Hypergeometric3F2

Notations

Traditional name

Generalized hypergeometric function ${}_3F_2$

Traditional notation

${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$

Mathematica StandardForm notation

HypergeometricPFQ[{a_1, a_2, a_3}, {b_1, b_2}, z]

Primary definition

07.27.02.0001.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k z^k}{(b_1)_k (b_2)_k k!} /; |z| < 1 \bigvee |z| = 1 \bigwedge \mathrm{Re}\left(\sum_{j=1}^{2} b_j - \sum_{j=1}^{3} a_j\right) > 0 $$

For $a_i = -n$, $b_j = -m$ /; $m \geq n$ being nonpositive integers and $\nexists_{a_k} (a_k > -n \bigwedge a_k \in \mathbb{N}) \bigwedge \nexists_{b_k} (b_k > -m \bigwedge b_k \in \mathbb{N})$ the function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a_i, b_j can approach nonpositive integers $-n$, $-m$; $m \geq n$ at different speeds. For the above conditions we define:

07.27.02.0002.01

$$ {}_3F_2(a_1, \ldots, a_i, \ldots, a_3; b_1, \ldots, b_j, \ldots, b_2; z) = \sum_{k=0}^{n} \frac{(a_1)_k (a_2)_k (a_3)_k z^k}{(b_1)_k (b_2)_k k!} /; a_i = -n \bigwedge b_j = -m \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge m \geq n $$

Specific values

Values at $z = 0$

07.27.03.0001.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 0) = 1 $$

Values at $z = 1$

For fixed a_1, a_2, a_3, b_1, b_2

07.27.03.0002.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = \frac{\Gamma(\psi_3)\prod_{k=1}^{2}\Gamma(b_k)}{\prod_{k=3}^{3}\Gamma(a_k)} \sum_{k=0}^{\infty} \frac{(\psi_3)_k\, \mathcal{E}_k^{(3)}(\{a_1, a_2, a_3\}, \{b_1, b_2\})}{\Gamma(k+a_1+\psi_3)\,\Gamma(k+a_2+\psi_3)} \,/;\, \psi_3 = b_1+b_2-a_1-a_2-a_3 \land \operatorname{Re}(\psi_3) > 0 \land \operatorname{Re}(a_3) > 0 $$

07.27.03.0003.01

$$ {}_3F_2(a, b, c; d, e; 1) = \sqrt{\Gamma(1-a)}\,\sqrt{\Gamma(1-b)}\,\sqrt{\Gamma(1-c)}\,\Gamma(d)\,\Gamma(e)\,\sqrt{\Gamma(-a-b-c+d+e)} \left\langle \frac{d-a-b-1}{2}\ \frac{e-a-c-1}{2}\ \frac{b+d-a-1}{2}\ \frac{a-c-e+1}{2} \,\middle|\, \frac{d-a-b-1}{2}\ \frac{e-a-c-1}{2}\ \frac{d+e-b-c}{2}-1\ \frac{b+d-c-e}{2} \right\rangle \Big/ \left(\sqrt{d+e-b-c-1}\,\sqrt{\Gamma(d-a)}\,\sqrt{\Gamma(d-b)}\,\sqrt{\Gamma(d-c)}\,\sqrt{\Gamma(e-a)}\,\sqrt{\Gamma(e-b)}\,\sqrt{\Gamma(e-c)}\right) /;\, \operatorname{Re}(d+e-a-b-c) > 0 $$

07.27.03.0004.01

$$ {}_3F_2(a, b, c; d, e; 1) = \left(\Gamma(d)\,\Gamma(e)\,\sqrt{\Gamma(1-a)}\,\sqrt{\Gamma(1-b)}\,\sqrt{\Gamma(1-c)}\,\sqrt{\Gamma(d+e-a-b-c)}\right) \Big/ \left(\sqrt{\Gamma(d-a)}\,\sqrt{\Gamma(d-b)}\,\sqrt{\Gamma(d-c)}\,\sqrt{\Gamma(e-a)}\,\sqrt{\Gamma(e-b)}\,\sqrt{\Gamma(e-c)}\right) i^{2(d-e)} \begin{pmatrix} \frac{d-a-b-1}{2} & \frac{e-a-c-1}{2} & \frac{d+e-b-c}{2}-1 \\ \frac{b+d-a-1}{2} & \frac{a-c-e+1}{2} & \frac{c+e-b-d}{2} \end{pmatrix} /;\, \operatorname{Re}(d+e-a-b-c) > 0 $$

For fixed a_1, a_2, a_3, b_1

07.27.03.0005.01

$$ {}_3F_2(a, b, c; d, a-1; 1) = \frac{\Gamma(d)\,\Gamma(d-b-c)}{\Gamma(d-b)\,\Gamma(d-c)} \left(1 - \frac{b\,c}{(a-1)\,(b+c-d+1)}\right) /;\, \operatorname{Re}(d-b-c) > 1 $$

For fixed a_1, a_2, a_3

07.27.03.0006.01

$$ {}_3F_2(a, b, c; a-b+1, a-c; 1) = \frac{\sqrt{\pi}\,\Gamma(a-c)\,\Gamma(a-b+1)}{2^a\,\Gamma(a-b-c+1)} \left(\frac{\Gamma\left(\frac{a}{2}-b-c+1\right)}{\Gamma\left(\frac{a}{2}-c\right)\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{2}-b+1\right)} + \frac{\Gamma\left(\frac{a+1}{2}-b-c\right)}{\Gamma\left(\frac{a+1}{2}-c\right)\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{a+1}{2}-b\right)} \right) /;\, \operatorname{Re}(a-2b-2c) > -1 $$

07.27.03.0007.01

$$ {}_3F_2(a, b, c; a-b+1, a-c+1; 1) = \left(\Gamma\left(\frac{a}{2}+1\right)\Gamma(a-b+1)\,\Gamma(a-c+1)\,\Gamma\left(\frac{a}{2}-b-c+1\right)\right) \Big/ \left(\Gamma(a+1)\,\Gamma\left(\frac{a}{2}-b+1\right)\Gamma\left(\frac{a}{2}-c+1\right)\Gamma(a-b-c+1)\right) /;\, \operatorname{Re}(a-2b-2c) > -2 $$

07.27.03.0008.01

$$ {}_3F_2(a, b, c; a-b+2, a-c+2; 1) = \frac{\Gamma(a-b+2)\,\Gamma(a-c+2)}{2\,(b-1)\,(c-1)\,\Gamma(a)\,\Gamma(a-b-c+2)} \left(\frac{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{a}{2}-b-c+2\right)}{\Gamma\left(\frac{a}{2}-b+1\right)\Gamma\left(\frac{a}{2}-c+1\right)} - \frac{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a+5}{2}-b-c\right)}{\Gamma\left(\frac{a+3}{2}-b\right)\Gamma\left(\frac{a+3}{2}-c\right)} \right) /;\, \operatorname{Re}(a-2b-2c) > -4 $$

07.27.03.0009.01

$$ {}_3F_2\left(a, b, c; \frac{a+b+1}{2}, 2c; 1\right) = \frac{\sqrt{\pi}\,\left(\Gamma\left(c+\frac{1}{2}\right)\Gamma\left(\frac{a+b+1}{2}\right)\Gamma\left(\frac{1-a-b}{2}+c\right)\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)\Gamma\left(\frac{1-a}{2}+c\right)\Gamma\left(\frac{1-b}{2}+c\right)} /;\, \operatorname{Re}(-a-b+2c) > -1 $$

07.27.03.0010.01

$$ {}_3F_2(a,b,c;a-n,b+m;1) = \frac{\Gamma(1-c)\,\Gamma(b+m)\,(b-a+1)_n}{\Gamma(b-c+1)\,(m-1)!\,(1-a)_n} \sum_{k=0}^{m-1} \frac{(b-a+n+1)_k\,(b)_k\,(1-m)_k}{k!\,(b-a+1)_k\,(b-c+1)_k} \;/;\; \mathrm{Re}(c) < m-n \land m \in \mathbb{N} \land n \in \mathbb{N} $$

07.27.03.0011.01

$$ {}_3F_2(a,b,c;a-n,a+1;1) = n!\,\frac{\Gamma(a+1)\,\Gamma(a-b-c-n+1)}{\Gamma(a-b+1)\,\Gamma(a-c+1)} \sum_{k=0}^{n} \frac{(a-b-n)_k\,(a-c-n)_k}{k!\,(a-n)_k} \;/; $$
$$ \mathrm{Re}(a) > n+1 \bigwedge \mathrm{Re}(a-b-c) > n-1 \bigwedge n \in \mathbb{N} $$

07.27.03.0012.01

$$ {}_3F_2(a,b,c;a-n,b+1;1) = \frac{\Gamma(1-c)\,\Gamma(b+1)\,(b-a+1)_n}{\Gamma(b-c+1)\,(1-a)_n} \;/;\; \mathrm{Re}(c) < 1-n \land n \in \mathbb{N} $$

07.27.03.0013.01

$$ {}_3F_2(a,b,c;a+1,b+1;1) = \frac{a\,b\,\Gamma(1-c)}{a-b}\left(\frac{\Gamma(b)}{\Gamma(b-c+1)} - \frac{\Gamma(a)}{\Gamma(a-c+1)}\right) /;\; \mathrm{Re}(c) < 2 \bigwedge a \neq b \bigwedge c \neq 1 $$

07.27.03.0014.01

$$ {}_3F_2(a,b,c;a+2,b+2;1) = $$
$$ \frac{1}{a-b}\left(\frac{2\,(a+1)\,(b+1)\,\Gamma(1-c)}{(a-b-1)\,(a-b+1)}\left(\frac{b\,\Gamma(a+1)}{\Gamma(a-c+1)} - \frac{a\,\Gamma(b+1)}{\Gamma(b-c+1)}\right) - \frac{b\,(b+1)\,\Gamma(2-c)\,\Gamma(a+2)}{(b-a-1)\,\Gamma(a-c+2)} + \frac{a\,(a+1)\,\Gamma(2-c)\,\Gamma(b+2)}{(a-b-1)\,\Gamma(b-c+2)}\right) /; $$
$$ \mathrm{Re}(c) < 4 \bigwedge a \neq b-1 \bigwedge a \neq b \bigwedge a \neq b+1 \bigwedge c \neq 1 \bigwedge c \neq 2 \bigwedge c \neq 3 $$

For fixed a_1, a_2, b_1

07.27.03.0015.01

$$ {}_3F_2(a,b,-n;d,a+b-d-n+1;1) = \frac{(d-a)_n\,(d-b)_n}{(d)_n\,(-a-b+d)_n} \;/;\; n \in \mathbb{N} $$

07.27.03.0016.01

$$ {}_3F_2(a,b,1-a;d,2b-d+1;1) = \frac{\pi\,2^{1-2b}\,\Gamma(d)\,\Gamma(2b-d+1)}{\Gamma\left(\frac{a-d+1}{2}+b\right)\Gamma\left(\frac{a+d}{2}\right)\Gamma\left(\frac{d-a+1}{2}\right)\Gamma\left(b-\frac{a+d}{2}+1\right)} \;/;\; \mathrm{Re}(b) > 0 $$

07.27.03.0017.01

$$ {}_3F_2(a,b,a-n;d,a-n+1;1) = \frac{(a-n)\,(b-a)_n\,(n-1)!\,\Gamma(d)\,\Gamma(d-a-b+1)}{(a-b)\,(1-a)_n\,(d-a)_n\,\Gamma(d-a)\,\Gamma(d-b)} \sum_{k=0}^{n-1} \frac{(1-a)_k\,(d-a)_k}{k!\,(b-a+1)_k} \;/;\; \mathrm{Re}(a+b-d) < 1 $$

07.27.03.0018.01

$$ {}_3F_2(a,b,1-a;d,2b-d+1;1) = $$
$$ \left(2^{1-2b}\,\pi\,\Gamma(d)\,\Gamma(2b-d+1)\right) \Big/ \left(\Gamma\left(\frac{a+d}{2}\right)\Gamma\left(b+\frac{a-d+1}{2}\right)\Gamma\left(\frac{d-a+1}{2}\right)\Gamma\left(b-\frac{a+d}{2}+1\right)\right) /;\; \mathrm{Re}(a+b-d) < 1 $$

07.27.03.0019.01

$$ {}_3F_2\left(a,b,a+\frac{1}{2};d,d+\frac{1}{2};1\right) = \frac{\Gamma(2d)\,\Gamma(2d-2a-b)}{\Gamma(2d-2a)\,\Gamma(2d-b)}\;{}_2F_1(2a,b;2d-b;-1) \;/;\; 0 < \mathrm{Re}(b) < 2\,\mathrm{Re}(d-a) $$

07.27.03.0020.01

$$ {}_3F_2(1,a,b;d,a+b-d+2;1) = \frac{a+b-d+1}{(a-d+1)\,(b-d+1)}\left(1-d+\frac{\Gamma(a+b-d+1)\,\Gamma(d)}{\Gamma(a)\,\Gamma(b)}\right) $$

07.27.03.0021.01

$$ {}_3F_2(1, a, b; 2, d; 1) = \frac{d-1}{(a-1)(b-1)} \left(\frac{\Gamma(d-1)\,\Gamma(d-a-b+1)}{\Gamma(d-a)\,\Gamma(d-b)} - 1 \right) /; \operatorname{Re}(d-a-b) > -1 \bigwedge a \neq 1 \bigwedge b \neq 1 \bigwedge c \neq 1 $$

07.27.03.0022.01

$$ {}_3F_2(1, a, b; 3, d; 1) = \frac{2(d-2)(d-1)}{(a-2)(a-1)(b-2)(b-1)} \left(\frac{\Gamma(d-2)\,\Gamma(d-a-b+2)}{\Gamma(d-a)\,\Gamma(d-b)} - 1 \right) - \frac{2(d-1)}{(a-1)(b-1)} /; $$
$$ \operatorname{Re}(d-a-b) > -2 \bigwedge a \neq 1 \bigwedge a \neq 2 \bigwedge b \neq 1 \bigwedge b \neq 2 \bigwedge c \neq 1 \bigwedge c \neq 2 $$

For fixed a_2, a_3, b_1

07.27.03.0023.01

$$ {}_3F_2(-n, b, c; d, b+c-d-n+1; 1) = \frac{(d-b)_n\,(d-c)_n}{(d)_n\,(-b-c+d)_n} /; n \in \mathbb{N} $$

07.27.03.0024.01

$$ {}_3F_2(-n, b, c; d, b+c-d-n+2; 1) = \frac{(d-b-1)_n\,(d-c)_n}{(d)_n\,(d-b-c-1)_n} \left(1 + \frac{c\,n}{(d-b-1)(c-d-n+1)} \right) /; n \in \mathbb{N} $$

For fixed a_1, a_2

07.27.03.0025.01

$$ {}_3F_2\left(a, b, \frac{a+b}{2}; a+b, \frac{a+b+1}{2}; 1\right) = \frac{\pi\,\Gamma\left(\frac{a+b+1}{2}\right)^2}{\Gamma\left(\frac{a+1}{2}\right)^2 \Gamma\left(\frac{b+1}{2}\right)^2} $$

07.27.03.0026.01

$$ {}_3F_2(a, b, 2a-b+1; a+1, 2a+1; 1) = \frac{\Gamma(2a+1)}{2\,\Gamma(b)\,\Gamma(2a-b+1)} \left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{b}{2}\right) - \psi\left(a + \frac{1-b}{2}\right) + \psi\left(a - \frac{b}{2} + 1\right) \right) $$

07.27.03.0027.01

$$ {}_3F_2(a, b, a; a+1, a+1; 1) = \frac{a\,\Gamma(a+1)\,\Gamma(1-b)\,(\psi(a-b+1) - \psi(a))}{\Gamma(a-b+1)} /; \operatorname{Re}(b) < 2 $$

07.27.03.0028.01

$$ {}_3F_2(a, b, a; a+2, a+2; 1) = \frac{\Gamma(a+2)\,\Gamma(1-b)}{\Gamma(a-b+2)} \left(2a^2 + (b-2a-1)\,a\,(a+1)\,(\psi(a+2) - \psi(a-b+2)) + \left(1 - 2a^2\right)(1-b) \right) /; $$
$$ b \neq 1 \bigwedge b \neq 2 \bigwedge b \neq 3 \bigwedge \operatorname{Re}(b) < 4 $$

07.27.03.0029.01

$$ {}_3F_2(a, b, a+1; a+2, a+3; 1) = \frac{(a+1)\,\Gamma(a+3)\,\Gamma(1-b)}{2\,\Gamma(a-b+2)} \left(\frac{(1-b)(2a-b+2)}{a-b+2} + 2a\,(\psi(a+1) - \psi(a-b+2)) \right) /; $$
$$ b \neq 1 \bigwedge b \neq 2 \bigwedge b \neq 3 \bigwedge \operatorname{Re}(b) < 4 $$

07.27.03.0030.01

$$ {}_3F_2\left(a, b, \frac{a}{2}; \frac{a}{2}+1, a-b+1; 1\right) = \frac{\sqrt{\pi}\;\Gamma(a-b+1)\,\Gamma(1-b)\,\Gamma\left(\frac{a}{2}+1\right)}{2^a\,\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{2}-b+1\right)^2} /; \operatorname{Re}(b) < 1 $$

07.27.03.0031.01

$$ {}_3F_2\left(a, b, \frac{a}{2}+1; \frac{a}{2}, a-b+1; 1\right) = 0 /; a \neq 0 \bigwedge \operatorname{Re}(b) < 0 $$

07.27.03.0032.01

$${}_3F_2\left(a, b, a+\frac{1}{2}; a+\frac{3}{2}, a+2; 1\right) = (2a+1)(a+1)\Gamma(1-b)\left(\frac{\Gamma(a+1)\,a}{\Gamma(a-b+2)} - \frac{2a\,\Gamma\left(a+\frac{1}{2}\right)}{\Gamma\left(a-b+\frac{3}{2}\right)} + \frac{\Gamma(a+1)}{\Gamma(a-b+1)}\right) /;$$

$$b \neq 1 \land b \neq 2 \land \mathrm{Re}(b) < 3$$

For fixed a_2, a_3

07.27.03.0033.01

$${}_3F_2(1, b, c; -b-m, -c-m; 1) = \left(2^{2m-1}\sqrt{\pi}\;\Gamma(1-b)\,\Gamma(1-c)\,\Gamma\left(-b-c-m-\frac{1}{2}\right)\right) \Big/ \left(\Gamma(-b-c-m)\,\Gamma\left(\frac{1}{2}-b-m\right)\Gamma\left(\frac{1}{2}-c-m\right)(2b+m+1)_m\,(2c+m+1)_m\right) + \frac{1}{2}\sum_{k=0}^{m+1}\frac{(b)_k\,(c)_k}{(-b-m)_k\,(-c-m)_k} /; \mathrm{Re}(b+c) < -m-\frac{1}{2} \bigwedge m+2 \in \mathbb{N}$$

