, -a+b+n+2; \frac{1}{z}\right)\right) \bigwedge n \in \mathbb{N}\right) \bigwedge \neg\, a-b \in \mathbb{Z} $$

Summed form of the truncated series expansion.

Case of double poles

07.24.06.0025.01

$$ {}_2\tilde{F}_1(a,\,a+n;\,c;\,z)=\frac{(-z)^{-a}}{\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,\Gamma(n-k)\,z^{-k}}{k!\,\Gamma(c-a-k)}+\frac{\sin((c-a)\,\pi)\,\Gamma(a-c+n+1)}{\pi\,n!\,\Gamma(a)}\log(-z)\,(-z)^{-a-n}\,{}_2F_1\!\left(a+n,\,a-c+n+1;\,n+1;\,\frac{1}{z}\right)+\frac{(-z)^{-a-n}}{\Gamma(a+n)\,\Gamma(c-a)}\sum_{k=0}^{\infty}\frac{(a)_{k+n}\,(a-c+1)_{k+n}}{k!\,(k+n)!}(\psi(k+1)+\psi(k+n+1)-\psi(c-a-k-n)-\psi(a+k+n))\,z^{-k}\;/;\;|z|>1\bigwedge n\in\mathbb{N}\bigwedge c-a\notin\mathbb{Z} $$

07.24.06.0026.01

$$ {}_2\tilde{F}_1(a,\,a+n;\,c;\,z)\propto\frac{(n-1)!\,(-z)^{-a}}{\Gamma(a+n)\,\Gamma(c-a)}\left(1+O\!\left(\frac{1}{z}\right)\right)+\frac{\sin((c-a)\,\pi)\;\Gamma(a-c+n+1)}{\pi\,n!\,\Gamma(a)}(\log(-z)-\psi(c-a-n)+\psi(n+1)-\psi(a+n)-\gamma)\,(-z)^{-a-n}\left(1+O\!\left(\frac{1}{z}\right)\right)\;/;\;(|z|\to\infty)\bigwedge n\in\mathbb{N}^+\bigwedge c-a\notin\mathbb{Z} $$

07.24.06.0027.01

$$ {}_2\tilde{F}_1(a,\,a+n;\,a+m;\,z)=\frac{\Gamma(n)\,(-z)^{-a}}{\Gamma(m)\,\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,(1-m)_k\,z^{-k}}{k!\,(1-n)_k}+\frac{(-1)^n\,\Gamma(a+m)}{\Gamma(a)\,\Gamma(a+n)}(-z)^{-a-m}\,{}_3\tilde{F}_2\!\left(1,\,1,\,a+m;\,m+1,\,m-n+1;\,\frac{1}{z}\right)+\frac{(-1)^n}{\Gamma(a)\,(m-n-1)!}(-z)^{-a-n}\sum_{k=0}^{m-n-1}\frac{(a+n)_k\,(1-m+n)_k}{k!\,(k+n)!}(\log(-z)-\psi(m-n-k)+\psi(k+1)+\psi(k+n+1)-\psi(a+k+n))\,z^{-k}\;/;\;n\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge m\geq n\bigwedge z\notin(0,\,1) $$

07.24.06.0028.01

$$ {}_2\tilde{F}_1(a,\,a+n;\,a+m;\,z)\propto\frac{\Gamma(n)\,(-z)^{-a}}{\Gamma(m)\,\Gamma(a+n)}\left(1+O\!\left(\frac{1}{z}\right)\right)+\frac{(-1)^n\,\Gamma(a+m)\,(-z)^{-a-m}}{\Gamma(a)\,\Gamma(a+n)\,m!\,(m-n)!}\left(1+O\!\left(\frac{1}{z}\right)\right)+\frac{(-1)^n}{\Gamma(a)\,n!\,(m-n-1)!}(\log(-z)-\psi(m-n)+\psi(n+1)-\psi(a+n)-\gamma)\,(-z)^{-a-n}\left(1+O\!\left(\frac{1}{z}\right)\right)/;\;(|z|\to\infty)\bigwedge n\in\mathbb{N}^+\bigwedge m\in\mathbb{N}^+\bigwedge m>n $$

07.24.06.0029.01

$$ {}_2\tilde{F}_1(a,\,a;\,a+m;\,z)\propto\frac{(\log(-z)-\psi(a)-\psi(m)-2\,\gamma)}{\Gamma(a)\,(m-1)!}(-z)^{-a}\left(1+O\!\left(\frac{1}{z}\right)\right)+\frac{\Gamma(a+m)}{\Gamma(a)^2\,m!^2}(-z)^{-a-m}\left(1+O\!\left(\frac{1}{z}\right)\right)/;\;(|z|\to\infty)\bigwedge m\in\mathbb{N}^+ $$

07.24.06.0065.01

$$ {}_2\tilde{F}_1(a,\,a+n;\,c;\,z)=F_\infty(z,\,a,\,a+n,\,c)\;/;\;\left(\left(F_n(z,\,a,\,a+n,\,c)=\frac{(-z)^{-a}}{\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,\Gamma(n-k)\,z^{-k}}{k!\,\Gamma(-a+c-k)}+\frac{(-z)^{-a-n}}{\Gamma(a+n)\,\Gamma(c-a)}\sum_{k=0}^{m}\frac{(a)_{k+n}\,(a-c+1)_{k+n}}{k!\,(k+n)!}(\log(-z)+\psi(k+1)+\psi(k+n+1)-\psi(-a+c-k-n)-\psi(a+k+n))\,z^{-k}={}_2\tilde{F}_1(a,\,a+n;\,c;\,z)-\frac{(-1)^n}{\Gamma(a)\,\Gamma(a+n)}G_{4,4}^{3,2}\!\left(-z\,\middle|\,\begin{matrix}-a-m-n,\,-a-m-n,\,1-a,\,-a-n+1\\0,\,-a-m-n,\,-a-m-n,\,1-c\end{matrix}\right)\right)\bigwedge m\in\mathbb{N}\right)\bigwedge n\in\mathbb{N}\bigwedge\neg\,c-a\in\mathbb{Z} $$

Summed form of the truncated series expansion.

Case of canceled double poles

07.24.06.0030.01

$${}_2\tilde{F}_1(a,\,a+n;\,a-m;\,z) = \frac{(-1)^m\,(m+n)!}{\Gamma(a)}\,(-z)^{-a-n}\,{}_2\tilde{F}_1\!\left(a+n,\,m+n+1;\,n+1;\,\frac{1}{z}\right)/;\,n\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge z\notin(0,\,1)$$

07.24.06.0031.01

$${}_2\tilde{F}_1(a,\,a+n;\,a-m;\,z) \propto \frac{(-1)^m\,(m+n)!}{\Gamma(a)\,n!}\,(-z)^{-a-n}\left(1+O\!\left(\frac{1}{z}\right)\right)/;\,(|z|\to\infty)\land n\in\mathbb{N}\land m\in\mathbb{N}$$

07.24.06.0032.01

$${}_2\tilde{F}_1(a,\,a+n;\,a+m;\,z) = \frac{\Gamma(n)\,(-z)^{-a}}{\Gamma(m)\,\Gamma(a+n)}\sum_{k=0}^{m-1}\frac{(a)_k\,(1-m)_k\,z^{-k}}{k!\,(1-n)_k} + (-1)^m\,\frac{(n-m)!}{\Gamma(a)}\,(-z)^{-a-n}\,{}_2\tilde{F}_1\!\left(a+n,\,-m+n+1;\,n+1;\,\frac{1}{z}\right)/;$$
$$n\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge m\le n\bigwedge z\notin(0,\,1)$$

07.24.06.0033.01

$${}_2\tilde{F}_1(a,\,a+n;\,a+m;\,z) \propto \frac{\Gamma(n)}{\Gamma(m)\,\Gamma(a+n)}\,(-z)^{-a}\left(1+O\!\left(\frac{1}{z}\right)\right)/;\,(|z|\to\infty)\bigwedge n\in\mathbb{N}^+\bigwedge m\in\mathbb{N}^+\bigwedge m\le n$$

Generic formulas for main term

07.24.06.0066.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) \propto$$
$$\begin{cases}
\frac{(b-a-1)!\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} + \frac{1}{\pi\,(b-a)!\,\Gamma(a)}\sin((c-a)\,\pi)\,\Gamma(b-c+1) \\ \quad (\log(-z)+\psi(a+b-a+1)-\psi(c-b)-\psi(b)-\gamma)\,(-z)^{-b} & b-a\in\mathbb{N}^+\bigwedge c-a\notin\mathbb{Z} \\
\frac{(-z)^{-a}\,(\log(-z)-\psi(c-a)-\psi(a)-2\,\gamma)}{\Gamma(a)\,\Gamma(c-a)} & b=a\bigwedge c-a\notin\mathbb{Z} \\
\frac{(a-b-1)!\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} + \frac{1}{\pi\,(a-b)!\,\Gamma(b)}\sin((c-b)\,\pi)\,\Gamma(a-c+1) \\ \quad (\log(-z)-\psi(c-a)-\psi(a)+\psi(a-b+1)-\gamma)\,(-z)^{-a} & a-b\in\mathbb{N}^+\bigwedge c-b\notin\mathbb{Z} \\
\frac{(-1)^{a-c}\,(b-c)!\,(-z)^{-b}}{\Gamma(a)\,(b-a)!} & b-a\in\mathbb{N}\bigwedge a-c\in\mathbb{N} \\
\frac{(-1)^{b-c}\,(a-c)!\,(-z)^{-a}}{\Gamma(b)\,(a-b)!} & a-b\in\mathbb{N}\bigwedge b-c\in\mathbb{N} \\
\frac{\Gamma(b-a)\,(-z)^{-a}}{\Gamma(c-a)\,\Gamma(b)} + \frac{(-1)^{b-a}\,(\log(-z)+\psi(b-a+1)-\psi(c-b)-\psi(b)-\gamma)\,(-z)^{-b}}{\Gamma(a)\,(b-a)!\,(c-b-1)!} & b-a\in\mathbb{N}^+\bigwedge c-a\in\mathbb{N}^+\bigwedge c-b>0 \\
\frac{\Gamma(a-b)\,(-z)^{-b}}{\Gamma(c-b)\,\Gamma(a)} + \frac{(-1)^{a-b}\,(\log(-z)-\psi(c-a)-\psi(a)+\psi(a-b+1)-\gamma)\,(-z)^{-a}}{\Gamma(b)\,(a-b)!\,(c-a-1)!} & a-b\in\mathbb{N}^+\bigwedge c-b\in\mathbb{N}^+\bigwedge c-a>0 \\
\frac{(\log(-z)-\psi(c-a)-\psi(a)-2\,\gamma)\,(-z)^{-a}}{\Gamma(a)\,(c-a-1)!} + \frac{\Gamma(c)\,(-z)^{-c}}{\Gamma(a)^2\,((c-a)!)^2} & b=a\bigwedge c-a\in\mathbb{N}^+ \\
0 & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a\le 0)\bigvee(-b\in\mathbb{N}\bigwedge c-b \\
\frac{(-1)^a\,(b)_{-a}\,z^{-a}}{(c-a-1)!} & -c\in\mathbb{N}\land -a\in\mathbb{N}\land c-a>0 \\
\frac{(-1)^b\,(a)_{-b}\,z^{-b}}{(c-b-1)!} & -c\in\mathbb{N}\bigwedge -b\in\mathbb{N}\bigwedge c-b>0 \\
\frac{\Gamma(b-a)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} + \frac{\Gamma(a-b)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} & \text{True}
\end{cases}$$

$$/;$$
$$(|z|\to\infty)$$

07.24.06.0067.01

$$ {}_2\tilde{F}_1(a, b; c; z) \propto \begin{cases} \frac{(-1)^{a-c} (b-c)! (-z)^{-b}}{\Gamma(a) (b-a)!} & b - a \in \mathbb{N} \bigwedge a - c \in \mathbb{N} \\ \frac{(-z)^{-a} \log(-z)}{\Gamma(a) (c-a-1)!} & b = a \bigwedge \neg (a - c \in \mathbb{Z} \land a - c \geq 0) \\ \frac{(-1)^{b-c} (a-c)! (-z)^{-a}}{\Gamma(b) (a-b)!} & a - b \in \mathbb{N} \bigwedge b - c \in \mathbb{N} \\ 0 & -c \in \mathbb{N} \bigwedge ((-a \in \mathbb{N} \land c - a \leq 0) \bigvee (-b \in \mathbb{N} \bigwedge c - b \leq 0)) \\ \frac{\Gamma(b-a) (-z)^{-a}}{\Gamma(b) \Gamma(c-a)} & \operatorname{Re}(b - a) > 0 \bigvee (-c \in \mathbb{N} \land -a \in \mathbb{N} \land c - a > 0) \\ \frac{\Gamma(a-b) (-z)^{-b}}{\Gamma(a) \Gamma(c-b)} & \operatorname{Re}(b - a) < 0 \bigvee (-c \in \mathbb{N} \bigwedge -b \in \mathbb{N} \bigwedge c - b > 0) \\ \frac{\Gamma(b-a) (-z)^{-a}}{\Gamma(b) \Gamma(c-a)} + \frac{\Gamma(a-b) (-z)^{-b}}{\Gamma(a) \Gamma(c-b)} & \text{True} \end{cases} \;/;\; (|z| \to \infty) $$

Expansions at $z = \infty$ for polynomial cases

07.24.06.0034.01

$$ {}_2\tilde{F}_1(-n, b; c; z) = \frac{\Gamma(1 - b)}{\Gamma(c + n)} z^n \, {}_2\tilde{F}_1\left(-n, 1 - c - n; 1 - b - n; \frac{1}{z}\right) /; n \in \mathbb{N} $$

Residue representations

General case

07.24.06.0035.01

$$ {}_2\tilde{F}_1(a, b; c; z) = \frac{1}{\Gamma(a) \Gamma(b)} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(a - s) \Gamma(b - s) (-z)^{-s}}{\Gamma(c - s)} \Gamma(s) \right)(-j) /; |z| < 1 $$

07.24.06.0036.01

$$ {}_2\tilde{F}_1(a, b; c; z) = -\frac{1}{\Gamma(a) \Gamma(b)} \left(\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s) \Gamma(b - s) (-z)^{-s}}{\Gamma(c - s)} \Gamma(a - s) \right)(a + j) + \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s) \Gamma(a - s) (-z)^{-s}}{\Gamma(c - s)} \Gamma(b - s) \right)(b + j) \right) /; $$

$$ |z| > 1 \bigwedge a - b \notin \mathbb{Z} $$

07.24.06.0068.01

$$ {}_2\tilde{F}_1(a, b; c; z) = \frac{1}{\Gamma(a) \Gamma(b) \Gamma(c - a) \Gamma(c - b)} \left(\sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(-a - b + c + s) \Gamma(a - s) \Gamma(b - s) (1 - z)^{-s}) \Gamma(s))(-j) + \right. $$

$$ \left. \sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(s) \Gamma(a - s) \Gamma(b - s) (1 - z)^{-s}) \Gamma(-a - b + c + s))(a + b - c - j) \right) /; |1 - z| < 1 \bigwedge c - a - b \notin \mathbb{Z} $$

Logarithmic cases

07.24.06.0069.01

$$ {}_2\tilde{F}_1(a, a + n; c; z) = -\frac{1}{\Gamma(a) \Gamma(b)} $$

$$ \left(\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s) \Gamma(a + n - s) (-z)^{-s}}{\Gamma(c - s)} \Gamma(a - s) \right)(a + j) + \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s) (-z)^{-s}}{\Gamma(c - s)} \Gamma(a - s) \Gamma(a + n - s) \right)(a + n + j) \right) /; |z| > 1 \land n \in \mathbb{N} $$

07.24.06.0070.01

$${}_2\tilde{F}_1(a,\, b;\, a+b+n;\, z) = \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(a+n)\,\Gamma(b+n)} \left(\sum_{j=0}^{n-1} \operatorname{res}_s((\Gamma(n+s)\,\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(s))\,(-j) + \sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(s)\,\Gamma(n+s))\,(-n-j) \right) /;\ |1-z| < 1 \land n \in \mathbb{N}$$

07.24.06.0071.01

$${}_2\tilde{F}_1(a,\, b;\, a+b-n;\, z) = \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(a-n)\,\Gamma(b-n)} \left(\sum_{j=0}^{n-1} \operatorname{res}_s((\Gamma(s)\,\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(s-n))\,(n-j) + \sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(s)\,\Gamma(s-n))\,(-j) \right) /;\ |1-z| < 1 \bigwedge n \in \mathbb{N}^+$$

Limit representations

07.24.09.0001.01

$${}_2\tilde{F}_1(a,\, b;\, c;\, z) = \lim_{p \to \infty} \frac{1}{\Gamma(c)}\, {}_3F_2(a,\, b,\, p\, z;\, c,\, p;\, 1) /;\ \operatorname{Re}(c-a-b+p\,(1-z)) > 0$$

07.24.09.0002.01

$${}_2\tilde{F}_1(a,\, b;\, c;\, z) = \lim_{p \to \infty} \Gamma(p)\, {}_2\tilde{F}_2(a,\, b;\, c,\, p;\, p\, z)$$

07.24.09.0003.01

$${}_2\tilde{F}_1(a,\, b;\, c;\, z) = \lim_{q \to \infty} \lim_{p \to \infty} \Gamma(p)\, \Gamma(q)\, {}_2\tilde{F}_3(a,\, b;\, c,\, p,\, q;\, p\, q\, z)$$

Continued fraction representations

07.24.10.0001.01

$${}_2\tilde{F}_1(a,\, b;\, c;\, z) = \frac{1}{\Gamma(c)} \left(1 + \frac{a\,b\,z}{c} \Bigg/ \left(1 - \frac{(a+1)\,(b+1)\,z}{(2\,(c+1)) \left(\frac{(a+1)\,(b+1)\,z}{2\,(c+1)} - \frac{(a+2)\,(b+2)\,z}{(3\,(c+2)) \left(\frac{(a+2)\,(b+2)\,z}{3\,(c+2)} + \ldots + 1 \right)} + 1 \right)} \right) \right)$$

07.24.10.0002.01

$${}_2\tilde{F}_1(a,\, b;\, c;\, z) = \frac{1}{\Gamma(c)} \left(1 + \frac{a\,b\,z}{c \left(1 + \mathrm{K}_k\left(-\frac{(a+k)\,(b+k)\,z}{(k+1)\,(c+k)},\, 1 + \frac{(a+k)\,(b+k)\,z}{(k+1)\,(c+k)} \right)_1^{\infty} \right)} \right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

Representation of fundamental system solutions near zero

07.24.13.0003.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\;w(z)=c_1\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\;+c_2\,G_{2,2}^{2,2}\!\left(z\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.24.13.0004.01

$$W_z\!\left({}_2\tilde{F}_1(a,\,b;\,c;\,z),\,G_{2,2}^{2,2}\!\left(z\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)\right)=-(1-z)^{-a-b+c-1}\,z^{-c}\,\Gamma(a-c+1)\,\Gamma(b-c+1)$$

07.24.13.0001.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;$$
$$w(z)=c_1\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)+c_2\,z^{1-c}\,{}_2\tilde{F}_1(a-c+1,\,b-c+1;\,2-c;\,z)\bigwedge c\notin\mathbb{Z}$$

07.24.13.0002.02

$$W_z\!\left({}_2\tilde{F}_1(a,\,b;\,c;\,z),\,z^{1-c}\,{}_2\tilde{F}_1(a-c+1,\,b-c+1;\,2-c;\,z)\right)=\frac{\sin(c\,\pi)}{\pi}\,(1-z)^{-a-b+c-1}\,z^{-c}$$

07.24.13.0005.01

$$w''(z)-\left(\frac{(c-(a+b+1)\,g(z))\,g'(z)}{(g(z)-1)\,g(z)}+\frac{g''(z)}{g'(z)}\right)w'(z)+\frac{a\,b\,g'(z)^2}{(g(z)-1)\,g(z)}\,w(z)=0\,/;$$
$$w(z)=\;c_1\,{}_2\tilde{F}_1(a,\,b;\,c;\,g(z))+c_2\,G_{2,2}^{2,2}\!\left(g(z)\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.24.13.0006.01

$$W_z\!\left({}_2\tilde{F}_1(a,\,b;\,c;\,g(z)),\,G_{2,2}^{2,2}\!\left(g(z)\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)\right)=-(1-g(z))^{c-a-b-1}\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,g'(z)\,g(z)^{-c}$$

07.24.13.0007.01

$$h(z)^2\,w''(z)\;-h(z)\left(h(z)\left(\frac{(c-(a+b+1)\,g(z))\,g'(z)}{(g(z)-1)\,g(z)}+\frac{g''(z)}{g'(z)}\right)+2\,h'(z)\right)w'(z)\;+$$
$$\left(\frac{a\,b\,h(z)^2\,g'(z)^2}{(g(z)-1)\,g(z)}+2\,h'(z)^2+h(z)\left(\frac{(c-(a+b+1)\,g(z))\,g'(z)\,h'(z)}{(g(z)-1)\,g(z)}+\frac{g''(z)\,h'(z)}{g'(z)}-h''(z)\right)\right)w(z)\;=0\,/;$$
$$w(z)=c_1\,h(z)\,{}_2\tilde{F}_1(a,\,b;\,c;\,g(z))\;+c_2\,h(z)\,G_{2,2}^{2,2}\!\left(g(z)\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.24.13.0008.01

$$W_z\!\left(h(z)\,{}_2\tilde{F}_1(a,\,b;\,c;\,g(z)),\,h(z)\,G_{2,2}^{2,2}\!\left(g(z)\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)\right)=-(1-g(z))^{-a-b+c-1}\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,h(z)^2\,g'(z)\,g(z)^{-c}$$

07.24.13.0009.01

$$z^2(1-d\,z^r)\,w''(z)\;+z\,((1-2\,s)\,(1-d\,z^r)-r\,((a+b)\,d\,z^r-c+1))\,w'(z)\,+$$
$$\left(-a\,b\,d\,r^2\,z^r+r\,s\,((a+b)\,d\,z^r-c+1)+s^2\,(1-d\,z^r)\right)w(z)=0\,/;$$
$$w(z)=c_1\,z^s\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,z^r)\;+c_2\,z^s\,G_{2,2}^{2,2}\!\left(d\,z^r\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.24.13.0010.01

$$W_z\!\left(z^s\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,z^r),\,z^s\,G_{2,2}^{2,2}\!\left(d\,z^r\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)\right)=-d\,r\,z^{r+2\,s-1}\,(d\,z^r)^{-c}\,(1-d\,z^r)^{-a-b+c-1}\,\Gamma(a-c+1)\,\Gamma(b-c+1)$$

07.24.13.0011.01

$$(1-d\,r^z)\,w''(z)-(((a+b)\,d\,r^z-c+1)\log(r)+2\,(1-d\,r^z)\log(s))\,w'(z)\,+$$
$$\left(-a\,b\,d\log^2(r)\,r^z+(1-d\,r^z)\log^2(s)+((a+b)\,d\,r^z-c+1)\log(r)\log(s)\right)w(z)=0\,/;$$
$$w(z)=c_1\,s^z\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,r^z)\;+c_2\,s^z\,G_{2,2}^{2,2}\!\left(d\,r^z\;\middle|\;\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.24.13.0012.01

$$W_z\left(s^z\,{}_2\tilde{F}_1(a,b;c;d\,r^z),\,s^z\,G_{2,2}^{2,2}\!\left(d\,r^z\,\middle|\,\begin{matrix}1-a,1-b\\0,1-c\end{matrix}\right)\right)=-d\,r^z\,(d\,r^z)^{-c}\,(1-d\,r^z)^{c-a-b-1}\,s^{2z}\,\Gamma(a-c+1)\,\Gamma(b-c+1)\log(r)$$

Representation of fundamental system solutions near unit

07.24.13.0013.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\; w(z)=c_1\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z)+c_2\,G_{2,2}^{2,2}\!\left(1-z\,\middle|\,\begin{matrix}1-a,1-b\\0,-a-b+c\end{matrix}\right)$$

07.24.13.0014.01

$$W_z\left({}_2\tilde{F}_1(a,b;a+b-c+1;1-z),\,G_{2,2}^{2,2}\!\left(1-z\,\middle|\,\begin{matrix}1-a,1-b\\0,c-a-b\end{matrix}\right)\right)=(1-z)^{c-a-b-1}\,z^{-c}\,\Gamma(c-a)\,\Gamma(c-b)$$

07.24.13.0015.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\; w(z)=c_1(1-z)^{c-a-b}\,{}_2\tilde{F}_1(c-a,c-b;-a-b+c+1;1-z)+c_2\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z)\bigwedge c-a-b\notin\mathbb{Z}$$

07.24.13.0016.01

$$W_z\left((1-z)^{-a-b+c}\,{}_2\tilde{F}_1(c-a,c-b;-a-b+c+1;1-z),\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z)\right)=\frac{\sin((c-a-b)\,\pi)}{\pi}\,(1-z)^{c-a-b-1}\,z^{-c}$$

Representation of fundamental system solutions near infinity

07.24.13.0017.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\; w(z)=c_1\,z^{-a}\,{}_2\tilde{F}_1\!\left(a,a-c+1;a-b+1;\frac{1}{z}\right)+c_2\,G_{2,2}^{2,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}c,1\\a,b\end{matrix}\right)$$

07.24.13.0018.01

$$W_z\left(z^{-a}\,{}_2\tilde{F}_1\!\left(a,a-c+1;a-b+1;\frac{1}{z}\right),\,G_{2,2}^{2,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}c,1\\a,b\end{matrix}\right)\right)=(z-1)^{-a-b+c-1}\,z^{-c}\,\Gamma(b-c+1)\,\Gamma(b)$$

07.24.13.0019.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\; w(z)=c_1\,z^{-a}\,{}_2\tilde{F}_1\!\left(a,a-c+1;a-b+1;\frac{1}{z}\right)+c_2\,z^{-b}\,{}_2\tilde{F}_1\!\left(b,b-c+1;-a+b+1;\frac{1}{z}\right)\bigwedge a-b\notin\mathbb{Z}$$

07.24.13.0020.01

$$W_z\left(z^{-a}\,{}_2\tilde{F}_1\!\left(a,a-c+1;a-b+1;\frac{1}{z}\right),\,z^{-b}\,{}_2\tilde{F}_1\!\left(b,b-c+1;-a+b+1;\frac{1}{z}\right)\right)=\frac{\sin((a-b)\,\pi)}{\pi}\,(z-1)^{-a-b+c-1}\,z^{-c}$$

List of solutions

07.24.13.0021.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\; w(z)=c_1\,u_{j,k}(z)+c_2\,u_{l,m}(z)\bigwedge 1\le j\le 6\bigwedge 1\le k\le 4\bigwedge 1\le l\le 6\bigwedge 1\le m\le 4\bigwedge j\ne l$$

where $u_{j,k}(z)$ are arbitrary functions from the following list of 24 functions

07.24.13.0022.01

$$u_{1,1}(z)={}_2\tilde{F}_1(a,b;c;z)$$

07.24.13.0023.01

$$u_{1,2}(z) = (1-z)^{-a-b+c}\, {}_2\tilde{F}_1(c-a, c-b; c; z)$$

07.24.13.0024.01

$$u_{1,3}(z) = (1-z)^{-a}\, {}_2\tilde{F}_1\left(a, c-b; c; \frac{z}{z-1}\right)$$

07.24.13.0025.01

$$u_{1,4}(z) = (1-z)^{-b}\, {}_2\tilde{F}_1\left(c-a, b; c; \frac{z}{z-1}\right)$$

07.24.13.0026.01

$$u_{2,1}(z) = {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z)$$

07.24.13.0027.01

$$u_{2,2}(z) = z^{1-c}\, {}_2\tilde{F}_1(b-c+1, a-c+1; a+b-c+1; 1-z)$$

07.24.13.0028.01

$$u_{2,3}(z) = z^{-a}\, {}_2\tilde{F}_1\left(a, a-c+1; a+b-c+1; \frac{z-1}{z}\right)$$