07.27.03.0034.01

$${}_3F_2(1, b, c; m-b, m-c; 1) = \left(\sqrt{\pi}\;(m-2b)_m\,(m-2c)_m\,(m-b-c)_m\,\Gamma(1-b)\,\Gamma(1-c)\,\Gamma\left(2m-b-c-\frac{1}{2}\right)\right) \Big/ \left(2^{2m+1}\left(m-b-c-\frac{1}{2}\right)_m\Gamma\left(m-b+\frac{1}{2}\right)\Gamma\left(m-c+\frac{1}{2}\right)\Gamma(2m-b-c)\right) - \frac{1}{2}\sum_{k=1}^{m-2}\frac{(b-m+1)_k\,(c-m+1)_k}{(1-b)_k\,(1-c)_k} /; \mathrm{Re}(b+c) < m-\frac{1}{2} \bigwedge m-2 \in \mathbb{N}$$

07.27.03.0035.01

$${}_3F_2(1, b, c; b+1, c+1; 1) = \frac{b\,c}{c-b}\,(\psi(c)-\psi(b)) /; c \neq b$$

07.27.03.0036.01

$${}_3F_2(1, b, c; b+2, c+2; 1) = \frac{(b+1)(c+1)((b-c)(b+c+1) - 2bc(\psi(b)-\psi(c)))}{(b-c-1)_3} /; c \neq b-1 \land c \neq b \land c \neq b+1$$

07.27.03.0037.01

$${}_3F_2(2, b, c; b+2, c+2; 1) = \frac{b(b+1)c(c+1)(b+c-1)}{(b-c-1)_3}\left(\psi(b)-\psi(c)-\frac{2(b-c)}{b+c-1}\right) /; c \neq b-1 \land c \neq b \land c \neq b+1$$

07.27.03.0038.01

$${}_3F_2(3, b, c; b+2, c+2; 1) = \frac{(b-1)_3\,(c-1)_3}{(b-c-1)_3}\left(\frac{(b+c-3)(b-c)}{2(b-1)(c-1)} - \psi(b)+\psi(c)\right) /; c \neq b-1 \land c \neq b \land c \neq b+1$$

07.27.03.0039.01

$${}_3F_2(-n, b, c; b-l, c-m; 1) = 0 /; n \in \mathbb{N} \land m \in \mathbb{N} \land l \in \mathbb{N}^+ \land 1-l \leq m \leq n-l-1$$

07.27.03.0040.01

$${}_3F_2(-n, b, c; b+1, c-m; 1) = \frac{n!\,(b-c+1)_m}{(b+1)_n\,(1-c)_m} /; n \in \mathbb{N} \land m \in \mathbb{N} \land m \leq n$$

07.27.03.0041.01

$${}_3F_2(-n, b, c; b+1, c-n-1; 1) = \frac{n!\,b}{(1-c)_{n+1}}\left(\frac{(b-c+1)_{n+1}}{(b)_{n+1}} - 1\right) /; n \in \mathbb{N}$$

07.27.03.0042.01

$${}_3F_2(-n, b, c; 1-b-n, 1-c-n; 1) = \frac{\left(n-2\left\lfloor\frac{n+1}{2}\right\rfloor+1\right)(-4)^{n/2}(1-b-c-n)_{\frac{n}{2}}\left(\frac{1}{2}\right)_{\frac{n}{2}}}{(1-b-n)_{\frac{n}{2}}(1-c-n)_{\frac{n}{2}}} /; n \in \mathbb{N}$$

07.27.03.0043.01

$${}_3F_2(-n, b, c; 2b-n+1, c-b+1; 1) = \frac{(2b-c-2n)(-b)_n(c-2b)_n}{(2b-c)(-2b)_n(c-b+1)_n} /; n \in \mathbb{N}$$

07.27.03.0044.01

$${}_3F_2(-n, b, c; b+n+1, b-c+1; 1) = \frac{(b+1)_n\left(\frac{b}{2}-c+1\right)_n}{\left(\frac{b}{2}+1\right)_n(b-c+1)_n} /; n \in \mathbb{N}$$

07.27.03.0045.01

$${}_3F_2\left(-n, b, c; 2b, \frac{c-n+1}{2}; 1\right) = \frac{\left(n-2\left\lfloor\frac{n+1}{2}\right\rfloor+1\right)\left(\frac{1}{2}\right)_{\frac{n}{2}}\left(b+\frac{1-c}{2}\right)_{\frac{n}{2}}}{\left(\frac{1-c}{2}\right)_{\frac{n}{2}}\left(b+\frac{1}{2}\right)_{\frac{n}{2}}} /; n \in \mathbb{N}$$

07.27.03.0046.01

$${}_3F_2\left(-n, b, c; -2n, \frac{b+c+1}{2}; 1\right) = \frac{\left(\frac{b+1}{2}\right)_n\left(\frac{c+1}{2}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{b+c+1}{2}\right)_n} /; n \in \mathbb{N}$$

07.27.03.0047.01

$${}_3F_2\left(-n, b, c; \frac{b-n}{2}, \frac{b-n+1}{2}; 1\right) = {}_2F_1(-n, 2c; b-n; 2) /; n \in \mathbb{N}$$

07.27.03.0048.01

$${}_3F_2\left(-n, b, c; \frac{b-n}{2}, \frac{b-n+1}{2}; 1\right) = \frac{(2c-b+1)_n}{(1-b)_n}\, {}_2F_1(-n, 2c; 2c-b+1; -1) /; n \in \mathbb{N}$$

For fixed a_2, b_1

07.27.03.0049.01

$${}_3F_2(-n, b, 1-b; d, -d-2n+1; 1) = \frac{2^{2n}}{(d)_{2n}}\left(\frac{b+d}{2}\right)_n\left(\frac{1+d-b}{2}\right)_n /; n \in \mathbb{N}$$

For fixed a_2, b_2

07.27.03.0050.01

$${}_3F_2\left(-n, b, \frac{b}{2}+1; \frac{b}{2}, e; 1\right) = \frac{(e-b-n-1)(e-b)_{n-1}}{(e)_n} /; n \in \mathbb{N}$$

For fixed a_3, b_1

07.27.03.0051.01

$${}_3F_2\left(-\frac{n}{2}, -\frac{n-1}{2}, c; d, d+\frac{1}{2}; 1\right) = \frac{(2d-c)_n}{(2d)_n}\, {}_2F_1(-n, c; 2d-c; -1) /; n \in \mathbb{N}$$

07.27.03.0052.01

$${}_3F_2\left(-\frac{n}{2}, -\frac{n-1}{2}, c; d, c-d-n+\frac{3}{2}; 1\right) = \frac{4^{-n}\,(2\,d-2\,c-1)_n\,(2\,d+n-1)_n}{(d)_n\left(d-c-\frac{1}{2}\right)_n} /; n \in \mathbb{N}$$

For fixed a_3, b_2

07.27.03.0053.01

$${}_3F_2(1, 1, c; 2, e; 1) = \frac{(e-1)\,(\psi(e-1)-\psi(e-c))}{c-1} /; c \neq 1 \land \operatorname{Re}(e-c) > 0$$

07.27.03.0055.01

$${}_3F_2(1, 1, c; 3, e; 1) = \frac{2\,(e-1)}{c-1} + \frac{(2\,(e-1)\,(e-c))\,(\psi(-c+e+1)-\psi(e-1))}{(c-2)\,(c-1)} /; c \neq 1 \land c \neq 2 \land \operatorname{Re}(e-c) > -1$$

07.27.03.0056.01

$${}_3F_2(1, 2, c; 3, e; 1) = \frac{2\,(e-2)\,(e-1)\,(\psi(e-2)-\psi(e-c))}{(c-2)\,(c-1)} - \frac{2\,(e-1)}{c-1} /; c \neq 1 \land c \neq 2 \land \operatorname{Re}(e-c) > 0$$

For fixed a_1

07.27.03.0057.01

$${}_3F_2(a, a-m, a-n; a-m+1, a-n+1; 1) = 0 /; m \neq n \bigwedge 1-a \notin \mathbb{N} \bigwedge \operatorname{Re}(a) < 2 \bigwedge m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$$

07.27.03.0058.01

$${}_3F_2(a, a, a; 1, 1; 1) = \frac{\Gamma\left(1-\frac{3\,a}{2}\right)\cos\left(\frac{\pi\,a}{2}\right)}{\Gamma\left(1-\frac{a}{2}\right)^3} /; \operatorname{Re}(a) < \frac{2}{3}$$

07.27.03.0059.01

$${}_3F_2(a, a, a; 2, 2; 1) = \frac{1}{(1-a)^3}\left(\frac{\Gamma\left(2-\frac{3\,a}{2}\right)\cos\left(\frac{\pi\,a}{2}\right)}{\Gamma\left(1-\frac{a}{2}\right)^3} - \frac{\Gamma\left(\frac{5-3\,a}{2}\right)\sin\left(\frac{\pi\,a}{2}\right)}{\Gamma\left(\frac{1-a}{2}\right)\Gamma\left(\frac{3-a}{2}\right)^2}\right) /; a \neq 1 \bigwedge \operatorname{Re}(a) < \frac{4}{3}$$

07.27.03.0060.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{1}{3}, \frac{2}{3}; 1\right) = 2\,3^{-\frac{3\,a}{2}-1}\cos\left(\frac{\pi\,a}{2}\right) /; \operatorname{Re}(a) < 0$$

07.27.03.0061.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{2}{3}, \frac{4}{3}; 1\right) = \frac{2}{1-3\,a}\,3^{-\frac{1}{2}(3\,a+1)}\cos\left(\frac{\pi}{6}\,(3\,a+1)\right) /; \operatorname{Re}(a) < \frac{1}{3}$$

07.27.03.0062.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; 1\right) = \frac{4}{(1-3\,a)\,(2-3\,a)}\,3^{-\frac{3\,a}{2}}\cos\left(\frac{\pi}{6}\,(3\,a+2)\right) /; a \neq \frac{1}{3} \bigwedge \operatorname{Re}(a) < \frac{2}{3}$$

For fixed a_2

07.27.03.0063.01

$${}_3F_2(1, b, b; b+1, b+1; 1) = b^2\,\psi^{(1)}(b)$$

07.27.03.0064.01

$${}_3F_2(1, b, -b; b+1, 1-b; 1) = \frac{1}{2}\,(b\,\pi\cot(\pi\,b)+1)$$

07.27.03.0065.01

$${}_3F_2(1,\,b,\,1-b;\,b+1,\,2-b;\,1)=\frac{(\pi\,b\,(1-b))\cot(\pi\,b)}{1-2\,b}$$

07.27.03.0066.01

$${}_3F_2(1,\,b,\,b;\,b+2,\,b+2;\,1)=(b+1)^2\left(2\,\psi^{(1)}(b)\,b^2-2\,b-1\right)$$

07.27.03.0067.01

$${}_3F_2\!\left(1,\,b,\,b+\frac{1}{2};\,b+\frac{3}{2},\,b+2;\,1\right)=(b+1)\,(2\,b+1)\left(2\,b\,\psi\!\left(b+\frac{1}{2}\right)-2\,b\,\psi(b)-1\right)$$

07.27.03.0068.01

$${}_3F_2(1,\,b,\,b+1;\,b+2,\,b+3;\,1)=\frac{1}{2}\,((b+1)\,(b+2))\left(-2\,(b+1)\,\psi^{(1)}(b+1)\,b+2\,b+1\right)$$

07.27.03.0069.01

$${}_3F_2(2,\,b,\,b;\,b+2,\,b+2;\,1)=b^2\,(b+1)^2\left(2-(2\,b-1)\,\psi^{(1)}(b)\right)$$

07.27.03.0070.01

$${}_3F_2\!\left(2,\,b,\,b+\frac{1}{2};\,b+\frac{3}{2},\,b+2;\,1\right)=b\,(2\,b+1)\,(b+1)\left(-(2\,b-1)\,\psi\!\left(b-\frac{1}{2}\right)+(2\,b-1)\,\psi(b)-1\right)$$

07.27.03.0071.01

$${}_3F_2(2,\,b,\,b+1;\,b+2,\,b+3;\,1)=\frac{(b+1)^2\,(b+2)}{2}\left(2\,\psi^{(1)}(b+1)\,b^2-2\,b+1\right)$$

07.27.03.0072.01

$${}_3F_2(3,\,b,\,b;\,b+2,\,b+2;\,1)=\frac{b^2\,(b+1)^2}{2}\left(2\,\psi^{(1)}(b)\,(b-1)^2-2\,b+3\right)$$

07.27.03.0073.01

$${}_3F_2\!\left(-n,\,b,\,\frac{b}{2}+1;\,\frac{b}{2},\,b+n+1;\,1\right)=0\;/;\;n\in\mathbb{N}^+$$

07.27.03.0074.01

$${}_3F_2\!\left(-n,\,b,\,b+\frac{1}{2};\,\frac{1}{3}\,(2\,b-n+1),\,\frac{1}{3}\,(2\,b-n+2);\,1\right)=\frac{(-1)^n\left(\frac{1}{3}\,(2\,b-n)\right)_n}{2^{2\,n}\,(1-2\,b)_n}\,{}_2F_1\!\left(-n,\,\frac{2\,(2\,b-n)}{3};\,1-\frac{2\,(b+n)}{3};\,-8\right)/;\;n\in\mathbb{N}$$

For fixed a_3

07.27.03.0075.01

$${}_3F_2\!\left(\frac{1}{2},\,1,\,c;\,\frac{3}{2},\,2-c;\,1\right)=\frac{\pi\,(1-c)\,\Gamma(1-c)^2}{4\,\Gamma\!\left(\frac{3}{2}-c\right)^2}\;/;\;\mathrm{Re}(c)<1$$

07.27.03.0076.01

$${}_3F_2(1,\,1,\,c;\,2,\,2;\,1)=\frac{\psi(2-c)+\gamma}{1-c}$$

07.27.03.0077.01

$${}_3F_2\!\left(-\frac{n}{2},\,-\frac{n-1}{2},\,c;\,c+1,\,-n-\frac{1}{2};\,1\right)=\frac{n!}{\left(\frac{1}{2}\right)_{n+1}}\left(\frac{\left(c+\frac{1}{2}\right)_{n+1}}{(2\,c+1)_n}-\frac{c}{2^n}\right)/;\;n\in\mathbb{N}$$

07.27.03.0078.01

$${}_3F_2\left(-\frac{n}{2}, -\frac{n-1}{2}, c; \frac{c-n+1}{2}, \frac{c-n}{2}+1; 1\right) = \frac{(-1)^n (c)_n}{(-c)_n} /; n \in \mathbb{N}$$

07.27.03.0079.01

$${}_3F_2\left(-\frac{n}{2}, -\frac{n-1}{2}, c; \frac{c-n+1}{3}, \frac{c-n+2}{3}; 1\right) = \frac{2^{-n}}{(1-c)_n} \left(\frac{2(c-n)}{3}\right)_n {}_2F_1\left(-n, \frac{c-n}{3}; 1-\frac{2c+n}{3}; -8\right) /; n \in \mathbb{N}$$

For fixed b_1

07.27.03.0080.01

$${}_3F_2\left(-\frac{n}{3}, -\frac{n-1}{3}, -\frac{n-2}{3}; d, d+\frac{1}{3}; 1\right) = \frac{3^n \left(d-\frac{1}{3}\right)_n}{(3d-1)_n} {}_2F_1\left(-\frac{n}{2}, -\frac{n-1}{2}; \frac{4}{3}-d-n; \frac{4}{3}\right) /; n \in \mathbb{N}$$

07.27.03.0081.01

$${}_3F_2\left(-\frac{n}{3}, -\frac{n-1}{3}, -\frac{n-2}{3}; d, 2-2d-n; 1\right) = 3^{-n} {}_2F_1\left(-n, d-\frac{1}{2}; 2-2d-n; 4\right) /; n \in \mathbb{N}$$

07.27.03.0082.01

$${}_3F_2\left(-\frac{2n}{3}, -\frac{2n-1}{3}, -\frac{2n-2}{3}; d, \frac{1}{2}-n; 1\right) = 9^{-n} {}_2F_1(-n, -2d-2n+2; d; 4) /; n \in \mathbb{N}$$

For fixed b_2

07.27.03.0083.01

$${}_3F_2(1, 1, 1; 2, e; 1) = (e-1)\psi^{(1)}(e-1) /; \mathrm{Re}(e) > 1$$

07.27.03.0084.01

$${}_3F_2(1, 1, 1; 3, e; 1) = 2\psi^{(1)}(e)(e-1)^2 + 2(2-e) /; \mathrm{Re}(e) > 0$$

07.27.03.0085.01

$${}_3F_2(1, 1, 2; 3, e; 1) = 2(e-1)\left(1-(e-2)\psi^{(1)}(e-1)\right) /; \mathrm{Re}(e) > 1$$

07.27.03.0086.01

$${}_3F_2(1, 2, 2; 3, e; 1) = 2(e-2)(e-1)\psi^{(1)}(e-2) - 2(e-1) /; c \neq 1 \land \mathrm{Re}(e) > 2$$

07.27.03.0087.01

$${}_3F_2(-n, 2, 2; 1, e; 1) = \frac{(1-e)(2-e)(e-n-3)}{(e+n-1)(e+n-2)(e+n-3)} /; n \in \mathbb{N}$$

For fixed z and integer parameters

07.27.03.0088.01

$${}_3F_2(1, m, n; m+1, n+1; 1) = \frac{m\,n}{m-n} \sum_{k=1}^{m-n} \frac{1}{k+n-1} /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} \bigwedge m > n$$

07.27.03.0089.01

$${}_3F_2\left(1, \frac{m}{n}, l+\frac{m}{n}; \frac{m}{n}+1, l+\frac{m}{n}+1; 1\right) = \frac{m(m+l\,n)}{n\,l} \sum_{k=0}^{l-1} \frac{1}{m+k\,n} /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+ \bigwedge l \in \mathbb{N}^+$$

07.27.03.0090.01

$${}_3F_2(1, 1, m; 2, m+1; 1) = \frac{m}{m-1} \sum_{k=1}^{m-1} \frac{1}{k} /; m-1 \in \mathbb{N}^+$$

07.27.03.0091.01

$${}_3F_2\left(1, 1, l+\frac{m}{n}; 2, l+\frac{m}{n}+1; 1\right) = \frac{m+l\,n}{m+l\,n-n}\left(\psi\left(\frac{m}{n}+1\right)+n\sum_{k=1}^{l-1}\frac{1}{m+k\,n}+\gamma\right) /; m\in\mathbb{N}^+ \bigwedge n\in\mathbb{N}^+ \bigwedge l\in\mathbb{N}^+$$