07.24.13.0029.01

$$u_{2,4}(z) = z^{-b}\, {}_2\tilde{F}_1\left(b-c+1, b; a+b-c+1; \frac{z-1}{z}\right)$$

07.24.13.0030.01

$$u_{3,1}(z) = (-z)^{-a}\, {}_2\tilde{F}_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right)$$

07.24.13.0031.01

$$u_{3,2}(z) = \left(\frac{z-1}{z}\right)^{-a-b+c} (-z)^{-a}\, {}_2\tilde{F}_1\left(1-b, c-b; a-b+1; \frac{1}{z}\right)$$

07.24.13.0032.01

$$u_{3,3}(z) = \left(\frac{z-1}{z}\right)^{-a} (-z)^{-a}\, {}_2\tilde{F}_1\left(a, c-b; a-b+1; \frac{1}{1-z}\right)$$

07.24.13.0033.01

$$u_{3,4}(z) = \left(\frac{z-1}{z}\right)^{-a+c-1} (-z)^{-a}\, {}_2\tilde{F}_1\left(1-b, a-c+1; a-b+1; \frac{1}{1-z}\right)$$

07.24.13.0034.01

$$u_{4,1}(z) = (-z)^{-b}\, {}_2\tilde{F}_1\left(b-c+1, b; -a+b+1; \frac{1}{z}\right)$$

07.24.13.0035.01

$$u_{4,2}(z) = \left(\frac{z-1}{z}\right)^{-a-b+c} (-z)^{-b}\, {}_2\tilde{F}_1\left(c-a, 1-a; -a+b+1; \frac{1}{z}\right)$$

07.24.13.0036.01

$$u_{4,3}(z) = \left(\frac{z-1}{z}\right)^{-b+c-1} (-z)^{-b}\, {}_2\tilde{F}_1\left(b-c+1, 1-a; -a+b+1; \frac{1}{1-z}\right)$$

07.24.13.0037.01

$$u_{4,4}(z) = \left(\frac{z-1}{z}\right)^{-b} (-z)^{-b}\, {}_2\tilde{F}_1\left(c-a, b; -a+b+1; \frac{1}{1-z}\right)$$

07.24.13.0038.01

$$u_{5,1}(z) = z^{1-c}\,{}_2\tilde{F}_1(a-c+1,\, b-c+1;\, 2-c;\, z)$$

07.24.13.0039.01

$$u_{5,2}(z) = (1-z)^{-a-b+c}\, z^{1-c}\,{}_2\tilde{F}_1(1-a,\, 1-b;\, 2-c;\, z)$$

07.24.13.0040.01

$$u_{5,3}(z) = (1-z)^{-a+c-1}\, z^{1-c}\,{}_2\tilde{F}_1\left(a-c+1,\, 1-b;\, 2-c;\, \frac{z}{z-1}\right)$$

07.24.13.0041.01

$$u_{5,4}(z) = (1-z)^{-b+c-1}\, z^{1-c}\,{}_2\tilde{F}_1\left(1-a,\, b-c+1;\, 2-c;\, \frac{z}{z-1}\right)$$

07.24.13.0042.01

$$u_{6,1}(z) = (1-z)^{-a-b+c}\,{}_2\tilde{F}_1(c-a,\, c-b;\, -a-b+c+1;\, 1-z)$$

07.24.13.0043.01

$$u_{6,2}(z) = (1-z)^{-a-b+c}\, z^{1-c}\,{}_2\tilde{F}_1(1-b,\, 1-a;\, -a-b+c+1;\, 1-z)$$

07.24.13.0044.01

$$u_{6,3}(z) = (1-z)^{-a-b+c}\, z^{a-c}\,{}_2\tilde{F}_1\left(c-a,\, 1-a;\, -a-b+c+1;\, \frac{z-1}{z}\right)$$

07.24.13.0045.01

$$u_{6,4}(z) = (1-z)^{-a-b+c}\, z^{b-c}\,{}_2\tilde{F}_1\left(1-b,\, c-b;\, -a-b+c+1;\, \frac{z-1}{z}\right)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.24.16.0001.01

$${}_2\tilde{F}_1(c-a,\, c-b;\, c;\, z) == (1-z)^{a+b-c}\,{}_2\tilde{F}_1(a,\, b;\, c;\, z)$$

07.24.16.0002.01

$${}_2\tilde{F}_1\left(a,\, c-b;\, c;\, \frac{z}{z-1}\right) == (1-z)^{a}\,{}_2\tilde{F}_1(a,\, b;\, c;\, z)\ /;\ z \notin (1,\, \infty)$$

07.24.16.0003.01

$${}_2\tilde{F}_1\left(c-a,\, b;\, c;\, \frac{z}{z-1}\right) == (1-z)^{b}\,{}_2\tilde{F}_1(a,\, b;\, c;\, z)\ /;\ z \notin (1,\, \infty)$$

07.24.16.0004.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, 2z-z^2\right) == \left(1-\frac{z}{2}\right)^{-2a}\,{}_2\tilde{F}_1\left(2a,\, 2a-c+1;\, c;\, \frac{z}{2-z}\right)\ /;\ \mathrm{Re}(z) < 1$$

07.24.16.0005.01

$${}_2\tilde{F}_1\left(a,\, b;\, a+b+\frac{1}{2};\, 4z(1-z)\right) == {}_2\tilde{F}_1\left(2a,\, 2b;\, a+b+\frac{1}{2};\, z\right)\ /;\ z \notin \left(\frac{1}{2},\, \infty\right)$$

07.24.16.0006.01

$${}_2\tilde{F}_1\left(a,\, b;\, 2b;\, \frac{4z}{(z+1)^2}\right) == \frac{\sqrt{\pi}\, 2^{1-2b}}{\Gamma(b)}\,(1+z)^{2a}\,{}_2\tilde{F}_1\left(a,\, a-b+\frac{1}{2};\, b+\frac{1}{2};\, z^2\right)\ /;\ |z| < 1$$

Products, sums, and powers of the direct function

Products of the direct function

07.24.16.0007.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,g\,z)\,{}_2\tilde{F}_1(\alpha,\,\beta;\,\gamma;\,h\,z) = \sum_{k=0}^{\infty} c_k\, z^k \,/;$$

$$ c_k = \frac{\Gamma(1-\alpha)\,\Gamma(1-\beta)\,h^k}{k!\,\Gamma(k+\gamma)}\, {}_4\tilde{F}_3\left(-k,\,-k-\gamma+1,\,a,\,b;\,-k-\alpha+1,\,-k-\beta+1,\,c;\,\frac{g}{h}\right) \bigvee $$

$$ c_k = \frac{\Gamma(1-a)\,\Gamma(1-b)\,g^k}{k!\,\Gamma(c+k)}\, {}_4\tilde{F}_3\left(-k,\,-c-k+1,\,\alpha,\,\beta;\,-a-k+1,\,-b-k+1,\,\gamma;\,\frac{h}{g}\right)$$

07.24.16.0008.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,g\,z)\,{}_2\tilde{F}_1(\alpha,\,\beta;\,\gamma;\,h\,z) = \sum_{k=0}^{\infty}\sum_{m=0}^{k} \frac{(a)_m\,(b)_m\,(\alpha)_{k-m}\,(\beta)_{k-m}\,g^m\,h^{k-m}\,z^k}{\Gamma(c+m)\,\Gamma(k-m+\gamma)\;m!\,(k-m)!}$$

07.24.16.0009.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,g\,z)\,{}_2\tilde{F}_1(\alpha,\,\beta;\,\gamma;\,h\,z) = \tilde{F}_{0;1;1}^{0:2;3}\left(\begin{matrix} :a,\,b;\,\alpha,\,\beta; \\ :c;\,\gamma; \end{matrix}\; g\,z,\,h\,z\right)$$

Identities

Recurrence identities

Consecutive neighbors

07.24.17.0001.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{2\,b-c+2+(a-b-1)\,z}{b-c+1}\,{}_2\tilde{F}_1(a,\,b+1;\,c;\,z) + \frac{(b+1)\,(z-1)}{b-c+1}\,{}_2\tilde{F}_1(a,\,b+2;\,c;\,z)$$

07.24.17.0002.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{2-2\,b+c+(b-a-1)\,z}{(b-1)\,(z-1)}\,{}_2\tilde{F}_1(a,\,b-1;\,c;\,z) + \frac{b-c-1}{(b-1)\,(z-1)}\,{}_2\tilde{F}_1(a,\,b-2;\,c;\,z)$$

07.24.17.0003.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{(2\,c-a-b+1)\,z-c}{z-1}\,{}_2\tilde{F}_1(a,\,b;\,c+1;\,z) + \frac{(a-c-1)\,(c-b+1)\,z}{z-1}\,{}_2\tilde{F}_1(a,\,b;\,c+2;\,z)$$

07.24.17.0004.01

$$ {}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{1-z}{(a-c+1)\,(b-c+1)\,z}\,{}_2\tilde{F}_1(a,\,b;\,c-2;\,z) + \frac{2-c-(a+b-2\,c+3)\,z}{(a-c+1)\,(b-c+1)\,z}\,{}_2\tilde{F}_1(a,\,b;\,c-1;\,z)$$

Consecutive neighbors (nine basic relations)

07.24.17.0130.01

$$ (a-c)\,{}_2\tilde{F}_1(a-1,\,b;\,c;\,z) + (c-2\,a+(a-b)\,z)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z) = a\,(z-1)\,{}_2\tilde{F}_1(a+1,\,b;\,c;\,z)$$

07.24.17.0131.01

$$ (b-c)\,{}_2\tilde{F}_1(a,\,b-1;\,c;\,z) + (c-2\,b+(b-a)\,z)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z) = b\,(z-1)\,{}_2\tilde{F}_1(a,\,b+1;\,c;\,z)$$

07.24.17.0132.01

$$ (1-z)\,{}_2\tilde{F}_1(a,\,b;\,c-1;\,z) + (-c+(2\,c-a-b-1)\,z+1)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z) = (a-c)\,(b-c)\,z\,{}_2\tilde{F}_1(a,\,b;\,c+1;\,z)$$

07.24.17.0133.01

$$(b-c)\,{}_2\tilde{F}_1(a,\,b-1;\,c;\,z)+(-a-b+c)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)=a\,(z-1)\,{}_2\tilde{F}_1(a+1,\,b;\,c;\,z)$$

07.24.17.0134.01

$$(a-c)\,{}_2\tilde{F}_1(a-1,\,b;\,c;\,z)+(c-a-b)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)=b\,(z-1)\,{}_2\tilde{F}_1(a,\,b+1;\,c;\,z)$$

07.24.17.0135.01

$$(c-b)\,(a-c)\,z\,{}_2\tilde{F}_1(a,\,b;\,c+1;\,z)+((c-b)\,z-a)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)=a\,(z-1)\,{}_2\tilde{F}_1(a+1,\,b;\,c;\,z)$$

07.24.17.0136.01

$$((c-b)\,z-a)\,{}_2\tilde{F}_1(a+1,\,b;\,c+1;\,z)+(a-c)\,{}_2\tilde{F}_1(a,\,b;\,c+1;\,z)=(z-1)\,{}_2\tilde{F}_1(a+1,\,b;\,c;\,z)$$

07.24.17.0137.01

$$(c-a)\,(b-c)\,z\,{}_2\tilde{F}_1(a,\,b;\,c+1;\,z)+((c-a)\,z-b)\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)=b\,(z-1)\,{}_2\tilde{F}_1(a,\,b+1;\,c;\,z)$$

07.24.17.0138.01

$$((c-a)\,z-b)\,{}_2\tilde{F}_1(a,\,b+1;\,c+1;\,z)+(b-c)\,{}_2\tilde{F}_1(a,\,b;\,c+1;\,z)=(z-1)\,{}_2\tilde{F}_1(a,\,b+1;\,c;\,z)$$

Distant neighbors

07.24.17.0139.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z)=C_n(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b+n;\,c;\,z)\;+\;\frac{(b+n)\,(z-1)}{b-c+n}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b+n+1;\,c;\,z)\;/;$$
$$C_0(a,\,b,\,c,\,z)=1\bigwedge C_1(a,\,b,\,c,\,z)=\frac{2\,a-b+z+2}{a-b+1}\bigwedge$$
$$C_n(a,\,b,\,c,\,z)=\frac{2\,b-c+2\,n+(a-b-n)\,z}{b-c+n}\,C_{n-1}(a,\,b,\,c,\,z)+\frac{(b+n-1)\,(z-1)}{b-c+n-1}\,C_{n-2}(a,\,b,\,c,\,z)\bigwedge n\in\mathbb{N}^+$$

07.24.17.0140.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z)=\frac{n-b+c}{(n-b)\,(z-1)}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b-n-1;\,c;\,z)+C_n(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b-n;\,c;\,z)\;/;$$
$$C_0(a,\,b,\,c,\,z)=1\bigwedge C_1(a,\,b,\,c,\,z)=\frac{2-2\,b+c+(b-a-1)\,z}{(b-1)\,(z-1)}\bigwedge$$
$$C_n(a,\,b,\,c,\,z)=-\frac{-2\,b+c+2\,n+(b-a-n)\,z}{(n-b)\,(z-1)}\,C_{n-1}(a,\,b,\,c,\,z)+\frac{n-b+c-1}{(n-b-1)\,(z-1)}\,C_{n-2}(a,\,b,\,c,\,z)\bigwedge n\in\mathbb{N}^+$$

07.24.17.0141.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z)=C_n(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b;\,c+n;\,z)-\frac{(a-c-n)\,(b-c-n)\,z}{z-1}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b;\,c+n+1;\,z)\;/;$$
$$C_0(a,\,b,\,c,\,z)=1\bigwedge C_1(a,\,b,\,c,\,z)=\frac{(-a-b+2\,c+1)\,z-c}{z-1}\bigwedge C_n(a,\,b,\,c,\,z)=$$
$$\frac{1-n-c+(2\,(n+c)-1-a-b)\,z}{z-1}\,C_{n-1}(a,\,b,\,c,\,z)-\frac{(a-c-n+1)\,(b-c-n+1)\,z}{z-1}\,C_{n-2}(a,\,b,\,c,\,z)\bigwedge n\in\mathbb{N}^+$$

07.24.17.0142.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z)=\frac{1-z}{(a-c+n)\,(b-c+n)\,z}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b;\,c-n-1;\,z)\;+C_n(a,\,b,\,c,\,z)\,{}_2\tilde{F}_1(a,\,b;\,c-n;\,z)\;/;$$
$$C_0(a,\,b,\,c,\,z)=1\bigwedge C_1(a,\,b,\,c,\,z)=\frac{2-c-(a+b-2\,c+3)\,z}{(a-c+1)\,(b-c+1)\,z}\bigwedge C_n(a,\,b,\,c,\,z)=$$
$$\frac{1-z}{(a-c+n-1)\,(b-c+n-1)\,z}\,C_{n-2}(a,\,b,\,c,\,z)+\frac{1-c+n-(a+b+2\,(n-c)+1)\,z}{(a-c+n)\,(b-c+n)\,z}\,C_{n-1}(a,\,b,\,c,\,z)\bigwedge n\in\mathbb{N}^+$$

07.24.17.0005.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{(-1)^m z^{-m}}{(1-a+c)_m} \sum_{k=0}^{m} \binom{m}{k} (z-1)^{m-k} {}_2\tilde{F}_1(a, b-k; c-m; z) \;/;\; m \in \mathbb{N}$$

07.24.17.0006.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{(-1)^m z^{-m}}{(1-a)_m} \sum_{k=0}^{m} (-1)^k \binom{m}{k} {}_2\tilde{F}_1(a-m, b-k; c-m; z) \;/;\; m \in \mathbb{N}$$

07.24.17.0007.01

$${}_2\tilde{F}_1(1, b; c; z) = \frac{(c-b)_m (1-z)^{-m}}{(1-b)_m} {}_2\tilde{F}_1(1, b-m; c; z) + \frac{1}{(b-1)\,\Gamma(c-1)} \sum_{k=0}^{m-1} \frac{(c-b)_k (1-z)^{-k-1}}{(2-b)_k} \;/;\; m \in \mathbb{N}^+$$

07.24.17.0008.01

$${}_2\tilde{F}_1(1, b; c; z) = \frac{(-1)^m}{(b-c+1)_m} \left(\frac{z-1}{z}\right)^m {}_2\tilde{F}_1(1, b; c-m; z) + \frac{1}{z\,\Gamma(c)} \sum_{k=1}^{m} \frac{(1-c)_k}{(b-c+1)_k} \left(\frac{z-1}{z}\right)^{k-1} \;/;\; m \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.24.17.0009.01

$$(a-c)\,{}_2F_1(a-1, b; c; z) - a\,(z-1)\,{}_2F_1(a+1, b; c; z) + (c-2\,a+(a-b)\,z)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0010.01

$$(a-c)\,{}_2F_1(a-1, b; c; z) + (c-b)\,{}_2F_1(a, b-1; c; z) + (z-1)\,(a-b)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0011.01

$$(b-c)\,{}_2F_1(a, b-1; c; z) - a\,(z-1)\,{}_2F_1(a+1, b; c; z) + (c-a-b)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0012.01

$$(a-c)\,{}_2F_1(a-1, b; c; z) - b\,(z-1)\,{}_2F_1(a, b+1; c; z) + (c-a-b)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0013.01

$$b\,{}_2F_1(a, b+1; c; z) - a\,{}_2F_1(a+1, b; c; z) + (a-b)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0014.01

$$(b-c)\,{}_2F_1(a, b-1; c; z) - b\,(z-1)\,{}_2F_1(a, b+1; c; z) + (c-2\,b+(b-a)\,z)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0015.01

$$(c-1)\,(z-1)\,c\,{}_2F_1(a, b; c-1; z) + (a-c)\,(b-c)\,z\,{}_2F_1(a, b; c+1; z) + c\,(c+(a+b-2\,c+1)\,z-1)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0016.01

$$(c-a)\,{}_2F_1(a-1, b; c; z) + (c-1)\,(z-1)\,{}_2F_1(a, b; c-1; z) + (a+(b-c+1)\,z-1)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0017.01

$$c\,{}_2F_1(a-1, b; c; z) + (b-c)\,z\,{}_2F_1(a, b; c+1; z) + (z-1)\,c\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0018.01

$$(c-1)\,{}_2F_1(a, b; c-1; z) - a\,{}_2F_1(a+1, b; c; z) + (a-c+1)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0019.01

$$(z-1)\,a\,c\,{}_2F_1(a+1, b; c; z) + (a-c)\,(b-c)\,z\,{}_2F_1(a, b; c+1; z) + c\,(a+(b-c)\,z)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0020.01

$$(c-b)\,{}_2F_1(a, b-1; c; z) + (c-1)\,(z-1)\,{}_2F_1(a, b; c-1; z) + (b+(a-c+1)\,z-1)\,{}_2F_1(a, b; c; z) = 0$$

07.24.17.0021.01

$$c\,{}_2F_1(a, b-1; c; z) + (a-c)\,z\,{}_2F_1(a, b; c+1; z) + (z-1)\,c\,{}_2F_1(a, b; c; z) == 0$$

07.24.17.0022.01

$$(c-1)\,{}_2F_1(a, b; c-1; z) - b\,{}_2F_1(a, b+1; c; z) + (b-c+1)\,{}_2F_1(a, b; c; z) == 0$$

07.24.17.0023.01

$$(z-1)\,b\,c\,{}_2F_1(a, b+1; c; z) + (a-c)\,(b-c)\,z\,{}_2F_1(a, b; c+1; z) + c\,(b+(a-c)\,z)\,{}_2F_1(a, b; c; z) == 0$$

07.24.17.0024.01

$$(a-b)\,(a-c)\,{}_2F_1(a-1, b; c; z) + a\,(c-a-b)\,{}_2F_1(a+1, b; c; z) + b\,(2\,a-c+(b-a)\,z)\,{}_2F_1(a, b+1; c; z) == 0$$

07.24.17.0025.01

$$a\,(c-2\,b+(b-a)\,z)\,{}_2F_1(a+1, b; c; z) + (a-b)\,(b-c)\,{}_2F_1(a, b-1; c; z) + b\,(a+b-c)\,{}_2F_1(a, b+1; c; z) == 0$$

07.24.17.0026.01

$$(a-c+1)\,(a-c)\,{}_2F_1(a-1, b; c; z) - a\,(a+(b-c+1)\,z-1)\,{}_2F_1(a+1, b; c; z) - (c-1)\,(c-2\,a+(a-b)\,z)\,{}_2F_1(a, b; c-1; z) == 0$$

07.24.17.0027.01

$$a\,c\,(1-c+(2\,c-a-b-1)\,z)\,{}_2F_1(a+1, b; c; z) + (c-1)\,c\,(a+(b-c)\,z)\,{}_2F_1(a, b; c-1; z) - (a-c)\,(a-c+1)\,(b-c)\,z\,{}_2F_1(a, b; c+1; z) == 0$$

07.24.17.0028.01

$$(b-c+1)\,(b-c)\,{}_2F_1(a, b-1; c; z) + b\,(1-b+(c-a-1)\,z)\,{}_2F_1(a, b+1; c; z) - (c-1)\,(z\,(b-a)-2\,b+c)\,{}_2F_1(a, b; c-1; z) == 0$$

07.24.17.0029.01

$$(c-1)\,c\,(b+(a-c)\,z)\,{}_2F_1(a, b; c-1; z) - b\,c\,(c+(a+b-2\,c+1)\,z-1)\,{}_2F_1(a, b+1; c; z) - (a-c)\,(b-c)\,(b-c+1)\,z\,{}_2F_1(a, b; c+1; z) == 0$$

07.24.17.0030.01

$$c\,{}_2F_1(a-1, b; c; z) - c\,{}_2F_1(a, b-1; c; z) - (a-b)\,z\,{}_2F_1(a, b; c+1; z) == 0$$

07.24.17.0031.01

$$(b-c+1)\,(a-c)\,{}_2F_1(a-1, b; c; z) + b\,(1-a+(c-b-1)\,z)\,{}_2F_1(a, b+1; c; z) + (c-1)\,(a+b-c)\,{}_2F_1(a, b; c-1; z) == 0$$

07.24.17.0032.01

$$a\,(1-b+(c-a-1)\,z)\,{}_2F_1(a+1, b; c; z) + (a-c+1)\,(b-c)\,{}_2F_1(a, b-1; c; z) + (c-1)\,(a+b-c)\,{}_2F_1(a, b; c-1; z) == 0$$

07.24.17.0033.01

$$(b-c+1)\,a\,{}_2F_1(a+1, b; c; z) - b\,(a-c+1)\,{}_2F_1(a, b+1; c; z) + (c-1)\,(a-b)\,{}_2F_1(a, b; c-1; z) == 0$$

07.24.17.0034.01

$$a\,c\,(b+(a-c)\,z)\,{}_2F_1(a+1, b; c; z) - b\,c\,(a+(b-c)\,z)\,{}_2F_1(a, b+1; c; z) + (a-b)\,(a-c)\,(b-c)\,z\,{}_2F_1(a, b; c+1; z) == 0$$

Additional relations between contiguous functions

07.24.17.0035.01

$$c\,{}_2F_1(a, b; c; z) - a\,{}_2F_1(a+1, b; c+1; z) + (a-c)\,{}_2F_1(a, b; c+1; z) == 0$$

07.24.17.0036.01

$$(a-b)\,c\,{}_2F_1(a, b; c; z) - a\,(c-b)\,{}_2F_1(a+1, b; c+1; z) + (c-a)\,b\,{}_2F_1(a, b+1; c+1; z) == 0$$

Relations for fixed z

07.24.17.0037.01

$${}_2F_1(a, b; c; -1) == 2^{-a}\,{}_2F_1\left(a, c-b; c; \frac{1}{2}\right)$$

07.24.17.0038.01

$${}_2F_1\left(a, b; c; \frac{1}{2}\right) = 2^a\, {}_2F_1(a, c-b; c; -1)$$

07.24.17.0039.01

$${}_2F_1(-n, b; 1; 2) = \frac{(-1)^n\, (b)_n}{n!}\, {}_2F_1(-n, 1-b; 1-b-n; -1) \;/;\; n \in \mathbb{N}$$

Relations of special kind

07.24.17.0040.01

$${}_2\tilde{F}_1(a, b; -n; z) = z^{n+1}\, (a)_{n+1}\, (b)_{n+1}\, {}_2\tilde{F}_1(a+n+1, b+n+1; n+2; z) \;/;\; n \in \mathbb{N}$$

07.24.17.0041.01

$${}_2F_1(a, b; a+1; z) + {}_2F_1(-a, b; 1-a; z) = 2\, {}_3F_2(a, -a, b; a+1, 1-a; z)$$

07.24.17.0042.01

$${}_2F_1(-a, a+1; c; z) + {}_2F_1(a, 1-a; c; z) = 2\, {}_2F_1(a, -a; c; z)$$

07.24.17.0043.01

$${}_2F_1(a, b; c; z)\, {}_2F_1(-a, -b; -c; z) + \frac{a\, b\, (a-c)\, (b-c)}{c^2\, \left(1-c^2\right)}\, z^2\, {}_2F_1(1-a, 1-b; 2-c; z)\, {}_2F_1(a+1, b+1; c+2; z) = 1$$

Reduction to polynomial

07.24.17.0044.01

$${}_2F_1(a, b; b-n; z) = (1-z)^{-a}\, {}_2F_1\left(-n, a; b-n; \frac{z}{z-1}\right) \;/;\; n \in \mathbb{N}$$

07.24.17.0045.01

$${}_2F_1(a, b; b-n; z) = \frac{(-1)^n\, (a)_n}{(1-b)_n}\, (1-z)^{-a-n}\, {}_2F_1(-n, b-a-n; 1-a-n; 1-z) \;/;\; n \in \mathbb{N}$$

07.24.17.0046.01

$${}_2F_1(a, b; b-n; z) = \frac{(-1)^n\, (a-b+1)_n}{(1-b)_n}\, z^n\, (1-z)^{-a-n}\, {}_2F_1\left(-n, 1-b; a-b+1; \frac{1}{z}\right) \;/;\; n \in \mathbb{N}$$

07.24.17.0047.01

$${}_2F_1(a, b; b-n; z) = \frac{(a-b+1)_n}{(1-b)_n}\, (1-z)^{-a}\, {}_2F_1\left(-n, a; a-b+1; \frac{1}{1-z}\right) \;/;\; n \in \mathbb{N}$$

07.24.17.0048.01

$${}_2F_1(a, b; b-n; z) = \frac{(a)_n}{(1-b)_n}\, (-z)^n\, (1-z)^{-a-n}\, {}_2F_1\left(-n, 1-b; 1-a-n; 1-\frac{1}{z}\right) \;/;\; n \in \mathbb{N}$$

07.24.17.0049.01

$${}_2F_1(a, b; b-n; z) = (1-z)^{-a-n}\, {}_2F_1(-n, -a+b-n; b-n; z) \;/;\; n \in \mathbb{N}$$

Division on even and odd parts and generalization

07.24.17.0050.01

$${}_2F_1(a, b; c; z) = A^+(z) + A^-(z) \;/;\; A^+(z) = \frac{1}{2}\left({}_2F_1(a, b; c; z) + {}_2F_1(a, b; c; -z)\right) \bigwedge A^-(z) = \frac{1}{2}\left({}_2F_1(a, b; c; z) - {}_2F_1(a, b; c; -z)\right)$$

07.24.17.0051.01

$${}_2F_1(a, b; c; z) = A^+(z) + A^-(z) \;/;$$
$$A^+(z) = {}_4F_3\left(\frac{a}{2}, \frac{b}{2}, \frac{a+1}{2}, \frac{b+1}{2}; \frac{1}{2}, \frac{c}{2}, \frac{c+1}{2}; z^2\right) \bigwedge A^-(z) = \frac{a\,b\,z}{c}\, {}_4F_3\left(\frac{a+1}{2}, \frac{b+1}{2}, \frac{a+2}{2}, \frac{b+2}{2}; \frac{3}{2}, \frac{c+1}{2}, \frac{c+2}{2}; z^2\right)$$

07.24.17.0052.01

$${}_2F_1(a, b; c; z) = \sum_{k=0}^{n-1} \frac{(a)_k\,(b)_k\,z^k}{k!\,(c)_k}\, {}_{2n+1}F_{2n}\left(1, \frac{a+k}{n}, \ldots, \frac{a+k+n-1}{n}, \frac{b+k}{n}, \ldots, \frac{b+k+n-1}{n}; \frac{k+1}{n}, \ldots, \frac{k+n}{n}, \frac{c+k}{n}, \ldots, \frac{c+k+n-1}{n}; z^n\right)$$

Major general cases

07.24.17.0053.01

$${}_2\tilde{F}_1(a, b; c; z) = (1-z)^{c-a-b}\, {}_2\tilde{F}_1(c-a, c-b; c; z)$$

07.24.17.0054.01

$${}_2\tilde{F}_1(a, b; c; z) = (1-z)^{-a}\, {}_2\tilde{F}_1\left(a, c-b; c; \frac{z}{z-1}\right) /;\, z \notin (1, \infty)$$

07.24.17.0055.01

$${}_2\tilde{F}_1(a, b; c; z) = (1-z)^{-b}\, {}_2\tilde{F}_1\left(c-a, b; c; \frac{z}{z-1}\right) /;\, z \notin (1, \infty)$$

07.24.17.0056.01

$${}_2\tilde{F}_1(a, b; c; z) = \pi \csc(\pi\,(b-a))\left(\frac{1}{\Gamma(b)\,\Gamma(c-a)}\,(-z)^{-a}\, {}_2\tilde{F}_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) - \frac{1}{\Gamma(a)\,\Gamma(c-b)}\,(-z)^{-b}\, {}_2\tilde{F}_1\left(b, b-c+1; -a+b+1; \frac{1}{z}\right)\right)$$

07.24.17.0057.01

$${}_2\tilde{F}_1(a, b; c; z) = \pi \csc(\pi\,(c-a-b))\left(\frac{1}{\Gamma(c-a)\,\Gamma(c-b)}\, {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z) - \frac{1}{\Gamma(a)\,\Gamma(b)}\,(1-z)^{-a-b+c}\, {}_2\tilde{F}_1(c-a, c-b; -a-b+c+1; 1-z)\right) /;\, c-a-b \notin \mathbb{Z}$$

07.24.17.0058.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{\pi}{\sin(\pi\,(b-a))}\left(\frac{1}{\Gamma(b)\,\Gamma(c-a)}\,(1-z)^{-a}\, {}_2\tilde{F}_1\left(a, c-b; a-b+1; \frac{1}{1-z}\right) - \frac{1}{\Gamma(a)\,\Gamma(c-b)}\,(1-z)^{-b}\, {}_2\tilde{F}_1\left(b, c-a; -a+b+1; \frac{1}{1-z}\right)\right) /;\, a-b \notin \mathbb{Z}$$

07.24.17.0059.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{\pi}{\sin(\pi\,(c-a-b))}\left(\frac{1}{\Gamma(c-a)\,\Gamma(c-b)}\, z^{-a}\, {}_2\tilde{F}_1\left(a, a-c+1; a+b-c+1; 1-\frac{1}{z}\right) - \frac{1}{\Gamma(a)\,\Gamma(b)}\, z^{a-c}\,(1-z)^{c-a-b}\, {}_2\tilde{F}_1\left(c-a, 1-a; c-a-b+1; 1-\frac{1}{z}\right)\right) /;\, c-a-b \notin \mathbb{Z} \bigwedge z \notin (-\infty, 0)$$

Relations including three Kummer's solutions

Below relations hold for $z \in \mathbb{C} \setminus \mathbb{R}$ and all values of parameters a, b, c, for which the gamma factors are finite.