07.27.03.0092.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{n}{2}, \frac{n+1}{2}; 1\right) = 2^{n-3}\,(n-1)\left(\log(2)+\sum_{k=1}^{n-3}\frac{(3-n)_k\left(1-2^{-k}\right)}{k\,k!}\right) /; n-3\in\mathbb{N}$$

07.27.03.0093.01

$${}_3F_2(1, 1, 1; n, n; 1) = \frac{(2\,n-4)!}{6}\left(\frac{n-1}{(n-2)!}\right)^2\left(\pi^2-18\sum_{k=0}^{n-3}\frac{k!^2}{(2\,k+2)!}\right) /; n-2\in\mathbb{N}$$

07.27.03.0094.01

$${}_3F_2\left(1, 1, \frac{3}{2}; \frac{n}{2}, \frac{n+1}{2}; 1\right) = (n-1)\,(n-2)\left(\frac{(-1)^n\left(2^{n-3}-1\right)+1}{2\,(n-3)}-2^{n-4}\log(2)-2^{n-4}\sum_{k=1}^{n-4}\frac{(3-n)_k\left(1-2^{-k}\right)}{k\,k!}\right) /; n-4\in\mathbb{N}$$

Values at $z = -1$

For fixed a_1, a_3

07.27.03.0095.01

$${}_3F_2\left(a, \frac{a}{2}+1, c; \frac{a}{2}, a-c+1; -1\right) = \frac{\sqrt{\pi}\;\Gamma(a-c+1)}{2^a\,\Gamma\left(\frac{a}{2}+1\right)\Gamma\left(\frac{a+1}{2}-c\right)}$$

For fixed a_2, a_3

07.27.03.0096.01

$${}_3F_2(1, b, c; 1-b-2\,n, 1-c-2\,n; -1) = \sum_{k=0}^{n}\frac{(-1)^k\,(b)_k\,(c)_k}{(1-b-2\,n)_k\,(1-c-2\,n)_k}+\frac{(-1)^n\,\Gamma(1-b)\,\Gamma(1-c)}{2\,(b+n)_n\,(c+n)_n\,\Gamma(1-b-c-2\,n)}-\frac{(-1)^n\,(b)_n\,(c)_n}{2\,(b+n)_n\,(c+n)_n} /; \mathrm{Re}(b+c)\le\frac{1}{2}-2\,n$$

07.27.03.0097.01

$${}_3F_2(1, b, c; b+1, c+1; -1) = \frac{b\,c}{2\,(b-c)}\left(\psi\left(\frac{b}{2}\right)-\psi\left(\frac{b+1}{2}\right)-\psi\left(\frac{c}{2}\right)+\psi\left(\frac{c+1}{2}\right)\right) /; b\ne c$$

For fixed a_1

07.27.03.0098.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{1}{3}, \frac{2}{3}; -1\right) = \frac{2}{3}\left(\cos(a\,\pi)+2^{-3\,a-1}\right) /; \mathrm{Re}(a)<\frac{1}{3}$$

07.27.03.0099.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{2}{3}, \frac{4}{3}; -1\right) = \frac{2}{3\,(1-3\,a)}\left(\cos\left(\left(a+\frac{1}{3}\right)\pi\right)+2^{-3\,a}\right) /; \mathrm{Re}(a)<0$$

07.27.03.0100.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; -1\right) = \frac{4}{3\,(1-3\,a)\,(2-3\,a)}\left(\cos\left(\left(a+\frac{2}{3}\right)\pi\right)+2^{1-3\,a}\right) /; \mathrm{Re}(a)<-\frac{1}{3}$$

For fixed a_2

07.27.03.0101.01

$${}_3F_2(1,\,b,\,b;\,b+1,\,b+1;\,-1) = \frac{b^2}{4}\left(\psi^{(1)}\!\left(\frac{b}{2}\right) - \psi^{(1)}\!\left(\frac{b+1}{2}\right)\right)$$

07.27.03.0102.01

$${}_3F_2(1,\,b,\,b;\,1-b,\,1-b;\,-1) = \frac{2^{2b-1}\sqrt{\pi}\;\Gamma(1-b)}{\Gamma\left(\frac{1}{2}-b\right)} + \frac{1}{2} \;/;\; \mathrm{Re}(b) < 0$$

07.27.03.0103.01

$${}_3F_2(1,\,b,\,1-b;\,b+1,\,2-b;\,-1) = \frac{b\,(1-b)}{2\,b-1}\left(\frac{\pi}{\sin(\pi\,b)} - \psi\!\left(\frac{b+1}{2}\right) + \psi\!\left(\frac{b}{2}\right)\right) /;\; b \neq \frac{1}{2}$$

07.27.03.0104.01

$${}_3F_2(2,\,b,\,2-b;\,b+1,\,3-b;\,-1) = \frac{b\,(b-2)}{4}\left(\psi\!\left(\frac{b}{2}\right) + \psi\!\left(1-\frac{b}{2}\right) - \psi\!\left(\frac{b-1}{2}\right) - \psi\!\left(\frac{1-b}{2}\right)\right)$$

07.27.03.0105.01

$${}_3F_2\!\left(-n,\,b,\,\frac{b}{2}+1;\,\frac{b}{2},\,b+n+1;\,-1\right) = \frac{(b+1)_n}{\left(\frac{b+1}{2}\right)_n} \;/;\; n \in \mathbb{N}$$

07.27.03.0106.01

$${}_3F_2(-n,\,b,\,-b-n;\,b+1,\,1-b-n;\,-1) = \frac{2\,(b+n)}{2\,b+n}\;{}_2F_1(-n,\,b;\,b+1;\,-1) \;/;\; n \in \mathbb{N}$$

For fixed a_3

07.27.03.0107.01

$${}_3F_2(1,\,1,\,c;\,2,\,3-c;\,-1) = \frac{2-c}{2\,(1-c)}\,(\psi(2-c)+\gamma)$$

07.27.03.0108.01

$${}_3F_2(1,\,1,\,c;\,2,\,c+1;\,-1) = \frac{c}{2\,(1-c)}\left(\psi\!\left(\frac{c+1}{2}\right) - \psi\!\left(\frac{c}{2}\right) - 2\log(2)\right)$$

For fixed z and integer parameters

07.27.03.0109.01

$${}_3F_2(1,\,m,\,n;\,m+1,\,n+1;\,-1) = \frac{((-1)^m - (-1)^n)\,m\,n\log(2)}{m-n} + \frac{(-1)^m\,m\,n}{m-n}\sum_{k=1}^{m-n}\frac{(-1)^k}{k} - (-1)^n\,m\,n\sum_{k=1}^{n-1}\frac{(-1)^k}{k\,(k+m-n)} \;/;$$
$$m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+ \bigwedge m > n$$

07.27.03.0110.01

$${}_3F_2\!\left(-n,\,-n-\frac{1}{2},\,\frac{1}{2};\,\frac{1}{2}-n,\,\frac{3}{2};\,-1\right) = \frac{2\,n+1}{n+1}\;{}_2F_1\!\left(-n,\,\frac{1}{2};\,\frac{3}{2};\,-1\right) /;\; n \in \mathbb{N}$$

07.27.03.0111.01

$${}_3F_2(-n,\,2,\,2;\,1,\,1;\,-1) = 2^{n-2}\,(n+1)\,(n+4) \;/;\; n \in \mathbb{N}$$

Values at $z = -\frac{1}{8}$

For fixed a_1

07.27.03.0112.01

$${}_3F_2\left(a, 1-a, 3a-1; 2a, a+\frac{1}{2}; -\frac{1}{8}\right) = \frac{2^{3a-3}}{\pi\, \Gamma\left(\frac{3a}{2}\right)^2} \Gamma\left(\frac{a}{2}\right)^2 \Gamma\left(a+\frac{1}{2}\right)^2$$

Values at $z = \frac{3}{4}$

For fixed a_2

07.27.03.0113.01

$${}_3F_2\left(-n, b, 3b+n-1; \frac{3b-1}{2}, \frac{3b}{2}; \frac{3}{4}\right) = \left(n - 3\left\lfloor\frac{n+2}{3}\right\rfloor + 1\right) \frac{n!\, (b)_{\left\lfloor\frac{n}{3}\right\rfloor}}{(3b-1)_n \left\lfloor\frac{n}{3}\right\rfloor!} \;/;\; n \in \mathbb{N}$$

07.27.03.0114.01

$${}_3F_2\left(-n, b, 3b+n; \frac{3b}{2}, \frac{3b+1}{2}; \frac{3}{4}\right) = \left(1 - \left|n - 3\left\lfloor\frac{n+1}{3}\right\rfloor\right|\right) \frac{n!\, (b+1)_{\left\lfloor\frac{n}{3}\right\rfloor}}{(3b+1)_n \left\lfloor\frac{n}{3}\right\rfloor!} \;/;\; n \in \mathbb{N}$$

Values at $z = \frac{4}{3}$

For fixed a_2

07.27.03.0115.01

$${}_3F_2\left(-n, b, b+\frac{1}{2}; 2b, \frac{2b-n+2}{3}; \frac{4}{3}\right) = \left(1 - \left|n - 3\left\lfloor\frac{n+1}{3}\right\rfloor\right|\right) \frac{\left(\frac{1}{3}\right)_{\left\lfloor\frac{n}{3}\right\rfloor} \left(\frac{2}{3}\right)_{\left\lfloor\frac{n}{3}\right\rfloor}}{\left(\frac{1}{3}(2b+2)\right)_{\left\lfloor\frac{n}{3}\right\rfloor} \left(\frac{1}{3}(1-2b)\right)_{\left\lfloor\frac{n}{3}\right\rfloor}} \;/;\; n \in \mathbb{N}$$

Specialized values

For fixed a_1, a_2, a_3, b_1, z

07.27.03.0116.01

$${}_3F_2(a, b, c; d, c; z) = {}_2F_1(a, b; d; z)$$

For fixed a_1, a_2, a_3, b_2, z

07.27.03.0117.01

$${}_3F_2(a, b, c; a-n, e; z) = \frac{1}{(1-a)_n} \sum_{k=0}^{n} \frac{(-1)^k (1-a)_{n-k} (b)_k (c)_k}{(e)_k} \binom{n}{k} {}_2F_1(b+k, c+k; e+k; z)\, z^k$$

07.27.03.0118.01

$${}_3F_2(a, b, c; a-1, e; z) = {}_2F_1(b, c; e; z) + \frac{b\, c\, z}{(a-1)\, e} {}_2F_1(b+1, c+1; e+1; z)$$

For fixed a_1, a_2, a_3, z

07.27.03.0119.01

$${}_3F_2(a, b, c; a+1, b+1; z) = \frac{1}{b-a} \left(b\, {}_2F_1(a, c; a+1; z) - a\, {}_2F_1(b, c; b+1; z)\right)$$

For fixed a_2, a_3, b_2, z

07.27.03.0120.01

$$ {}_3F_2(1,b,c;2,e;z)=\frac{e-1}{(b-1)(c-1)z}\left({}_2F_1(b-1,c-1;e-1;z)-1\right) $$

07.27.03.0121.01

$$ {}_3F_2(1,b,c;3,e;z)=\frac{2(e-2)(e-1)}{(b-2)(b-1)(c-2)(c-1)z^2}\left({}_2F_1(b-2,c-2;e-2;z)-1\right)-\frac{2(e-1)}{(b-1)(c-1)z} $$

For fixed a_1, a_2, z

07.27.03.0122.01

$$ {}_3F_2\left(a,b,\frac{a+b-1}{2};\frac{a+b}{2},a+b-1;z\right)={}_2F_1\left(\frac{a-1}{2},\frac{b}{2};\frac{a+b}{2};z\right){}_2F_1\left(\frac{a+1}{2},\frac{b}{2};\frac{a+b}{2};z\right) $$

07.27.03.0123.01

$$ {}_3F_2\left(a,b,\frac{a+b-1}{2};\frac{a+b}{2},a+b-1;z\right)={}_2F_1\left(\frac{a}{2},\frac{b-1}{2};\frac{a+b}{2};z\right){}_2F_1\left(\frac{a}{2},\frac{b+1}{2};\frac{a+b}{2};z\right) $$

07.27.03.0124.01

$$ {}_3F_2\left(a,b,\frac{a+b}{2};\frac{a+b-1}{2},a+b;z\right)=2\,{}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b-1}{2};z\right){}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b+1}{2};z\right)-{}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b+1}{2};z\right)^2 $$

07.27.03.0125.01

$$ {}_3F_2\left(a,b,\frac{a+b}{2};\frac{a+b+1}{2},a+b-1;z\right)={}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b-1}{2};z\right){}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b+1}{2};z\right) $$

07.27.03.0126.01

$$ {}_3F_2\left(a,b,\frac{a+b}{2};\frac{a+b+1}{2},a+b-1;z\right)={}_2F_1\left(\frac{a-1}{2},\frac{b-1}{2};\frac{a+b-1}{2};z\right){}_2F_1\left(\frac{a+1}{2},\frac{b+1}{2};\frac{a+b+1}{2};z\right) $$

07.27.03.0127.01

$$ {}_3F_2\left(a,b,\frac{a+b}{2};\frac{a+b+1}{2},a+b;z\right)={}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b+1}{2};z\right)^2 $$

07.27.03.0128.01

$$ {}_3F_2\left(a,b,\frac{a+b}{2};\frac{a+b+1}{2},a+b;z\right)={}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b+1}{2};z\right)^2 $$

07.27.03.0129.01

$$ {}_3F_2\left(a,b,\frac{a+b}{2};\frac{a+b+1}{2},a+b;z\right)=\sqrt{1-z}\;{}_2F_1\left(\frac{a}{2},\frac{b}{2};\frac{a+b+1}{2};z\right){}_2F_1\left(\frac{a+1}{2},\frac{b+1}{2};\frac{a+b+1}{2};z\right) $$

07.27.03.0130.01

$$ {}_3F_2\left(a,b,\frac{a+b+1}{2};\frac{a+b}{2}+1,a+b;z\right)= $$

$$ \frac{a}{a-b}\,{}_2F_1\left(\frac{a+1}{2},\frac{b}{2};\frac{a+b}{2};z\right){}_2F_1\left(\frac{a+1}{2},\frac{b}{2};\frac{a+b}{2}+1;z\right)-\frac{b}{a-b}\,{}_2F_1\left(\frac{a}{2},\frac{b+1}{2};\frac{a+b}{2};z\right){}_2F_1\left(\frac{a}{2},\frac{b+1}{2};\frac{a+b}{2}+1;z\right) $$

07.27.03.0131.01

$$ {}_3F_2\left(a,b,\frac{a+b+1}{2};\frac{a+b}{2}+1,a+b+1;z\right)=\frac{a}{a-b}\,{}_2F_1\left(\frac{a+1}{2},\frac{b}{2};\frac{a+b}{2}+1;z\right)^2-\frac{b}{a-b}\,{}_2F_1\left(\frac{a}{2},\frac{b+1}{2};\frac{a+b}{2}+1;z\right)^2 $$

07.27.03.0132.01

$${}_3F_2\left(a, b, \frac{a+b}{2}+1; \frac{a+b+1}{2}, a+b; z\right) = \frac{2(a+b-1)}{a+b}\, {}_2F_1\left(\frac{a}{2}, \frac{b}{2}; \frac{a+b-1}{2}; z\right) {}_2F_1\left(\frac{a}{2}, \frac{b}{2}; \frac{a+b+1}{2}; z\right) - \frac{a+b-2}{a+b}\, {}_2F_1\left(\frac{a}{2}, \frac{b}{2}; \frac{a+b+1}{2}; z\right)^2$$

07.27.03.0133.01

$${}_3F_2\left(a, b, a+\frac{1}{2}; 2a, b+1; z\right) = \left(\frac{2}{z}\left(1-\sqrt{1-z}\right)\right)^{2b} {}_2F_1\left(b, 2b-2a+1; b+1; 1-\frac{2}{z}\left(1-\sqrt{1-z}\right)\right)$$

07.27.03.0134.01

$${}_3F_2\left(a, b, a+\frac{1}{2}; 2a, b+1; z\right) = \left(\frac{1+\sqrt{1-z}}{2}\right)^{-b} {}_2F_1\left(b, 2a-b; b+1; \frac{1-\sqrt{1-z}}{2}\right)$$

07.27.03.0135.01

$${}_3F_2\left(a, b, \frac{a-1}{2}; a-1, \frac{a+b+1}{2}; z\right) = (1-z)^{-\frac{b}{2}} {}_2F_1\left(\frac{b}{2}, \frac{a-b-1}{2}; \frac{a+b+1}{2}; z\right)$$

07.27.03.0136.01

$${}_3F_2\left(a, b, \frac{a}{2}+1; \frac{a}{2}, a-b+1; z\right) = (1-z)\, {}_2F_1(a+1, b+1; a-b+1; z)$$

For fixed a_2, b_1, z

07.27.03.0137.01

$${}_3F_2\left(\frac{1}{2}, b, 1-b; d, 2-d; z\right) = {}_2F_1\left(b, 1-b; 2-d; \frac{1-\sqrt{1-z}}{2}\right) {}_2F_1\left(b, 1-b; d; \frac{1-\sqrt{1-z}}{2}\right)$$

For fixed a_1, z

07.27.03.0138.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{3a}{2}, \frac{3a+1}{2}; -\frac{27z}{4(1-z)^3}\right) = \frac{(1-z)^{3a}}{2z+1}$$

07.27.03.0139.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{3a+1}{2}, \frac{3a}{2}+1; -\frac{27z}{4(1-z)^3}\right) = (1-z)^{3a}$$

07.27.03.0140.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{1}{3}, \frac{2}{3}; z\right) = \frac{1}{3}\left(\left(1-\sqrt[3]{z}\right)^{-3a} + \left(1-e^{\frac{2\pi i}{3}}\sqrt[3]{z}\right)^{-3a} + \left(1-e^{\frac{4\pi i}{3}}\sqrt[3]{z}\right)^{-3a}\right)$$

07.27.03.0141.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{2}{3}, \frac{4}{3}; z\right) = \frac{1}{3(3a-1)\sqrt[3]{z}}\left(\left(1-\sqrt[3]{z}\right)^{1-3a} + e^{-\frac{2\pi i}{3}}\left(1-e^{\frac{2\pi i}{3}}\sqrt[3]{z}\right)^{1-3a} + e^{-\frac{4\pi i}{3}}\left(1-e^{\frac{4\pi i}{3}}\sqrt[3]{z}\right)^{1-3a}\right)$$

07.27.03.0142.01

$${}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; z\right) = \frac{2}{3(3a-1)(3a-2)z^{2/3}}\left(\left(1-\sqrt[3]{z}\right)^{2-3a} + e^{-\frac{4\pi i}{3}}\left(1-e^{\frac{2\pi i}{3}}\sqrt[3]{z}\right)^{2-3a} + e^{-\frac{2\pi i}{3}}\left(1-e^{\frac{4\pi i}{3}}\sqrt[3]{z}\right)^{2-3a}\right)$$

For fixed a_2, z

07.27.03.0143.01

$$ {}_3F_2\left(-n,\, b,\, 2\,b+n;\, b+\frac{1}{2},\, 2\,b;\, z\right) = \left(\frac{n!}{(2\,a)_n}\, C_n^b\left(\sqrt{1-z}\right)\right)^2 \;/;\; n \in \mathbb{N} $$

For fixed z and with symbolical integers in parameters

07.27.03.0144.01

$$ {}_3F_2\left(\frac{m}{p},\, \frac{n}{q},\, 1;\, \frac{m}{p}+1,\, \frac{n}{q}+1;\, z\right) = $$

$$ \frac{m\,n}{n\,p-m\,q}\left(-\sum_{k=0}^{p-1}\exp\left(-\frac{2\,\pi\,i\,k\,m}{p}\right)\log\left(1-z^{1/p}\exp\left(\frac{2\,\pi\,i\,k}{p}\right)\right)z^{-\frac{m}{p}} + \left(\sum_{k=0}^{q-1}\exp\left(-\frac{2\,\pi\,i\,k\,n}{q}\right)\log\left(1-z^{1/q}\exp\left(\frac{2\,\pi\,i\,k}{q}\right)\right)\right)z^{-\frac{n}{q}} + \right. $$

$$ \left. q\sum_{k=1}^{-\left\lfloor -\frac{n}{q}\right\rfloor -1}\frac{z^{-k}}{n-k\,q}\right) /;\; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land p \in \mathbb{N}^+ \land q \in \mathbb{N}^+ \land p \neq m\,q \land m \leq p $$

07.27.03.0145.01

$$ {}_3F_2(1,\, m,\, n;\, m+1,\, n+1;\, z) = \frac{m\,n}{(n-m)\,z^n}\left((1-z^{n-m})\log(1-z) - z^{n-m}\sum_{k=1}^{m-1}\frac{z^k}{k} + \sum_{k=1}^{n-1}\frac{z^k}{k}\right) /;\; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ $$

07.27.03.0146.01

$$ {}_3F_2\left(-n,\, \frac{1}{2},\, n+1;\, 1,\, 1;\, z\right) = P_n\left(\sqrt{1-z}\right)^2 \;/;\; n \in \mathbb{N} $$

For integer and half-integer parameters and fixed z

For some numeric parameters and fixed z

For fixed z and $a_1 = 1$, $a_2 = \frac{3+i}{2}$, $a_3 = \frac{3-i}{2}$

07.27.03.0147.01

$$ {}_3F_2\left(1,\, \frac{3+i}{2},\, \frac{3-i}{2};\, \frac{3}{2},\, 2;\, z\right) = \frac{1}{z}\left(\frac{\cosh\left(\sin^{-1}\left(\sqrt{z}\right)\right)}{\cos\left(\sin^{-1}\left(\sqrt{z}\right)\right)} - 1\right) $$

For fixed z and $a_1 = \frac{1}{8}$, $a_2 = \frac{m}{8}$, $a_3 = 1$

07.27.03.0335.01

$$ {}_3F_2\left(\frac{1}{8},\, \frac{3}{8},\, 1;\, \frac{9}{8},\, \frac{11}{8};\, z\right) = $$

$$ \frac{3}{16\,x^3}\left(\left(x^2+1\right)\left(2\tan^{-1}(x) + \frac{1}{\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)\right) - \left(1-x^2\right)\left(\sqrt{2}\,\tan^{-1}\left(1-x^2,\, \sqrt{2}\,x\right) + \log\left(\frac{1+x}{1-x}\right)\right)\right) /;\; x = \sqrt[8]{z} $$

07.27.03.0336.01

$$ {}_3F_2\left(\frac{1}{8},\, \frac{3}{8},\, 1;\, \frac{9}{8},\, \frac{11}{8};\, -z\right) = \frac{3}{32\,x^3}\left(2\left(a\,x^2-b\right)\tan^{-1}\left(1-x^2,\, a\,x\right) + 2\left(b\,x^2+a\right)\tan^{-1}\left(1-x^2,\, b\,x\right) + \right. $$

$$ \left. \left(a\,x^2+b\right)\log\left(\frac{x^2+a\,x+1}{x^2-a\,x+1}\right) + \left(b\,x^2-a\right)\log\left(\frac{x^2+b\,x+1}{x^2-b\,x+1}\right)\right) /;\; a = \sqrt{2-\sqrt{2}} \bigwedge b = \sqrt{2+\sqrt{2}} \bigwedge x = \sqrt[8]{z} $$

07.27.03.0337.01

$${}_3F_2\left(\frac{1}{8},\frac{3}{8},1;\frac{9}{8},\frac{11}{8};z\right)=$$
$$-\frac{3}{16z^{3/8}}\Big(-\log\left(\sqrt[8]{z}+1\right)\sqrt[4]{z}+(-1)^{3/4}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)\sqrt[4]{z}+\sqrt[4]{-1}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)\sqrt[4]{z}+i\log\left(i\sqrt[8]{z}+1\right)\sqrt[4]{z}-$$
$$(-1)^{3/4}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)\sqrt[4]{z}-\sqrt[4]{-1}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)\sqrt[4]{z}+\left(\sqrt[4]{z}-1\right)\log\left(1-\sqrt[8]{z}\right)+$$
$$\log\left(\sqrt[8]{z}+1\right)-\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)-(-1)^{3/4}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)+i\log\left(i\sqrt[8]{z}+1\right)+$$
$$\sqrt[4]{-1}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)+(-1)^{3/4}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)-i\left(\sqrt[4]{z}+1\right)\log\left(1-i\sqrt[8]{z}\right)\Big)$$

07.27.03.0338.01

$${}_3F_2\left(\frac{1}{8},\frac{5}{8},1;\frac{9}{8},\frac{13}{8};z\right)=$$
$$\frac{5}{32x^5}\left(\frac{x^4+1}{\sqrt{2}}\left(2\tan^{-1}\left(1-x^2,\sqrt{2}\,x\right)+\log\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)\right)-\left(1-x^4\right)\left(2\tan^{-1}(x)+\log\left(\frac{1+x}{1-x}\right)\right)\right)/;\ x=\sqrt[8]{z}$$

07.27.03.0339.01

$${}_3F_2\left(\frac{1}{8},\frac{5}{8},1;\frac{9}{8},\frac{13}{8};z\right)=-\frac{5}{32z^{5/8}}\Big(\left(\sqrt{z}-1\right)\log\left(1-\sqrt[8]{z}\right)-\sqrt{z}\log\left(\sqrt[8]{z}+1\right)+\log\left(\sqrt[8]{z}+1\right)+(-1)^{3/4}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)+$$
$$(-1)^{3/4}\sqrt{z}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)+\sqrt[4]{-1}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)+\sqrt[4]{-1}\sqrt{z}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)-$$
$$i\log\left(i\sqrt[8]{z}+1\right)+i\sqrt{z}\log\left(i\sqrt[8]{z}+1\right)-(-1)^{3/4}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)-(-1)^{3/4}\sqrt{z}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)-$$
$$\sqrt[4]{-1}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)-\sqrt[4]{-1}\sqrt{z}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)-i\left(\sqrt{z}-1\right)\log\left(1-i\sqrt[8]{z}\right)\Big)$$

07.27.03.0340.01

$${}_3F_2\left(\frac{1}{8},\frac{7}{8},1;\frac{9}{8},\frac{15}{8};z\right)=$$
$$\frac{7}{48x^7}\left(\left(x^6+1\right)\left(2\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(1-x^2,\sqrt{2}\,x\right)\right)-\left(1-x^6\right)\left(\log\left(\frac{1+x}{1-x}\right)+\frac{1}{\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)\right)\right)/;\ x=\sqrt[8]{z}$$

07.27.03.0341.01

$${}_3F_2\left(\frac{1}{8},\frac{7}{8},1;\frac{9}{8},\frac{15}{8};z\right)=$$
$$-\frac{7}{48z^{7/8}}\Big(-\log\left(\sqrt[8]{z}+1\right)z^{3/4}+(-1)^{3/4}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)z^{3/4}+\sqrt[4]{-1}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)z^{3/4}+i\log\left(i\sqrt[8]{z}+1\right)z^{3/4}-$$
$$(-1)^{3/4}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)z^{3/4}-\sqrt[4]{-1}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)z^{3/4}+\left(z^{3/4}-1\right)\log\left(1-\sqrt[8]{z}\right)+$$
$$\log\left(\sqrt[8]{z}+1\right)+\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)+(-1)^{3/4}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)+i\log\left(i\sqrt[8]{z}+1\right)-$$
$$\sqrt[4]{-1}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)-(-1)^{3/4}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)-i\left(z^{3/4}+1\right)\log\left(1-i\sqrt[8]{z}\right)\Big)$$

For fixed z and $a_1=\frac{1}{6}$, $a_2=\frac{m}{3}$, $a_3=1$

07.27.03.0342.01

$$ {}_3F_2\left(\frac{1}{6}, \frac{1}{3}, 1; \frac{7}{6}, \frac{4}{3}; z\right) = \frac{1}{3\sqrt[3]{z}} \left(\log\left(\sqrt[6]{z}+1\right)\sqrt[6]{z} - (-1)^{2/3}\log\left(\sqrt[3]{-1}\sqrt[6]{z}+1\right)\sqrt[6]{z} - \sqrt[3]{-1}\log\left((-1)^{2/3}\sqrt[6]{z}+1\right)\sqrt[6]{z} + (-1)^{2/3}\log\left(1-\sqrt[3]{-1}\sqrt[6]{z}\right)\sqrt[6]{z} + \sqrt[3]{-1}\log\left(1-(-1)^{2/3}\sqrt[6]{z}\right)\sqrt[6]{z} + \log\left(1-\sqrt[3]{z}\right) + (-1)^{2/3}\log\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right) - \sqrt[3]{-1}\log\left(1-(-1)^{2/3}\sqrt[3]{z}\right) - \sqrt[6]{z}\log\left(1-\sqrt[6]{z}\right)\right) $$

07.27.03.0343.01

$$ {}_3F_2\left(\frac{1}{6}, \frac{2}{3}, 1; \frac{7}{6}, \frac{5}{3}; z\right) = -\frac{2}{9z^{2/3}} \left(-\log\left(1-\sqrt[3]{z}\right) + \sqrt[3]{-1}\log\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right) - (-1)^{2/3}\log\left(1-(-1)^{2/3}\sqrt[3]{z}\right) + \sqrt{z}\log\left(1-\sqrt[6]{z}\right) - \sqrt{z}\log\left(\sqrt[6]{z}+1\right) + (-1)^{2/3}\sqrt{z}\log\left(\sqrt[3]{-1}\sqrt[6]{z}+1\right) + \sqrt[3]{-1}\sqrt{z}\log\left((-1)^{2/3}\sqrt[6]{z}+1\right) - (-1)^{2/3}\sqrt{z}\log\left(1-\sqrt[3]{-1}\sqrt[6]{z}\right) - \sqrt[3]{-1}\sqrt{z}\log\left(1-(-1)^{2/3}\sqrt[6]{z}\right)\right) $$

07.27.03.0344.01

$$ {}_3F_2\left(\frac{1}{6}, \frac{2}{3}, 1; \frac{5}{3}, \frac{13}{6}; z\right) = \frac{7}{27z^{7/6}} \left(-6\sqrt[6]{z} + 2\left(\log\left(1-\sqrt[3]{z}\right) - \sqrt[3]{-1}\log\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right) + (-1)^{2/3}\log\left(1-(-1)^{2/3}\sqrt[3]{z}\right)\right)\sqrt{z} - (z+1)\left(\log\left(1-\sqrt[6]{z}\right) - \log\left(\sqrt[6]{z}+1\right) + \sqrt[3]{-1}\left(\sqrt[3]{-1}\log\left(\sqrt[3]{-1}\sqrt[6]{z}+1\right) + \log\left((-1)^{2/3}\sqrt[6]{z}+1\right) - \sqrt[3]{-1}\log\left(1-\sqrt[3]{-1}\sqrt[6]{z}\right) - \log\left(1-(-1)^{2/3}\sqrt[6]{z}\right)\right)\right)\right) $$

For fixed z and $a_1 = \frac{1}{4}, a_2 = \frac{m}{4}, a_3 = 1$

07.27.03.0345.01

$$ {}_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}; z\right) = -\frac{3}{8z^{3/4}} \left(\left(\sqrt{z}-1\right)\log\left(1-\sqrt[4]{z}\right) - \sqrt{z}\log\left(\sqrt[4]{z}+1\right) + \log\left(\sqrt[4]{z}+1\right) + i\log\left(i\sqrt[4]{z}+1\right) + i\sqrt{z}\log\left(i\sqrt[4]{z}+1\right) - i\left(\sqrt{z}+1\right)\log\left(1-i\sqrt[4]{z}\right)\right) $$

07.27.03.0346.01

$$ {}_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{7}{4}, \frac{9}{4}; z\right) = -\frac{15}{32z^{5/4}} \left(\left(\sqrt{z}-1\right)^2\log\left(1-\sqrt[4]{z}\right) - i\left(-\log\left(i\sqrt[4]{z}+1\right)\left(\sqrt{z}+1\right)^2 + \log\left(1-i\sqrt[4]{z}\right)\left(\sqrt{z}+1\right)^2 - i\left(\left(\sqrt{z}-1\right)^2\log\left(\sqrt[4]{z}+1\right) - 4\sqrt[4]{z}\right)\right)\right) $$

07.27.03.0347.01

$$ {}_3F_2\left(\frac{1}{4}, 1, 1; \frac{5}{4}, 2; z\right) = \frac{1}{3z} \left(\log\left(\sqrt[4]{z}+1\right)z^{3/4} - i\log\left(i\sqrt[4]{z}+1\right)z^{3/4} + i\log\left(1-i\sqrt[4]{z}\right)z^{3/4} + \log(1-z) - z^{3/4}\log\left(1-\sqrt[4]{z}\right)\right) $$

For fixed z and $a_1 = \frac{1}{3}, a_2 = \frac{m}{n}, a_3 = 1$

07.27.03.0348.01

$$ {}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{4}{3}, \frac{5}{3}; z\right) = -\frac{2}{3z^{2/3}} \left(\left(\sqrt[3]{z}-1\right)\log\left(1-\sqrt[3]{z}\right) + \sqrt[3]{-1}\left(\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right)\log\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right) - \left(\sqrt[3]{z}+\sqrt[3]{-1}\right)\log\left(1-(-1)^{2/3}\sqrt[3]{z}\right)\right)\right) $$

07.27.03.0349.01

$$ {}_3F_2\left(\frac{1}{3}, \frac{5}{6}, 1; \frac{4}{3}, \frac{11}{6}; z\right) == -\frac{5}{9 z^{5/6}} \left(\sqrt{z} \log\left(1-\sqrt[3]{z}\right) + (-1)^{2/3}\sqrt{z} \log\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right) - \sqrt[3]{-1}\sqrt{z}\log\left(1-(-1)^{2/3}\sqrt[3]{z}\right) - \log\left(1-\sqrt[6]{z}\right) + \log\left(\sqrt[6]{z}+1\right) + \sqrt[3]{-1}\log\left(\sqrt[3]{-1}\sqrt[6]{z}+1\right) + (-1)^{2/3}\log\left((-1)^{2/3}\sqrt[6]{z}+1\right) - \sqrt[3]{-1}\log\left(1-\sqrt[3]{-1}\sqrt[6]{z}\right) - (-1)^{2/3}\log\left(1-(-1)^{2/3}\sqrt[6]{z}\right)\right) $$

07.27.03.0350.01

$$ {}_3F_2\left(\frac{1}{3}, 1, 1; \frac{4}{3}, 2; z\right) == \frac{1}{2z}\left(-(-1)^{2/3}\log\left(\sqrt[3]{-1}\sqrt[3]{z}+1\right)z^{2/3} + \sqrt[3]{-1}\log\left(1-(-1)^{2/3}\sqrt[3]{z}\right)z^{2/3} + \log(1-z) - z^{2/3}\log\left(1-\sqrt[3]{z}\right)\right) $$

For fixed z and $a_1 = \frac{3}{8}, a_2 = \frac{m}{8}, a_3 = 1$

07.27.03.0351.01

$$ {}_3F_2\left(\frac{3}{8}, \frac{5}{8}, 1; \frac{11}{8}, \frac{13}{8}; z\right) == \frac{15}{16x^5}\left(2\left(x^2+1\right)\left(\frac{1}{\sqrt{2}}\tan^{-1}\left(1-x^2, \sqrt{2}\,x\right) - \tan^{-1}(x)\right) - \left(1-x^2\right)\left(\log\left(\frac{1+x}{1-x}\right) - \frac{1}{\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)\right)\right) /; x == z^{1/8} \bigwedge z \notin (1, \infty) $$

07.27.03.0352.01

$$ {}_3F_2\left(\frac{3}{8}, \frac{5}{8}, 1; \frac{11}{8}, \frac{13}{8}; -z\right) == \frac{15}{32x^5}\left(a\left(2\left(1-x^2\right)\tan^{-1}\left(1-x^2, b\,x\right) + \left(x^2+1\right)\log\left(\frac{x^2+b\,x+1}{x^2-b\,x+1}\right)\right) - b\left(2\left(1-x^2\right)\tan^{-1}\left(1-x^2, a\,x\right) + \left(1+x^2\right)\log\left(\frac{x^2+a\,x+1}{x^2-a\,x+1}\right)\right)\right) /; a == \sqrt{2-\sqrt{2}} \bigwedge b == \sqrt{2+\sqrt{2}} \bigwedge x == z^{1/8} $$

07.27.03.0353.01

$$ {}_3F_2\left(\frac{3}{8}, \frac{5}{8}, 1; \frac{11}{8}, \frac{13}{8}; z\right) == -\frac{15}{16z^{5/8}}\left(-\log\left(\sqrt[8]{z}+1\right)\sqrt[4]{z} + \sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right)\sqrt[4]{z} + (-1)^{3/4}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right)\sqrt[4]{z} - i\log\left(i\sqrt[8]{z}+1\right)\sqrt[4]{z} - \sqrt[4]{-1}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right)\sqrt[4]{z} - (-1)^{3/4}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right)\sqrt[4]{z} + \left(\sqrt[4]{z}-1\right)\log\left(1-\sqrt[8]{z}\right) + \log\left(\sqrt[8]{z}+1\right) + (-1)^{3/4}\log\left(\sqrt[4]{-1}\sqrt[8]{z}+1\right) + \sqrt[4]{-1}\log\left((-1)^{3/4}\sqrt[8]{z}+1\right) - i\log\left(i\sqrt[8]{z}+1\right) - (-1)^{3/4}\log\left(1-\sqrt[4]{-1}\sqrt[8]{z}\right) - \sqrt[4]{-1}\log\left(1-(-1)^{3/4}\sqrt[8]{z}\right) + i\left(\sqrt[4]{z}+1\right)\log\left(1-i\sqrt[8]{z}\right)\right) $$