07.24.17.0060.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(1-b)\,\Gamma(a-c+1)}{\Gamma(c)\,\Gamma(1-c)} (-z)^{-a} {}_2\tilde{F}_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(1-b)\,\Gamma(a-c+1)}{\Gamma(a)\,\Gamma(c-b)} (-z)^{1-c} {}_2\tilde{F}_1(a-c+1, b-c+1; 2-c; z) \,/;\, z \notin (0,1) $$

07.24.17.0061.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(1-b)}{\Gamma(a)} z^{c-a}(-z)^{a-c} (1-z)^{c-a-b} {}_2\tilde{F}_1(c-a, c-b; c-a-b+1; 1-z) + \frac{\Gamma(1-b)}{\Gamma(c-a)} z^{-a} {}_2\tilde{F}_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) /;\, z \notin (0,1) $$

07.24.17.0062.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(a-c+1)}{\Gamma(b)} z^{b-c}(-z)^{c-a-b} {}_2\tilde{F}_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(a-c+1)}{\Gamma(c-b)} z^{b}(-z)^{-b} {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z) \,/;\, z \notin (0,1) $$

07.24.17.0063.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(b-c+1)}{\Gamma(c-a)} z^{a}(-z)^{-a} {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z) + \frac{\Gamma(b-c+1)}{\Gamma(a)} z^{a-c}(-z)^{c-a-b} {}_2\tilde{F}_1\left(b-c+1, b; b-a+1; \frac{1}{z}\right) /;\, z \notin (0,1) $$

07.24.17.0064.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(a-c+1)\,\Gamma(b-c+1)}{\Gamma(a)\,\Gamma(b)} z^{1-c} {}_2\tilde{F}_1(a-c+1, b-c+1; 2-c; z) + \frac{\Gamma(a-c+1)\,\Gamma(b-c+1)}{\Gamma(c)\,\Gamma(1-c)} {}_2\tilde{F}_1(a, b; a+b-c+1; 1-z) $$

07.24.17.0065.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(c)\,\Gamma(1-c)} (-z)^{-b} {}_2\tilde{F}_1\left(b-c+1, b; b-a+1; \frac{1}{z}\right) + \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(b)\,\Gamma(c-a)} (-z)^{1-c} {}_2\tilde{F}_1(a-c+1, b-c+1; 2-c; z) \,/;\, z \notin (0,1) $$

07.24.17.0066.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(1-a)}{\Gamma(b)} (-z)^{b-c} z^{c-b}(1-z)^{c-a-b} {}_2\tilde{F}_1(c-a, c-b; c-a-b+1; 1-z) + \frac{\Gamma(1-a)}{\Gamma(c-b)} z^{-b} {}_2\tilde{F}_1\left(b-c+1, b; b-a+1; \frac{1}{z}\right) /;\, z \notin (0,1) $$

07.24.17.0067.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c)\,\Gamma(1-c)} (1-z)^{c-a-b} {}_2\tilde{F}_1(c-a, c-b; -a-b+c+1; 1-z) + \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c-a)\,\Gamma(c-b)} z^{1-c} {}_2\tilde{F}_1(a-c+1, b-c+1; 2-c; z) $$

07.24.17.0068.01

$$ {}_2\tilde{F}_1(a,b;c;z) = \frac{\Gamma(1-b)}{\Gamma(a)} (z-1)^{c-a} (1-z)^{-b} {}_2\tilde{F}_1(c-b, c-a; -a-b+c+1; 1-z) + \frac{\Gamma(1-b)}{\Gamma(c-a)} (z-1)^{-a} {}_2\tilde{F}_1\left(a, c-b; a-b+1; \frac{1}{1-z}\right) /;\, z \notin (0,1) $$

07.24.17.0069.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)}{\Gamma(b)}\,(1-z)^{-a}\,(z-1)^{c-b}\,{}_2\tilde{F}_1(c-b,\,c-a;\,c-a-b+1;\,1-z) + \frac{\Gamma(1-a)}{\Gamma(c-b)}\,(z-1)^{-b}\,{}_2\tilde{F}_1\!\left(c-a,\,b;\,b-a+1;\,\frac{1}{1-z}\right) /;\, z\notin(0,\,1)$$

07.24.17.0070.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-b)\,\Gamma(a-c+1)}{\Gamma(c)\,\Gamma(1-c)}\,(1-z)^{-a}\,{}_2\tilde{F}_1\!\left(a,\,c-b;\,a-b+1;\,\frac{1}{1-z}\right) + \frac{\Gamma(1-b)\,\Gamma(a-c+1)}{\Gamma(a)\,\Gamma(c-b)}\,(1-z)^{1-c}\,(z-1)^{c-1}\,z^{1-c}\,{}_2\tilde{F}_1(b-c+1,\,a-c+1;\,2-c;\,z)\ /;\, z\notin(0,\,1)$$

07.24.17.0071.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(c)\,\Gamma(1-c)}\,(1-z)^{-b}\,{}_2\tilde{F}_1\!\left(c-a,\,b;\,b-a+1;\,\frac{1}{1-z}\right) - \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(b)\,\Gamma(c-a)}\,z^{1-c}(1-z)^{-c}\,(z-1)^{c}\,{}_2\tilde{F}_1(b-c+1,\,a-c+1;\,2-c;\,z)\ /;\, z\notin(0,\,1)$$

07.24.17.0072.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c-a)\,\Gamma(c-b)}\,z^{1-c}\,{}_2\tilde{F}_1(b-c+1,\,a-c+1;\,2-c;\,z) + \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c)\,\Gamma(1-c)}\,(1-z)^{c-a-b}\,{}_2\tilde{F}_1(c-b,\,c-a;\,c-a-b+1;\,1-z)$$

07.24.17.0073.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(c)\,\Gamma(1-c)}\left(-\frac{1}{z}\right)^{b}\,{}_2\tilde{F}_1\!\left(b,\,b-c+1;\,b-a+1;\,\frac{1}{z}\right) + \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(b)\,\Gamma(c-a)}\left(-\frac{1}{z}\right)^{c-1}\,{}_2\tilde{F}_1(b-c+1,\,a-c+1;\,2-c;\,z)\ /;\, z\notin(1,\,\infty)$$

07.24.17.0074.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c)\,\Gamma(1-c)}\left(\frac{1}{z}\right)^{c-b}\left(-\frac{1}{z}\right)^{a+b-c}\left(1-\frac{1}{z}\right)^{c-a-b}\,{}_2\tilde{F}_1\!\left(1-b,\,c-b;\,-a-b+c+1;\,1-\frac{1}{z}\right) + \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c-a)\,\Gamma(c-b)}\left(\frac{1}{z}\right)^{c-1}\,{}_2\tilde{F}_1(b-c+1,\,a-c+1;\,2-c;\,z) /;\, z\notin(1,\,\infty)$$

07.24.17.0075.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)}{\Gamma(b)}\left(-\frac{1}{z}\right)^{a}\left(1-\frac{1}{z}\right)^{c-a-b}\,{}_2\tilde{F}_1\!\left(1-b,\,c-b;\,-a-b+c+1;\,1-\frac{1}{z}\right) + \frac{\Gamma(1-a)}{\Gamma(c-b)}\left(\frac{1}{z}\right)^{b}\,{}_2\tilde{F}_1\!\left(b,\,b-c+1;\,-a+b+1;\,\frac{1}{z}\right) /;\, z\notin(1,\,\infty)$$

07.24.17.0076.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-b)}{\Gamma(c-a)}\left(\frac{1}{z}\right)^{a}\,{}_2\tilde{F}_1\!\left(a-c+1,\,a;\,a-b+1;\,\frac{1}{z}\right) + \frac{\Gamma(1-b)}{\Gamma(a)}\left(1-\frac{1}{z}\right)^{c-a-b}\left(-\frac{1}{z}\right)^{b}\,{}_2\tilde{F}_1\!\left(c-a,\,1-a;\,c-a-b+1;\,1-\frac{1}{z}\right) /;\, z\notin(1,\,\infty)$$

Quadratic transformations with fixed *a, b, z*

07.24.17.0077.01

$${}_2\tilde{F}_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}}\, {}_2\tilde{F}_1\left(2a-1, 2b-1; a+b-\frac{1}{2}; \frac{1}{2}\left(1-\sqrt{1-z}\right)\right)$$

07.24.17.0078.01

$${}_2\tilde{F}_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}} \left(\frac{\sqrt{1-z}+1}{2}\right)^{1-2a} {}_2\tilde{F}_1\left(2a-1, a-b+\frac{1}{2}; a+b-\frac{1}{2}; \frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right)$$

07.24.17.0079.01

$${}_2\tilde{F}_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{2^{2a+2b-3}\,\Gamma(a+b-1)}{\sqrt{\pi}} \frac{\left(\sqrt{1-z}+\sqrt{-z}\right)^{1-2a}}{\sqrt{1-z}}\, {}_2\tilde{F}_1\left(2a-1, a+b-1; 2a+2b-2; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0080.01

$${}_2\tilde{F}_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{2^{2a+2b-3}\,\Gamma(a+b-1)}{\sqrt{\pi}} \frac{\left(\sqrt{1-z}-\sqrt{-z}\right)^{1-2a}}{\sqrt{1-z}}\, {}_2\tilde{F}_1\left(2a-1, a+b-1; 2a+2b-2; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) > \frac{1}{2}$$

07.24.17.0081.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; z\right) = {}_2\tilde{F}_1\left(2a, 2b; a+b+\frac{1}{2}; \frac{1-\sqrt{1-z}}{2}\right)$$

07.24.17.0082.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; z\right) = \left(\frac{\sqrt{1-z}+1}{2}\right)^{-2a} {}_2\tilde{F}_1\left(2a, a-b+\frac{1}{2}; a+b+\frac{1}{2}; \frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right)$$

07.24.17.0083.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; z\right) = \frac{2^{2a+2b-1}\,\Gamma(a+b)}{\sqrt{\pi}} \left(\sqrt{1-z}+\sqrt{-z}\right)^{-2a} {}_2\tilde{F}_1\left(2a, a+b; 2a+2b; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0084.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; z\right) = \frac{2^{2a+2b-1}\,\Gamma(a+b)}{\sqrt{\pi}} \left(\sqrt{1-z}-\sqrt{-z}\right)^{-2a} {}_2\tilde{F}_1\left(2a, a+b; 2a+2b; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) > \frac{1}{2}$$

07.24.17.0085.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; 4z(1-z)\right) = {}_2\tilde{F}_1\left(2a, 2b; a+b+\frac{1}{2}; z\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0086.01

$${}_2\tilde{F}_1(a, b; a-b+1; z) = (1-z)^{-a}\, {}_2\tilde{F}_1\left(\frac{a}{2}, \frac{a+1}{2}-b; a-b+1; -\frac{4z}{(1-z)^2}\right) /; |z| < 1$$

07.24.17.0087.01

$${}_2\tilde{F}_1(a, b; a-b+1; z) = \frac{z+1}{(1-z)^{a+1}}\, {}_2\tilde{F}_1\left(\frac{a+1}{2}, \frac{a}{2}-b+1; a-b+1; -\frac{4z}{(1-z)^2}\right) /; |z| < 1$$

07.24.17.0088.01

$${}_2\tilde{F}_1(a, b; a-b+1; z) = (z+1)^{-a}\, {}_2\tilde{F}_1\left(\frac{a}{2}, \frac{a+1}{2}; a-b+1; \frac{4z}{(z+1)^2}\right) /; |z| < 1$$

07.24.17.0089.01

$${}_2\tilde{F}_1(a, b; a-b+1; z) = (1-z)^{1-2b} (z+1)^{2b-a-1}\, {}_2\tilde{F}_1\left(\frac{a+1}{2} - b, \frac{a}{2} - b + 1; a-b+1; \frac{4z}{(z+1)^2}\right) /; |z| < 1$$

07.24.17.0090.01

$${}_2\tilde{F}_1(a, b; a-b+1; z) = \frac{2^{2a-2b}\, \Gamma\left(a-b+\frac{1}{2}\right)}{\sqrt{\pi}} \left(\sqrt{z}+1\right)^{-2a} {}_2\tilde{F}_1\left(a, a-b+\frac{1}{2}; 2a-2b+1; \frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right) /; |z| < 1$$

07.24.17.0091.01

$${}_2\tilde{F}_1(a, b; a-b+1; z) = \frac{2^{2a-2b}\, \Gamma\left(a-b+\frac{1}{2}\right)}{\sqrt{\pi}} \left(1-\sqrt{z}\right)^{-2a} {}_2\tilde{F}_1\left(a, a-b+\frac{1}{2}; 2a-2b+1; -\frac{4\sqrt{z}}{\left(1-\sqrt{z}\right)^2}\right) /; |z| < 1$$

07.24.17.0092.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; z\right) = {}_2\tilde{F}_1\left(\frac{a}{2}, \frac{b}{2}; \frac{a+b+1}{2}; 4z(1-z)\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0093.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; z\right) = (1-2z)\, {}_2\tilde{F}_1\left(\frac{a+1}{2}, \frac{b+1}{2}; \frac{a+b+1}{2}; 4z(1-z)\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0094.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; z\right) = (1-2z)^{-a}\, {}_2\tilde{F}_1\left(\frac{a}{2}, \frac{a+1}{2}; \frac{a+b+1}{2}; \frac{4z(z-1)}{(2z-1)^2}\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0095.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; z\right) = \frac{2^{a+b-1}\, \Gamma\left(\frac{a+b}{2}\right)}{\sqrt{\pi}} \left(\sqrt{1-z}+\sqrt{-z}\right)^{-2a} {}_2\tilde{F}_1\left(a, \frac{a+b}{2}; a+b; \frac{4\sqrt{z^2-z}}{\left(\sqrt{1-z}+\sqrt{-z}\right)^2}\right) /; \operatorname{Re}(z) < \frac{1}{2}$$

07.24.17.0096.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; z\right) = \frac{\pi(2z-1)}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{b}{2}\right)}\, {}_2\tilde{F}_1\left(\frac{a+1}{2}, \frac{b+1}{2}; \frac{3}{2}; (2z-1)^2\right) + \frac{\pi}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)}\, {}_2\tilde{F}_1\left(\frac{a}{2}, \frac{b}{2}; \frac{1}{2}; (2z-1)^2\right)$$

07.24.17.0097.01

$${}_2\tilde{F}_1(a, b; 2b; z) = \frac{2^{1-2b}\sqrt{\pi}}{\Gamma(b)} (1-z)^{-\frac{a}{2}}\, {}_2\tilde{F}_1\left(\frac{a}{2}, b-\frac{a}{2}; b+\frac{1}{2}; \frac{z^2}{4(z-1)}\right)$$

07.24.17.0098.01

$${}_2\tilde{F}_1(a, b; 2b; z) = \frac{2^{-2b}\sqrt{\pi}}{\Gamma(b)} (2-z)(1-z)^{-\left(\frac{a+1}{2}\right)}\, {}_2\tilde{F}_1\left(\frac{1-a}{2}+b, \frac{a+1}{2}; b+\frac{1}{2}; \frac{z^2}{4(z-1)}\right)$$

07.24.17.0099.01

$${}_2\tilde{F}_1(a, b; 2b; z) = \frac{2^{a-2b+1}\sqrt{\pi}}{\Gamma(b)} (2-z)^{-a}\, {}_2\tilde{F}_1\left(\frac{a}{2}, \frac{a+1}{2}; b+\frac{1}{2}; \frac{z^2}{(2-z)^2}\right) /; z \notin (2, \infty)$$

07.24.17.0100.01

$${}_2\tilde{F}_1(a,\,b;\,2\,b;\,z) = \frac{2^{1-a}\sqrt{\pi}}{\Gamma(b)}\,(1-z)^{b-a}\,(2-z)^{a-2\,b}\,{}_2\tilde{F}_1\!\left(b-\frac{a}{2},\,\frac{1-a}{2}+b;\,b+\frac{1}{2};\,\frac{z^2}{(2-z)^2}\right)/;\,z\notin(2,\,\infty)$$

07.24.17.0101.01

$${}_2\tilde{F}_1(a,\,b;\,2\,b;\,z) = \frac{2^{1-2\,b}\sqrt{\pi}}{\Gamma(b)}\,(1-z)^{-\frac{a}{2}}\,{}_2\tilde{F}_1\!\left(a,\,2\,b-a;\,b+\frac{1}{2};\,-\frac{\left(1-\sqrt{1-z}\right)^2}{4\sqrt{1-z}}\right)$$

07.24.17.0102.01

$${}_2\tilde{F}_1(a,\,b;\,2\,b;\,z) = \frac{2^{2\,a-2\,b+1}\sqrt{\pi}}{\Gamma(b)}\left(1+\sqrt{1-z}\right)^{-2\,a}\,{}_2\tilde{F}_1\!\left(a,\,a-b+\frac{1}{2};\,b+\frac{1}{2};\,\left(\frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)^2\right)$$

07.24.17.0103.01

$${}_2\tilde{F}_1\!\left(a,\,b;\,\frac{1}{2};\,z\right) = \frac{\Gamma\!\left(a+\frac{1}{2}\right)\Gamma\!\left(b+\frac{1}{2}\right)}{2\,\pi}\left({}_2\tilde{F}_1\!\left(2\,a,\,2\,b;\,a+b+\frac{1}{2};\,\frac{1-\sqrt{z}}{2}\right)+{}_2\tilde{F}_1\!\left(2\,a,\,2\,b;\,a+b+\frac{1}{2};\,\frac{1+\sqrt{z}}{2}\right)\right)$$

07.24.17.0104.01

$${}_2\tilde{F}_1\!\left(a,\,b;\,\frac{1}{2};\,z\right) = \frac{\Gamma\!\left(a+\frac{1}{2}\right)\Gamma(1-b)}{2\,\pi}\,(1-z)^{-a}$$
$$\left({}_2\tilde{F}_1\!\left(2\,a,\,1-2\,b;\,a-b+1;\,\frac{\sqrt{1-z}-\sqrt{-z}}{2\sqrt{1-z}}\right)+{}_2\tilde{F}_1\!\left(2\,a,\,1-2\,b;\,a-b+1;\,\frac{\sqrt{1-z}+\sqrt{-z}}{2\sqrt{1-z}}\right)\right)/;\,z\notin(1,\,\infty)$$

07.24.17.0105.01

$${}_2\tilde{F}_1\!\left(a,\,b;\,\frac{3}{2};\,z\right) = \frac{\Gamma\!\left(a-\frac{1}{2}\right)\Gamma\!\left(b-\frac{1}{2}\right)}{2\,\pi}\,z^{-\frac{1}{2}}\left({}_2\tilde{F}_1\!\left(2\,a-1,\,2\,b-1;\,a+b-\frac{1}{2};\,\frac{1+\sqrt{z}}{2}\right)-{}_2\tilde{F}_1\!\left(2\,a-1,\,2\,b-1;\,a+b-\frac{1}{2};\,\frac{1-\sqrt{z}}{2}\right)\right)$$

Quadratic transformations with fixed *a*, *c*, *z*

07.24.17.0106.01

$${}_2\tilde{F}_1(a,\,1-a;\,c;\,z) = (1-z)^{c-1}\,{}_2\tilde{F}_1\!\left(\frac{c-a}{2},\,\frac{a+c-1}{2};\,c;\,4\,z\,(1-z)\right)/;\,\mathrm{Re}(z)<\frac{1}{2}$$

07.24.17.0107.01

$${}_2\tilde{F}_1(a,\,1-a;\,c;\,z) = (1-z)^{c-1}\,(1-2\,z)\,{}_2\tilde{F}_1\!\left(\frac{a+c}{2},\,\frac{1+c-a}{2};\,c;\,4\,z\,(1-z)\right)/;\,\mathrm{Re}(z)<\frac{1}{2}$$

07.24.17.0108.01

$${}_2\tilde{F}_1(a,\,1-a;\,c;\,z) = (1-z)^{c-1}\,(1-2\,z)^{a-c}\,{}_2\tilde{F}_1\!\left(\frac{c-a}{2},\,\frac{1+c-a}{2};\,c;\,\frac{4\,z\,(z-1)}{(1-2\,z)^2}\right)/;\,\mathrm{Re}(z)<\frac{1}{2}$$

07.24.17.0109.01

$${}_2\tilde{F}_1(a,\,1-a;\,c;\,z) =$$
$$\frac{2^{2\,c-2}\,\Gamma\!\left(c-\frac{1}{2}\right)}{\sqrt{\pi}}\,(1-z)^{c-1}\left(\sqrt{1-z}+\sqrt{-z}\right)^{2-2\,a-2\,c}\,{}_2\tilde{F}_1\!\left(a+c-1,\,c-\frac{1}{2};\,2\,c-1;\,\frac{4\sqrt{z\,(z-1)}}{\left(\sqrt{1-z}+\sqrt{-z}\right)^2}\right)/;\,\mathrm{Re}(z)<\frac{1}{2}$$

07.24.17.0110.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, z\right) = (1-z)^{-a}\, {}_2\tilde{F}_1\left(2\,a,\, -2\,a+2\,c-1;\, c;\, \frac{\sqrt{1-z}\,-1}{2\,\sqrt{1-z}}\right)$$

07.24.17.0111.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, z\right) = \left(\frac{\sqrt{1-z}\,+1}{2}\right)^{-2\,a} {}_2\tilde{F}_1\left(2\,a,\, 2\,a-c+1;\, c;\, \frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)$$

07.24.17.0112.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, z\right) = \frac{2^{2\,c-2}\,\Gamma\left(c-\frac{1}{2}\right)}{\sqrt{\pi}}\left(1+\sqrt{z}\right)^{-2\,a} {}_2\tilde{F}_1\left(2\,a,\, c-\frac{1}{2};\, 2\,c-1;\, \frac{2\,\sqrt{z}}{\sqrt{z}\,+1}\right)$$

07.24.17.0113.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, z\right) = \frac{2^{2\,c-2}\,\Gamma\left(c-\frac{1}{2}\right)}{\sqrt{\pi}}\left(1-\sqrt{z}\right)^{-2\,a} {}_2\tilde{F}_1\left(2\,a,\, c-\frac{1}{2};\, 2\,c-1;\, -\frac{2\,\sqrt{z}}{1-\sqrt{z}}\right) /;\, z \notin (1,\, \infty)$$

Cubic transformations

07.24.17.0114.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, \frac{4\,a+2}{3};\, z\right) = \frac{2^{\frac{2\,a+1}{3}}\,\sqrt{\pi}}{\Gamma\left(\frac{2\,a+1}{3}\right)}\,(8-9\,z)^{-\frac{2\,a}{3}}\, {}_2\tilde{F}_1\left(\frac{a}{3},\, \frac{a}{3}+\frac{1}{2};\, \frac{4\,a+5}{6};\, -\frac{27\,z^2\,(1-z)}{(8-9\,z)^2}\right) /;\, |z-1| > \frac{1}{3}$$

07.24.17.0115.01

$${}_2\tilde{F}_1\left(a,\, a-\frac{1}{2};\, \frac{4\,a}{3};\, z\right) = \frac{2^{\frac{2\,a}{3}}\,\sqrt{\pi}}{\Gamma\left(\frac{2\,a}{3}\right)}\,(8-9\,z)^{\frac{1-2\,a}{3}}\, {}_2\tilde{F}_1\left(\frac{2\,a-1}{6},\, \frac{a+1}{3};\, \frac{4\,a+3}{6};\, \frac{27\,(z-1)\,z^2}{(8-9\,z)^2}\right) /;\, |z-1| > \frac{1}{3}$$

07.24.17.0116.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, \frac{4\,a+5}{6};\, z\right) = (1-9\,z)^{-\frac{2\,a}{3}}\, {}_2\tilde{F}_1\left(\frac{a}{3},\, \frac{a}{3}+\frac{1}{2};\, \frac{4\,a+5}{6};\, -\frac{27\,z\,(1-z)^2}{(1-9\,z)^2}\right) /;\, |z| < \frac{1}{9}$$