07.27.03.0354.01

$$ {}_3F_2\left(\frac{3}{8}, \frac{7}{8}, 1; \frac{11}{8}, \frac{15}{8}; z\right) == \frac{21}{32x^7}\left(\frac{x^4+1}{\sqrt{2}}\left(2\tan^{-1}\left(1-x^2, \sqrt{2}\,x\right) - \log\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)\right) - \left(1-x^4\right)\left(\log\left(\frac{1+x}{1-x}\right) - 2\tan^{-1}(x)\right)\right) /; x == z^{1/8} \bigwedge z \notin (1, \infty) $$

07.27.03.0355.01

$${}_3F_2\left(\frac{3}{8}, \frac{7}{8}, 1; \frac{11}{8}, \frac{15}{8}; -z\right) = \frac{21}{64 x^7}\left(\left(a x^4 + b\right)\left(\log\left(\frac{x^2 + b x + 1}{x^2 - b x + 1}\right) - 2\tan^{-1}\left(1 - x^2, b x\right)\right) - \left(b x^4 - a\right)\left(\log\left(\frac{x^2 + a x + 1}{x^2 - a x + 1}\right) - 2\tan^{-1}\left(1 - x^2, a x\right)\right)\right) /; a = \sqrt{2 - \sqrt{2}} \bigwedge b = \sqrt{2 + \sqrt{2}} \bigwedge x = \sqrt[8]{z}$$

07.27.03.0356.01

$${}_3F_2\left(\frac{3}{8}, \frac{7}{8}, 1; \frac{11}{8}, \frac{15}{8}; z\right) = -\frac{21}{32 z^{7/8}} \left(\left(\sqrt{z} - 1\right)\log\left(1 - \sqrt[8]{z}\right) - \sqrt{z}\log\left(\sqrt[8]{z} + 1\right) + \log\left(\sqrt[8]{z} + 1\right) + \sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[8]{z} + 1\right) + \sqrt[4]{-1}\sqrt{z}\log\left(\sqrt[4]{-1}\sqrt[8]{z} + 1\right) + (-1)^{3/4}\log\left((-1)^{3/4}\sqrt[8]{z} + 1\right) + (-1)^{3/4}\sqrt{z}\log\left((-1)^{3/4}\sqrt[8]{z} + 1\right) + i\log\left(i\sqrt[8]{z} + 1\right) - i\sqrt{z}\log\left(i\sqrt[8]{z} + 1\right) - \sqrt[4]{-1}\log\left(1 - \sqrt[4]{-1}\sqrt[8]{z}\right) - \sqrt[4]{-1}\sqrt{z}\log\left(1 - \sqrt[4]{-1}\sqrt[8]{z}\right) - (-1)^{3/4}\log\left(1 - (-1)^{3/4}\sqrt[8]{z}\right) - (-1)^{3/4}\sqrt{z}\log\left(1 - (-1)^{3/4}\sqrt[8]{z}\right) + i\left(\sqrt{z} - 1\right)\log\left(1 - i\sqrt[8]{z}\right)\right)$$

For fixed z and $a_1 = \frac{1}{2}, a_2 = 1, a_3 = 1$

07.27.03.0409.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{1}{4}, \frac{3}{4}; z\right) = \frac{1}{2}\sqrt[4]{z}\left(\left(1 - \sqrt{z}\right)^{-3/2}\sin^{-1}\left(\sqrt[4]{z}\right) - \left(\sqrt{z} + 1\right)^{-3/2}\log\left(\sqrt[4]{z} + \sqrt{\sqrt{z} + 1}\right)\right) + \frac{1}{1 - z}$$

For fixed z and $a_1 = \frac{2}{3}, a_2 = 1, a_3 = \frac{m}{n}$

07.27.03.0498.01

$${}_3F_2\left(\frac{2}{3}, 1, 1; \frac{5}{3}, 2; z\right) = \frac{1}{z}\left(2\left(\sqrt[3]{-1}\log\left(\sqrt[3]{-1}\sqrt[3]{z} + 1\right)\sqrt[3]{z} - (-1)^{2/3}\log\left(1 - (-1)^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z} + \log(1 - z) - \sqrt[3]{z}\log\left(1 - \sqrt[3]{z}\right)\right)\right)$$

07.27.03.0499.01

$${}_3F_2\left(\frac{2}{3}, 1, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; z\right) = \frac{4}{z^{4/3}}\left(\sqrt[3]{z} - \frac{z^{2/3} + 1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{z}}{\sqrt[3]{z} + 2}\right) + \frac{1}{6}\left(1 - z^{2/3}\right)\left(3\log\left(1 - \sqrt[3]{z}\right) - \log(1 - z)\right)\right)$$

For fixed z and $a_1 = \frac{3}{4}, a_2 = 1, a_3 = \frac{m}{n}$

07.27.03.0500.01

$${}_3F_2\left(\frac{3}{4}, 1, 1; \frac{7}{4}, 2; z\right) = \frac{3}{z}\left(\log\left(\sqrt[4]{z} + 1\right)\sqrt[4]{z} + i\log\left(i\sqrt[4]{z} + 1\right)\sqrt[4]{z} - i\log\left(1 - i\sqrt[4]{z}\right)\sqrt[4]{z} + \log(1 - z) - \sqrt[4]{z}\log\left(1 - \sqrt[4]{z}\right)\right)$$

07.27.03.0501.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; z\right) = \frac{15}{8 z^{5/4}}\left(4\sqrt[4]{z} - 2\left(\sqrt{z} + 1\right)\tan^{-1}\left(\sqrt[4]{z}\right) - \left(1 - \sqrt{z}\right)\log\left(\frac{1 + \sqrt[4]{z}}{1 - \sqrt[4]{z}}\right)\right)$$

For fixed z and $a_1 = \frac{5}{6}, a_2 = 1, a_3 = \frac{7}{6}$

07.27.03.0502.01

$${}_3F_2\left(\frac{5}{6}, 1, \frac{7}{6}; \frac{11}{6}, \frac{13}{6}; z\right) = \frac{35}{24 z^{7/6}} \left(12 \sqrt[6]{z} - 2\sqrt{3}\left(\sqrt[3]{z}+1\right)\tan^{-1}\left(1-\sqrt[3]{z}, \sqrt{3}\sqrt[6]{z}\right) + \left(\sqrt[3]{z}-1\right)\left(\log\left(\frac{\sqrt[3]{z}+\sqrt[6]{z}+1}{\sqrt[3]{z}-\sqrt[6]{z}+1}\right) + 2\log\left(\frac{\sqrt[6]{z}+1}{1-\sqrt[6]{z}}\right)\right)\right) /; z \notin (1, \infty)$$

For fixed z and $a_1 = \frac{7}{8}, a_2 = 1, a_3 = \frac{9}{8}$

07.27.03.0503.01

$${}_3F_2\left(\frac{7}{8}, 1, \frac{9}{8}; \frac{15}{8}, \frac{17}{8}; -z\right) = \frac{63}{32 x^9}\left(-16x + 2\left(x^2+1\right)\left(a\tan^{-1}\left(1-x^2, a x\right) + b\tan^{-1}\left(1-x^2, b x\right)\right) + \left(1-x^2\right)\left(a\log\left(\frac{x^2+a x+1}{x^2-a x+1}\right) + b\log\left(\frac{x^2+b x+1}{x^2-b x+1}\right)\right)\right) /; x = \sqrt[8]{z}$$

For fixed z and $a_1 = 1, a_2 = \frac{5}{4}, a_3 = \frac{7}{4}$

07.27.03.0544.01

$${}_3F_2\left(1, \frac{5}{4}, \frac{7}{4}; 2, \frac{5}{2}; z\right) = \frac{8}{z}\left(\frac{\sqrt{2}}{\sqrt{\sqrt{1-z}+1}} - 1\right)$$

Values at fixed points

Values at $z = 1$

07.27.03.0777.01

$${}_3F_2\left(\frac{1}{8}, \frac{3}{8}, 1; \frac{9}{8}, \frac{11}{8}; 1\right) = \frac{3}{16}\left(2\sqrt{2}\log\left(1+\sqrt{2}\right)+\pi\right)$$

07.27.03.0778.01

$${}_3F_2\left(\frac{1}{8}, \frac{1}{2}, 1; \frac{9}{8}, \frac{3}{2}; 1\right) = \frac{1}{12}\left(\pi\left(1+\sqrt{2}\right)+\log(16)+2\sqrt{2}\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0779.01

$${}_3F_2\left(\frac{1}{8}, \frac{5}{8}, 1; \frac{9}{8}, \frac{13}{8}; 1\right) = \frac{5\sqrt{2}}{32}\left(2\log\left(1+\sqrt{2}\right)+\pi\right)$$

07.27.03.0780.01

$${}_3F_2\left(\frac{1}{8}, \frac{3}{4}, 1; \frac{9}{8}, \frac{7}{4}; 1\right) = \frac{3\sqrt{2}}{40}\left(\pi\left(1+\sqrt{2}\right)+\sqrt{2}\log(2)+2\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0781.01

$${}_3F_2\left(\frac{1}{8}, \frac{7}{8}, 1; \frac{9}{8}, \frac{15}{8}; 1\right) = \frac{7}{48}\pi\left(1+\sqrt{2}\right)$$

07.27.03.0782.01

$${}_3F_2\left(\frac{1}{8}, 1, 1; \frac{9}{8}, 2; 1\right) = \frac{1}{14}\left(\pi\left(1+\sqrt{2}\right)+8\log(2)+2\sqrt{2}\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0783.01

$$_3F_2\left(\frac{1}{6}, \frac{1}{3}, 1; \frac{7}{6}, \frac{4}{3}; 1\right) = 3^{-3/2}\left(2\sqrt{3}\log(2)+\pi\right)$$

07.27.03.0784.01

$$_3F_2\left(\frac{1}{6}, \frac{2}{3}, 1; \frac{7}{6}, \frac{5}{3}; 1\right) = \frac{4\sqrt{3}}{27}\left(\sqrt{3}\log(2)+\pi\right)$$

07.27.03.0785.01

$$_3F_2\left(\frac{1}{6}, \frac{5}{6}, 1; \frac{7}{6}, \frac{11}{6}; 1\right) = \frac{5\sqrt{3}}{24}\pi$$

07.27.03.0786.01

$$_3F_2\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}; \frac{5}{4}, \frac{5}{4}; 1\right) = \frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{4}\right)^2}{16\sqrt{2}}$$

07.27.03.0787.01

$$_3F_2\left(\frac{1}{4}, \frac{1}{4}, 1; \frac{5}{4}, \frac{5}{4}; 1\right) = \frac{1}{16}\left(8C+\pi^2\right)$$

07.27.03.0788.01

$$_3F_2\left(\frac{1}{4}, \frac{3}{8}, 1; \frac{5}{4}, \frac{11}{8}; 1\right) = \frac{3\sqrt{2}}{8}\left(\pi\left(\sqrt{2}-1\right)-\sqrt{2}\log(2)+2\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0789.01

$$_3F_2\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, \frac{3}{2}; 1\right) = \frac{1}{4}(2\log(2)+\pi)$$

07.27.03.0790.01

$$_3F_2\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, \frac{5}{2}; 1\right) = \frac{3}{10}(2\log(2)+\pi-1)$$

07.27.03.0791.01

$$_3F_2\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{3}{2}, \frac{9}{4}; 1\right) = \frac{5}{12}(2\log(2)+\pi-2)$$

07.27.03.0792.01

$$_3F_2\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{9}{4}, \frac{5}{2}; 1\right) = \frac{1}{2}(4\log(2)+2\pi-7)$$

07.27.03.0793.01

$$_3F_2\left(\frac{1}{4}, \frac{5}{8}, 1; \frac{5}{4}, \frac{13}{8}; 1\right) = \frac{1}{24}\left(5\sqrt{2}\right)\left(-\sqrt{2}\log(2)+2\log\left(1+\sqrt{2}\right)+\pi\right)$$

07.27.03.0794.01

$$_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}; 1\right) = \frac{3\pi}{8}$$

07.27.03.0795.01

$$_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{11}{4}; 1\right) = \frac{7}{48}(3\pi-2)$$

07.27.03.0796.01

$${}_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{7}{4}, \frac{9}{4}; 1\right) = \frac{15(\pi - 2)}{16}$$

07.27.03.0797.01

$${}_3F_2\left(\frac{1}{4}, \frac{7}{8}, 1; \frac{5}{4}, \frac{15}{8}; 1\right) = \frac{7\sqrt{2}}{40}\left(\pi\left(1+\sqrt{2}\right) - \sqrt{2}\,\log(2) - 2\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0798.01

$${}_3F_2\left(\frac{1}{4}, 1, 1; \frac{5}{4}, 2; 1\right) = \log(2) + \frac{\pi}{6}$$

07.27.03.0799.01

$${}_3F_2\left(\frac{1}{4}, 1, 1; 2, \frac{9}{4}; 1\right) = \frac{5}{6}(6\log(2) + \pi - 6)$$

07.27.03.0800.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{3}{2}; \frac{5}{4}, \frac{5}{2}; 1\right) = \frac{3}{20}(2\log(2) + \pi + 4)$$

07.27.03.0801.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{7}{4}; \frac{5}{4}, \frac{11}{4}; 1\right) = \frac{7}{72}(3\pi + 4)$$

07.27.03.0802.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{35}{144}(-3\pi + 14)$$

07.27.03.0803.01

$${}_3F_2\left(\frac{1}{3}, \frac{1}{2}, 1; \frac{4}{3}, \frac{3}{2}; 1\right) = \frac{\sqrt{3}}{6}\left(-4\sqrt{3}\,\log(2) + 3\sqrt{3}\,\log(3) + \pi\right)$$

07.27.03.0804.01

$${}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{4}{3}, \frac{5}{3}; 1\right) = \frac{2\pi}{3\sqrt{3}}$$

07.27.03.0805.01

$${}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{4}{3}, \frac{8}{3}; 1\right) = \frac{5\sqrt{3}}{36}\left(2\pi - \sqrt{3}\right)$$

07.27.03.0806.01

$${}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{5}{3}, \frac{7}{3}; 1\right) = \frac{4\sqrt{3}}{9}\left(\pi - \sqrt{3}\right)$$

07.27.03.0807.01

$${}_3F_2\left(\frac{1}{3}, \frac{5}{6}, 1; \frac{4}{3}, \frac{11}{6}; 1\right) = \frac{10\sqrt{3}}{27}\left(\pi - \sqrt{3}\,\log(2)\right)$$

07.27.03.0808.01

$${}_3F_2\left(\frac{1}{3}, 1, 1; \frac{4}{3}, 2; 1\right) = \frac{\sqrt{3}}{12}\left(3\sqrt{3}\,\log(3) + \pi\right)$$

07.27.03.0809.01

$${}_3F_2\left(\frac{1}{3}, 1, \frac{5}{3}; \frac{4}{3}, \frac{8}{3}; 1\right) = \frac{5\sqrt{3}}{72}\left(2\pi + 3\sqrt{3}\right)$$

07.27.03.0810.01

$${}_3F_2\left(\frac{3}{8}, \frac{1}{2}, 1; \frac{11}{8}, \frac{3}{2}; 1\right) = \frac{3}{4}\left(\pi\left(\sqrt{2}-1\right) + 4\log(2) - 2\sqrt{2}\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0811.01

$${}_3F_2\left(\frac{3}{8}, \frac{5}{8}, 1; \frac{11}{8}, \frac{13}{8}; 1\right) = \frac{15}{16}\pi\left(\sqrt{2}-1\right)$$

07.27.03.0812.01

$${}_3F_2\left(\frac{3}{8}, \frac{3}{4}, 1; \frac{11}{8}, \frac{7}{4}; 1\right) = \frac{3\sqrt{2}}{8}\left(\sqrt{2}\log(2) - 2\log\left(\sqrt{2}+1\right) + \pi\right)$$

07.27.03.0813.01

$${}_3F_2\left(\frac{3}{8}, \frac{7}{8}, 1; \frac{11}{8}, \frac{15}{8}; 1\right) = \frac{21\sqrt{2}}{32}\left(\pi - 2\log\left(\sqrt{2}+1\right)\right)$$

07.27.03.0814.01

$${}_3F_2\left(\frac{3}{8}, 1, 1; \frac{11}{8}, 2; 1\right) = \frac{3}{10}\left(\pi\left(\sqrt{2}-1\right) + 8\log(2) - 2\sqrt{2}\log\left(\sqrt{2}+1\right)\right)$$

07.27.03.0815.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, 1; 1\right) = \frac{1}{4\pi^3}\Gamma\left(\frac{1}{4}\right)^4$$

07.27.03.0816.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; 1\right) = \frac{4C}{\pi}$$

07.27.03.0817.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; 1\right) = \frac{\pi}{2}\log(2)$$

07.27.03.0818.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; 1\right) = \frac{\pi^2}{8}$$

07.27.03.0819.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{5}{2}; 1\right) = \frac{3}{16}\left(-4 + \pi^2\right)$$

07.27.03.0820.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; 2, 2; 1\right) = \frac{4(4-\pi)}{\pi}$$

07.27.03.0821.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{5}{2}, \frac{5}{2}; 1\right) = \frac{9}{16}\left(-8 + \pi^2\right)$$

07.27.03.0822.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{7}{2}, \frac{7}{2}; 1\right) = \frac{25}{256}\left(-256 + 27\pi^2\right)$$

07.27.03.0823.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}; 1, \frac{5}{2}; 1\right) = \frac{3}{2\pi}(2C+1)$$

07.27.03.0824.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 2; 1, 3; 1\right) = \frac{40}{9\pi}$$

07.27.03.0825.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 2; \frac{5}{2}, \frac{5}{2}; 1\right) = \frac{9}{8}$$

07.27.03.0826.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{5}{2}; 1, \frac{7}{2}; 1\right) = \frac{5}{32\pi}(18C+13)$$

07.27.03.0827.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 3; 1, 4; 1\right) = \frac{356}{75\pi}$$

07.27.03.0828.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{7}{2}; 1, \frac{9}{2}; 1\right) = \frac{7}{128\pi}(50C+43)$$

07.27.03.0829.01

$${}_3F_2\left(\frac{1}{2}, \frac{5}{8}, 1; \frac{3}{2}, \frac{13}{8}; 1\right) = \frac{5}{4}\left(\pi\left(\sqrt{2}-1\right) - 4\log(2) + 2\sqrt{2}\log\left(\sqrt{2}+1\right)\right)$$

07.27.03.0830.01

$${}_3F_2\left(\frac{1}{2}, \frac{2}{3}, 1; \frac{3}{2}, \frac{5}{3}; 1\right) = \frac{4\sqrt{3}\log(2) - 3\sqrt{3}\log(3) + \pi}{\sqrt{3}}$$

07.27.03.0831.01

$${}_3F_2\left(\frac{1}{2}, \frac{3}{4}, 1; \frac{3}{2}, \frac{7}{4}; 1\right) = \frac{3}{4}(\pi - 2\log(2))$$

07.27.03.0832.01

$${}_3F_2\left(\frac{1}{2}, \frac{3}{4}, 1; \frac{3}{2}, \frac{11}{4}; 1\right) = \frac{7}{20}(-6\log(2) + 3\pi - 2)$$

07.27.03.0833.01

$${}_3F_2\left(\frac{1}{2}, \frac{3}{4}, 1; \frac{7}{4}, \frac{5}{2}; 1\right) = \frac{3}{2}(-2\log(2) + \pi - 1)$$

07.27.03.0834.01

$${}_3F_2\left(\frac{1}{2}, \frac{7}{8}, 1; \frac{3}{2}, \frac{15}{8}; 1\right) = \frac{7}{12}\left(\pi\left(1+\sqrt{2}\right) - 4\log(2) - 2\sqrt{2}\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0835.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{3}{2}, 2; 1\right) = 2\log(2)$$

07.27.03.0836.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; 2, \frac{5}{2}; 1\right) = 3(2\log(2) - 1)$$

07.27.03.0837.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{5}{2}, 3; 1\right) = 2\,(8\log(2) - 5)$$

07.27.03.0838.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{5}{4}; \frac{3}{2}, \frac{9}{4}; 1\right) = \frac{5}{12}\,(-2\log(2) - \pi + 8)$$

07.27.03.0839.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{5}{4}; \frac{9}{4}, \frac{5}{2}; 1\right) = \frac{5}{2}\,(-2\log(2) - \pi + 5)$$

07.27.03.0840.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{3}{2}; 2, 2; 1\right) = \frac{4}{\pi}\,(\pi - 2)$$

07.27.03.0841.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{7}{4}; \frac{3}{2}, \frac{11}{4}; 1\right) = \frac{7}{60}\,(3\pi + 8 - 6\log(2))$$

07.27.03.0842.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{7}{4}; \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{10}\,(6\log(2) - 3\pi + 7)$$

07.27.03.0843.01

$${}_3F_2\left(\frac{1}{2}, 1, 2; \frac{3}{2}, 3; 1\right) = \frac{2}{3}\,(2\log(2) + 1)$$

07.27.03.0844.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{7}{2}; \frac{5}{2}, \frac{9}{2}; 1\right) = \frac{77}{60}$$

07.27.03.0845.01

$${}_3F_2\left(\frac{5}{8}, \frac{7}{8}, 1; \frac{13}{8}, \frac{15}{8}; 1\right) = \frac{35}{16}\left(\pi - 2\sqrt{2}\,\log\left(1 + \sqrt{2}\right)\right)$$

07.27.03.0846.01

$${}_3F_2\left(\frac{5}{8}, 1, 1; \frac{13}{8}, 2; 1\right) = \frac{5}{6}\left(-\pi\left(\sqrt{2} - 1\right) + 8\log(2) - 2\sqrt{2}\,\log\left(1 + \sqrt{2}\right)\right)$$

07.27.03.0847.01

$${}_3F_2\left(\frac{2}{3}, 1, 1; \frac{5}{3}, 2; 1\right) = 3\log(3) - \frac{\pi}{\sqrt{3}}$$

07.27.03.0848.01

$${}_3F_2\left(\frac{2}{3}, 1, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; 1\right) = \frac{4\sqrt{3}}{9}\left(3\sqrt{3} - \pi\right)$$

07.27.03.0849.01

$${}_3F_2\left(\frac{3}{4}, \frac{3}{4}, 1; \frac{11}{4}, \frac{11}{4}; 1\right) = \frac{49}{128}\left(-20 - 72\,C + 9\,\pi^2\right)$$

07.27.03.0850.01

$${}_3F_2\left(\frac{3}{4}, \frac{7}{8}, 1; \frac{7}{4}, \frac{15}{8}; 1\right) = \frac{21\sqrt{2}}{8}\left(-\sqrt{2}\,\log(2) - 2\log\left(\sqrt{2} + 1\right) + \pi\right)$$

07.27.03.0851.01

$${}_3F_2\left(\frac{3}{4}, 1, 1; \frac{7}{4}, 2; 1\right) = \frac{3}{2}(6\log(2) - \pi)$$

07.27.03.0852.01

$${}_3F_2\left(\frac{3}{4}, 1, 1; 2, \frac{11}{4}; 1\right) = \frac{7}{6}(18\log(2) - 3\pi - 2)$$

07.27.03.0853.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; 1\right) = \frac{15(4-\pi)}{8}$$

07.27.03.0854.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{5}{4}; \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{35}{16}(10 - 3\pi)$$

07.27.03.0855.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{3}{2}; \frac{7}{4}, \frac{5}{2}; 1\right) = \frac{3}{4}(2\log(2) - \pi + 4)$$

07.27.03.0856.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{3}{2}; \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{21}{4}(2\log(2) - \pi + 2)$$

07.27.03.0857.01

$${}_3F_2\left(\frac{4}{5}, 1, \frac{6}{5}; \frac{9}{5}, \frac{11}{5}; 1\right) = \frac{12}{5}\left(5 - \sqrt{1 + \frac{2}{\sqrt{5}}}\ \pi\right)$$

07.27.03.0858.01

$${}_3F_2\left(\frac{5}{6}, 1, \frac{7}{6}; \frac{11}{6}, \frac{13}{6}; 1\right) = \frac{35}{12}\left(6 - \pi\sqrt{3}\right)$$

07.27.03.0859.01

$${}_3F_2\left(\frac{7}{8}, 1, 1; \frac{15}{8}, 2; 1\right) = \frac{7}{2}\left(8\log(2) - \pi\left(\sqrt{2} + 1\right) + 2\sqrt{2}\log\left(1 + \sqrt{2}\right)\right)$$

07.27.03.0860.01

$${}_3F_2\left(\frac{7}{8}, 1, \frac{9}{8}; \frac{15}{8}, \frac{17}{8}; 1\right) = \frac{63}{16}\left(8 - \pi\left(\sqrt{2} + 1\right)\right)$$

07.27.03.0861.01

$${}_3F_2\left(1, 1, 1; \frac{3}{2}, 2; 1\right) = \frac{\pi^2}{4}$$

07.27.03.0862.01

$${}_3F_2\left(1, 1, 1; \frac{3}{2}, \frac{5}{2}; 1\right) = \frac{3(\pi - 2)}{2}$$

07.27.03.0863.01

$${}_3F_2(1, 1, 1; 2, 2; 1) = \frac{\pi^2}{6}$$

07.27.03.0864.01

$${}_3F_2(1,1,1;2,3;1)=\frac{1}{3}\left(\pi^2-6\right)$$

07.27.03.0865.01

$${}_3F_2\left(1,1,1;\frac{5}{2},\frac{5}{2};1\right)=9\,(\pi-3)$$

07.27.03.0866.01

$${}_3F_2(1,1,1;3,3;1)=\frac{4}{3}\left(-9+\pi^2\right)$$

07.27.03.0867.01

$${}_3F_2(1,1,1;4,4;1)=\frac{9}{4}\left(-39+4\,\pi^2\right)$$

07.27.03.0868.01

$${}_3F_2(1,1,1;5,5;1)=\frac{8}{3}\left(-197+20\,\pi^2\right)$$

07.27.03.0869.01

$${}_3F_2\left(1,1,\frac{9}{8};2,\frac{17}{8};1\right)=\frac{9}{2}\left(16-\pi\left(1+\sqrt{2}\right)-8\log(2)-2\sqrt{2}\,\log\left(\sqrt{2}+1\right)\right)$$

07.27.03.0870.01

$${}_3F_2\left(1,1,\frac{5}{4};2,\frac{9}{4};1\right)=\frac{5}{2}\,(-6\log(2)-\pi+8)$$

07.27.03.0871.01

$${}_3F_2\left(1,1,\frac{5}{4};\frac{9}{4},3;1\right)=\frac{10}{3}\,(-12\log(2)-2\,\pi+15)$$

07.27.03.0872.01

$${}_3F_2\left(1,1,\frac{4}{3};\frac{7}{3},3;1\right)=32-18\log(3)-2\,\pi\sqrt{3}$$

07.27.03.0873.01

$${}_3F_2\left(1,1,\frac{11}{8};2,\frac{19}{8};1\right)=\frac{11}{18}\left(16-3\,\pi\left(\sqrt{2}-1\right)-24\log(2)+6\sqrt{2}\,\log\left(\sqrt{2}+1\right)\right)$$

07.27.03.0874.01

$${}_3F_2\left(1,1,\frac{3}{2};2,2;1\right)=4\log(2)$$

07.27.03.0875.01

$${}_3F_2\left(1,1,\frac{3}{2};2,\frac{5}{2};1\right)=6\,(1-\log(2))$$

07.27.03.0876.01

$${}_3F_2\left(1,1,\frac{3}{2};2,\frac{7}{2};1\right)=\frac{5}{3}\,(5-6\log(2))$$

07.27.03.0877.01

$${}_3F_2\left(1,1,\frac{3}{2};\frac{5}{2},3;1\right)=6\,(3-4\log(2))$$

07.27.03.0878.01

$${}_3F_2\left(1, 1, \frac{3}{2}; 3, \frac{7}{2}; 1\right) == \frac{10}{3} (17 - 24 \log(2))$$

07.27.03.0879.01

$${}_3F_2\left(1, 1, \frac{13}{8}; 2, \frac{21}{8}; 1\right) == \frac{13}{50} \left(5 \pi \left(\sqrt{2} - 1\right) - 40 \log(2) + 10 \sqrt{2} \log\left(1 + \sqrt{2}\right) + 16\right)$$

07.27.03.0880.01

$${}_3F_2\left(1, 1, \frac{5}{3}; 2, \frac{8}{3}; 1\right) == \frac{5}{12} \left(9 - 9 \log(3) + \sqrt{3} \pi\right)$$

07.27.03.0881.01

$${}_3F_2\left(1, 1, \frac{5}{3}; \frac{8}{3}, 3; 1\right) == \frac{5}{2} \left(5 - 9 \log(3) + \sqrt{3} \pi\right)$$

07.27.03.0882.01

$${}_3F_2\left(1, 1, \frac{7}{4}; 2, \frac{11}{4}; 1\right) == \frac{7}{18} (8 - 18 \log(2) + 3 \pi)$$

07.27.03.0883.01

$${}_3F_2\left(1, 1, \frac{7}{4}; \frac{11}{4}, 3; 1\right) == \frac{14}{9} (7 - 36 \log(2) + 6 \pi)$$

07.27.03.0884.01

$${}_3F_2\left(1, 1, \frac{15}{8}; 2, \frac{23}{8}; 1\right) == \frac{15}{98} \left(7 \left(1 + \sqrt{2}\right) \pi - 2 \left(28 \log(2) + 7 \sqrt{2} \log\left(1 + \sqrt{2}\right) - 8\right)\right)$$

07.27.03.0885.01

$${}_3F_2(1, 1, 2; 3, 3; 1) == \frac{2}{3} \left(12 - \pi^2\right)$$

07.27.03.0886.01

$${}_3F_2\left(1, 1, \frac{5}{2}; 2, \frac{7}{2}; 1\right) == \frac{5}{9} (8 - 3 \log(4))$$

07.27.03.0887.01

$${}_3F_2\left(1, 1, \frac{5}{2}; 3, \frac{7}{2}; 1\right) == \frac{10}{9} (6 \log(4) - 7)$$

07.27.03.0888.01

$${}_3F_2(1, 1, 3; 2, 4; 1) == \frac{9}{4}$$

07.27.03.0889.01

$${}_3F_2(1, 1, 3; 2, 5; 1) == \frac{5}{3}$$

07.27.03.0890.01

$${}_3F_2(1, 1, 4; 2, 5; 1) == \frac{22}{9}$$

07.27.03.0891.01

$${}_3F_2(1, 1, 4; 3, 5; 1) == \frac{14}{9}$$

07.27.03.0892.01

$${}_3F_2(1,\,1,\,5;\,2,\,6;\,1)=\frac{125}{48}$$

07.27.03.0893.01

$${}_3F_2\left(1,\,\frac{3}{2},\,\frac{3}{2};\,3,\,3;\,1\right)=\frac{16\,(16-5\,\pi)}{\pi}$$

07.27.03.0894.01

$${}_3F_2(1,\,2,\,2;\,3,\,3;\,1)=\frac{2}{3}\left(\pi^2-6\right)$$

Values at $z = -1$

07.27.03.0895.01

$${}_3F_2\left(1,\,\frac{1+i}{2},\,\frac{1-i}{2};\,\frac{3+i}{2},\,\frac{3-i}{2};\,-1\right)=\frac{i}{4}\left(\psi\left(\frac{3+i}{4}\right)+\psi\left(\frac{1-i}{4}\right)-\psi\left(\frac{1+i}{4}\right)-\psi\left(\frac{3-i}{4}\right)\right)$$

07.27.03.0896.01

$${}_3F_2\left(1,\,\frac{2+i}{4},\,\frac{2-i}{4};\,\frac{6+i}{4},\,\frac{6-i}{4};\,-1\right)=\frac{5}{16}\,i\left(\psi\left(\frac{1}{4}-\frac{i}{8}\right)-\psi\left(\frac{3}{4}-\frac{i}{8}\right)-\psi\left(\frac{1}{4}+\frac{i}{8}\right)+\psi\left(\frac{3}{4}+\frac{i}{8}\right)\right)$$

07.27.03.0897.01

$${}_3F_2(2,\,1+i,\,1-i;\,2+i,\,2-i;\,-1)=\frac{1}{2}\left(\psi\left(1+\frac{i}{2}\right)+\psi\left(1-\frac{i}{2}\right)-\psi\left(\frac{1-i}{2}\right)-\psi\left(\frac{1+i}{2}\right)\right)$$

07.27.03.0898.01

$${}_3F_2\left(2,\,1+\frac{i}{2},\,1-\frac{i}{2};\,2+\frac{i}{2},\,2-\frac{i}{2};\,-1\right)=\frac{5}{16}\left(\psi\left(1+\frac{i}{4}\right)-\psi\left(\frac{1}{2}+\frac{i}{4}\right)+\psi\left(1-\frac{i}{4}\right)-\psi\left(\frac{1}{2}-\frac{i}{4}\right)\right)$$

07.27.03.0899.01

$${}_3F_2\left(\frac{1}{6},\,\frac{1}{3},\,1;\,\frac{7}{6},\,\frac{4}{3};\,-1\right)=\frac{1}{9}\left(\pi\left(3-\sqrt{3}\right)-3\log(2)+3\sqrt{3}\,\log\left(2+\sqrt{3}\right)\right)$$

07.27.03.0900.01

$${}_3F_2\left(\frac{1}{6},\,\frac{2}{3},\,1;\,\frac{7}{6},\,\frac{5}{3};\,-1\right)=\frac{2}{27}\left(\pi\left(3-\sqrt{3}\right)+3\log(2)+3\sqrt{3}\,\log\left(2+\sqrt{3}\right)\right)$$

07.27.03.0901.01

$${}_3F_2\left(\frac{1}{6},\,\frac{5}{6},\,1;\,\frac{7}{6},\,\frac{11}{6};\,-1\right)=\frac{1}{12}\left(5\sqrt{3}\right)\log\left(2+\sqrt{3}\right)$$

07.27.03.0902.01

$${}_3F_2\left(\frac{1}{4},\,\frac{1}{2},\,1;\,\frac{5}{4},\,\frac{3}{2};\,-1\right)=\frac{1}{4}\left(\left(\sqrt{2}-1\right)\pi-2\sqrt{2}\,\log\left(\sqrt{2}-1\right)\right)$$

07.27.03.0903.01

$${}_3F_2\left(\frac{1}{4},\,\frac{1}{2},\,1;\,\frac{5}{4},\,\frac{5}{2};\,-1\right)=\frac{3}{20}\left(\pi\left(-3+2\sqrt{2}\right)+4\sqrt{2}\,\log\left(1+\sqrt{2}\right)+2\right)$$

07.27.03.0904.01

$${}_3F_2\left(\frac{1}{4},\,\frac{1}{2},\,1;\,\frac{3}{2},\,\frac{9}{4};\,-1\right)=\frac{1}{24}\left(5\sqrt{2}\right)\left(\pi\left(1-\sqrt{2}\right)+2\log\left(1+\sqrt{2}\right)+2\sqrt{2}\right)$$

07.27.03.0905.01

$${}_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{11}{4}; -1\right) = \frac{7\sqrt{2}}{96}\left(18\log\left(1+\sqrt{2}\right)+2\sqrt{2}-3\pi\right)$$

07.27.03.0906.01

$${}_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{7}{4}, \frac{9}{4}; -1\right) = \frac{15\sqrt{2}}{32}\left(2\log\left(1+\sqrt{2}\right)+2\sqrt{2}-\pi\right)$$

07.27.03.0907.01

$${}_3F_2\left(\frac{1}{4}, 1, 1; \frac{5}{4}, 2; -1\right) = \frac{\sqrt{2}}{6}\left(\pi-\sqrt{2}\log(2)+2\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0908.01

$${}_3F_2\left(\frac{1}{4}, 1, 1; 2, \frac{9}{4}; -1\right) = \frac{5}{24}\left(24-8\log(2)-4\sqrt{2}\log\left(1+\sqrt{2}\right)-2\sqrt{2}\pi\right)$$

07.27.03.0909.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{3}{2}; \frac{5}{4}, \frac{5}{2}; -1\right) = \frac{3\sqrt{2}}{40}\left(\pi\left(2+\sqrt{2}\right)+4\log\left(1+\sqrt{2}\right)-4\sqrt{2}\right)$$

07.27.03.0910.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{3}{2}; \frac{9}{4}, \frac{5}{2}; -1\right) = \frac{3\sqrt{2}}{8}\left(\pi\left(3-\sqrt{2}\right)+6\log\left(1+\sqrt{2}\right)-6\sqrt{2}\right)$$

07.27.03.0911.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{7}{4}; \frac{5}{4}, \frac{11}{4}; -1\right) = \frac{7\sqrt{2}}{72}\left(3\pi-2\sqrt{2}\right)$$

07.27.03.0912.01

$${}_3F_2\left(\frac{1}{4}, 1, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; -1\right) = \frac{35\sqrt{2}}{288}\left(18\log\left(1+\sqrt{2}\right)-14\sqrt{2}+3\pi\right)$$