07.24.17.0117.01

$${}_2\tilde{F}_1\left(a,\, a-\frac{1}{2};\, \frac{4\,a+3}{6};\, z\right) = (1-9\,z)^{\frac{1}{3}(1-2\,a)}\, {}_2\tilde{F}_1\left(\frac{2\,a-1}{6},\, \frac{a+1}{3};\, \frac{4\,a+3}{6};\, -\frac{27\,(z-1)^2\,z}{(1-9\,z)^2}\right) /;\, |z| < \frac{1}{9}$$

07.24.17.0118.01

$${}_2\tilde{F}_1\left(a,\, \frac{2\,a+1}{6};\, \frac{4\,a+2}{3};\, z\right) = \frac{2^{\frac{2\,a+1}{3}}\,\sqrt{\pi}}{\Gamma\left(\frac{2\,a+1}{3}\right)}\,(4-z)^{-a}\, {}_2\tilde{F}_1\left(\frac{a}{3},\, \frac{a+1}{3};\, \frac{4\,a+5}{6};\, -\frac{27\,z^2}{(z-4)^3}\right) /;\, z \notin (4,\, \infty)$$

07.24.17.0119.01

$${}_2\tilde{F}_1\left(a,\, 3\,a-\frac{1}{2};\, 4\,a;\, z\right) = \frac{2^{2\,a}\,\sqrt{\pi}}{\Gamma(2\,a)}\,(4-z)^{\frac{1}{2}-3\,a}\, {}_2\tilde{F}_1\left(a-\frac{1}{6},\, a+\frac{1}{6};\, 2\,a+\frac{1}{2};\, -\frac{27\,z^2}{(z-4)^3}\right) /;\, z \notin (4,\, \infty)$$

07.24.17.0120.01

$${}_2\tilde{F}_1\left(a,\, \frac{1-a}{3};\, \frac{4\,a+5}{6};\, z\right) = (1-4\,z)^{-a}\, {}_2\tilde{F}_1\left(\frac{a}{3},\, \frac{a+1}{3};\, \frac{4\,a+5}{6};\, \frac{27\,z}{(4\,z-1)^3}\right) /;\, |z| < \frac{1}{8}$$

07.24.17.0121.01

$$ {}_2\tilde{F}_1\left(a,\, 1-3a;\, \frac{3}{2}-2a;\, z\right) = (1-4z)^{3a-1}\, {}_2\tilde{F}_1\left(\frac{1}{3}-a,\, \frac{2}{3}-a;\, \frac{3}{2}-2a;\, \frac{27z}{(4z-1)^3}\right) /;\, |z| < \frac{1}{8} $$

07.24.17.0122.01

$$ {}_2\tilde{F}_1\left(a,\, \frac{2a+1}{6};\, \frac{1}{2};\, z\right) = (1-z)^{-\frac{2a}{3}}\, {}_2\tilde{F}_1\left(\frac{a}{3},\, \frac{1-2a}{6};\, \frac{1}{2};\, -\frac{z(z-9)^2}{27(1-z)^2}\right) /;\, |z| < 1 $$

07.24.17.0123.01

$$ {}_2\tilde{F}_1\left(a,\, 3a-\frac{1}{2};\, \frac{1}{2};\, z\right) = (1-z)^{\frac{1}{3}-2a}\, {}_2\tilde{F}_1\left(a-\frac{1}{6},\, \frac{1}{3}-a;\, \frac{1}{2};\, -\frac{(z-9)^2 z}{27(z-1)^2}\right) /;\, |z| < 1 $$

07.24.17.0124.01

$$ {}_2\tilde{F}_1\left(a,\, \frac{1-a}{3};\, \frac{1}{2};\, z\right) = (1-z)^{-\frac{a}{3}}\, {}_2\tilde{F}_1\left(\frac{a}{3},\, \frac{1-2a}{6};\, \frac{1}{2};\, \frac{z(9-8z)^2}{27(1-z)}\right) /;\, \mathrm{Re}(z) < \frac{5}{8} \bigvee |z| < \frac{3}{4} $$

07.24.17.0125.01

$$ {}_2\tilde{F}_1\left(a,\, 1-3a;\, \frac{1}{2};\, z\right) = (1-z)^{a-\frac{1}{3}}\, {}_2\tilde{F}_1\left(\frac{1}{3}-a,\, a-\frac{1}{6};\, \frac{1}{2};\, \frac{(9-8z)^2 z}{27(1-z)}\right) /;\, \mathrm{Re}(z) < \frac{5}{8} \bigvee |z| < \frac{3}{4} $$

07.24.17.0126.01

$$ {}_2\tilde{F}_1\left(a,\, \frac{2a+3}{6};\, \frac{3}{2};\, z\right) = \left(1-\frac{z}{9}\right)\left(\frac{z}{3}+1\right)^{-a-1}\, {}_2\tilde{F}_1\left(\frac{a+1}{3},\, \frac{a+2}{3};\, \frac{3}{2};\, \frac{(z-9)^2 z}{(z+3)^3}\right) /;\, |z| < 1 $$

07.24.17.0127.01

$$ {}_2\tilde{F}_1\left(a,\, 3a-\frac{3}{2};\, \frac{3}{2};\, z\right) = \left(1-\frac{z}{9}\right)\left(\frac{z}{3}+1\right)^{\frac{1}{2}-3a}\, {}_2\tilde{F}_1\left(a-\frac{1}{6},\, a+\frac{1}{6};\, \frac{3}{2};\, \frac{z(z-9)^2}{(z+3)^3}\right) /;\, |z| < 1 $$

07.24.17.0128.01

$$ {}_2\tilde{F}_1\left(a,\, 1-\frac{a}{3};\, \frac{3}{2};\, z\right) = \left(1-\frac{4z}{3}\right)^{-a-1}\left(1-\frac{8z}{9}\right) {}_2\tilde{F}_1\left(\frac{a+1}{3},\, \frac{a+2}{3};\, \frac{3}{2};\, \frac{z(9-8z)^2}{(4z-3)^3}\right) /;\, |z| < \frac{3}{4} \bigvee \mathrm{Re}(z) < \frac{5}{8} $$

07.24.17.0129.01

$$ {}_2\tilde{F}_1\left(a,\, 3-3a;\, \frac{3}{2};\, z\right) = \left(1-\frac{8z}{9}\right)\left(1-\frac{4z}{3}\right)^{3a-4}\, {}_2\tilde{F}_1\left(\frac{4}{3}-a,\, \frac{5}{3}-a;\, \frac{3}{2};\, \frac{z(9-8z)^2}{(4z-3)^3}\right) /;\, |z| < \frac{3}{4} \bigvee \mathrm{Re}(z) < \frac{5}{8} $$

Differentiation

Low-order differentiation

With respect to *a*

07.24.20.0001.01

$$ {}_2\tilde{F}_1^{(1,0,0,0)}(a,\, b;\, c;\, z) = \sum_{k=0}^{\infty} \frac{(a)_k\, (b)_k\, \psi(a+k)\, z^k}{\Gamma(c+k)\, k!} - \psi(a)\, {}_2\tilde{F}_1(a,\, b;\, c;\, z) /;\, |z| < 1 $$

07.24.20.0002.01

$$ {}_2\tilde{F}_1^{(1,0,0,0)}(a,\, b;\, c;\, z) = z\, b\, \Gamma(a+1)\, \tilde{F}_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix} a+1,\, b+1;\, 1;\, 1,\, a; \\ 2,\, c+1;;\, a+1; \end{matrix}\; z,\, z\right) $$

With respect to *b*

07.24.20.0003.01

$${}_2\tilde{F}_1^{(0,1,0,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k \psi(b+k) z^k}{\Gamma(c+k) k!} - \psi(b)\, {}_2\tilde{F}_1(a, b; c; z) \;/;\; |z| < 1$$

07.24.20.0004.01

$${}_2\tilde{F}_1^{(0,1,0,0)}(a, b; c; z) = z\, a\, \Gamma(b+1)\, \tilde{F}_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix} a+1, b+1; 1; 1, b; \\ 2, c+1;; b+1; \end{matrix} \; z, z \right)$$

With respect to *c*

07.24.20.0005.01

$${}_2\tilde{F}_1^{(0,0,1,0)}(a, b; c; z) = -\sum_{k=0}^{\infty} \frac{(a)_k (b)_k \psi(c+k) z^k}{\Gamma(c+k) k!} \;/;\; |z| < 1$$

07.24.20.0006.01

$${}_2\tilde{F}_1^{(0,0,1,0)}(a, b; c; z) = -z\, a\, b\, \Gamma(c)\, \tilde{F}_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix} a+1, b+1; 1; 1, c; \\ 2, c+1;; c+1; \end{matrix} \; z, z \right) - \psi(c)\, {}_2\tilde{F}_1(a, b; c; z)$$

With respect to element of parameters ||| With respect to element of parameters

07.24.20.0046.01

$$\frac{\partial\, {}_2\tilde{F}_1(a, b; a+1; z)}{\partial a} =$$

$$n!\, z\, b \left(\Gamma(a+1)\, {}_3\tilde{F}_2(a+1, a+1, b+1; a+2, a+2; z) + {}_2\tilde{F}_1(a+1, b+1; a+2; z)\, \psi(a+1)\right) - \frac{\psi(a+1)(1-z)^{-b}}{\Gamma(a+1)}$$

07.24.20.0047.01

$$\frac{\partial\, {}_2\tilde{F}_1(a+1, b; a; z)}{\partial a} = \frac{(1-z)^{-b}}{\Gamma(a)} \left(\frac{b\, z\, \psi(a+1)}{(z-1)\, a} - \psi(a) \right)$$

With respect to *z*

07.24.20.0007.01

$$\frac{\partial\, {}_2\tilde{F}_1(a, b; c; z)}{\partial z} = a\, b\, {}_2\tilde{F}_1(a+1, b+1; c+1; z)$$

07.24.20.0008.01

$$\frac{\partial^2\, {}_2\tilde{F}_1(a, b; c; z)}{\partial z^2} = a(a+1)\, b(b+1)\, {}_2\tilde{F}_1(a+2, b+2; c+2; z)$$

Symbolic differentiation

With respect to *a*

07.24.20.0009.02

$${}_2\tilde{F}_1^{(n,0,0,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(b)_k}{\Gamma(c+k) k!} \frac{\partial^n (a)_k}{\partial a^n} z^k \;/;\; |z| < 1 \land n \in \mathbb{N}$$

With respect to *b*

07.24.20.0010.02

$${}_2\tilde{F}_1^{(0,n,0,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k}{\Gamma(c+k) k!} \frac{\partial^n (b)_k}{\partial b^n} z^k \;/;\; |z| < 1 \land n \in \mathbb{N}$$

With respect to c

07.24.20.0011.02

$$ {}_2\tilde{F}_1^{(0,0,n,0)}(a,\,b;\,c;\,z) = \sum_{k=0}^{\infty} \frac{(a)_k\,(b)_k}{k!} \frac{\partial^n \frac{1}{\Gamma(c+k)}}{\partial c^n}\, z^k \;/;\; |z| < 1 \land n \in \mathbb{N} $$

With respect to element of parameters ||| With respect to element of parameters

07.24.20.0048.02

$$ \frac{\partial^n {}_2\tilde{F}_1(a,\,b;\,a+1;\,z)}{\partial a^n} = $$

$$ \frac{\partial^n \frac{1}{\Gamma(a+1)}}{\partial a^n} (1-z)^{-b} - n!\,z\,b \sum_{k=0}^{n} \frac{(-1)^k\,\Gamma(a+1)^{k+1}}{(n-k)!} \frac{\partial^{n-k} \frac{1}{\Gamma(a+1)}}{\partial a^{n-k}}\, {}_{k+2}\tilde{F}_{k+1}(a+1,\,\ldots,\,a+1,\,b+1;\,a+2,\,\ldots,\,a+2;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0049.02

$$ \frac{\partial^n {}_2\tilde{F}_1(a+1,\,b;\,a;\,z)}{\partial a^n} = b\,z \frac{\partial^n \frac{1}{\Gamma(a+1)}}{\partial a^n} (1-z)^{-b-1} + \frac{\partial^n \frac{1}{\Gamma(a)}}{\partial a^n} (1-z)^{-b} \;/;\; n \in \mathbb{N} $$

With respect to z

07.24.20.0012.02

$$ \frac{\partial^n {}_2\tilde{F}_1(a,\,b;\,c;\,z)}{\partial z^n} = (a)_n\,(b)_n\,{}_2\tilde{F}_1(a+n,\,b+n;\,c+n;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0013.02

$$ \frac{\partial^n {}_2\tilde{F}_1(a,\,b;\,c;\,z)}{\partial z^n} = z^{-n}\,{}_3\tilde{F}_2(1,\,a,\,b;\,1-n,\,c;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0014.02

$$ \frac{\partial^n \left(z^{\alpha}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = z^{\alpha-n}\,\Gamma(\alpha+1)\,{}_3\tilde{F}_2(\alpha+1,\,a,\,b;\,\alpha+1-n,\,c;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0015.02

$$ \frac{\partial^n \left(z^{a+n-1}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = (a)_n\,z^{a-1}\,{}_2\tilde{F}_1(a+n,\,b;\,c;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0016.02

$$ \frac{\partial^n \left(z^{c-1}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = z^{c-n-1}\,{}_2\tilde{F}_1(a,\,b;\,c-n;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0017.02

$$ \frac{\partial^n \left((1-z)^{a+n-1}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = (-1)^n\,(a)_n\,(c-b)_n\,(1-z)^{a-1}\,{}_2\tilde{F}_1(a+n,\,b;\,c+n;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0018.02

$$ \frac{\partial^n \left((1-z)^{a+b-c}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = (c-a)_n\,(c-b)_n\,(1-z)^{a+b-c-n}\,{}_2\tilde{F}_1(a,\,b;\,c+n;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0019.02

$$ \frac{\partial^n \left(z^{c-1}\,(1-z)^{b-c+n}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = z^{c-n-1}\,(1-z)^{b-c}\,{}_2\tilde{F}_1(a-n,\,b;\,c-n;\,z) \;/;\; n \in \mathbb{N} $$

07.24.20.0020.02

$$\frac{\partial^n \left(z^{c-1} (1-z)^{a+b-c} \, {}_2\tilde{F}_1(a, b; c; z)\right)}{\partial z^n} = z^{c-n-1} (1-z)^{a+b-c-n} \, {}_2\tilde{F}_1(a-n, b-n; c-n; z) \,/;\, n \in \mathbb{N}$$

07.24.20.0021.02

$$\frac{\partial^n \left(z^{c-a+n-1} (1-z)^{a+b-c} \, {}_2\tilde{F}_1(a, b; c; z)\right)}{\partial z^n} = (c-a)_n \, z^{c-a-1} (1-z)^{a+b-c-n} \, {}_2\tilde{F}_1(a-n, b; c; z) \,/;\, n \in \mathbb{N}$$

07.24.20.0022.02

$$\frac{\partial^n \left(z^n \, {}_2\tilde{F}_1\left(-n, b; \frac{1}{2}; z\right)\right)}{\partial z^n} = n! \, {}_3\tilde{F}_2\left(-n, n+1, b; \frac{1}{2}, 1; z\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0023.02

$$\frac{\partial^n \left(z^\alpha \, {}_2\tilde{F}_1(-n, b; c; z)\right)}{\partial z^n} = z^{\alpha-n} \, \Gamma(\alpha+1) \, {}_3\tilde{F}_2(-n, \alpha+1, b; \alpha-n+1, c; z) \,/;\, n \in \mathbb{N}$$

07.24.20.0024.02

$$\frac{\partial^n \left(z^{-a} \, {}_2\tilde{F}_1\left(a, b; c; \frac{1}{z}\right)\right)}{\partial z^n} = (-1)^n (a)_n \, z^{-a-n} \, {}_2\tilde{F}_1\left(a+n, b; c; \frac{1}{z}\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0025.02

$$\frac{\partial^n \left(z^{-a} (z-1)^{a+b-c} \, {}_2\tilde{F}_1\left(a, b; c; \frac{1}{z}\right)\right)}{\partial z^n} = (-1)^n (c-b)_n \, z^{-a} (z-1)^{a+b-c-n} \, {}_2\tilde{F}_1\left(a, b-n; c; \frac{1}{z}\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0026.02

$$\frac{\partial^n \left(z^{-a} (z-1)^{a+n-1} \, {}_2\tilde{F}_1\left(a, b; c; \frac{1}{z}\right)\right)}{\partial z^n} = (a)_n (c-b)_n \, z^{-a-n} (z-1)^{a-1} \, {}_2\tilde{F}_1\left(a+n, b; c+n; \frac{1}{z}\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0027.02

$$\frac{\partial^n \left(z^{-a} (z-1)^{a-c+n} \, {}_2\tilde{F}_1\left(a, b; c; \frac{1}{z}\right)\right)}{\partial z^n} = (-1)^n z^{-a} (z-1)^{a-c} \, {}_2\tilde{F}_1\left(a, b-n; c-n; \frac{1}{z}\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0028.02

$$\frac{\partial^n \, {}_2\tilde{F}_1\left(\frac{1}{2} - \left\lfloor \frac{n}{2} \right\rfloor, b; c; z^2\right)}{\partial z^n} =$$

$$(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{2\left\lfloor \frac{n}{2} \right\rfloor} \left(\frac{1}{2}\right)_{\left\lfloor \frac{n}{2} \right\rfloor} \left(n - 2\left\lfloor \frac{n}{2} \right\rfloor + \frac{1}{2}\right)_{\left\lfloor \frac{n}{2} \right\rfloor} (b)_{n-\left\lfloor \frac{n}{2} \right\rfloor} \, z^{n-2\left\lfloor \frac{n}{2} \right\rfloor} \, {}_2\tilde{F}_1\left(n - \left\lfloor \frac{n}{2} \right\rfloor + \frac{1}{2}, b+n-\left\lfloor \frac{n}{2} \right\rfloor; c+n-\left\lfloor \frac{n}{2} \right\rfloor; z^2\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0029.02

$$\frac{\partial^n \left(z \, {}_2\tilde{F}_1\left(-n + \left\lfloor \frac{n}{2} \right\rfloor + \frac{3}{2}, b; c; z^2\right)\right)}{\partial z^n} =$$

$$(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{2\left\lfloor \frac{n}{2} \right\rfloor} \left(\frac{3}{2}\right)_{\left\lfloor \frac{n}{2} \right\rfloor} \left(n - 2\left\lfloor \frac{n}{2} \right\rfloor - \frac{1}{2}\right)_{\left\lfloor \frac{n}{2} \right\rfloor} (b)_{\left\lfloor \frac{n}{2} \right\rfloor} \, z^{-n+2\left\lfloor \frac{n}{2} \right\rfloor+1} \, {}_2\tilde{F}_1\left(\left\lfloor \frac{n}{2} \right\rfloor + \frac{3}{2}, b+\left\lfloor \frac{n}{2} \right\rfloor; c+\left\lfloor \frac{n}{2} \right\rfloor; z^2\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0030.02

$$\frac{\partial^n \left(z^{2c-1} \, {}_2\tilde{F}_1\left(c - n + \left\lfloor \frac{n}{2} \right\rfloor + \frac{1}{2}, b; c; z^2\right)\right)}{\partial z^n} = \frac{(-1)^{n-\left\lfloor \frac{n}{2} \right\rfloor} (1-2c)_n}{(1-c)_{\left\lfloor \frac{n}{2} \right\rfloor}} \, z^{2c-n-1} \, {}_2\tilde{F}_1\left(c + \frac{1}{2}, b; c - \left\lfloor \frac{n}{2} \right\rfloor; z^2\right) \,/;\, n \in \mathbb{N}$$

07.24.20.0031.02

$$\frac{\partial^n \left(z^{2c-2}\,{}_2\tilde{F}_1\left(c-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},\,b;\,c;\,z^2\right)\right)}{\partial z^n}=\frac{(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}(2-2c)_n}{(1-c)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}\,z^{2c-n-2}\,{}_2\tilde{F}_1\left(c-\frac{1}{2},\,b;\,c-n+\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0032.02

$$\frac{\partial^n\left(\left(1-z^2\right)^{b+\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2\tilde{F}_1\left(c+\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},\,b;\,c;\,z^2\right)\right)}{\partial z^n}=$$

$$(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\,2^{2\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}(c-b)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{b-n+\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2\tilde{F}_1\left(c-\frac{1}{2},\,b;\,c+n-\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0033.02

$$\frac{\partial^n\left(z\left(1-z^2\right)^{b+n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{3}{2}}\,{}_2\tilde{F}_1\left(c+n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{3}{2},\,b;\,c;\,z^2\right)\right)}{\partial z^n}=$$

$$(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\,2^{2\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{3}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}(c-b)_{\left\lfloor\frac{n}{2}\right\rfloor}\,z^{-n+2\left\lfloor\frac{n}{2}\right\rfloor+1}\left(1-z^2\right)^{b-\left\lfloor\frac{n}{2}\right\rfloor-\frac{3}{2}}\,{}_2\tilde{F}_1\left(c-\frac{3}{2},\,b;\,c+\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0034.02

$$\frac{\partial^n\left(z^{2c-1}\left(1-z^2\right)^{b-c+n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2\tilde{F}_1\left(n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},\,b;\,c;\,z^2\right)\right)}{\partial z^n}=$$

$$\frac{(-1)^{n-\left\lfloor\frac{n}{2}\right\rfloor}(1-2c)_n\,z^{2c-n-1}\left(1-z^2\right)^{b-c-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}}{(1-c)_{\left\lfloor\frac{n}{2}\right\rfloor}}\,{}_2\tilde{F}_1\left(-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},\,b-\left\lfloor\frac{n}{2}\right\rfloor;\,c-\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0035.02

$$\frac{\partial^n\left(z^{2c-2}\left(1-z^2\right)^{b-c+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}}\,{}_2\tilde{F}_1\left(\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2},\,b;\,c;\,z^2\right)\right)}{\partial z^n}=$$

$$\frac{(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}(2-2c)_n}{(1-c)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}\,z^{2c-n-2}\left(1-z^2\right)^{b-c-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}}\,{}_2\tilde{F}_1\left(-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2},\,b-n+\left\lfloor\frac{n}{2}\right\rfloor;\,c-n+\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0036.02

$$\frac{\partial^n\,{}_2\tilde{F}_1\left(a,\,b;\,\frac{1}{2};\,z^2\right)}{\partial z^n}=2^n\,(a)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,(b)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2\tilde{F}_1\left(a+n-\left\lfloor\frac{n}{2}\right\rfloor,\,b+n-\left\lfloor\frac{n}{2}\right\rfloor;\,n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0037.02

$$\frac{\partial^n\left(z\,{}_2\tilde{F}_1\left(a,\,b;\,\frac{3}{2};\,z^2\right)\right)}{\partial z^n}=2^n\,(a)_{\left\lfloor\frac{n}{2}\right\rfloor}\,(b)_{\left\lfloor\frac{n}{2}\right\rfloor}\,z^{2\left\lfloor\frac{n}{2}\right\rfloor-n+1}\,{}_2\tilde{F}_1\left(a+\left\lfloor\frac{n}{2}\right\rfloor,\,b+\left\lfloor\frac{n}{2}\right\rfloor;\,-n+2\left\lfloor\frac{n}{2}\right\rfloor+\frac{3}{2};\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0038.02

$$\frac{\partial^n\left(\left(1-z^2\right)^{a+b-\frac{1}{2}}\,{}_2\tilde{F}_1\left(a,\,b;\,\frac{1}{2};\,z^2\right)\right)}{\partial z^n}=$$

$$2^n\left(\frac{1}{2}-a\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{1}{2}-b\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{a+b-n-\frac{1}{2}}\,{}_2\tilde{F}_1\left(a-\left\lfloor\frac{n}{2}\right\rfloor,\,b-\left\lfloor\frac{n}{2}\right\rfloor;\,n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};\,z^2\right)\,/;\,n\in\mathbb{N}$$

07.24.20.0039.02

$$\frac{\partial^n \left(z\left(1-z^2\right)^{a+b-\frac{3}{2}}\,{}_2\tilde{F}_1\left(a,b;\frac{3}{2};z^2\right)\right)}{\partial z^n} = 2^n \left(\frac{3}{2}-a\right)_{\left\lfloor\frac{n}{2}\right\rfloor} \left(\frac{3}{2}-b\right)_{\left\lfloor\frac{n}{2}\right\rfloor} z^{2\left\lfloor\frac{n}{2}\right\rfloor-n+1} \left(1-z^2\right)^{a+b-n-\frac{3}{2}}\,{}_2\tilde{F}_1\left(a-n+\left\lfloor\frac{n}{2}\right\rfloor, b-n+\left\lfloor\frac{n}{2}\right\rfloor; -n+2\left\lfloor\frac{n}{2}\right\rfloor+\frac{3}{2}; z^2\right) /; n\in\mathbb{N}$$

07.24.20.0040.02

$$\frac{\partial^n \left(\left(1-z^2\right)^{a+n-\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2\tilde{F}_1\left(a,a+n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2};\frac{1}{2};z^2\right)\right)}{\partial z^n} = (-1)^{n-\left\lfloor\frac{n}{2}\right\rfloor} 2^n (a)_{n-\left\lfloor\frac{n}{2}\right\rfloor} \left(2\left\lfloor\frac{n}{2}\right\rfloor-n-a+1\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor} z^{n-2\left\lfloor\frac{n}{2}\right\rfloor} \left(1-z^2\right)^{a-\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2\tilde{F}_1\left(a+n-2\left\lfloor\frac{n}{2}\right\rfloor, a+n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}; n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}; z^2\right) /; n\in\mathbb{N}$$

07.24.20.0041.02

$$\frac{\partial^n \left(z\left(1-z^2\right)^{a+\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2\tilde{F}_1\left(a,a-n+2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};\frac{3}{2};z^2\right)\right)}{\partial z^n} = (-1)^{\left\lfloor\frac{n}{2}\right\rfloor} 2^n (a)_{\left\lfloor\frac{n}{2}\right\rfloor} \left(n-a-2\left\lfloor\frac{n}{2}\right\rfloor+1\right)_{\left\lfloor\frac{n}{2}\right\rfloor} z^{2\left\lfloor\frac{n}{2}\right\rfloor-n+1} \left(1-z^2\right)^{a-n+\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2\tilde{F}_1\left(a-n+2\left\lfloor\frac{n}{2}\right\rfloor, a-n+2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}; 2\left\lfloor\frac{n}{2}\right\rfloor-n+\frac{3}{2}; z^2\right) /; n\in\mathbb{N}$$

07.24.20.0042.02

$$\frac{\partial^n \left(\left(1-z^2\right)^{n-\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2\tilde{F}_1\left(1,a;\frac{1}{2};z^2\right)\right)}{\partial z^n} = (-1)^{n-\left\lfloor\frac{n}{2}\right\rfloor} 2^n \left(n-\left\lfloor\frac{n}{2}\right\rfloor\right)! \left(\frac{1}{2}-a\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor} z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2\tilde{F}_1\left(a, n-\left\lfloor\frac{n}{2}\right\rfloor+1; n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}; z^2\right) /; n\in\mathbb{N}$$

07.24.20.0043.02

$$\frac{\partial^n \left(z^\alpha\,{}_2\tilde{F}_1\left(-\frac{n}{2},\frac{1-n}{2};c;z^m\right)\right)}{\partial z^n} = m^{n-\alpha-\frac{1}{2}} \Gamma(\alpha+1) (2\pi)^{\frac{m-1}{2}} z^{\alpha-n}\,{}_{m+2}\tilde{F}_{m+1}\left(-\frac{n}{2}, \frac{1-n}{2}, \frac{\alpha+1}{m}, \frac{\alpha+2}{m}, \ldots, \frac{\alpha+m}{m}; \frac{\alpha-n+1}{m}, \frac{\alpha-n+2}{m}, \ldots, \frac{\alpha-n+m}{m}, c; z^m\right) /; m\in\mathbb{N}^+ \land n\in\mathbb{N}$$