07.27.03.0913.01

$${}_3F_2\left(\frac{1}{3}, \frac{1}{2}, 1; \frac{4}{3}, \frac{3}{2}; -1\right) = \pi\left(\frac{\sqrt{3}}{3}-\frac{1}{2}\right)+\log(2)$$

07.27.03.0914.01

$${}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{4}{3}, \frac{5}{3}; -1\right) = \frac{4}{3}\log(2)$$

07.27.03.0915.01

$${}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{4}{3}, \frac{8}{3}; -1\right) = \frac{5}{108}\left(48\log(2)-4\sqrt{3}\pi+9\right)$$

07.27.03.0916.01

$${}_3F_2\left(\frac{1}{3}, \frac{2}{3}, 1; \frac{5}{3}, \frac{7}{3}; -1\right) = \frac{4\sqrt{3}}{27}\left(4\sqrt{3}\log(2)+3\sqrt{3}-2\pi\right)$$

07.27.03.0917.01

$${}_3F_2\left(\frac{1}{3}, \frac{5}{6}, 1; \frac{4}{3}, \frac{11}{6}; -1\right) = \frac{5}{27}\left(\pi\left(-3+\sqrt{3}\right)+3\log(2)+3\sqrt{3}\log\left(2+\sqrt{3}\right)\right)$$

07.27.03.0918.01

$${}_3F_2\left(\frac{1}{3}, 1, 1; \frac{4}{3}, 2; -1\right) = \frac{\pi}{2\sqrt{3}}$$

07.27.03.0919.01

$${}_3F_2\left(\frac{1}{3}, 1, \frac{5}{3}; \frac{4}{3}, \frac{8}{3}; -1\right) = \frac{5\sqrt{3}}{72}\left(4\pi - 3\sqrt{3}\right)$$

07.27.03.0920.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; -1\right) = C$$

07.27.03.0921.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{5}{2}, \frac{5}{2}; -1\right) = \frac{9(4-\pi)}{8}$$

07.27.03.0922.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{7}{2}, \frac{7}{2}; -1\right) = \frac{25}{64}(19 - 18\,C)$$

07.27.03.0923.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{9}{2}, \frac{9}{2}; -1\right) = \frac{49}{576}(75\,\pi - 224)$$

07.27.03.0924.01

$${}_3F_2\left(\frac{1}{2}, \frac{2}{3}, 1; \frac{3}{2}, \frac{5}{3}; -1\right) = 2\log(2) - \frac{2\pi}{\sqrt{3}} + \pi$$

07.27.03.0925.01

$${}_3F_2\left(\frac{1}{2}, \frac{3}{4}, 1; \frac{3}{2}, \frac{7}{4}; -1\right) = \frac{3}{4}\left(\pi\left(1 - \sqrt{2}\right) + 2\sqrt{2}\,\log\left(1 + \sqrt{2}\right)\right)$$

07.27.03.0926.01

$${}_3F_2\left(\frac{1}{2}, \frac{3}{4}, 1; \frac{7}{4}, \frac{5}{2}; -1\right) = \frac{3}{4}\left(\pi\left(1 - 2\sqrt{2}\right) + 4\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + 2\right)$$

07.27.03.0927.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{3}{2}, \frac{3}{2}; -1\right) = \frac{1}{8}\left(\pi^2 - 4\log^2\left(1 + \sqrt{2}\right)\right)$$

07.27.03.0928.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{3}{2}, 2; -1\right) = \frac{\pi}{2} - \log(2)$$

07.27.03.0929.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; 2, \frac{5}{2}; -1\right) = 3(1 - \log(2))$$

07.27.03.0930.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{5}{2}, 3; -1\right) = 10 - 4\log(2) - 2\pi$$

07.27.03.0931.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{5}{4}; \frac{3}{2}, \frac{9}{4}; -1\right) = \frac{5}{12}\left(\pi\left(1 + \sqrt{2}\right) + 2\sqrt{2}\,\log\left(1 + \sqrt{2}\right) - 8\right)$$

07.27.03.0932.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{5}{4}; \frac{9}{4}, \frac{5}{2}; -1\right) = \frac{5}{4}\left(\pi\left(2\sqrt{2} - 1\right) + 4\sqrt{2}\log\left(1+\sqrt{2}\right) - 10\right)$$

07.27.03.0933.01

$${}_3F_2\left(\frac{1}{2}, 1, 2; \frac{3}{2}, 3; -1\right) = \frac{1}{3}\left(2\log(2) + \pi - 2\right)$$

07.27.03.0934.01

$${}_3F_2\left(\frac{1}{2}, 1, \frac{7}{2}; \frac{5}{2}, \frac{9}{2}; -1\right) = \frac{7}{120}\left(15\pi - 32\right)$$

07.27.03.0935.01

$${}_3F_2\left(\frac{2}{3}, 1, 1; \frac{5}{3}, 2; -1\right) = \frac{2\sqrt{3}}{3}\left(\pi - 2\sqrt{3}\log(2)\right)$$

07.27.03.0936.01

$${}_3F_2\left(\frac{2}{3}, 1, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; -1\right) = \frac{4\sqrt{3}}{9}\left(2\pi - 3\sqrt{3}\right)$$

07.27.03.0937.01

$${}_3F_2\left(\frac{3}{4}, 1, 1; \frac{7}{4}, 2; -1\right) = \frac{3\sqrt{2}}{2}\left(\pi - \sqrt{2}\log(2) - 2\log\left(1+\sqrt{2}\right)\right)$$

07.27.03.0938.01

$${}_3F_2\left(\frac{3}{4}, 1, 1; 2, \frac{11}{4}; -1\right) = \frac{7\sqrt{2}}{12}\left(3\pi - 6\sqrt{2}\log(2) - 6\log\left(1+\sqrt{2}\right) + 2\sqrt{2}\right)$$

07.27.03.0939.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; -1\right) = \frac{15\sqrt{2}}{8}\left(\pi - 2\sqrt{2}\right)$$

07.27.03.0940.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{5}{4}; \frac{9}{4}, \frac{11}{4}; -1\right) = \frac{35\sqrt{2}}{32}\left(6\log\left(1+\sqrt{2}\right) - 10\sqrt{2} + 3\pi\right)$$

07.27.03.0941.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{3}{2}; \frac{7}{4}, \frac{5}{2}; -1\right) = \frac{3\sqrt{2}}{8}\left(\pi\left(2+\sqrt{2}\right) - 4\log\left(1+\sqrt{2}\right) - 4\sqrt{2}\right)$$

07.27.03.0942.01

$${}_3F_2\left(\frac{3}{4}, 1, \frac{3}{2}; \frac{5}{2}, \frac{11}{4}; -1\right) = \frac{21}{8}\left(\pi\left(2-\sqrt{2}\right) + 2\sqrt{2}\log\left(1+\sqrt{2}\right) - 4\right)$$

07.27.03.0943.01

$${}_3F_2\left(\frac{4}{5}, 1, \frac{6}{5}; \frac{9}{5}, \frac{11}{5}; -1\right) = \frac{12}{25}\left(\sqrt{50 + 10\sqrt{5}}\,\pi - 25\right)$$

07.27.03.0944.01

$${}_3F_2\left(\frac{5}{6}, 1, \frac{7}{6}; \frac{11}{6}, \frac{13}{6}; -1\right) = \frac{35(\pi - 3)}{6}$$

07.27.03.0945.01

$${}_3F_2\left(\frac{7}{8}, 1, \frac{9}{8}; \frac{15}{8}, \frac{17}{8}; -1\right) = \frac{63}{8}\left(\frac{\sqrt{1+\sqrt{2}}\ \pi}{\sqrt[4]{2}} - 4\right)$$

07.27.03.0946.01

$${}_3F_2(1, 1, 1; 2, 2; -1) = \frac{\pi^2}{12}$$

07.27.03.0947.01

$${}_3F_2(1, 1, 1; 2, 3; -1) = -4\log(2) + \frac{\pi^2}{6} + 2$$

07.27.03.0948.01

$${}_3F_2(1, 1, 1; 3, 3; -1) = 12 - 16\log(2)$$

07.27.03.0949.01

$${}_3F_2(1, 1, 1; 4, 4; -1) = \frac{63}{4} - \frac{3\pi^2}{2}$$

07.27.03.0950.01

$${}_3F_2\left(1, 1, \frac{5}{4}; 2, \frac{9}{4}; -1\right) = \frac{5\sqrt{2}}{2}\left(\sqrt{2}\log(2) + 2\log\left(1+\sqrt{2}\right) - 4\sqrt{2} + \pi\right)$$

07.27.03.0951.01

$${}_3F_2\left(1, 1, \frac{5}{4}; \frac{9}{4}, 3; -1\right) = \frac{10}{3}\left(2\log(2) + 4\sqrt{2}\log\left(1+\sqrt{2}\right) + 2\sqrt{2}\,\pi - 15\right)$$

07.27.03.0952.01

$${}_3F_2\left(1, 1, \frac{4}{3}; 2, \frac{7}{3}; -1\right) = \frac{4\sqrt{3}}{3}\left(2\sqrt{3}\log(2) - 3\sqrt{3} + \pi\right)$$

07.27.03.0953.01

$${}_3F_2\left(1, 1, \frac{4}{3}; \frac{7}{3}, 3; -1\right) = 16\log(2) + 4\sqrt{3}\,\pi - 32$$

07.27.03.0954.01

$${}_3F_2\left(1, 1, \frac{3}{2}; 2, \frac{5}{2}; -1\right) = \frac{3}{2}(2\log(2) + \pi - 4)$$

07.27.03.0955.01

$${}_3F_2\left(1, 1, \frac{3}{2}; 2, \frac{7}{2}; -1\right) = \frac{5}{3}(3\log(2) + 3\pi - 11)$$

07.27.03.0956.01

$${}_3F_2\left(1, 1, \frac{3}{2}; \frac{5}{2}, 3; -1\right) = 6(\pi - 3)$$

07.27.03.0957.01

$${}_3F_2\left(1, 1, \frac{3}{2}; 3, \frac{7}{2}; -1\right) = \frac{10}{3}(3\pi - 6\log(2) - 5)$$

07.27.03.0958.01

$${}_3F_2\left(1, 1, \frac{5}{3}; 2, \frac{8}{3}; -1\right) = \frac{5\sqrt{3}}{12}\left(2\pi - 3\sqrt{3}\right)$$

07.27.03.0959.01

$${}_3F_2\left(1,1,\frac{5}{3};\frac{8}{3},3;-1\right)=\frac{5}{2}\left(2\sqrt{3}\,\pi-8\log(2)-5\right)$$

07.27.03.0960.01

$${}_3F_2\left(1,1,\frac{7}{4};2,\frac{11}{4};-1\right)=\frac{7}{18}\left(6\log(2)-6\sqrt{2}\log\left(1+\sqrt{2}\right)+3\sqrt{2}\,\pi-8\right)$$

07.27.03.0961.01

$${}_3F_2\left(1,1,\frac{7}{4};\frac{11}{4},3;-1\right)=\frac{14}{9}\left(6\sqrt{2}\,\pi-6\log(2)-12\sqrt{2}\log\left(1+\sqrt{2}\right)-7\right)$$

07.27.03.0962.01

$${}_3F_2(1,1,2;3,3;-1)=8\log(2)+\frac{\pi^2}{3}-8$$

07.27.03.0963.01

$${}_3F_2\left(1,1,\frac{5}{2};2,\frac{7}{2};-1\right)=\frac{5}{18}(6\log(2)-3\pi+8)$$

07.27.03.0964.01

$${}_3F_2\left(1,1,\frac{5}{2};3,\frac{7}{2};-1\right)=\frac{10}{9}(12\log(2)+3\pi-17)$$

07.27.03.0965.01

$${}_3F_2(1,1,3;2,4;-1)=\frac{3}{4}$$

07.27.03.0966.01

$${}_3F_2(1,1,4;2,5;-1)=\frac{2}{9}(6\log(4)-5)$$

07.27.03.0967.01

$${}_3F_2(1,1,5;2,6;-1)=\frac{35}{48}$$

07.27.03.0968.01

$${}_3F_2\left(\frac{5}{4},\frac{7}{4},2;\frac{9}{4},\frac{11}{4};-1\right)=\frac{35}{16\sqrt{2}}\left(4\log\left(1+\sqrt{2}\right)-\pi\right)$$

07.27.03.0969.01

$${}_3F_2(2,2,2;3,3;-1)=\frac{1}{3}\left(\pi^2-12\log(2)\right)$$

Values at $z=\frac{1}{2}$

07.27.03.0970.01

$${}_3F_2\left(1,1,1;2,2;\frac{1}{2}\right)=\frac{\pi^2}{6}-\log^2(2)$$

Values at $z=-\frac{1}{4}$

07.27.03.0971.01

$${}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{4}\right)=\frac{\pi^2}{10}$$

07.27.03.0972.01

$${}_3F_2\left(\frac{1}{2}, 1, 1; \frac{3}{2}, \frac{3}{2}; -\frac{1}{4}\right) = \frac{\pi^2}{6} - 3\log^2\left(\frac{\sqrt{5}-1}{2}\right)$$

Values at $z = -\frac{1}{8}$

07.27.03.0973.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{1}{4}\right) = \frac{\pi^2}{10}$$

Values at other z

07.27.03.0974.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; 9-4\sqrt{5}\right) = \frac{1}{24}\left(\sqrt{5}+2\right)\left(\pi^2 - 2\log^2\left(\sqrt{5}-2\right)\right)$$

07.27.03.0975.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; 3-2\sqrt{2}\right) = \frac{1}{16}\left(\sqrt{2}+1\right)\left(\pi^2 - 4\log^2\left(\sqrt{2}-1\right)\right)$$

07.27.03.0976.01

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; \frac{1}{2}\left(3-\sqrt{5}\right)\right) = \frac{1}{24}\left(\sqrt{5}+1\right)\left(\pi^2 - 9\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\right)$$

07.27.03.0978.01

$${}_3F_2\left(1, 1, 1; 2, 2; -\frac{\sqrt{5}-1}{2}\right) = \frac{\sqrt{5}+1}{2}\left(\frac{\pi^2}{15} - \frac{1}{2}\log^2\left(\frac{\sqrt{5}-1}{2}\right)\right)$$

07.27.03.0980.01

$${}_3F_2\left(1, 1, 1; 2, 2; \frac{3-\sqrt{5}}{2}\right) = \frac{\sqrt{5}+3}{2}\left(\frac{\pi^2}{15} - \log^2\left(\frac{\sqrt{5}-1}{2}\right)\right)$$

07.27.03.0982.01

$${}_3F_2\left(1, 1, 1; 2, 2; \frac{\sqrt{5}-1}{2}\right) = \frac{\sqrt{5}+1}{2}\left(\frac{\pi^2}{10} - \log^2\left(\frac{\sqrt{5}-1}{2}\right)\right)$$

General characteristics

Some abbreviations

07.27.04.0001.01

$$\mathcal{NT}(\{a_1, a_2, a_3\}) = \neg\,(-a_1 \in \mathbb{N} \lor -a_2 \in \mathbb{N} \lor -a_3 \in \mathbb{N})$$

Domain and analyticity

${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ is an analytical function of a_1, a_2, a_3, b_1, b_2 and z which is defined in $\mathbb{C}^6$. If parameters $a_k$include negative integers, the function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ degenerates to a polynomial in z.

07.27.04.0002.01

$$(\{a_1 * a_2 * a_3\} * \{b_1 * b_2\} * z) \longrightarrow {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) :: (\{\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}\} \otimes \{\mathbb{C} \otimes \mathbb{C}\} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.27.04.0003.02

$${}_3F_2\left(\overline{a_1}, \overline{a_2}, \overline{a_3}; \overline{b_1}, \overline{b_2}; \bar{z}\right) = \overline{{}_3F_2(a_1, a_2, a_3; b_1, b_2; z)} \;/;\; z \notin (1, \infty)$$

Permutation symmetry

07.27.04.0004.01

$${}_3F_2\left(a_1, a_2, \ldots, a_k, \ldots, a_j, \ldots, a_3; b_1, b_2; z\right) = {}_3F_2\left(a_1, a_2, \ldots, a_j, \ldots, a_k, \ldots, a_3; b_1, b_2; z\right) \;/;\; a_k \neq a_j \bigwedge k \neq j$$

07.27.04.0005.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = {}_3F_2(a_1, a_2, a_3; b_2, b_1; z)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed a_l, b_j in nonpolynomial cases (when $\neg\,(-a_1 \in \mathbb{N} \bigvee -a_2 \in \mathbb{N} \bigvee -a_3 \in \mathbb{N})$), the function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ does not have poles and essential singularities.

07.27.04.0006.01

$$\mathcal{Sing}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\} \;/;\; \mathcal{NT}(\{a_1, a_2, a_3\})$$

If parameters a_k include r negative integers α_k, the function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ is a polynomial and has pole of order $\min(-\alpha_1, \ldots, -\alpha_r\}$ at $z = \tilde{\infty}$.

07.27.04.0007.01

$$\mathcal{Sing}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\{\tilde{\infty}, -\alpha\}\} \;/;\; \neg\,(\mathcal{NT}(\{a_1, a_2, a_3\})) \bigwedge \alpha = \min\left(-a_{s_1}, \ldots, -a_{s_r}\right\} \bigwedge -a_{s_k} \in \mathbb{N}^+$$

With respect to a_l

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of a_l, $1 \leq l \leq 3$, has only one singular point at $a_l = \tilde{\infty}$. It is an essential singular point.

07.27.04.0008.01

$$\mathcal{Sing}_{a_l}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\{\tilde{\infty}, \infty\}\} \;/;\; 1 \leq l \leq 3$$

With respect to b_j

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of b_2 has an infinite set of singular points:

a) $b_2 = -k \;/;\; k \in \mathbb{N}$, are the simple poles with residues $\frac{(-1)^k}{k!}\, {}_3\tilde{F}_2(a_1, a_2, a_3; b_1, -k; z)$;

b) $b_2 = \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of b_1 has an infinite set of singular points:

a) $b_1 = -k \;/;\; k \in \mathbb{N}$, are the simple poles with residues $\frac{(-1)^k}{k!}\, {}_3\tilde{F}_2(a_1, a_2, a_3; -k, b_2; z)$;

b) $b_1 = \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.27.04.0009.01

$$\mathcal{S}ing_{b_j}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\{\{-k, 1\} /; k \in \mathbb{N}\}, \{\tilde{\infty}, \infty\}\} /; j \in \{1, 2\}$$

07.27.04.0010.01

$$\mathrm{res}_{b_2}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z))(-k) = \frac{(-1)^k}{k!}\, {}_3\tilde{F}_2(a_1, a_2, a_3; b_1, -k; z) /; k \in \mathbb{N}$$

07.27.04.0011.01

$$\mathrm{res}_{b_1}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z))(-k) = \frac{(-1)^k}{k!}\, {}_3\tilde{F}_2(a_1, a_2, a_3; -k, b_2; z) /; k \in \mathbb{N}$$

Branch points

With respect to z

For all a_k, not being negative integer, the function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ has two branch points: $z = 1$, $z = \tilde{\infty}$.