07.24.20.0044.02

$$\frac{\partial^n \left(e^{-z}\,{}_2\tilde{F}_1(-n,b;c;z)\right)}{\partial z^n} = (-1)^n e^{-z} \sum_{k=0}^{n} \frac{(-n)_k z^k}{k!}\,{}_3\tilde{F}_1(-n, k-n, b+k; c+k; z) /; n\in\mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.24.20.0045.01

$$\frac{\partial^\alpha\,{}_2\tilde{F}_1(a,b;c;z)}{\partial z^\alpha} = z^{-\alpha}\,{}_3\tilde{F}_2(1,a,b;1-\alpha,c;z)$$

Integration

Indefinite integration

Involving only one direct function

07.24.21.0001.01

$$\int {}_2\tilde{F}_1(a, b; c; z)\, dz = \frac{1}{(a-1)(b-1)}\, {}_2\tilde{F}_1(a-1, b-1; c-1; z)$$

Involving one direct function and elementary functions

Involving power function

07.24.21.0002.01

$$\int z^{\alpha-1}\, {}_2\tilde{F}_1(a, b; c; z)\, dz = z^{\alpha}\, \Gamma(\alpha)\, {}_3\tilde{F}_2(a, b, \alpha; c, \alpha+1; z)$$

Definite integration

For the direct function itself

07.24.21.0003.01

$$\int_0^{\infty} t^{\alpha-1}\, {}_2\tilde{F}_1(a, b; c; -t)\, dt = \frac{\Gamma(\alpha)\,\Gamma(a-\alpha)\,\Gamma(b-\alpha)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-\alpha)} \;/;\; 0 < \mathrm{Re}(\alpha) < \min(\mathrm{Re}(a), \mathrm{Re}(b))$$

Involving the direct function

07.24.21.0004.01

$$\int_0^{\infty} t^{\alpha-1}\, e^{-d\,t}\, {}_2\tilde{F}_1(a, b; c; -t)\, dt = \frac{\Gamma(a-\alpha)\,\Gamma(b-\alpha)\,\Gamma(\alpha)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-\alpha)}\, {}_2F_2(\alpha, 1-c+\alpha; 1-a+\alpha, 1-b+\alpha; d) + \frac{\Gamma(a-b)\,\Gamma(\alpha-b)}{\Gamma(a)\,\Gamma(c-b)}\, d^{b-\alpha}\, {}_2F_2(b, b-c+1; 1-a+b, b-\alpha+1; d) + \frac{\Gamma(b-a)\,\Gamma(\alpha-a)}{\Gamma(b)\,\Gamma(c-a)}\, d^{a-\alpha}\, {}_2F_2(a, a-c+1; a-b+1, a-\alpha+1; d) \;/;\; \mathrm{Re}(\alpha) > 0 \bigwedge \mathrm{Re}(d) > 0$$

Integral transforms

Laplace transforms

07.24.22.0001.01

$$\mathcal{L}_t\left[{}_2\tilde{F}_1(a, b; c; -t)\right](z) = \frac{\pi}{\sin(\pi\,(b-a))}\left(\frac{\Gamma(1-a)}{\Gamma(b)\,\Gamma(c-a)}\, z^{a-1}\, {}_1\tilde{F}_1(a-c+1; a-b+1; z) - \frac{\Gamma(1-b)}{\Gamma(a)\,\Gamma(c-b)}\, z^{b-1}\, {}_1\tilde{F}_1(b-c+1; -a+b+1; z)\right) + \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(c-1)}\, {}_2\tilde{F}_2(1, 2-c; 2-a, 2-b; z) \;/;\; \mathrm{Re}(z) > 0 \;/;\; \mathrm{Re}(z) > 0$$

Summation

Finite summation

07.24.23.0001.01

$$\sum_{k=0}^{m} \binom{m}{k} (z-1)^{-k}\, {}_2\tilde{F}_1(a, b-k; c; z) = (c-a)_m \left(\frac{z}{z-1}\right)^m {}_2\tilde{F}_1(a, b; c+m; z) \;/;\; m \in \mathbb{N}$$

07.24.23.0002.01

$$\sum_{k=0}^{m}(-1)^k \binom{m}{k} {}_2\tilde{F}_1(a, b-k; c; z) == (a)_m z^m {}_2\tilde{F}_1(a+m, b; c+m; z) /; m \in \mathbb{N}$$

Infinite summation

07.24.23.0003.01

$$\sum_{k=0}^{\infty} \frac{(a)_k z^k}{k!} {}_2\tilde{F}_1(-k, b; c; w) == \left(\frac{1}{1-z}\right)^a {}_2\tilde{F}_1\left(a, b; c; \frac{z w}{z-1}\right)$$

Operations

Limit operation

07.24.25.0001.01

$$\lim_{z\to 1} (1-z)^{a+b-c} {}_2\tilde{F}_1(a, b; c; z) == \frac{\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)} /; \operatorname{Re}(-a-b+c) < 0$$

07.24.25.0002.01

$$\lim_{a\to\infty} {}_2\tilde{F}_1\left(a, b; c; \frac{z}{a}\right) == {}_1\tilde{F}_1(b; c; z)$$

07.24.25.0003.01

$$\lim_{b\to\infty} \lim_{a\to\infty} {}_2\tilde{F}_1\left(a, b; c; \frac{z}{a\,b}\right) == {}_0\tilde{F}_1(; c; z)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

07.24.26.0001.01

$${}_2\tilde{F}_1(a, b; c; z) == {}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z) /; p == 2 \land q == 1 \land a_1 == a \land a_2 == b \land b_1 == c$$

07.24.26.0002.01

$${}_2\tilde{F}_1(a, b; c; z) == {}_3\tilde{F}_2(a, b, a_3; c, a_3; z)$$

Involving ${}_pF_q$

07.24.26.0003.01

$${}_2\tilde{F}_1(a, b; c; z) == \frac{1}{\Gamma(c)} {}_2F_1(a, b; c; z) /; -c \notin \mathbb{N}$$

Through Meijer *G*

Classical cases for the direct function itself

07.24.26.0004.01

$${}_2\tilde{F}_1(a, b; c; z) == \frac{1}{\Gamma(a)\,\Gamma(b)} G_{2,2}^{1,2}\left(-z \,\middle|\, \begin{matrix} 1-a, 1-b \\ 0, 1-c \end{matrix}\right)$$

07.24.26.0005.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) \text{==} \frac{\pi}{\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{1,2}\!\left(z \,\middle|\, \begin{array}{c} 1-a,\,1-b,\,\frac{1}{2} \\ 0,\,1-c,\,\frac{1}{2} \end{array}\right) /;\, |z|<1$$

07.24.26.0006.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) \text{==} \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{2,2}^{2,2}\!\left(1-z \,\middle|\, \begin{array}{c} 1-a,\,1-b \\ 0,\,c-a-b \end{array}\right)$$

07.24.26.0007.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,1-z) \text{==} \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{2,2}^{2,2}\!\left(z \,\middle|\, \begin{array}{c} 1-a,\,1-b \\ 0,\,c-a-b \end{array}\right)$$

07.24.26.0008.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) - {}_2\tilde{F}_1(a,\,b;\,c;\,-z) \text{==} \frac{2^{a+b-c}\,z}{\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\!\left(-z^2 \,\middle|\, \begin{array}{c} -\frac{a}{2},\,-\frac{b}{2},\,\frac{1-a}{2},\,\frac{1-b}{2} \\ 0,\,-\frac{1}{2},\,-\frac{c}{2},\,\frac{1-c}{2} \end{array}\right)$$

07.24.26.0009.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) + {}_2\tilde{F}_1(a,\,b;\,c;\,-z) \text{==} \frac{2^{a+b-c}}{\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\!\left(-z^2 \,\middle|\, \begin{array}{c} \frac{1-a}{2},\,\frac{1-b}{2},\,1-\frac{a}{2},\,1-\frac{b}{2} \\ 0,\,\frac{1}{2},\,\frac{1-c}{2},\,1-\frac{c}{2} \end{array}\right)$$

07.24.26.0010.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) \text{==} \frac{1}{\Gamma(c)} - \frac{1}{\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{1,3}\!\left(-z \,\middle|\, \begin{array}{c} 1,\,1-a,\,1-b \\ 1,\,0,\,1-c \end{array}\right)$$

07.24.26.0011.01

$${}_2\tilde{F}_1(a,\,b;\,c;\,z) \text{==} \frac{1}{\Gamma(c)} - \frac{\pi}{\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,3}\!\left(z \,\middle|\, \begin{array}{c} 1,\,1-a,\,1-b,\,\frac{1}{2} \\ 1,\,0,\,\frac{1}{2},\,1-c \end{array}\right) /;\, |z|<1$$

Classical cases involving algebraic functions with linear arguments

07.24.26.0012.01

$$(1-z)^{a+b-c}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z) \text{==} \frac{1}{\Gamma(c-a)\,\Gamma(c-b)}\,G_{2,2}^{1,2}\!\left(-z \,\middle|\, \begin{array}{c} a-c+1,\,b-c+1 \\ 0,\,1-c \end{array}\right)$$

07.24.26.0013.01

$$(1-z)^{a+b-c}\,{}_2\tilde{F}_1(a,\,b;\,c;\,z) \text{==} \frac{\pi}{\Gamma(c-a)\,\Gamma(c-b)}\,G_{3,3}^{1,2}\!\left(z \,\middle|\, \begin{array}{c} a-c+1,\,b-c+1,\,\frac{1}{2} \\ 0,\,1-c,\,\frac{1}{2} \end{array}\right) /;\, |z|<1$$

07.24.26.0014.01

$$(1-z)\,{}_2\tilde{F}_1(a,\,b;\,1+a-b;\,z) \text{==} \frac{2}{\Gamma(a)\,\Gamma(b-1)}\,G_{3,3}^{1,3}\!\left(-z \,\middle|\, \begin{array}{c} 2-a,\,2-b,\,\frac{1-a}{2} \\ 0,\,\frac{3-a}{2},\,b-a \end{array}\right)$$

07.24.26.0015.01

$$(1-z)\,{}_2\tilde{F}_1\!\left(a,\,b;\,a+\frac{b-1}{2};\,z\right) \text{==} \frac{1}{2\,\Gamma(a)\,\Gamma(b-1)}\,G_{3,3}^{1,3}\!\left(-z \,\middle|\, \begin{array}{c} 2-2\,a,\,2-a,\,2-b \\ 0,\,3-2\,a,\,\frac{3-b}{2}-a \end{array}\right)$$

07.24.26.0016.01

$$(1-z)^{2\,b}\,{}_2\tilde{F}_1(a,\,b;\,1+a-b;\,z) \text{==} \frac{2}{\Gamma(a-2\,b+1)\,\Gamma(-b)}\,G_{3,3}^{1,3}\!\left(-z \,\middle|\, \begin{array}{c} b+1,\,b-\frac{a}{2},\,1-a+2\,b \\ 0,\,b-a,\,1-\frac{a}{2}+b \end{array}\right)$$

07.24.26.0017.01

$$(1-z)\,{}_2\tilde{F}_1(a,\,b;\,1+a-b;\,z) = \frac{2\,\pi}{\Gamma(a)\,\Gamma(b-1)}\,G_{4,4}^{1,3}\!\left(z \,\middle|\, \begin{matrix} 2-a,\,2-b,\,\frac{1-a}{2},\,\frac{1}{2} \\ 0,\,\frac{3-a}{2},\,b-a,\,\frac{1}{2} \end{matrix}\right) /;\ |z|<1$$

07.24.26.0018.01

$$(1-z)^{-b}\,{}_2\tilde{F}_1(a,\,b;\,1+a+2\,b;\,z) = \frac{1}{2\,\Gamma(2\,b)\,\Gamma(a+b+1)}\,G_{3,3}^{1,3}\!\left(-z \,\middle|\, \begin{matrix} -2\,(a+b),\,1-2\,b,\,1-a-b \\ 0,\,1-2\,a-2\,b,\,-a-2\,b \end{matrix}\right)$$

07.24.26.0019.01

$$(1-z)^{c-1}\,{}_2\tilde{F}_1(a,\,b;\,c;\,1-z) = (z-1)^{1-c}\,(1-z)^{c-1}\,G_{2,2}^{0,2}\!\left(z \,\middle|\, \begin{matrix} c-a,\,c-b \\ 0,\,c-a-b \end{matrix}\right) + G_{2,2}^{2,0}\!\left(z \,\middle|\, \begin{matrix} c-a,\,c-b \\ 0,\,c-a-b \end{matrix}\right) /;\ z \notin (-1,\,0)$$

Classical cases involving algebraic functions with rational arguments

07.24.26.0020.01

$$(1-z)^{c-1}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{z-1}{z}\right) = G_{2,2}^{2,0}\!\left(z \,\middle|\, \begin{matrix} a+b,\,c \\ a,\,b \end{matrix}\right) - (1-z)^c\,(z-1)^{-c}\,G_{2,2}^{0,2}\!\left(z \,\middle|\, \begin{matrix} a+b,\,c \\ a,\,b \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

07.24.26.0021.01

$$(1+z)^{-a}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{z}{z+1}\right) = \frac{1}{\Gamma(a)\,\Gamma(c-b)}\,G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1-a,\,b-c+1 \\ 0,\,1-c \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

07.24.26.0022.01

$$(1+z)^{-b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{z}{z+1}\right) = \frac{1}{\Gamma(b)\,\Gamma(c-a)}\,G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1-b,\,a-c+1 \\ 0,\,1-c \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

07.24.26.0023.01

$$(z+1)^{1-a}\,{}_2\tilde{F}_1\!\left(a,\,1-a;\,c;\,\frac{z}{z+1}\right) = \frac{2}{\Gamma(a-1)\,\Gamma(a+c-1)}\,G_{3,3}^{1,3}\!\left(z \,\middle|\, \begin{matrix} 2-a,\,3-a-c,\,1-\frac{a+c}{2} \\ 0,\,1-c,\,2-\frac{a+c}{2} \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

07.24.26.0024.01

$$(z+1)^{1-a}\,{}_2\tilde{F}_1\!\left(a,\,b;\,\frac{a+b+1}{2};\,\frac{z}{z+1}\right) = \frac{2}{\Gamma(a)\,\Gamma\!\left(\frac{a-b-1}{2}\right)}\,G_{3,3}^{1,3}\!\left(z \,\middle|\, \begin{matrix} 2-a,\,\frac{1-a}{2},\,\frac{3-a+b}{2} \\ 0,\,\frac{3-a}{2},\,\frac{1-a-b}{2} \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

07.24.26.0025.01

$$(z+1)^{1-b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,2\,b-a-1;\,\frac{z}{z+1}\right) = \frac{1}{2\,\Gamma(b)\,\Gamma(2\,b-2\,a-2)}\,G_{3,3}^{1,3}\!\left(z \,\middle|\, \begin{matrix} 2-2\,b,\,2\,a-2\,b+3,\,2-b \\ 0,\,3-2\,b,\,a-2\,b+2 \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

07.24.26.0026.01

$$(z+1)^{-2\,a}\,{}_2\tilde{F}_1\!\left(a,\,2\,a+1;\,c;\,\frac{z}{z+1}\right) = \frac{1}{2\,\Gamma(2\,a)\,\Gamma(c-a)}\,G_{3,3}^{1,3}\!\left(z \,\middle|\, \begin{matrix} 1-2\,a,\,2\,(a-c+1),\,a-c+2 \\ 0,\,2\,a-2\,c+3,\,1-c \end{matrix}\right) /;\ z \notin (-\infty,\,-1)$$

Classical cases involving algebraic functions with quadratic arguments

07.24.26.0027.01

$$(z+1)^{-2\,a}\,{}_2\tilde{F}_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{4\,z}{(z+1)^2}\right) = \frac{\Gamma(c-2\,a)}{\Gamma(2\,a)}\,G_{2,2}^{1,1}\!\left(z \,\middle|\, \begin{matrix} 1-2\,a,\,c-2\,a \\ 0,\,1-c \end{matrix}\right) /;\ z \notin (-1,\,0)$$

07.24.26.0028.01

$$(z+1)^{2\,a-2\,c+1}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{2\,a-c+\frac{1}{2}}\,{}_2\tilde{F}_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{4\,z}{(z+1)^2}\right) = \frac{\Gamma(2\,a-c+1)}{\Gamma(2\,c-2\,a-1)}\,G_{2,2}^{1,1}\!\left(z \,\middle|\, \begin{matrix} 2\,(a-c+1),\,2\,a-c+1 \\ 0,\,1-c \end{matrix}\right)$$

07.24.26.0029.01

$$(z+1)^{1-2a}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{-a} {}_2\tilde{F}_1\left(a, b; a+b-\frac{1}{2}; -\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(2a-1)} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{array}{c} 2-2a, b-a+\frac{1}{2} \\ 0, \frac{3}{2}-a-b \end{array}\right)$$

07.24.26.0030.01

$$(z+1)^{-2a}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{-a} {}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(2a)} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{array}{c} 1-2a, -a+b+\frac{1}{2} \\ 0, -a-b+\frac{1}{2} \end{array}\right)$$

07.24.26.0031.01

$$\left(\sqrt{z}+1\right)^{-2a} {}_2\tilde{F}_1\left(a, b; 2b; \frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\,\Gamma(b)} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{array}{c} 1-a, b-a+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0032.01

$$\left(\sqrt{z}+1\right)^{2(a-2b)}\left(\frac{\left(\sqrt{z}-1\right)^2}{\left(\sqrt{z}+1\right)^2}\right)^{a-b} {}_2\tilde{F}_1\left(a, b; 2b; \frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\,\Gamma(b)} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{array}{c} a-2b+1, a-b+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{array}\right)$$

07.24.26.0033.01

$$\left(\sqrt{z}+1\right)^{-2a}\left(\frac{\left(\sqrt{z}-1\right)^2}{\left(\sqrt{z}+1\right)^2}\right)^{-a} {}_2\tilde{F}_1\left(a, b; 2b; -\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\,\Gamma(b)} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{array}{c} 1-a, b-a+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{array}\right)$$

07.24.26.0034.01

$$\left(\sqrt{z}+1\right)^{2(a-2b)}\left(\frac{\left(\sqrt{z}-1\right)^2}{\left(\sqrt{z}+1\right)^2}\right)^{-b} {}_2\tilde{F}_1\left(a, b; 2b; -\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\,\Gamma(b)} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{array}{c} a-2b+1, a-b+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{array}\right)$$

Classical cases involving algebraic functions with squares in arguments

07.24.26.0035.01

$$\left(\sqrt{1-z}+1\right)^{-b} {}_2\tilde{F}_1\left(a, b; b+1; \frac{1-\sqrt{1-z}}{2}\right) = \frac{2^{a-1}}{\sqrt{\pi}\,\Gamma(b)} G_{3,3}^{1,3}\left(-z \,\middle|\, \begin{array}{c} \frac{1-a-b}{2}, 1-\frac{a+b}{2}, 1-b \\ 0, 1-a-b, -b \end{array}\right)$$

07.24.26.0036.01

$$\left(\sqrt{1-z}+1\right)^{b-2c+2} {}_2\tilde{F}_1\left(b, 1; c; \frac{1-\sqrt{1-z}}{2}\right) = \frac{1}{\sqrt{\pi}\,\Gamma(c-1)} G_{3,3}^{1,3}\left(-z \,\middle|\, \begin{array}{c} \frac{b}{2}+1-c, \frac{b+3}{2}-c, 2-c \\ 0, b-2c+2, 1-c \end{array}\right)$$

07.24.26.0037.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-b} {}_2\tilde{F}_1\left(a, b; b+1; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a-1}}{\sqrt{\pi}\,\Gamma(b)} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1-\frac{b}{2}, \frac{b+2}{2}, \frac{b}{2}+a \\ \frac{a+1}{2}, \frac{a}{2}, \frac{b}{2} \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0038.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{b-2c+2} {}_2\tilde{F}_1\left(1, b; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{1}{\sqrt{\pi}\,\Gamma(c-1)} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} \frac{b}{2}-c+2, \frac{b}{2}+1, c-\frac{b}{2} \\ \frac{1}{2}, 1, \frac{b}{2} \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0039.01

$$\left(\sqrt{z+1}-1\right)^{b+c-1}\,{}_2\tilde{F}_1\left(1,b;c;\frac{z-2\sqrt{z+1}+2}{z}\right)=\frac{1}{2\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{b+c}{2},\frac{b+c+1}{2},b+1\\ b+c-1,1,b\end{matrix}\right)$$

07.24.26.0040.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{2a}\,{}_2\tilde{F}_1\left(a,b;a+1;2z-2\sqrt{z+1}\sqrt{z}+1\right)=\frac{2^{-b}}{\sqrt{\pi}\,\Gamma(a)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-a,1,a-b+1\\ \frac{1-b}{2},1-\frac{b}{2},0\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0041.01

$$\left(\sqrt{z+1}-1\right)^{2b}\,{}_2\tilde{F}_1\left(a,b;b+1;\frac{z-2\sqrt{z+1}+2}{z}\right)=\frac{2^{-a}}{\sqrt{\pi}\,\Gamma(b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{a}{2}+b,\frac{a+1}{2}+b,b+1\\ 2b,a,b\end{matrix}\right)$$

07.24.26.0042.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{b+c-1}\,{}_2\tilde{F}_1\left(1,b;c;1+2z-2\sqrt{z}\sqrt{z+1}\right)=\frac{1}{2\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{3-b-c}{2},\frac{c-b+1}{2},\frac{b+c-1}{2}\\ 0,\frac{1}{2},\frac{c-b-1}{2}\end{matrix}\right)$$

07.24.26.0043.01

$$\left(\sqrt{1-z}+1\right)^{1-b-c}\,{}_2\tilde{F}_1\left(1,b;c;\frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right)=\frac{1}{2\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{1,3}\left(-z\,\middle|\,\begin{matrix}1-\frac{b+c}{2},\frac{3-b-c}{2},2-c\\ 0,2-b-c,1-c\end{matrix}\right)$$

07.24.26.0044.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-b-c}\,{}_2\tilde{F}_1\left(1,b;c;\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right)=\frac{1}{2\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{3-b-c}{2},\frac{1+c-b}{2},\frac{b+c-1}{2}\\ 0,\frac{1}{2},\frac{c-b-1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0045.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-2b}\,{}_2\tilde{F}_1\left(a,b;b+1;\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right)=\frac{2^{-a}}{\sqrt{\pi}\,\Gamma(b)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-b,1,1-a+b\\ 0,\frac{1-a}{2},1-\frac{a}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0046.01

$$\left(\sqrt{z+1}-1\right)^{b}\,{}_2\tilde{F}_1\left(a,b;b+1;\frac{z-2\sqrt{z+1}+2}{2-2\sqrt{z+1}}\right)=\frac{2^{a-1}}{\sqrt{\pi}\,\Gamma(b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1+b-a}{2},\frac{b-a}{2}+1,1\\ b,0,1-a\end{matrix}\right)$$

07.24.26.0047.01

$$\left(\sqrt{z+1}-1\right)^{2c-b-2}\,{}_2\tilde{F}_1\left(1,b;c;\frac{z-2\sqrt{z+1}+2}{2-2\sqrt{z+1}}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}c-\frac{b}{2}-1,c-\frac{b+1}{2}+c,c-b\\ 2c-b-2,0,c-b-1\end{matrix}\right)$$

07.24.26.0048.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{a}\,{}_2\tilde{F}_1\left(a,b;a+1;\frac{2z-2\sqrt{z+1}\sqrt{z}+1}{2z-2\sqrt{z}\sqrt{z+1}}\right)=\frac{2^{b-1}}{\sqrt{\pi}\,\Gamma(a)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-\frac{a}{2},\frac{a+2}{2},\frac{a}{2}+b\\ \frac{b}{2},\frac{b+1}{2},\frac{a}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0049.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{2c-b-2}\,{}_2\tilde{F}_1\left(1,b;c;\frac{2z-2\sqrt{z+1}\sqrt{z}+1}{2z-2\sqrt{z}\sqrt{z+1}}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{b}{2}-c+2,\frac{b}{2}+1,c-\frac{b}{2}\\ \frac{1}{2},1,\frac{b}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

Classical cases involving algebraic functions with cubics in arguments

07.24.26.0050.01

$$(4z+1)^{-3a}\,{}_2\tilde{F}_1\left(a, a+\frac{1}{3}; 2a+\frac{5}{6}; \frac{27z}{(4z+1)^3}\right) = \frac{2^{-2a-\frac{1}{3}}\,\Gamma\left(2a+\frac{1}{3}\right)}{\sqrt{\pi}\,\Gamma(3a)\,\Gamma\left(a+\frac{1}{6}\right)} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 1-3a, a+\frac{2}{3} \\ \frac{1}{6}-2a, 0 \end{matrix}\right)$$

07.24.26.0051.01

$$(z+4)^{-3a}\,{}_2\tilde{F}_1\left(a, a+\frac{1}{3}; 2a+\frac{5}{6}; \frac{27z^2}{(z+4)^3}\right) = \frac{2^{-2a-\frac{1}{3}}\,\Gamma\left(2a+\frac{1}{3}\right)}{\sqrt{\pi}\,\Gamma(3a)\,\Gamma\left(a+\frac{1}{6}\right)} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} \frac{5}{6}-a, 1-3a \\ 0, \frac{1}{3}-4a \end{matrix}\right)$$

07.24.26.0052.01

$$(3z+4)^{-3a}\,(9z+8)\,{}_2\tilde{F}_1\left(a, a+\frac{1}{3}; \frac{3}{2}; \frac{(9z+8)^2}{(3z+4)^3}\right) = \frac{3^{2-3a}}{\Gamma\left(\frac{4}{3}-a\right)\Gamma(3a-1)} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 2-3a, \frac{5}{2}-3a \\ \frac{7}{3}-4a, 0 \end{matrix}\right) /;\ |z|>1 \lor \mathrm{Re}(z)\geq 0$$

07.24.26.0053.01

$$(3z-1)^{-3a}\,(9z+1)\,{}_2\tilde{F}_1\left(a, a+\frac{1}{3}; \frac{3}{2}; \frac{(9z+1)^2}{(1-3z)^3}\right) = \frac{1}{3^{3a-2}\,\Gamma\left(a+\frac{1}{6}\right)\Gamma(3a-1)} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 2-3a, \frac{5}{2}-3a \\ \frac{7}{6}-2a, 0 \end{matrix}\right) /;\ |z|>1 \lor \mathrm{Re}(z)>0$$

07.24.26.0054.01

$$(3-z)^{-3a}\,(z+9)\,{}_2\tilde{F}_1\left(a, a+\frac{1}{3}; \frac{3}{2}; \frac{z(z+9)^2}{(z-3)^3}\right) = \frac{3^{2-3a}}{\Gamma\left(a+\frac{1}{6}\right)\Gamma(3a-1)} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} \frac{5}{6}-a, 2-3a \\ 0, -\frac{1}{2} \end{matrix}\right) /;\ |z|<1 \lor \mathrm{Re}(z)>0$$

07.24.26.0055.01

$$(4z+3)^{-3a}\,(8z+9)\,{}_2\tilde{F}_1\left(a, a+\frac{1}{3}; \frac{3}{2}; \frac{z(8z+9)^2}{(4z+3)^3}\right) = \frac{3^{2-3a}}{\Gamma\left(\frac{4}{3}-a\right)\Gamma(3a-1)} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} a-\frac{1}{3}, 2-3a \\ 0, -\frac{1}{2} \end{matrix}\right) /;\ |z|<1 \lor \mathrm{Re}(z)\geq 0$$

07.24.26.0056.01

$$(z+1)^{-a}\,{}_2\tilde{F}_1\left(a, \frac{1}{6}-a; \frac{1}{2}; -\frac{(9z+8)^2}{27z^2(z+1)}\right) = \frac{1}{\Gamma\left(\frac{1}{3}-a\right)\Gamma(3a)} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 1-a, \frac{1}{2}-a \\ \frac{1}{3}-2a, 2a \end{matrix}\right) /;\ |z|>1 \lor \mathrm{Re}(z)>0$$

07.24.26.0057.01

$$(z+1)^{-a}\,{}_2\tilde{F}_1\left(a, \frac{1}{6}-a; \frac{1}{2}; -\frac{z(8z+9)^2}{27(z+1)}\right) = \frac{1}{\Gamma\left(\frac{1}{3}-a\right)\Gamma(3a)} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} a+\frac{2}{3}, 1-3a \\ 0, \frac{1}{2} \end{matrix}\right) /;\ |z|<1 \lor \mathrm{Re}(z)>0$$

Classical cases with rational arguments and unit step θ

07.24.26.0058.01

$$\theta(1-|z|)\,{}_2\tilde{F}_1(a, b; c; -z) = \Gamma(1-a)\,\Gamma(1-b)\,G_{2,2}^{1,0}\left(z \,\middle|\, \begin{matrix} 1-a, 1-b \\ 0, 1-c \end{matrix}\right)$$

07.24.26.0059.01

$$\theta(1-|z|)\,{}_2\tilde{F}_1(a, b; c; z) = \pi\,\Gamma(1-a)\,\Gamma(1-b)\,G_{3,3}^{1,0}\left(z \,\middle|\, \begin{matrix} 1-a, 1-b, \frac{1}{2} \\ 0, 1-c, \frac{1}{2} \end{matrix}\right)$$

07.24.26.0060.01

$$\theta(|z|-1)\,{}_2\tilde{F}_1\left(a, b; c; -\frac{1}{z}\right) = \Gamma(1-a)\,\Gamma(1-b)\,G_{2,2}^{0,1}\left(z \,\middle|\, \begin{matrix} 1, c \\ a, b \end{matrix}\right)$$

07.24.26.0061.01

$$\theta(|z|-1)\,{}_2\tilde{F}_1\left(a,b;c;\frac{1}{z}\right) = \pi\,\Gamma(1-a)\,\Gamma(1-b)\,G_{3,3}^{0,1}\left(z\,\middle|\,\begin{matrix}1,\,c,\,\frac{1}{2}\\ a,\,b,\,\frac{1}{2}\end{matrix}\right)$$

Classical cases involving algebraic functions with linear arguments and unit step θ

07.24.26.0062.01

$$\theta(1-|z|)\,(z+1)^{a+b-c}\,{}_2\tilde{F}_1(a,b;c;-z) = \Gamma(a-c+1)\,\Gamma(b-c+1)\,G_{2,2}^{1,0}\left(z\,\middle|\,\begin{matrix}a-c+1,\,b-c+1\\ 0,\,1-c\end{matrix}\right)$$

07.24.26.0063.01

$$\theta(1-|z|)\,(1-z)^{a+b-c}\,{}_2\tilde{F}_1(a,b;c;z) = \pi\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,G_{3,3}^{1,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,a-c+1,\,b-c+1\\ 0,\,\frac{1}{2},\,1-c\end{matrix}\right)$$

07.24.26.0064.01

$$\theta(1-|z|)\,(1-z)^{c-1}\,{}_2\tilde{F}_1(a,b;c;1-z) = G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}c-a,\,c-b\\ 0,\,c-a-b\end{matrix}\right) /;\ \notin(-1,0)\land \operatorname{Re}(c)>0$$

07.24.26.0065.01

$$\theta(|z|-1)\,(z-1)^{c-1}\,{}_2\tilde{F}_1(a,b;c;1-z) = G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}c-a,\,c-b\\ 0,\,c-a-b\end{matrix}\right) /; \operatorname{Re}(c)>0$$

Classical cases involving algebraic functions with rational arguments and unit step θ

07.24.26.0066.01

$$\theta(|z|-1)\,(z+1)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;-\frac{1}{z}\right) = \Gamma(a-c+1)\,\Gamma(b-c+1)\,G_{2,2}^{0,1}\left(z\,\middle|\,\begin{matrix}a+b-c+1,\,a+b\\ a,\,b\end{matrix}\right)$$

07.24.26.0067.01

$$\theta(|z|-1)\,(z-1)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{1}{z}\right) = \pi\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,G_{3,3}^{0,1}\left(z\,\middle|\,\begin{matrix}a+b-c+1,\,a+b-c+\frac{1}{2},\,a+b\\ a+b-c+\frac{1}{2},\,a,\,b\end{matrix}\right)$$

07.24.26.0068.01

$$\theta(1-|z|)\,(1-z)^{c-1}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z-1}{z}\right) = G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}a+b,\,c\\ a,\,b\end{matrix}\right) /; \operatorname{Re}(c)>0$$

07.24.26.0069.01

$$\theta(1-|z|)\,(z+1)^{-a}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z}{z+1}\right) = \Gamma(1-a)\,\Gamma(b-c+1)\,G_{2,2}^{1,0}\left(z\,\middle|\,\begin{matrix}1-a,\,b-c+1\\ 0,\,1-c\end{matrix}\right)$$

07.24.26.0070.01

$$\theta(1-|z|)\,(1-z)^{-a}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z}{z-1}\right) = \pi\,\Gamma(1-a)\,\Gamma(b-c+1)\,G_{3,3}^{1,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,1-a,\,b-c+1\\ 0,\,\frac{1}{2},\,1-c\end{matrix}\right)$$

07.24.26.0071.01

$$\theta(1-|z|)\,(z+1)^{-b}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z}{1+z}\right) = \Gamma(1-b)\,\Gamma(a-c+1)\,G_{2,2}^{1,0}\left(z\,\middle|\,\begin{matrix}1-b,\,a-c+1\\ 0,\,1-c\end{matrix}\right)$$

07.24.26.0072.01

$$\theta(1-|z|)\,(1-z)^{-b}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z}{z-1}\right) = \pi\,\Gamma(1-b)\,\Gamma(a-c+1)\,G_{3,3}^{1,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,1-b,\,a-c+1\\ 0,\,\frac{1}{2},\,1-c\end{matrix}\right)$$

07.24.26.0073.01

$$\theta(|z|-1)\,(z-1)^{c-1}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{z-1}{z}\right) = G_{2,2}^{0,2}\!\left(z\,\middle|\,\begin{matrix}a+b,\,c\\ a,\,b\end{matrix}\right)\,/;\ \notin(-\infty,\,-1)\bigwedge \operatorname{Re}(c)>0$$

07.24.26.0074.01

$$\theta(|z|-1)\,(z+1)^{-a}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{1}{z+1}\right) = \Gamma(1-a)\,\Gamma(b-c+1)\,G_{2,2}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-a,\,c-a\\ 0,\,c-a-b\end{matrix}\right)$$

07.24.26.0075.01

$$\theta(|z|-1)\,(z+1)^{-b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{1}{z+1}\right) = \Gamma(1-b)\,\Gamma(a-c+1)\,G_{2,2}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-b,\,c-b\\ 0,\,c-a-b\end{matrix}\right)$$

07.24.26.0076.01

$$\theta(|z|-1)\,(z-1)^{-a}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{1}{1-z}\right) = \pi\,\Gamma(1-a)\,\Gamma(b-c+1)\,G_{3,3}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-a,\,\frac{1}{2}-a,\,c-a\\ \frac{1}{2}-a,\,0,\,c-a-b\end{matrix}\right)$$

07.24.26.0077.01

$$\theta(|z|-1)\,(z-1)^{-b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,c;\,\frac{1}{1-z}\right) = \pi\,\Gamma(1-b)\,\Gamma(a-c+1)\,G_{3,3}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-b,\,\frac{1}{2}-b,\,c-b\\ \frac{1}{2}-b,\,0,\,c-a-b\end{matrix}\right)$$

Classical cases involving sgn

07.24.26.0078.01

$$(z+1)^{-2a}\,((1-z)\operatorname{sgn}(1-|z|))^{4a-2c+1}\,{}_2\tilde{F}_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{4z}{(z+1)^2}\right) = \frac{\Gamma(2a-c+1)}{\Gamma(2c-2a-1)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}2(a-c+1),\,2a-c+1\\ 0,\,1-c\end{matrix}\right)$$

07.24.26.0079.01

$$((1-z)\operatorname{sgn}(1-|z|))^{-2a}\,{}_2\tilde{F}_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\!\left(b-a+\frac{1}{2}\right)}{\Gamma(2a)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}1-2a,\,b-a+\frac{1}{2}\\ 0,\,\frac{1}{2}-a-b\end{matrix}\right)$$

07.24.26.0080.01

$$((1-z)\operatorname{sgn}(1-|z|))^{-2b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\!\left(a-b+\frac{1}{2}\right)}{\Gamma(2b)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}1-2b,\,a-b+\frac{1}{2}\\ 0,\,\frac{1}{2}-a-b\end{matrix}\right)$$

07.24.26.0081.01

$$(z+1)\,((1-z)\operatorname{sgn}(1-|z|))^{-2a}\,{}_2\tilde{F}_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\!\left(b-a+\frac{1}{2}\right)}{\Gamma(2a-1)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}2-2a,\,b-a+\frac{1}{2}\\ 0,\,\frac{3}{2}-a-b\end{matrix}\right)$$

07.24.26.0082.01

$$(z+1)\,((1-z)\operatorname{sgn}(1-|z|))^{-2b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\!\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-1)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}2-2b,\,a-b+\frac{1}{2}\\ 0,\,\frac{3}{2}-a-b\end{matrix}\right)$$

07.24.26.0083.01

$$\left(1+\sqrt{z}\right)^{-2b}\left(\left(1-\sqrt{z}\right)\operatorname{sgn}(1-|z|)\right)^{2a-2b}\,{}_2\tilde{F}_1\!\left(a,\,b;\,2b;\,\frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\!\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\,\Gamma(b)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}a-2b+1,\,a-b+\frac{1}{2}\\ 0,\,\frac{1}{2}-b\end{matrix}\right)$$

07.24.26.0084.01

$$\left(\left(1-\sqrt{z}\right)\operatorname{sgn}(1-|z|)\right)^{-2a}\,{}_2\tilde{F}_1\left(a,b;2b;-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=\frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\,\Gamma(b)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{matrix}1-a,\,b-a+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{matrix}\right)$$

07.24.26.0085.01

$$\left(\sqrt{z}+1\right)^{2a-2b}\left(\left(1-\sqrt{z}\right)\operatorname{sgn}(1-|z|)\right)^{-2b}\,{}_2\tilde{F}_1\left(a,b;2b;-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=$$
$$\frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\,\Gamma(b)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{matrix}a-2b+1,\,a-b+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{matrix}\right)$$

Classical cases involving powers of ${}_2\tilde{F}_1$

07.24.26.0086.01

$${}_2\tilde{F}_1\left(a,b;a+b+\frac{1}{2};-z\right)^2=\frac{2^{2a+2b-1}}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-2a,\,1-2b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2a-2b\end{matrix}\right)$$

07.24.26.0087.01

$$(1+z)\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right)^2=\frac{2^{2a+2b-3}}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}2-2a,\,2-2b,\,2-a-b\\0,\,\frac{3}{2}-a-b,\,3-2a-2b\end{matrix}\right)$$

07.24.26.0088.01

$$(z+1)^{-2a}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{1}{z+1}\right)^2=\frac{4^{c-1}}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-2a,\,c-2a,\,2c-2a-1\\0,\,2c-4a-1,\,c-2a-\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0089.01

$$(1+z)^{-2a}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{z}{z+1}\right)^2=\frac{4^{c-1}}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-2a,\,2a-2c+2,\,\frac{3}{2}-c\\0,\,1-c,\,2-2c\end{matrix}\right)/;\,z\notin(-\infty,-1)$$

07.24.26.0090.01

$${}_2\tilde{F}_1\left(a,b;\frac{a+b+1}{2};\frac{1-\sqrt{1+z}}{2}\right)^2=\frac{2^{a+b-1}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-a,\,1-b,\,1-\frac{a+b}{2}\\0,\,\frac{1-a-b}{2},\,1-a-b\end{matrix}\right)$$

07.24.26.0091.01

$$\left(\sqrt{1+z}+1\right)^{2-2c}\,{}_2\tilde{F}_1\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right)^2=\frac{1}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}a-c+1,\,2-a-c,\,\frac{3}{2}-c\\0,\,1-c,\,2-2c\end{matrix}\right)$$

07.24.26.0092.01

$${}_2\tilde{F}_1\left(a,b;\frac{a+b+1}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)^2=\frac{2^{a+b-1}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1,\,\frac{1+a+b}{2},\,a+b\\b,\,a,\,\frac{a+b}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0093.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2-2c}\,{}_2\tilde{F}_1\left(a,1-a;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)^2=\frac{1}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}2-c,\,1,\,c\\a,\,1-a,\,\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.24.26.0094.01

$$\left(\sqrt{1+z}+1\right)^{-2a}\,{}_2\tilde{F}_1\!\left(a,b;a-b+1;\left(\frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right)^2\right)=\frac{4^{-b}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}1-a,\,2b-a,\,\frac{1}{2}-a+b\\0,\,b-a,\,2b-2a\end{matrix}\right)$$

07.24.26.0095.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-2a}\,{}_2\tilde{F}_1\!\left(a,b;a-b+1;\left(\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right)^2\right)=\frac{4^{-b}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{3,1}\!\left(z\,\middle|\,\begin{matrix}1-a,\,1-b,\,a-2b+1\\0,\,1-2b,\,\frac{1}{2}-b\end{matrix}\right)/;\,z\notin(-1,0)$$

Classical cases for products of ${}_2\tilde{F}_1$ with linear arguments

07.24.26.0096.01

$${}_2\tilde{F}_1\!\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-2}}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}1-2a,\,1-2b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,2(1-a-b)\end{matrix}\right)$$

07.24.26.0097.01

$${}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(a+1,b;a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-1}}{\sqrt{\pi}\,\Gamma(2b)\,\Gamma(2a+1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}-2a,\,1-2b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2a-2b\end{matrix}\right)$$

07.24.26.0098.01

$${}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(a,b+1;a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-1}}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b+1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}1-2a,\,-2b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2a-2b\end{matrix}\right)$$

07.24.26.0099.01

$${}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(a+1,b+1;a+b+\frac{3}{2};-z\right)=\frac{2^{2a+2b}}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}-2a,\,-2b,\,-a-b\\0,\,-\frac{1}{2}-a-b,\,-2(a+b)\end{matrix}\right)$$

07.24.26.0100.01

$${}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(\frac{1}{2}-a,\frac{1}{2}-b;\frac{3}{2}-a-b;-z\right)=\frac{\cos(\pi(a-b))}{\pi^{3/2}}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,a-b+\frac{1}{2},\,\frac{1}{2}-a+b\\0,\,\frac{1}{2}-a-b,\,a+b-\frac{1}{2}\end{matrix}\right)$$

Classical cases involving products of ${}_2\tilde{F}_1$ with linear arguments

07.24.26.0101.01

$$\frac{1}{\sqrt{z+1}}\,{}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(a+\frac{1}{2},b-\frac{1}{2};a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-1}}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}-2a,\,1-2b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2a-2b\end{matrix}\right)$$

07.24.26.0102.01

$$\frac{1}{\sqrt{z+1}}\,{}_2\tilde{F}_1\!\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\!\left(a+\frac{1}{2},b+\frac{1}{2};a+b+\frac{3}{2};-z\right)=\frac{2^{2a+2b}}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}-2a,\,-2b,\,-a-b\\0,\,-a-b-\frac{1}{2},\,-2(a+b)\end{matrix}\right)$$

07.24.26.0103.01

$$\sqrt{z+1}\ {}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a-\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}}{\sqrt{\pi}\ \Gamma(2a-1)\Gamma(2b-1)}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.24.26.0104.01

$$\sqrt{z+1}\ {}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a-\frac{1}{2},b+\frac{1}{2};a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}}{\sqrt{\pi}\ \Gamma(2a-1)\Gamma(2b)}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2a,1-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.24.26.0105.01

$$\sqrt{z+1}\ {}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(1-a,1-b;\frac{5}{2}-a-b;-z\right)=\frac{\cos((a-b)\pi)}{\pi^{3/2}}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},a-b+\frac{1}{2},\frac{1}{2}-a+b\\0,\frac{3}{2}-a-b,a+b-\frac{3}{2}\end{array}\right)$$

07.24.26.0106.01

$$\sqrt{z+1}\ {}_2\tilde{F}_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a+\frac{1}{2},b+\frac{1}{2};a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-1}}{\sqrt{\pi}\ \Gamma(2a)\Gamma(2b)}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}1-2a,1-2b,1-a-b\\0,\frac{1}{2}-a-b,1-2a-2b\end{array}\right)$$

07.24.26.0107.01

$$\sqrt{z+1}\ {}_2\tilde{F}_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\left(1-a,1-b;\frac{3}{2}-a-b;-z\right)=\frac{\cos((a-b)\pi)}{\pi^{3/2}}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},a-b+\frac{1}{2},b-a+\frac{1}{2}\\0,\frac{1}{2}-a-b,a+b-\frac{1}{2}\end{array}\right)$$

07.24.26.0108.01

$$(1+z)\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(\frac{3}{2}-a,\frac{3}{2}-b;\frac{5}{2}-a-b;-z\right)=\frac{\cos((a-b)\pi)}{\pi^{3/2}}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},a-b+\frac{1}{2},b-a+\frac{1}{2}\\0,\frac{3}{2}-a-b,a+b-\frac{3}{2}\end{array}\right)$$

Classical cases for products of ${}_2\tilde{F}_1$ with algebraic arguments

07.24.26.0109.01

$${}_2\tilde{F}_1\left(a,b;c;-2\left(z+\sqrt{z+1}\ \sqrt{z}\right)\right){}_2\tilde{F}_1\left(a,b;c;-2\left(z-\sqrt{z}\ \sqrt{z+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right)$$

07.24.26.0110.01

$${}_2\tilde{F}_1\left(a,b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right){}_2\tilde{F}_1\left(a,b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}1,\frac{c}{2},\frac{c+1}{2},c\\a,b,c-a,c-b\end{array}\right)/;\ z\notin(-1,0)$$

07.24.26.0111.01

$${}_2\tilde{F}_1\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a,b;a+b-c+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-1}}{\sqrt{\pi}\ \Gamma(a)\Gamma(b)}G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}1-a,1-b,1-\frac{a+b}{2},\frac{1-a-b}{2}\\0,1-a-b,1-c,c-a-b\end{array}\right)$$

07.24.26.0112.01

$${}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a,a+\frac{1}{2};2a-c+\frac{3}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{4a-\frac{3}{2}}}{\pi\,\Gamma(2a)}G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}\frac{1}{4}-a,\frac{1}{2}-a,\frac{3}{4}-a,1-a\\0,1-c,\frac{1}{2}-2a,\frac{1}{2}-2a+c\end{array}\right)$$

07.24.26.0113.01

$${}_2\tilde{F}_1\left(a,\, b;\, \frac{a+b+1}{2};\, \frac{1-\sqrt{1+z}}{2}\right) {}_2\tilde{F}_1\left(1-a,\, 1-b;\, \frac{3-a-b}{2};\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)}{\pi^{3/2}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{a-b+1}{2}, \frac{b-a+1}{2} \\ 0, \frac{1-a-b}{2}, \frac{a+b-1}{2} \end{matrix}\right)$$

07.24.26.0114.01

$${}_2\tilde{F}_1\left(a,\, 1-a;\, c;\, \frac{1-\sqrt{1+z}}{2}\right) {}_2\tilde{F}_1\left(a,\, 1-a;\, 2-c;\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{\sin(a\pi)}{\pi^{3/2}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 1-a, a \\ 0, 1-c, c-1 \end{matrix}\right)$$

07.24.26.0115.01

$${}_2\tilde{F}_1\left(a,\, b;\, c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2\tilde{F}_1\left(a,\, b;\, a+b-c+1;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a+b-1}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)} G_{4,4}^{4,1}\left(z \,\middle|\, \begin{matrix} 1, c, a+b, a+b-c+1 \\ a, b, \frac{a+b}{2}, \frac{a+b+1}{2} \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.24.26.0116.01

$${}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, 2a-c+\frac{3}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{4a-\frac{3}{2}}}{\pi\,\Gamma(2a)} G_{4,4}^{4,1}\left(z \,\middle|\, \begin{matrix} 1, c, 2a+\frac{1}{2}, 2a-c+\frac{3}{2} \\ a, a+\frac{1}{4}, a+\frac{1}{2}, a+\frac{3}{4} \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.24.26.0117.01

$${}_2\tilde{F}_1\left(a,\, b;\, \frac{a+b+1}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2\tilde{F}_1\left(1-a,\, 1-b;\, \frac{3-a-b}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)}{\pi^{3/2}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 1, \frac{1+a+b}{2}, \frac{3-a-b}{2} \\ \frac{1}{2}, \frac{1+a-b}{2}, \frac{1-a+b}{2} \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.24.26.0118.01

$${}_2\tilde{F}_1\left(a,\, 1-a;\, c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2\tilde{F}_1\left(a,\, 1-a;\, 2-c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\sin(a\pi)}{\pi^{3/2}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 1, c, 2-c \\ a, 1-a, \frac{1}{2} \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

Classical cases involving products of ${}_2\tilde{F}_1$ with algebraic arguments

07.24.26.0119.01

$$\left(2z+2\sqrt{z+1}\sqrt{z}+1\right)^{c-a-b} {}_2\tilde{F}_1\left(a,\, b;\, c;\, -2\left(z-\sqrt{z}\sqrt{z+1}\right)\right) {}_2\tilde{F}_1\left(c-a,\, c-b;\, c;\, -2\left(z+\sqrt{z}\sqrt{z+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{1,4}\left(z \,\middle|\, \begin{matrix} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{matrix}\right)$$

07.24.26.0120.01

$$\left(2z-2\sqrt{z+1}\sqrt{z}+1\right)^{c-a-b} {}_2\tilde{F}_1\left(a,\, b;\, c;\, -2\left(z+\sqrt{z}\sqrt{z+1}\right)\right) {}_2\tilde{F}_1\left(c-a,\, c-b;\, c;\, -2\left(z-\sqrt{z}\sqrt{z+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{1,4}\left(z \,\middle|\, \begin{matrix} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{matrix}\right)$$

07.24.26.0121.01

$$\left(z+2\sqrt{z+1}+2\right)^{c-a-b}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)\,{}_2\tilde{F}_1\left(c-a,\,c-b;\,c;\,-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}c-a-b+1,\,\frac{3c}{2}-a-b,\,\frac{3c+1}{2}-a-b,\,2c-a-b\\ c-a,\,c-b,\,2c-2a-b,\,2c-2b-a\end{array}\right)/;\,z\notin(-1,\,0)$$

07.24.26.0122.01

$$\left(z-2\sqrt{z+1}+2\right)^{c-a-b}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)\,{}_2\tilde{F}_1\left(c-a,\,c-b;\,c;\,-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}c-a-b+1,\,\frac{3c}{2}-a-b,\,\frac{3c+1}{2}-a-b,\,2c-a-b\\ c-a,\,c-b,\,2c-2a-b,\,2c-2b-a\end{array}\right)/;\,z\notin(-1,\,0)$$

07.24.26.0123.01

$$\left(1-2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)^{a}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)\,{}_2\tilde{F}_1\left(a,\,c-b;\,c;\,2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(-z\,\middle|\,\begin{array}{c}1-a,\,a-c+1,\,1-b,\,b-c+1\\ 0,\,1-\frac{c}{2},\,\frac{1-c}{2},\,1-c\end{array}\right)$$

07.24.26.0124.01

$$\left(1-2z-2\sqrt{z}\,\sqrt{z-1}\right)^{b}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)\,{}_2\tilde{F}_1\left(c-a,\,b;\,c;\,2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(-z\,\middle|\,\begin{array}{c}1-a,\,1-b,\,a-c+1,\,b-c+1\\ 0,\,1-\frac{c}{2},\,\frac{1-c}{2},\,1-c\end{array}\right)$$

07.24.26.0125.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^{a}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)\,{}_2\tilde{F}_1\left(a,\,c-b;\,c;\,\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^{a}\,G_{4,4}^{4,1}\left(-z\,\middle|\,\begin{array}{c}1,\,\frac{c}{2},\,\frac{c+1}{2},\,c\\ a,\,b,\,c-a,\,c-b\end{array}\right)$$

07.24.26.0126.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^{b}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)\,{}_2\tilde{F}_1\left(c-a,\,b;\,c;\,\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^{a}\,G_{4,4}^{4,1}\left(-z\,\middle|\,\begin{array}{c}1,\,\frac{c}{2},\,\frac{c+1}{2},\,c\\ a,\,b,\,c-a,\,c-b\end{array}\right)$$

07.24.26.0127.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2\tilde{F}_1\left(a,\,b;\,c;\,\frac{1-\sqrt{1+z}}{2}\right)\,{}_2\tilde{F}_1\left(a-c+1,\,b-c+1;\,a+b-c+1;\,\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-c}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}1-a,\,1-b,\,\frac{1-a-b}{2},\,1-\frac{a+b}{2}\\ 0,\,1-a-b,\,1-c,\,c-a-b\end{array}\right)$$

07.24.26.0128.01

$$\left(\sqrt{1+z}+1\right)^{a+b-2c+1}\,{}_2\tilde{F}_1\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(1-a,1-b;-a-b+c+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}a-c+1,\,b-c+1,\,\frac{a+b+1}{2}-c,\,\frac{a+b}{2}-c+1\\0,\,a+b-2c+1,\,1-c,\,a+b-c\end{array}\right)$$

07.24.26.0129.01

$$\left(\sqrt{1+z}+1\right)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(c-a,c-b;c-a-b+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{c-1}}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}a-c+1,\,b-c+1,\,\frac{a+b}{2}-c+1,\,\frac{a+b+1}{2}-c\\0,\,a+b-2c+1,\,1-c,\,a+b-c\end{array}\right)$$

07.24.26.0130.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{4a-c-\frac{1}{2}}}{\pi\,\Gamma(2a)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}\frac{1}{4}-a,\,\frac{1}{2}-a,\,\frac{3}{4}-a,\,1-a\\0,\,c-2a-\frac{1}{2},\,\frac{1}{2}-2a,\,1-c\end{array}\right)$$

07.24.26.0131.01

$$\left(\sqrt{1+z}+1\right)^{2a-2c+\frac{3}{2}}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(1-a,\frac{1}{2}-a;-2a+c+\frac{1}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{4^{c-a-1}}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}a-c+\frac{3}{4},\,a-c+1,\,a-c+\frac{5}{4},\,a-c+\frac{3}{2}\\0,\,1-c,\,2a-2c+\frac{3}{2},\,2a-c+\frac{1}{2}\end{array}\right)$$

07.24.26.0132.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2\tilde{F}_1\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(2-a-c,a-c+1;2-c;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{1-c}\sin(a\pi)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\,1-a,\,a\\0,\,c-1,\,1-c\end{array}\right)$$

07.24.26.0133.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a-c+1,b-c+1;a+b-c+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{a+b-c}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}\frac{3-c}{2},\,a+b+\frac{1-c}{2},\,\frac{c+1}{2},\,a+b+\frac{3-3c}{2}\\a+\frac{1-c}{2},\,b+\frac{1-c}{2},\,\frac{a+b-c+1}{2},\,\frac{a+b-c}{2}+1\end{array}\right)\,/;\,z\notin(-1,0)$$

07.24.26.0134.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-2c+1}\,{}_2\tilde{F}_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(1-a,1-b;c-a-b+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}\frac{a+b+3}{2}-c,\,\frac{1-a-b}{2}+c,\,\frac{1+a+b}{2},\,\frac{3-a-b}{2}\\\frac{1}{2},\,1,\,\frac{1-a+b}{2},\,\frac{1+a-b}{2}\end{array}\right)\,/;\,z\notin(-1,0)$$

07.24.26.0135.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(c-a,c-b;c-a-b+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{c-1}}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}\frac{a+b-c}{2}+1,\frac{3c-a-b}{2},\frac{a+b+c}{2},\frac{c-a-b}{2}+1\\ \frac{b-a+c}{2},\frac{a-b+c}{2},\frac{c}{2},\frac{c+1}{2}\end{matrix}\right)/;\ z\notin(-1,0)$$

07.24.26.0136.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{4a-c-\frac{1}{2}}}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}\frac{3-c}{2},2a-\frac{3c}{2}+2,2a-\frac{c}{2}+1,\frac{c+1}{2}\\ a+\frac{1-c}{2},a+\frac{3-2c}{4},a-\frac{c}{2}+1,a+\frac{5-2c}{4}\end{matrix}\right)/;\ z\notin(-1,0)$$

07.24.26.0137.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2a-2c+\frac{3}{2}}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(1-a,\frac{1}{2}-a;c-2a+\frac{1}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{4^{c-a-1}}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}a-c+\frac{7}{4},a+\frac{3}{4},c-a+\frac{1}{4},\frac{5}{4}-a\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1\end{matrix}\right)/;\ z\notin(-1,0)$$

07.24.26.0138.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2\tilde{F}_1\left(a,1-a;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(2-a-c,a-c+1;2-c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{1-c}\sin(a\pi)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1-a,a\\ 0,c-1,1-c\end{matrix}\right)/;\ z\notin(-1,0)$$

Classical cases involving ${}_2F_1$ with linear arguments

07.24.26.0139.01

$${}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a-1,b-1;a+b-\frac{3}{2};-z\right)=\frac{4^{a+b-2}\,\Gamma\left(a+b-\frac{3}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}2-2a,2-2b,2-a-b\\ 0,\frac{3}{2}-a-b,2(2-a-b)\end{matrix}\right)$$

07.24.26.0140.01

$${}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a-1,b;a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}2-2a,1-2b,2-a-b\\ 0,\frac{3}{2}-a-b,3-2a-2b\end{matrix}\right)$$

07.24.26.0141.01

$${}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a,b-1;a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-2a,2-2b,2-a-b\\ 0,\frac{3}{2}-a-b,3-2a-2b\end{matrix}\right)$$

07.24.26.0142.01

$${}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a,b;a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-2}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-2a,1-2b,1-a-b\\ 0,\frac{1}{2}-a-b,2(1-a-b)\end{matrix}\right)$$

07.24.26.0143.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a, b; a+b-\frac{1}{2}; -z\right) = \frac{2^{2a+2b-2}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} 1-2a, 1-2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 2(1-a-b) \end{array} \right.\right)$$

07.24.26.0144.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a, b; a+b+\frac{1}{2}; -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} 1-2a, 1-2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 1-2a-2b \end{array} \right.\right)$$

07.24.26.0145.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a+1, b; a+b+\frac{1}{2}; -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2b)\,\Gamma(2a+1)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} -2a, 1-2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 1-2a-2b \end{array} \right.\right)$$

07.24.26.0146.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a, b+1; a+b+\frac{1}{2}; -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b+1)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} 1-2a, -2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 1-2a-2b \end{array} \right.\right)$$

07.24.26.0147.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a+1, b+1; a+b+\frac{3}{2}; -z\right) = \frac{2^{2a+2b}\,\Gamma\left(a+b+\frac{3}{2}\right)}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} -2a, -2b, -a-b \\ 0, -a-b-\frac{1}{2}, -2(a+b) \end{array} \right.\right)$$

07.24.26.0148.01

$${}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(\frac{1}{2}-a, \frac{1}{2}-b; \frac{3}{2}-a-b; -z\right) = \frac{\cos(\pi(a-b))\,\Gamma\left(\frac{3}{2}-a-b\right)}{\pi^{3/2}} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} \frac{1}{2}, a-b+\frac{1}{2}, b-a+\frac{1}{2} \\ 0, \frac{1}{2}-a-b, a+b-\frac{1}{2} \end{array} \right.\right)$$

07.24.26.0149.01

$$(z+1)\,{}_2\tilde{F}_1\left(a, b; a+b-\frac{1}{2}; -z\right) {}_2F_1\left(a, b; a+b-\frac{1}{2}; -z\right) = \frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} 2-2a, 2-2b, 2-a-b \\ 0, \frac{3}{2}-a-b, 3-2a-2b \end{array} \right.\right)$$

Classical cases involving algebraic functions and ${}_2F_1$ with linear arguments

07.24.26.0150.01

$$\frac{1}{\sqrt{z+1}}\,{}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a-\frac{1}{2}, b-\frac{1}{2}; a+b-\frac{1}{2}; -z\right) = \frac{4^{a+b-1}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} 1-2a, 1-2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 2(1-a-b) \end{array} \right.\right)$$

07.24.26.0151.01

$$\frac{1}{\sqrt{z+1}}\,{}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a-\frac{1}{2}, b+\frac{1}{2}; a+b+\frac{1}{2}; -z\right) =$$
$$\frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2b+1)\,\Gamma(2a)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} 1-2a, -2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 1-2a-2b \end{array} \right.\right)$$

07.24.26.0152.01

$$\frac{1}{\sqrt{z+1}}\,{}_2\tilde{F}_1\left(a, b; a+b+\frac{1}{2}; -z\right) {}_2F_1\left(a+\frac{1}{2}, b-\frac{1}{2}; a+b+\frac{1}{2}; -z\right) =$$
$$\frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b)} G_{3,3}^{1,3}\left(z \left| \begin{array}{c} -2a, 1-2b, 1-a-b \\ 0, \frac{1}{2}-a-b, 1-2a-2b \end{array} \right.\right)$$

07.24.26.0153.01

$$\frac{1}{\sqrt{z+1}}\,{}_2\tilde{F}_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2F_1\left(a+\frac{1}{2},b+\frac{1}{2};a+b+\frac{3}{2};-z\right)=\frac{2^{2a+2b}\,\Gamma\left(a+b+\frac{3}{2}\right)}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}-2a,-2b,-a-b\\0,-a-b-\frac{1}{2},-2(a+b)\end{array}\right)$$

07.24.26.0154.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a-\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.24.26.0155.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a-\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.24.26.0156.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a-\frac{1}{2},b+\frac{1}{2};a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2a,1-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.24.26.0157.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(a+\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)=\frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}1-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.24.26.0158.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2F_1\left(a+\frac{1}{2},b+\frac{1}{2};a+b+\frac{1}{2};-z\right)=\frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}1-2a,1-2b,1-a-b\\0,\frac{1}{2}-a-b,1-2a-2b\end{array}\right)$$

07.24.26.0159.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2F_1\left(1-a,1-b;\frac{5}{2}-a-b;-z\right)=\frac{\cos((b-a)\pi)\,\Gamma\left(\frac{5}{2}-a-b\right)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}-a+b,a-b+\frac{1}{2}\\0,a+b-\frac{3}{2},\frac{3}{2}-a-b\end{array}\right)$$

07.24.26.0160.01

$$\sqrt{z+1}\,{}_2\tilde{F}_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2F_1\left(1-a,1-b;\frac{3}{2}-a-b;-z\right)=\frac{\cos((b-a)\pi)\,\Gamma\left(\frac{3}{2}-a-b\right)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},b-a+\frac{1}{2},a-b+\frac{1}{2}\\0,a+b-\frac{1}{2},\frac{1}{2}-a-b\end{array}\right)$$

07.24.26.0161.01

$$(z+1)\,{}_2\tilde{F}_1\left(a,\, b;\, a+b-\frac{1}{2};\, -z\right){}_2F_1\left(\frac{3}{2}-a,\, \frac{3}{2}-b;\, \frac{5}{2}-a-b;\, -z\right) = \frac{\cos((a-b)\,\pi)\,\Gamma\left(\frac{5}{2}-a-b\right)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\, a-b+\frac{1}{2},\, b-a+\frac{1}{2}\\ 0,\, \frac{3}{2}-a-b,\, a+b-\frac{3}{2}\end{array}\right)$$

Classical cases involving ${}_2F_1$ with algebraic arguments

07.24.26.0162.01

$$ {}_2\tilde{F}_1\left(a,\, b;\, c;\, -2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right){}_2F_1\left(a,\, b;\, c;\, -2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}1-a,\, 1-b,\, a-c+1,\, b-c+1\\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c\end{array}\right)$$

07.24.26.0163.01

$$ {}_2\tilde{F}_1\left(a,\, b;\, c;\, -2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right){}_2F_1\left(a,\, b;\, c;\, -2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}1-a,\, 1-b,\, a-c+1,\, b-c+1\\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c\end{array}\right)$$

07.24.26.0164.01

$$ {}_2\tilde{F}_1\left(a,\, b;\, c;\, \frac{1-\sqrt{1+z}}{2}\right){}_2F_1\left(a,\, b;\, a+b-c+1;\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{a+b-1}\,\Gamma(a+b-c+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}1-a,\, 1-b,\, 1-\frac{a+b}{2},\, \frac{1-a-b}{2}\\ 0,\, 1-a-b,\, 1-c,\, c-a-b\end{array}\right)$$

07.24.26.0165.01

$$ {}_2\tilde{F}_1\left(a,\, b;\, c;\, -\frac{2\left(1+\sqrt{z+1}\right)}{z}\right){}_2F_1\left(a,\, b;\, c;\, -\frac{2\left(1-\sqrt{z+1}\right)}{z}\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}1,\, \frac{c}{2},\, \frac{c+1}{2},\, c\\ a,\, b,\, c-a,\, c-b\end{array}\right) /;\, z \notin (-1,\, 0)$$

07.24.26.0166.01

$$ {}_2\tilde{F}_1\left(a,\, b;\, c;\, -\frac{2\left(1-\sqrt{z+1}\right)}{z}\right){}_2F_1\left(a,\, b;\, c;\, -\frac{2\left(1+\sqrt{z+1}\right)}{z}\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}1,\, \frac{c}{2},\, \frac{c+1}{2},\, c\\ a,\, b,\, c-a,\, c-b\end{array}\right) /;\, z \notin (-1,\, 0)$$

07.24.26.0167.01

$$ {}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{1-\sqrt{1+z}}{2}\right){}_2F_1\left(a,\, a+\frac{1}{2};\, 2\,a-c+\frac{3}{2};\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{4\,a-\frac{3}{2}}\,\Gamma\left(2\,a-c+\frac{3}{2}\right)}{\pi\,\Gamma(2\,a)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{array}{c}\frac{1}{4}-a,\, \frac{1}{2}-a,\, \frac{3}{4}-a,\, 1-a\\ 0,\, 1-c,\, \frac{1}{2}-2\,a,\, \frac{1}{2}-2\,a+c\end{array}\right)$$

07.24.26.0168.01

$$ {}_2\tilde{F}_1\left(a,\, 1-a;\, c;\, \frac{1-\sqrt{1+z}}{2}\right){}_2F_1\left(a,\, 1-a;\, 2-c;\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{\sin(a\,\pi)\,\Gamma(2-c)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\, 1-a,\, a\\ 0,\, 1-c,\, c-1\end{array}\right)$$

07.24.26.0169.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(a, b; \frac{a+b+1}{2}; \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)} G_{3,3}^{1,3}\left(z \middle| \begin{array}{c} 1-a, 1-b, 1-\frac{a+b}{2} \\ 0, \frac{1-a-b}{2}, 1-a-b \end{array}\right)$$

07.24.26.0170.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(1-a, 1-b; \frac{3-a-b}{2}; \frac{1-\sqrt{1+z}}{2}\right) = \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)\Gamma\left(\frac{3-a-b}{2}\right)}{\pi^{3/2}} G_{3,3}^{1,3}\left(z \middle| \begin{array}{c} \frac{1}{2}, \frac{a-b+1}{2}, \frac{b-a+1}{2} \\ 0, \frac{1-a-b}{2}, \frac{a+b-1}{2} \end{array}\right)$$

07.24.26.0171.01

$${}_2\tilde{F}_1\left(a, b; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(a, b; a+b-c+1; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a+b-1}\,\Gamma(1+a+b-c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)} G_{4,4}^{4,1}\left(z \middle| \begin{array}{c} 1, c, a+b, a+b-c+1 \\ a, b, \frac{a+b}{2}, \frac{a+b+1}{2} \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0172.01

$${}_2\tilde{F}_1\left(a, a+\frac{1}{2}; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(a, a+\frac{1}{2}; 2a-c+\frac{3}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{4a-\frac{3}{2}}\,\Gamma\left(\frac{3}{2}+2a-c\right)}{\pi\,\Gamma(2a)} G_{4,4}^{4,1}\left(z \middle| \begin{array}{c} 1, c, 2a+\frac{1}{2}, 2a-c+\frac{3}{2} \\ a, a+\frac{1}{4}, a+\frac{1}{2}, a+\frac{3}{4} \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0173.01

$${}_2\tilde{F}_1\left(a, 1-a; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(a, 1-a; 2-c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\sin(a\pi)\,\Gamma(2-c)}{\pi^{3/2}} G_{3,3}^{3,1}\left(z \middle| \begin{array}{c} 1, c, 2-c \\ a, 1-a, \frac{1}{2} \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0174.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(a, b; \frac{a+b+1}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)} G_{3,3}^{3,1}\left(z \middle| \begin{array}{c} 1, \frac{1+a+b}{2}, a+b \\ b, a, \frac{a+b}{2} \end{array}\right) /; z \notin (-1, 0)$$

07.24.26.0175.01

$${}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(1-a, 1-b; \frac{3-a-b}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)\Gamma\left(\frac{3-a-b}{2}\right)}{\pi^{3/2}} G_{3,3}^{3,1}\left(z \middle| \begin{array}{c} 1, \frac{1+a+b}{2}, \frac{3-a-b}{2} \\ \frac{1}{2}, \frac{1+a-b}{2}, \frac{1-a+b}{2} \end{array}\right) /; z \notin (-1, 0)$$

Classical cases involving algebraic functions and ${}_2F_1$ with algebraic arguments

07.24.26.0176.01

$$(z+1)^{-2a}\,{}_2\tilde{F}_1\left(a, a+\frac{1}{2}; c; \frac{1}{z+1}\right){}_2F_1\left(a, a+\frac{1}{2}; c; \frac{1}{z+1}\right) = \frac{4^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-2a, c-2a, 2c-2a-1\\ 0, 2c-4a-1, c-2a-\frac{1}{2}\end{matrix}\right)\,/;\, z\notin(-1,0)$$

07.24.26.0177.01

$$(z+1)^{-2a}\,{}_2\tilde{F}_1\left(a, a+\frac{1}{2}; c; \frac{z}{z+1}\right){}_2F_1\left(a, a+\frac{1}{2}; c; \frac{z}{z+1}\right) = \frac{4^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-2a, 2a-2c+2, \frac{3}{2}-c\\ 0, 1-c, 2-2c\end{matrix}\right)\,/;\, z\notin(-\infty,-1)$$

07.24.26.0178.01

$$\left(\sqrt{z+1}+1\right)^{2-2c}\,{}_2\tilde{F}_1\left(a, 1-a; c; \frac{1-\sqrt{1+z}}{2}\right){}_2F_1\left(a, 1-a; c; \frac{1-\sqrt{1+z}}{2}\right) = \frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}a-c+1, 2-a-c, \frac{3}{2}-c\\ 0, 1-c, 2-2c\end{matrix}\right)$$

07.24.26.0179.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2-2c}\,{}_2\tilde{F}_1\left(a, 1-a; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(a, 1-a; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}2-c, 1, c\\ a, 1-a, \frac{1}{2}\end{matrix}\right)\,/;\, z\notin(-1,0)$$

07.24.26.0180.01

$$\left(\sqrt{z+1}+1\right)^{-2a}\,{}_2\tilde{F}_1\left(a, b; a-b+1; \frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right){}_2F_1\left(a, b; a-b+1; \frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right) = \frac{4^{-b}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-a, 2b-a, \frac{1}{2}-a+b\\ 0, b-a, 2b-2a\end{matrix}\right)$$

07.24.26.0181.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-2a}\,{}_2\tilde{F}_1\left(a, b; a-b+1; \frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right){}_2F_1\left(a, b; a-b+1; \frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right) = \frac{4^{-b}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-a, 1-b, a-2b+1\\ 0, 1-2b, \frac{1}{2}-b\end{matrix}\right)\,/;\, z\notin(-1,0)$$

07.24.26.0182.01

$$\left(2z+2\sqrt{z+1}\,\sqrt{z}+1\right)^{c-a-b}\,{}_2\tilde{F}_1\left(a, b; c; -2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right){}_2F_1\left(c-a, c-b; c; -2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z\,\middle|\,\begin{matrix}1-a, 1-b, a-c+1, b-c+1\\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c\end{matrix}\right)$$

07.24.26.0183.01

$$\left(2z-2\sqrt{z+1}\sqrt{z}+1\right)^{c-a-b}{}_2\tilde{F}_1\!\left(a,b;c;-2\left(z+\sqrt{z}\sqrt{z+1}\right)\right){}_2F_1\!\left(c-a,c-b;c;-2\left(z-\sqrt{z}\sqrt{z+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{1,4}\!\left(z\left|\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right.\right)$$

07.24.26.0184.01

$$\left(2z+2\sqrt{z+1}\sqrt{z}+1\right)^{a+b-c}{}_2\tilde{F}_1\!\left(a,b;c;-2z-2\sqrt{z+1}\sqrt{z}\right){}_2F_1\!\left(c-a,c-b;c;2\sqrt{z}\sqrt{z+1}-2z\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{1,4}\!\left(z\left|\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right.\right)$$

07.24.26.0185.01

$$\left(2z-2\sqrt{z+1}\sqrt{z}+1\right)^{a+b-c}{}_2\tilde{F}_1\!\left(a,b;c;2\sqrt{z}\sqrt{z+1}-2z\right){}_2F_1\!\left(c-a,c-b;c;-2\sqrt{z}\sqrt{z+1}-2z\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{1,4}\!\left(z\left|\begin{array}{c}a-c+1,b-c+1,1-a,1-b\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right.\right)$$

07.24.26.0186.01

$$\left(z+2\sqrt{z+1}+2\right)^{c-a-b}{}_2\tilde{F}_1\!\left(a,b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right){}_2F_1\!\left(c-a,c-b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}c-a-b+1,\frac{3c}{2}-a-b,\frac{3c+1}{2}-a-b,2c-a-b\\c-a,c-b,2c-2a-b,2c-2b-a\end{array}\right.\right)/;z\notin(-1,0)$$

07.24.26.0187.01

$$\left(z-2\sqrt{z+1}+2\right)^{c-a-b}{}_2\tilde{F}_1\!\left(a,b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right){}_2F_1\!\left(c-a,c-b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}c-a-b+1,\frac{3c}{2}-a-b,\frac{3c+1}{2}-a-b,2c-a-b\\c-a,c-b,2c-2a-b,2c-2b-a\end{array}\right.\right)/;z\notin(-1,0)$$

07.24.26.0188.01

$$\left(z+2\sqrt{z+1}+2\right)^{a+b-c}{}_2\tilde{F}_1\!\left(a,b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right){}_2F_1\!\left(c-a,c-b;c;-\frac{2\left(1-\sqrt{1+z}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}a+b-c+1,a+b-\frac{c}{2},a+b+\frac{1-c}{2},a+b\\a,b,2a+b-c,a+2b-c\end{array}\right.\right)/;z\notin(-1,0)$$

07.24.26.0189.01

$$\left(z-2\sqrt{z+1}+2\right)^{a+b-c}{}_2\tilde{F}_1\!\left(a,b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right){}_2F_1\!\left(c-a,c-b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}a+b-c+1,a+b-\frac{c}{2},a+b+\frac{1-c}{2},a+b\\a,b,2a+b-c,a+2b-c\end{array}\right.\right)/;z\notin(-1,0)$$

07.24.26.0190.01

$$\left(1-2z-2\sqrt{z}\,\sqrt{z-1}\right)^a\,{}_2\tilde{F}_1\!\left(a,b;c;2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right){}_2F_1\!\left(a,c-b;c;2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(-z\,\middle|\,\begin{matrix}1-a,a-c+1,1-b,b-c+1\\0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)$$

07.24.26.0191.01

$$\left(1-2z-2\sqrt{z}\,\sqrt{z-1}\right)^b\,{}_2\tilde{F}_1\!\left(a,b;c;2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right){}_2F_1\!\left(c-a,b;c;2\left(z+\sqrt{z}\,\sqrt{z-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(-z\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)$$

07.24.26.0192.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^a\,{}_2\tilde{F}_1\!\left(a,b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right){}_2F_1\!\left(a,c-b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}\,z^a\,G_{4,4}^{4,1}\!\left(-z\,\middle|\,\begin{matrix}1,\frac{c}{2},\frac{c+1}{2},c\\a,b,c-a,c-b\end{matrix}\right)$$

07.24.26.0193.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^b\,{}_2\tilde{F}_1\!\left(a,b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right){}_2F_1\!\left(c-a,b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}\,z^a\,G_{4,4}^{4,1}\!\left(-z\,\middle|\,\begin{matrix}1,\frac{c}{2},\frac{c+1}{2},c\\a,b,c-a,c-b\end{matrix}\right)$$

07.24.26.0194.01

$$\left(\sqrt{z+1}+1\right)^{1-c}\,{}_2\tilde{F}_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(a-c+1,b-c+1;a+b-c+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-c}\,\Gamma(a+b-c+1)}{\sqrt{\pi}\,\Gamma(a)\Gamma(b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}1-a,1-b,\frac{1-a-b}{2},1-\frac{a+b}{2}\\0,1-a-b,1-c,c-a-b\end{matrix}\right)$$

07.24.26.0195.01

$$\left(\sqrt{z+1}+1\right)^{a+b-c}\,{}_2\tilde{F}_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(c-b,c-a;c-a-b+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{c-1}\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+1,b-c+1,\frac{a+b+1}{2}-c,\frac{a+b}{2}-c+1\\0,a+b-2c+1,a+b-c,1-c\end{matrix}\right)$$

07.24.26.0196.01

$$\left(\sqrt{z+1}+1\right)^{a+b-2c+1}\,{}_2\tilde{F}_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(1-a,1-b;c-a-b+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+1,b-c+1,\frac{a+b+1}{2}-c,\frac{a+b}{2}-c+1\\0,a+b-2c+1,1-c,a+b-c\end{matrix}\right)$$

07.24.26.0197.01

$$\left(\sqrt{z+1}+1\right)^{1-c}\,{}_2\tilde{F}_1\!\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{4a-c-\frac{1}{2}}\,\Gamma\!\left(2a-c+\frac{3}{2}\right)}{\pi\,\Gamma(2a)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{4}-a,\frac{1}{2}-a,\frac{3}{4}-a,1-a\\0,c-2a-\frac{1}{2},\frac{1}{2}-2a,1-c\end{array}\right)$$

07.24.26.0198.01

$$\left(\sqrt{z+1}+1\right)^{2a-c+\frac{1}{2}}\,{}_2\tilde{F}_1\!\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(c-a-\frac{1}{2},c-a;c-2a+\frac{1}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{3c-2a-3}\,\Gamma\!\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}a-c+\frac{3}{4},a-c+1,a-c+\frac{5}{4},a-c+\frac{3}{2}\\0,1-c,2a-2c+\frac{3}{2},2a-c+\frac{1}{2}\end{array}\right)$$

07.24.26.0199.01

$$\left(\sqrt{z+1}+1\right)^{2a-2c+\frac{3}{2}}\,{}_2\tilde{F}_1\!\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(\frac{1}{2}-a,1-a;c-2a+\frac{1}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{4^{c-a-1}\,\Gamma\!\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}a-c+\frac{3}{4},a-c+1,a-c+\frac{5}{4},a-c+\frac{3}{2}\\0,1-c,2a-2c+\frac{3}{2},2a-c+\frac{1}{2}\end{array}\right)$$

07.24.26.0200.01

$$\left(\sqrt{z+1}+1\right)^{1-c}\,{}_2\tilde{F}_1\!\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(2-a-c,a-c+1;2-c;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{1-c}\sin(a\pi)\,\Gamma(2-c)}{\pi^{3/2}}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},1-a,a\\0,c-1,1-c\end{array}\right)$$

07.24.26.0201.01

$$\left(\sqrt{z+1}+1\right)^{\frac{a+b-1}{2}}\,{}_2\tilde{F}_1\!\left(a,b;\frac{a+b+1}{2};\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(\frac{a-b+1}{2},\frac{b-a+1}{2};\frac{3-a-b}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{\frac{a+b-1}{2}}}{\pi^{3/2}}\cos\!\left(\frac{1}{2}(a-b)\pi\right)\Gamma\!\left(\frac{1}{2}(3-a-b)\right)G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1-a+b}{2},\frac{a-b+1}{2}\\0,\frac{1-a-b}{2},\frac{a+b-1}{2}\end{array}\right)$$

07.24.26.0202.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2\tilde{F}_1\!\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\!\left(a-c+1,b-c+1;a+b-c+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{a+b-c}\,\Gamma(a+b-c+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}\frac{3-c}{2},a+b+\frac{1-c}{2},\frac{c+1}{2},a+b+\frac{3-3c}{2}\\a+\frac{1-c}{2},b+\frac{1-c}{2},\frac{a+b-c+1}{2},\frac{a+b-c}{2}+1\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0203.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(c-a,c-b;c-a-b+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{c-1}\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}\frac{a+b-c}{2}+1,\frac{3c-a-b}{2},\frac{a+b+c}{2},\frac{c-a-b}{2}+1\\ \frac{b-a+c}{2},\frac{a-b+c}{2},\frac{c}{2},\frac{c+1}{2}\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0204.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-2c+1}\,{}_2\tilde{F}_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(1-a,1-b;c-a-b+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}\frac{a+b+3}{2}-c,\frac{1-a-b}{2}+c,\frac{1+a+b}{2},\frac{3-a-b}{2}\\ \frac{1}{2},1,\frac{1-a+b}{2},\frac{1+a-b}{2}\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0205.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{4a-c-\frac{1}{2}}\,\Gamma\left(2a-c+\frac{3}{2}\right)}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}\frac{3-c}{2},2a-\frac{3c}{2}+2,2a-\frac{c}{2}+1,\frac{c+1}{2}\\ a+\frac{1-c}{2},a+\frac{3-2c}{4},a-\frac{c}{2}+1,a+\frac{5-2c}{4}\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0206.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2a-c+\frac{1}{2}}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(c-a-\frac{1}{2},c-a;c-2a+\frac{1}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{3c-2a-3}\,\Gamma\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}a+\frac{5-2c}{4},a+\frac{2c+1}{4},\frac{6c-1}{4}-a,\frac{2c+3}{4}-a\\ \frac{2c-1}{4},\frac{c}{2},\frac{2c+1}{4},\frac{c+1}{2}\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0207.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2a-2c+\frac{3}{2}}\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(\frac{1}{2}-a,1-a;c-2a+\frac{1}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{4^{c-a-1}\,\Gamma\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}a-c+\frac{7}{4},a+\frac{3}{4},c-a+\frac{1}{4},\frac{5}{4}-a\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0208.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,1-a;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(2-a-c,a-c+1;2-c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{1-c}\,\sin(a\pi)\,\Gamma(2-c)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},1-a,a\\ 0,c-1,1-c\end{array}\right)/;\,z\notin(-1,0)$$

07.24.26.0209.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{\frac{a+b-1}{2}} {}_2\tilde{F}_1\left(a, b; \frac{a+b+1}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(\frac{b-a+1}{2}, \frac{a-b+1}{2}; \frac{3-a-b}{2}; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{\frac{a+b-1}{2}}}{\pi^{3/2}} \cos\left(\frac{1}{2}(a-b)\pi\right) \Gamma\left(\frac{1}{2}(3-a-b)\right) G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{a+b+3}{4}, \frac{3a+3b+1}{4}, \frac{5-a-b}{4} \\ \frac{a+b+1}{4}, \frac{3a-b+1}{4}, \frac{3b-a+1}{4} \end{matrix}\right) /; z \notin (-1, 0)$$

Generalized cases for the direct function itself

07.24.26.0210.01

$${}_2\tilde{F}_1(a, b; c; z) - {}_2\tilde{F}_1(a, b; c; -z) = \frac{2^{a+b-c}\pi}{\Gamma(a)\Gamma(b)} G_{5,5}^{1,4}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1-a}{2}, \frac{1-b}{2}, 1-\frac{a}{2}, 1-\frac{b}{2}, 1 \\ \frac{1}{2}, 0, 1, \frac{1-c}{2}, 1-\frac{c}{2} \end{matrix}\right) /; |z| < 1$$

07.24.26.0211.01

$${}_2\tilde{F}_1(a, b; c; z) + {}_2\tilde{F}_1(a, b; c; -z) = \frac{2^{a+b-c}\pi}{\Gamma(a)\Gamma(b)} G_{5,5}^{1,4}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1-a}{2}, \frac{1-b}{2}, 1-\frac{a}{2}, 1-\frac{b}{2}, \frac{1}{2} \\ 0, \frac{1}{2}, \frac{1}{2}, \frac{1-c}{2}, 1-\frac{c}{2} \end{matrix}\right) /; |z| < 1$$

Generalized cases involving algebraic functions with quadratic arguments

07.24.26.0212.01

$$(z+1)^{-2a} {}_2\tilde{F}_1\left(a, b; 2b; \frac{4z}{(z+1)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\Gamma(b)} G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-a, b-a+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{matrix}\right)$$

07.24.26.0213.01

$$(z+1)^{2(a-2b)} \left(\frac{(z-1)^2}{(z+1)^2}\right)^{a-b} {}_2\tilde{F}_1\left(a, b; 2b; \frac{4z}{(z+1)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\Gamma(b)} G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a-2b+1, a-b+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{matrix}\right)$$

07.24.26.0214.01

$$(z+1)^{-2a} \left(\frac{(z-1)^2}{(z+1)^2}\right)^{-a} {}_2\tilde{F}_1\left(a, b; 2b; -\frac{4z}{(z-1)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\Gamma(b)} G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-a, b-a+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{matrix}\right)$$

07.24.26.0215.01

$$(z+1)^{2(a-2b)} \left(\frac{(z-1)^2}{(z+1)^2}\right)^{-b} {}_2\tilde{F}_1\left(a, b; 2b; -\frac{4z}{(z-1)^2}\right) = \frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\Gamma(b)} G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a-2b+1, a-b+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{matrix}\right)$$

Generalized cases involving algebraic functions with squares in arguments

07.24.26.0216.01

$$\left(z+\sqrt{z^2+1}\right)^{-b} {}_2\tilde{F}_1\left(a, b; b+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{a-1}}{\sqrt{\pi}\,\Gamma(b)} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-\frac{b}{2}, \frac{b+2}{2}, a+\frac{b}{2} \\ \frac{a+1}{2}, \frac{a}{2}, \frac{b}{2} \end{matrix}\right) /; \mathrm{Re}(z) > 0$$

07.24.26.0217.01

$$\left(z+\sqrt{z^2+1}\right)^{a-2b} {}_2\tilde{F}_1\left(a, 1; b+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{1}{\sqrt{\pi}\,\Gamma(b)} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{a}{2}-b+1, \frac{a}{2}+1, b-\frac{a}{2}+1 \\ 1, \frac{1}{2}, \frac{a}{2} \end{matrix}\right) /; \mathrm{Re}(z) > 0$$

07.24.26.0218.01

$$\left(\sqrt{z^2+1}-z\right)^{2a}\,{}_2\tilde{F}_1\left(a,b;a+1;2z^2-2z\sqrt{z^2+1}+1\right)=\frac{2^{-b}}{\sqrt{\pi}\,\Gamma(a)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1,a-b+1\\ \frac{1-b}{2},1-\frac{b}{2},0\end{matrix}\right)/;\operatorname{Re}(z)>0$$

07.24.26.0219.01

$$\left(\sqrt{z^2+1}-z\right)^{b+c-1}\,{}_2\tilde{F}_1\left(1,b;c;2z^2-2z\sqrt{z^2+1}+1\right)=\frac{1}{2\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-b-c}{2},\frac{c-b+1}{2},\frac{b+c-1}{2}\\ 0,\frac{1}{2},\frac{c-b-1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

07.24.26.0220.01

$$\left(z+\sqrt{z^2+1}\right)^{1-b-c}\,{}_2\tilde{F}_1\left(1,b;c;\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)=\frac{1}{2\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-b-c}{2},\frac{1+c-b}{2},\frac{b+c-1}{2}\\ 0,\frac{1}{2},\frac{c-b-1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

07.24.26.0221.01

$$\left(z+\sqrt{z^2+1}\right)^{-2b}\,{}_2\tilde{F}_1\left(a,b;b+1;\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)=\frac{2^{-a}}{\sqrt{\pi}\,\Gamma(b)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-b,1,b-a+1\\ 0,\frac{1-a}{2},1-\frac{a}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

07.24.26.0222.01

$$\left(\sqrt{z^2+1}-z\right)^{a}\,{}_2\tilde{F}_1\left(a,b;a+1;\frac{2z^2-2z\sqrt{z^2+1}+1}{2z^2-2z\sqrt{z^2+1}}\right)=\frac{2^{b-1}}{\sqrt{\pi}\,\Gamma(a)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{a}{2},\frac{a+2}{2},\frac{a}{2}+b\\ \frac{b}{2},\frac{b+1}{2},\frac{a}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

07.24.26.0223.01

$$\left(\sqrt{z^2+1}-z\right)^{2c-b-2}\,{}_2\tilde{F}_1\left(1,b;c;\frac{2z^2-2z\sqrt{z^2+1}+1}{2z^2-2z\sqrt{z^2+1}}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(c-1)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{b}{2}-c+2,\frac{b}{2}+1,c-\frac{b}{2}\\ \frac{1}{2},1,\frac{b}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

Classical cases involving sgn

07.24.26.0224.01

$$(z+1)^{-2b}\left((1-z)\operatorname{sgn}(1-|z|)\right)^{2a-2b}\,{}_2\tilde{F}_1\left(a,b;2b;\frac{4z}{(z+1)^2}\right)=\frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\,\Gamma(b)}\,G_{2,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}a-2b+1,a-b+\frac{1}{2}\\ 0,\frac{1}{2}-b\end{matrix}\right)$$

07.24.26.0225.01

$$\left((1-z)\operatorname{sgn}(1-|z|)\right)^{-2a}\,{}_2\tilde{F}_1\left(a,b;2b;-\frac{4z}{(z-1)^2}\right)=\frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\,\Gamma(b)}\,G_{2,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,b-a+\frac{1}{2}\\ 0,\frac{1}{2}-b\end{matrix}\right)$$

07.24.26.0226.01

$$(z+1)^{2a-2b}\left((1-z)\operatorname{sgn}(1-|z|)\right)^{-2b}\,{}_2\tilde{F}_1\left(a,b;2b;-\frac{4z}{(z-1)^2}\right)=\frac{2^{1-2b}\sqrt{\pi}\,\Gamma\left(a-b+\frac{1}{2}\right)}{\Gamma(2b-a)\,\Gamma(b)}\,G_{2,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}a-2b+1,a-b+\frac{1}{2}\\ 0,\frac{1}{2}-b\end{matrix}\right)$$

Generalized cases involving powers of ${}_2\tilde{F}_1$

07.24.26.0227.01

$${}_2\tilde{F}_1\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)^2=\frac{2^{a+b-1}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,\frac{a+b+1}{2},a+b\\ a,b,\frac{a+b}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

07.24.26.0228.01

$$\left(z+\sqrt{z^2+1}\right)^{2-2c}\,{}_2\tilde{F}_1\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right)^2 = \frac{1}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}2-c,1,c\\a,1-a,\frac{1}{2}\end{matrix}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0229.01

$$\left(z+\sqrt{z^2+1}\right)^{-2a}\,{}_2\tilde{F}_1\left(a,b;a-b+1;\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)^2 = \frac{4^{-b}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-2b+1\\0,1-2b,\frac{1}{2}-b\end{matrix}\right)\;/;\;\mathrm{Re}(z)>0$$

Generalized cases for products of ${}_2\tilde{F}_1$ with algebraic arguments

07.24.26.0230.01

$${}_2\tilde{F}_1\left(a,b;c;-2\left(z^2+z\sqrt{z^2+1}\right)\right)\,{}_2\tilde{F}_1\left(a,b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right)$$

07.24.26.0231.01

$${}_2\tilde{F}_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right)\,{}_2\tilde{F}_1\left(a,b;a+b-c+1;\frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{a+b-1}}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,c,a+b,a+b-c+1\\a,b,\frac{a+b}{2},\frac{a+b+1}{2}\end{matrix}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0232.01

$${}_2\tilde{F}_1\left(a,a+\frac{1}{2};c;\frac{z-\sqrt{z^2+1}}{2z}\right)\,{}_2\tilde{F}_1\left(a,a+\frac{1}{2};2a-c+\frac{3}{2};\frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{4a-\frac{3}{2}}}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,c,2a+\frac{1}{2},2a-c+\frac{3}{2}\\a,a+\frac{1}{4},a+\frac{1}{2},a+\frac{3}{4}\end{matrix}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0233.01

$${}_2\tilde{F}_1\left(a,b;\frac{1+a+b}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)\,{}_2\tilde{F}_1\left(1-a,1-b;\frac{3-a-b}{2};\frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)}{\pi^{3/2}}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,\frac{a+b+1}{2},\frac{3-a-b}{2}\\\frac{1}{2},\frac{1+a-b}{2},\frac{1-a+b}{2}\end{matrix}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0234.01

$${}_2\tilde{F}_1\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right)\,{}_2\tilde{F}_1\left(a,1-a;2-c;\frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{\sin(a\pi)}{\pi^{3/2}}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,c,2-c\\a,1-a,\frac{1}{2}\end{matrix}\right)\;/;\;\mathrm{Re}(z)>0$$

Generalized cases involving products of ${}_2\tilde{F}_1$ with algebraic arguments

07.24.26.0235.01

$$\left(2 z^2+2 z \sqrt{z^2+1}+1\right)^{c-a-b} {}_2\tilde{F}_1\left(a, b; c; -2\left(z^2-z \sqrt{z^2+1}\right)\right) {}_2\tilde{F}_1\left(c-a, c-b; c; -2\left(z^2+z \sqrt{z^2+1}\right)\right) = \frac{2^{1-c} \sqrt{\pi}}{\Gamma(a) \Gamma(b) \Gamma(c-a) \Gamma(c-b)} G_{4,4}^{1,4}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{array}\right)$$

07.24.26.0236.01

$$\left(1+2 z^2-2 z \sqrt{z^2+1}\right)^{c-a-b} {}_2\tilde{F}_1\left(a, b; c; -2\left(z^2+z \sqrt{z^2+1}\right)\right) {}_2\tilde{F}_1\left(c-a, c-b; c; -2\left(z^2-z \sqrt{z^2+1}\right)\right) = \frac{2^{1-c} \sqrt{\pi}}{\Gamma(a) \Gamma(b) \Gamma(c-a) \Gamma(c-b)} G_{4,4}^{1,4}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{array}\right)$$

07.24.26.0237.01

$$\left(1-2\left(z^2+z \sqrt{z^2-1}\right)\right)^{a} {}_2\tilde{F}_1\left(a, b; c; 2\left(z^2+z \sqrt{z^2-1}\right)\right) {}_2\tilde{F}_1\left(a, c-b; c; 2\left(z^2+z \sqrt{z^2-1}\right)\right) = \frac{2^{1-c} \sqrt{\pi}}{\Gamma(a) \Gamma(b) \Gamma(c-a) \Gamma(c-b)} G_{4,4}^{1,4}\left(i z, \frac{1}{2} \middle| \begin{array}{c} 1-a, a-c+1, 1-b, b-c+1 \\ 0, 1-\frac{c}{2}, \frac{1-c}{2}, 1-c \end{array}\right) /; z \notin (-\infty, -1)$$

07.24.26.0238.01

$$\left(1-2\left(z^2+z \sqrt{z^2-1}\right)\right)^{b} {}_2\tilde{F}_1\left(a, b; c; 2\left(z^2+z \sqrt{z^2-1}\right)\right) {}_2\tilde{F}_1\left(c-a, b; c; 2\left(z^2+z \sqrt{z^2-1}\right)\right) = \frac{2^{1-c} \sqrt{\pi}}{\Gamma(a) \Gamma(b) \Gamma(c-a) \Gamma(c-b)} G_{4,4}^{1,4}\left(i z, \frac{1}{2} \middle| \begin{array}{c} 1-a, a-c+1, 1-b, b-c+1 \\ 0, 1-\frac{c}{2}, \frac{1-c}{2}, 1-c \end{array}\right) /; z \notin (-\infty, -1)$$

07.24.26.0239.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c} {}_2\tilde{F}_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2 z}\right) {}_2\tilde{F}_1\left(a-c+1, b-c+1; a+b-c+1; \frac{z-\sqrt{z^2+1}}{2 z}\right) = \frac{2^{a+b-c}}{\sqrt{\pi}\, \Gamma(a) \Gamma(b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{3-c}{2}, a+b+\frac{1-c}{2}, \frac{c+1}{2}, a+b+\frac{3-3 c}{2} \\ a+\frac{1-c}{2}, b+\frac{1-c}{2}, \frac{a+b-c+1}{2}, \frac{a+b-c}{2}+1 \end{array}\right) /; \operatorname{Re}(z)>0$$

07.24.26.0240.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-2 c+1} {}_2\tilde{F}_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2 z}\right) {}_2\tilde{F}_1\left(1-a, 1-b; c-a-b+1; \frac{z-\sqrt{z^2+1}}{2 z}\right) = \frac{1}{\sqrt{\pi}\, \Gamma(c-a) \Gamma(c-b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{a+b+3}{2}-c, \frac{1-a-b}{2}+c, \frac{a+b+1}{2}, \frac{3-a-b}{2} \\ \frac{1}{2}, 1, \frac{1-a+b}{2}, \frac{a-b+1}{2} \end{array}\right) /; \operatorname{Re}(z)>0$$

07.24.26.0241.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-c} {}_2\tilde{F}_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2 z}\right) {}_2\tilde{F}_1\left(c-a, c-b; c-a-b+1; \frac{z-\sqrt{z^2+1}}{2 z}\right) = \frac{2^{c-1}}{\sqrt{\pi}\, \Gamma(c-a) \Gamma(c-b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{a+b-c}{2}+1, \frac{3 c-a-b}{2}, \frac{a+b+c}{2}, \frac{c-a-b}{2}+1 \\ \frac{b-a+c}{2}, \frac{a-b+c}{2}, \frac{c}{2}, \frac{c+1}{2} \end{array}\right) /; \operatorname{Re}(z)>0$$

07.24.26.0242.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(a-c+1,\,a-c+\frac{3}{2};\,2\,a-c+\frac{3}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=$$
$$\frac{2^{4\,a-c-\frac{1}{2}}}{\pi\,\Gamma(2\,a)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},\,2\,a-\frac{3\,c}{2}+2,\,2\,a-\frac{c}{2}+1,\,\frac{c+1}{2}\\ a+\frac{1-c}{2},\,a+\frac{3-2\,c}{4},\,a-\frac{c}{2}+1,\,a+\frac{5-2\,c}{4}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.24.26.0243.01

$$\left(z+\sqrt{z^2+1}\right)^{2\,a-2\,c+\frac{3}{2}}\,{}_2\tilde{F}_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(1-a,\,\frac{1}{2}-a;\,c-2\,a+\frac{1}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=$$
$$\frac{4^{c-a-1}}{\pi\,\Gamma(2\,c-2\,a-1)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}a-c+\frac{7}{4},\,a+\frac{3}{4},\,c-a+\frac{1}{4},\,\frac{5}{4}-a\\ \frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,1\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.24.26.0244.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,\,1-a;\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(2-a-c,\,a-c+1;\,2-c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=$$
$$\frac{2^{1-c}\sin(a\,\pi)}{\pi^{3/2}}\,G_{3,3}^{3,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},\,\frac{5-3\,c}{2},\,\frac{c+1}{2}\\ 1-\frac{c}{2},\,\frac{3-c}{2}-a,\,a+\frac{1-c}{2}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

Generalized cases involving ${}_2F_1$ with algebraic arguments

07.24.26.0245.01

$${}_2\tilde{F}_1\left(a,\,b;\,c;\,-2\left(z^2+z\sqrt{z^2+1}\right)\right)\,{}_2F_1\left(a,\,b;\,c;\,-2\left(z^2-z\sqrt{z^2+1}\right)\right)=$$
$$\frac{2^{1-c}\sqrt{\pi}\;\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,\,1-b,\,a-c+1,\,b-c+1\\ 0,\,\frac{1-c}{2},\,1-\frac{c}{2},\,1-c\end{matrix}\right)$$

07.24.26.0246.01

$${}_2\tilde{F}_1\left(a,\,b;\,c;\,-2\left(z^2-z\sqrt{z^2+1}\right)\right)\,{}_2F_1\left(a,\,b;\,c;\,-2\left(z^2+z\sqrt{z^2+1}\right)\right)=$$
$$\frac{2^{1-c}\sqrt{\pi}\;\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,\,1-b,\,a-c+1,\,b-c+1\\ 0,\,\frac{1-c}{2},\,1-\frac{c}{2},\,1-c\end{matrix}\right)$$

07.24.26.0247.01

$${}_2\tilde{F}_1\left(a,\,b;\,\frac{a+b+1}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2F_1\left(a,\,b;\,\frac{a+b+1}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=$$
$$\frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)}{\sqrt{\pi}\;\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{3,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,\frac{a+b+1}{2},\,a+b\\ a,\,b,\,\frac{a+b}{2}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.24.26.0248.01

$$ {}_2\tilde{F}_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(a, b; a+b-c+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{a+b-1}\,\Gamma(a+b-c+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1, c, a+b, a+b-c+1 \\ a, b, \frac{a+b}{2}, \frac{1}{2}(a+b+1) \end{array}\right) /; \operatorname{Re}(z) > 0 $$

07.24.26.0249.01

$$ {}_2\tilde{F}_1\left(a, a+\frac{1}{2}; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(a, a+\frac{1}{2}; 2a-c+\frac{3}{2}; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{4a-\frac{3}{2}}\,\Gamma\left(2a-c+\frac{3}{2}\right)}{\pi\,\Gamma(2a)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1, c, 2a+\frac{1}{2}, 2a-c+\frac{3}{2} \\ a, a+\frac{1}{4}, a+\frac{1}{2}, a+\frac{3}{4} \end{array}\right) /; \operatorname{Re}(z) > 0 $$

07.24.26.0250.01

$$ {}_2\tilde{F}_1\left(a, b; \frac{1+a+b}{2}; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(1-a, 1-b; \frac{3-a-b}{2}; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{1}{\pi^{3/2}} \cos\left(\frac{1}{2}(a-b)\pi\right) \Gamma\left(\frac{1}{2}(3-a-b)\right) G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1, \frac{a+b+1}{2}, \frac{3-a-b}{2} \\ \frac{1}{2}, \frac{1+a-b}{2}, \frac{1-a+b}{2} \end{array}\right) /; \operatorname{Re}(z) > 0 $$

07.24.26.0251.01

$$ {}_2\tilde{F}_1\left(a, 1-a; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(a, 1-a; 2-c; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{\sin(a\pi)\,\Gamma(2-c)}{\pi^{3/2}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1, c, 2-c \\ a, 1-a, \frac{1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0 $$

Generalized cases involving algebraic functions and ${}_2F_1$ with algebraic arguments

07.24.26.0252.01

$$ \left(2z^2+2z\sqrt{z^2+1}+1\right)^{c-a-b} {}_2\tilde{F}_1\left(a, b; c; -2\left(z^2-z\sqrt{z^2+1}\right)\right) {}_2F_1\left(c-a, c-b; c; -2\left(z^2+z\sqrt{z^2+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{1,4}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{array}\right) $$

07.24.26.0253.01

$$ \left(2z^2-2z\sqrt{z^2+1}+1\right)^{c-a-b} {}_2\tilde{F}_1\left(a, b; c; -2\left(z^2+z\sqrt{z^2+1}\right)\right) {}_2F_1\left(c-a, c-b; c; -2\left(z^2-z\sqrt{z^2+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{1,4}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{array}\right) $$

07.24.26.0254.01

$$ \left(2z^2+2z\sqrt{z^2+1}+1\right)^{a+b-c} {}_2\tilde{F}_1\left(a, b; c; -2z\left(z+\sqrt{z^2+1}\right)\right) {}_2F_1\left(c-a, c-b; c; -2z\left(z-\sqrt{z^2+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{1,4}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1-a, 1-b, a-c+1, b-c+1 \\ 0, \frac{1-c}{2}, 1-\frac{c}{2}, 1-c \end{array}\right) $$

07.24.26.0255.01

$$\left(2z^2-2z\sqrt{z^2+1}+1\right)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;-2z\left(z-\sqrt{z^2+1}\right)\right){}_2F_1\left(c-a,c-b;c;-2z\left(z+\sqrt{z^2+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right)$$

07.24.26.0256.01

$$\left(1-2z^2-2z\sqrt{z^2-1}\right)^{a}\,{}_2\tilde{F}_1\left(a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right){}_2F_1\left(a,c-b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,1+a-c,1+b-c\\0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)/;\,z\notin(-\infty,-1)$$

07.24.26.0257.01

$$\left(1-2z^2-2z\sqrt{z^2-1}\right)^{b}\,{}_2\tilde{F}_1\left(a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right){}_2F_1\left(c-a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)/;\,z\notin(-\infty,-1)$$

07.24.26.0258.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(a-c+1,b-c+1;a+b-c+1;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{2^{a+b-c}\,\Gamma(a+b-c+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},a+b+\frac{1-c}{2},\frac{c+1}{2},a+b+\frac{3-3c}{2}\\a+\frac{1-c}{2},b+\frac{1-c}{2},\frac{a+b-c+1}{2},\frac{a+b-c}{2}+1\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

07.24.26.0259.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(c-a,c-b;c-a-b+1;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{2^{c-1}\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a+b-c}{2}+1,\frac{3c-a-b}{2},\frac{a+b+c}{2},\frac{c-a-b}{2}+1\\\frac{b-a+c}{2},\frac{a-b+c}{2},\frac{c}{2},\frac{c+1}{2}\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

07.24.26.0260.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-2c+1}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(1-a,1-b;c-a-b+1;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a+b+3}{2}-c,\frac{1-a-b}{2}+c,\frac{a+b+1}{2},\frac{3-a-b}{2}\\\frac{1}{2},1,\frac{1-a+b}{2},\frac{a-b+1}{2}\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

07.24.26.0261.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2\tilde{F}_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(a-c+1,b-c+1;a+b-c+1;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{2^{a+b-c}\,\Gamma(a+b-c+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},a+b+\frac{1-c}{2},a+b+\frac{3-3c}{2},\frac{c+1}{2}\\a+\frac{1-c}{2},b+\frac{1-c}{2},\frac{a+b-c+1}{2},\frac{a+b-c}{2}+1\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

07.24.26.0262.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2\tilde{F}_1\!\left(a,a+\frac{1}{2};c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=$$

$$\frac{2^{4a-c-\frac{1}{2}}\,\Gamma\!\left(2a-c+\frac{3}{2}\right)}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{3-c}{2},2a-\frac{3c}{2}+2,2a-\frac{c}{2}+1,\frac{c+1}{2}\\ a+\frac{1-c}{2},a+\frac{3-2c}{4},a-\frac{c}{2}+1,a+\frac{5-2c}{4}\end{array}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0263.01

$$\left(z+\sqrt{z^2+1}\right)^{2a-c+\frac{1}{2}}\,{}_2\tilde{F}_1\!\left(a,a+\frac{1}{2};c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(c-a-\frac{1}{2},c-a;c-2a+\frac{1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=$$

$$\frac{2^{3c-2a-3}\,\Gamma\!\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}a+\frac{5-2c}{4},a+\frac{2c+1}{4},\frac{6c-1}{4}-a,\frac{2c+3}{4}-a\\ \frac{2c-1}{4},\frac{c}{2},\frac{2c+1}{4},\frac{c+1}{2}\end{array}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0264.01

$$\left(z+\sqrt{z^2+1}\right)^{2a-2c+\frac{3}{2}}\,{}_2\tilde{F}_1\!\left(a,a+\frac{1}{2};c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(\frac{1}{2}-a,1-a;c-2a+\frac{1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=$$

$$\frac{4^{c-a-1}\,\Gamma\!\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}a-c+\frac{7}{4},a+\frac{3}{4},c-a+\frac{1}{4},\frac{5}{4}-a\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1\end{array}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0265.01

$$\left(z+\sqrt{z^2+1}\right)^{2-2c}\,{}_2\tilde{F}_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right)=$$

$$\frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}2-c,1,c\\ a,1-a,\frac{1}{2}\end{array}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0266.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2\tilde{F}_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(2-a-c,a-c+1;2-c;\frac{z-\sqrt{z^2+1}}{2z}\right)=$$

$$\frac{2^{1-c}\sin(a\pi)\,\Gamma(2-c)}{\pi^{3/2}}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{3-c}{2},\frac{5-3c}{2},\frac{c+1}{2}\\ 1-\frac{c}{2},\frac{3-c}{2}-a,a+\frac{1-c}{2}\end{array}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0267.01

$$\left(z+\sqrt{z^2+1}\right)^{\frac{a+b-1}{2}}\,{}_2\tilde{F}_1\!\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(\frac{b-a+1}{2},\frac{a-b+1}{2};\frac{3-a-b}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=$$

$$\frac{2^{\frac{a+b-1}{2}}}{\pi^{3/2}}\cos\!\left(\frac{1}{2}(a-b)\pi\right)\Gamma\!\left(\frac{1}{2}(3-a-b)\right)G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{a+b+3}{4},\frac{3a+3b+1}{4},\frac{5-a-b}{4}\\ \frac{a+b+1}{4},\frac{3a-b+1}{4},\frac{3b-a+1}{4}\end{array}\right)\;/;\;\mathrm{Re}(z)>0$$

07.24.26.0268.01

$$\left(z+\sqrt{z^2+1}\right)^{-2a} {}_2\tilde{F}_1\left(a, b; a-b+1; \frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right) {}_2F_1\left(a, b; a-b+1; \frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right) = \frac{4^{-b}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-a, 1-b, a-2b+1 \\ 0, 1-2b, \frac{1}{2}-b \end{array}\right) /; \operatorname{Re}(z) > 0$$

Through other functions

Involving some hypergeometric-type functions

07.24.26.0269.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{1}{\Gamma(c)} F_1(a; b, b_2; c; z, 0) /; -c \notin \mathbb{N}$$

07.24.26.0270.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{1}{\Gamma(c)} F_1(a; b, 0; c; z, z_2)$$

07.24.26.0271.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{1}{\Gamma(c)} F_1(a; d, b-d; c; z, z)$$

07.24.26.0272.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{\Gamma(p-a+c)}{\Gamma(c+p)\,\Gamma(c-a)} F_1(a; b, p; c+p; z, 1) /; \operatorname{Re}(c-a) > 0$$

Representations through equivalent functions

With related functions

07.24.27.0001.01

$${}_2\tilde{F}_1(a, b; c; z) = \frac{\Gamma(1-a)}{\Gamma(c-a)} P_{-a}^{(c-1, a+b-c)}(1-2z)$$

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