07.27.04.0012.01

$$\mathcal{BP}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{1, \tilde{\infty}\} /; \mathcal{NT}(\{a_1, a_2, a_3\})$$

07.27.04.0013.01

$$\mathcal{R}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z), 1) = \log /; \psi_2 \in \mathbb{Z} \bigvee \psi_2 \notin \mathbb{Q} \bigwedge \psi_2 = \sum_{j=1}^{2} b_j - \sum_{j=1}^{3} a_j \bigwedge \mathcal{NT}(\{a_1, a_2, a_3\})$$

07.27.04.0014.01

$$\mathcal{R}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z), 1) = s /; \psi_2 = \sum_{j=1}^{2} b_j - \sum_{j=1}^{3} a_j = \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) = 1 \bigwedge \mathcal{NT}(\{a_1, a_2, a_3\})$$

07.27.04.0015.01

$$\mathcal{R}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z), \tilde{\infty}) = \log /; \exists_{a_i, a_j} \left(a_i - a_j \in \mathbb{Z} \land 1 \le i \le 3 \land 1 \le j \le 3 \land i \ne j\right) \bigwedge (a_1 \notin \mathbb{Q} \lor a_2 \notin \mathbb{Q} \lor a_3 \notin \mathbb{Q})$$

07.27.04.0016.01

$$\mathcal{R}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z), \tilde{\infty}) = \mathrm{lcm}(s_1, s_2, s_3) /; \\ a_l = \frac{r_l}{s_l} \bigwedge \{r_l, s_l\} \in \mathbb{Z} \bigwedge s_l > 1 \bigwedge \gcd(r_l, s_l) = 1 \bigwedge 1 \le l \le 3 \bigwedge \mathcal{NT}(\{s_1, s_2, s_3\})$$

With respect to a_l

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of a_l, $1 \le l \le 3$, does not have branch points.

07.27.04.0017.01

$$\mathcal{BP}_{a_l}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\} /; 1 \le l \le 3$$

With respect to b_j

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of b_j, $1 \le j \le 2$, does not have branch points.

07.27.04.0018.01

$$\mathcal{BP}_{b_j}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\} /; 1 \le j \le 2$$

Branch cuts

With respect to z

For all a_k, not being negative integer, the function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ is a single-valued function on the z-plane cut along the interval $(1, \infty)$, where it is continuous from below.

07.27.04.0019.01

$$\mathcal{BC}_z({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\{(1, \infty), i\}\} /; \mathcal{NT}(\{a_1, a_2, a_3\})$$

07.27.04.0020.01

$$\lim_{\epsilon \to +0} {}_3F_2(a_1, a_2, a_3; b_1, b_2; x - i\,\epsilon) = {}_3F_2(a_1, a_2, a_3; b_1, b_2; x) /; x > 1$$

07.27.04.0021.01

$$\lim_{\epsilon \to +0} {}_3F_2(a_1, a_2, a_3; b_1, b_2; x + i\,\epsilon) = \frac{\Gamma(a_2 - a_1)\,\Gamma(a_3 - a_1)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1 - a_1)\,\Gamma(b_2 - a_1)} \left(-\frac{1}{x}\right)^{a_1} {}_3F_2\left(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1; a_1 - a_2 + 1, a_1 - a_3 + 1; \frac{1}{x}\right) + \frac{\Gamma(a_1 - a_2)\,\Gamma(a_3 - a_2)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_3)\,\Gamma(b_1 - a_2)\,\Gamma(b_2 - a_2)} \left(-\frac{1}{x}\right)^{a_2} {}_3F_2\left(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1; -a_1 + a_2 + 1, a_2 - a_3 + 1; \frac{1}{x}\right) + \frac{\Gamma(a_1 - a_3)\,\Gamma(a_2 - a_3)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(b_1 - a_3)\,\Gamma(b_2 - a_3)} \left(-\frac{1}{x}\right)^{a_3} {}_3F_2\left(a_3, a_3 - b_1 + 1, a_3 - b_2 + 1; -a_1 + a_3 + 1, -a_2 + a_3 + 1; \frac{1}{x}\right) /; \forall_{\{j,k\},\{j,k\} \in \mathbb{Z} \land j \neq k \land 1 \leq j \leq 3 \land 1 \leq k \leq 3} \left(a_j - a_k \notin \mathbb{Z}\right) \land x > 1$$

With respect to a_l

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of a_l, $1 \leq l \leq 3$, does not have branch cuts.

07.27.04.0022.01

$$\mathcal{BC}_{a_l}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\} /; 1 \leq l \leq 3$$

With respect to b_j

The function ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ as a function of b_j, $1 \leq j \leq 2$, does not have branch cuts.

07.27.04.0023.01

$$\mathcal{BC}_{b_j}({}_3F_2(a_1, a_2, a_3; b_1, b_2; z)) = \{\} /; 1 \leq j \leq 2$$

Limit representations

07.27.09.0001.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \lim_{p \to \infty} {}_4F_3(a_1, a_2, a_3, p\,z; b_1, b_2, p; 1) /; \mathrm{Re}(p\,(1 - z) - a_1 - a_2 - a_3 + b_1 + b_2) > 0$$

07.27.09.0002.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \lim_{p \to \infty} {}_3F_3(a_1, a_2, a_3; b_1, b_2, p; p\,z)$$

Continued fraction representations

07.27.10.0001.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = 1 + (a_1\, a_2\, a_3\, z/(b_1\, b_2)) \Bigg/ \left(1 + -\frac{z\,(1+a_1)\,(1+a_2)\,(1+a_3)}{2\,(1+b_1)\,(1+b_2)} \Bigg/ \left(1 + \frac{z\,(1+a_1)\,(1+a_2)\,(1+a_3)}{2\,(1+b_1)\,(1+b_2)} + \frac{-\frac{z\,(2+a_1)\,(2+a_2)\,(2+a_3)}{3\,(2+b_1)\,(2+b_2)}}{1 + \frac{z\,(2+a_1)\,(2+a_2)\,(2+a_3)}{3\,(2+b_1)\,(2+b_2)} + \ldots}\right)\right)$$

07.27.10.0002.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = 1 + \frac{a_1\, a_2\, a_3\, z}{b_1\, b_2 \left(1 + \mathrm{K}_k\left(-\frac{(k+a_1)\,(k+a_2)\,(k+a_3)\,z}{(k+1)\,(k+b_1)\,(k+b_2)}, \frac{(k+a_1)\,(k+a_2)\,(k+a_3)\,z}{(k+1)\,(k+b_1)\,(k+b_2)} + 1\right)_1^{\infty}\right)}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

Representation of fundamental system solutions near zero

07.27.13.0002.01

$$(1-z)\,z^2\, w^{(3)}(z) + (-(a_1+a_2+a_3+3)\,z + b_1 + b_2 + 1)\,z\,w''(z) + (b_1\, b_2 - (a_2\, a_1 + a_3\, a_1 + a_1 + a_2 + a_2\, a_3 + a_3 + 1)\,z)\,w'(z) - a_1\, a_2\, a_3\, w(z) = 0 \;/;\; w(z) = c_1\, {}_3\tilde{F}_2(a_1, a_2, a_3; b_1, b_2; z) + c_2\left(G_{3,3}^{2,3}\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right) + G_{3,3}^{2,3}\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_2, 1-b_1 \end{matrix}\right)\right) + c_3\, G_{3,3}^{3,3}\left(-z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right)$$

07.27.13.0003.01

$$W_z\left({}_3\tilde{F}_2(a_1, a_2, a_3; b_1, b_2; z), G_{3,3}^{2,3}\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right) + G_{3,3}^{2,3}\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_2, 1-b_1 \end{matrix}\right), G_{3,3}^{3,3}\left(-z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right)\right) = (1-z)^{-a_1-a_2-a_3+b_1+b_2-2}\, (-z)^{-b_1-b_2}\, z^{-b_1-b_2-1} \left(z^{b_2}\, (-z)^{b_1} + z^{b_1}\, (-z)^{b_2}\right) \Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(a_1-b_2+1)\,\Gamma(a_2-b_2+1)\,\Gamma(a_3-b_2+1)$$

07.27.13.0004.01

$$(1-z)\,w^{(3)}(z)\,z^2 + (-(a_1+a_2+a_3+3)\,z + b_1 + b_2 + 1)\,z\,w''(z) + (b_1\, b_2 - (a_2\, a_1 + a_3\, a_1 + a_1 + a_2 + a_2\, a_3 + a_3 + 1)\,z)\,w'(z) - a_1\, a_2\, a_3\, w(z) = 0 \;/;\; w(z) = c_1\, {}_3\tilde{F}_2(a_1, a_2, a_3; b_1, b_2; z) + c_2\, z^{1-b_1}\, {}_3\tilde{F}_2(a_1-b_1+1, a_2-b_1+1, a_3-b_1+1; 2-b_1, 1-b_1+b_2; z) + c_3\, z^{1-b_2}\, {}_3\tilde{F}_2(a_1-b_2+1, a_2-b_2+1, a_3-b_2+1; 2-b_2, b_1-b_2+1; z) \bigwedge b_1 \notin \mathbb{Z} \bigwedge b_2 \notin \mathbb{Z} \bigwedge b_1 - b_2 \notin \mathbb{Z}$$

07.27.13.0005.01

$$W_z\left({}_3\tilde{F}_2(a_1, a_2, a_3; b_1, b_2; z), z^{1-b_1}\, {}_3\tilde{F}_2(a_1-b_1+1, a_2-b_1+1, a_3-b_1+1; 2-b_1, -b_1+b_2+1; z), z^{1-b_2}\, {}_3\tilde{F}_2(a_1-b_2+1, a_2-b_2+1, a_3-b_2+1; 2-b_2, b_1-b_2+1; z)\right) = \frac{\sin(\pi\, b_1)\,\sin(\pi\,(b_1-b_2))\,\sin(\pi\, b_2)}{\pi^3}\,(1-z)^{-a_1-a_2-a_3+b_1+b_2-2}\, z^{-b_1-b_2-1}$$

07.27.13.0001.01

$$(1-z)\,z^2\,w^{(3)}(z)\;+(-(a_1+a_2+a_3+3)\,z+b_1+b_2+1)\,z\,w''(z)+ (b_1\,b_2-(a_2\,a_1+a_3\,a_1+a_1+a_2+a_2\,a_3+a_3+1)\,z)\,w'(z)-a_1\,a_2\,a_3\,w(z)=0\;/;\; w(z)=c_1\,{}_3F_2(a_1,a_2,a_3;b_1,b_2;z)+\;c_2\,z^{1-b_1}\,{}_3F_2(a_1-b_1+1,a_2-b_1+1,a_3-b_1+1;2-b_1,-b_1+b_2+1;z)+ c_3\,z^{1-b_2}\,{}_3F_2(a_1-b_2+1,a_2-b_2+1,a_3-b_2+1;2-b_2,b_1-b_2+1;z)\;\bigwedge b_1\notin\mathbb{Z}\bigwedge b_2\notin\mathbb{Z}\bigwedge b_1-b_2\notin\mathbb{Z}$$

07.27.13.0006.01

$$W_z\big({}_3F_2(a_1,a_2,a_3;b_1,b_2;z),\,z^{1-b_1}\,{}_3F_2(a_1-b_1+1,a_2-b_1+1,a_3-b_1+1;2-b_1,-b_1+b_2+1;z), z^{1-b_2}\,{}_3F_2(a_1-b_2+1,a_2-b_2+1,a_3-b_2+1;2-b_2,b_1-b_2+1;z)\big)= (b_1-1)\,(b_2-1)\,(b_1-b_2)\,(1-z)^{-a_1-a_2-a_3+b_1+b_2-2}\,z^{-b_1-b_2-1}$$

07.27.13.0007.01

$$w^{(3)}(z)+\left(\frac{(a_1+a_2+a_3+3)\,g'(z)}{g(z)-1}-\frac{(b_1+b_2+1)\,g'(z)}{(g(z)-1)\,g(z)}-\frac{3\,g''(z)}{g'(z)}\right)w''(z)+ \left(\frac{b_1\,b_2\,g'(z)^2}{(1-g(z))\,g(z)^2}+\frac{3\,g''(z)^2}{g'(z)^2}+\frac{((a_2+1)\,(a_3+1)+a_1\,(a_2+a_3+1))\,g'(z)^2+(b_1+b_2+1)\,g''(z)}{(g(z)-1)\,g(z)}- \frac{(a_1+a_2+a_3+3)\,g''(z)}{g(z)-1}-\frac{g^{(3)}(z)}{g'(z)}\right)w'(z)-\frac{a_1\,a_2\,a_3\,g'(z)^3}{(1-g(z))\,g(z)^2}\,w(z)=0\;/;$$

$$w(z)=c_1\,{}_3\tilde{F}_2(a_1,a_2,a_3;b_1,b_2;g(z))\;+c_2\left(G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)+G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_2,1-b_1\end{matrix}\right)\right)+ c_3\,G_{3,3}^{3,3}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)$$

07.27.13.0008.01

$$W_z\Bigg({}_3\tilde{F}_2(a_1,a_2,a_3;b_1,b_2;g(z)), G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)+G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_2,1-b_1\end{matrix}\right),\;G_{3,3}^{3,3}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)\Bigg)= (1-g(z))^{-a_1-a_2-a_3+b_1+b_2-2}\,(-g(z))^{-b_1-b_2}\,g(z)^{-b_1-b_2-1}\left(g(z)^{b_2}\,(-g(z))^{b_1}+g(z)^{b_1}\,(-g(z))^{b_2}\right) \Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(a_1-b_2+1)\,\Gamma(a_2-b_2+1)\,\Gamma(a_3-b_2+1)\,g'(z)^3$$

07.27.13.0009.01

$$w^{(3)}(z)+\left(\frac{(a_1+a_2+a_3+3)\,g'(z)}{g(z)-1}-\frac{(b_1+b_2+1)\,g'(z)}{(g(z)-1)\,g(z)}-\frac{3\,h'(z)}{h(z)}-\frac{3\,g''(z)}{g'(z)}\right)w''(z)+$$
$$\left(\frac{b_1\,b_2\,g'(z)^2}{(1-g(z))\,g(z)^2}+\frac{2\,(b_1+b_2+1)\,h'(z)\,g'(z)}{(g(z)-1)\,g(z)\,h(z)}-\frac{2\,(a_1+a_2+a_3+3)\,h'(z)\,g'(z)}{(g(z)-1)\,h(z)}+\frac{6\,h'(z)^2}{h(z)^2}+\frac{3\,g''(z)^2}{g'(z)^2}+\frac{6\,h'(z)\,g''(z)}{h(z)\,g'(z)}+\right.$$
$$\left.\frac{((a_2+1)\,(a_3+1)+a_1\,(a_2+a_3+1))\,g'(z)^2+(b_1+b_2+1)\,g''(z)}{(g(z)-1)\,g(z)}-\frac{(a_1+a_2+a_3+3)\,g''(z)}{g(z)-1}-\frac{3\,h''(z)}{h(z)}-\frac{g^{(3)}(z)}{g'(z)}\right)$$
$$w'(z)+w(z)\left(-\frac{a_1\,a_2\,a_3\,g'(z)^3}{(1-g(z))\,g(z)^2}-\frac{b_1\,b_2\,h'(z)\,g'(z)^2}{(1-g(z))\,g(z)^2\,h(z)}+\frac{2\,(a_1+a_2+a_3+3)\,h'(z)^2\,g'(z)}{(g(z)-1)\,h(z)^2}-\right.$$
$$\frac{2\,(b_1+b_2+1)\,h'(z)^2\,g'(z)}{(g(z)-1)\,g(z)\,h(z)^2}+\frac{6\,h'(z)\,h''(z)}{h(z)^2}+\frac{1}{(g(z)-1)\,g(z)\,h(z)}$$
$$\left((b_1+b_2+1)\,g'(z)\,h''(z)-h'(z)\left(((a_2+1)\,(a_3+1)+a_1\,(a_2+a_3+1))\,g'(z)^2+(b_1+b_2+1)\,g''(z)\right)\right)+$$
$$\frac{(a_1+a_2+a_3+3)\,(h'(z)\,g''(z)-g'(z)\,h''(z))}{(g(z)-1)\,h(z)}+\frac{3\,g''(z)\,h''(z)+h'(z)\,g^{(3)}(z)}{h(z)\,g'(z)}-$$
$$\left.\frac{h^{(3)}(z)}{h(z)}-\frac{6\,h'(z)^3}{h(z)^3}-\frac{6\,h'(z)^2\,g''(z)}{h(z)^2\,g'(z)}-\frac{3\,h'(z)\,g''(z)^2}{h(z)\,g'(z)^2}\right)=0\;/;$$
$$w(z)=c_1\,h(z)\,{}_3\tilde{F}_2(a_1,a_2,a_3;b_1,b_2;g(z))\;+c_2\,h(z)\left(G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)+G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_2,1-b_1\end{matrix}\right)\right)+$$
$$c_3\,h(z)\,G_{3,3}^{3,3}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)$$

07.27.13.0010.01

$$W_z\Bigg(h(z)\,{}_3\tilde{F}_2(a_1,a_2,a_3;b_1,b_2;g(z)),$$
$$h(z)\left(G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)+G_{3,3}^{2,3}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_2,1-b_1\end{matrix}\right)\right),\,h(z)\,G_{3,3}^{3,3}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)\Bigg)=$$
$$(1-g(z))^{-a_1-a_2-a_3+b_1+b_2-2}\,(-g(z))^{-b_1-b_2}\,g(z)^{-b_1-b_2-1}\left(g(z)^{b_2}\,(-g(z))^{b_1}+g(z)^{b_1}\,(-g(z))^{b_2}\right)\Gamma(a_1-b_1+1)$$
$$\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(a_1-b_2+1)\,\Gamma(a_2-b_2+1)\,\Gamma(a_3-b_2+1)\,h(z)^3\,g'(z)^3$$

07.27.13.0011.01

$$w^{(3)}(z) + \left(\frac{a\, r\, (a_1 + a_2 + a_3 + 3)\, z^{r-1}}{a\, z^r - 1} - \frac{r\, (b_1 + b_2 + 1)}{z\, (a\, z^r - 1)} - \frac{3\, (r + s - 1)}{z}\right) w''(z) + \left(\frac{a\, r\, (-2\, r - 6\, s - 2\, s\, a_3 + a_3 + a_2\, (-2\, s + r\, a_3 + 1) + a_1\, (-2\, s + r\, a_2 + r\, a_3 + 1) + 3)\, z^{r-2}}{a\, z^r - 1} + \frac{2\, r^2 + 6\, s\, r - 3\, r + 3\, s^2 - 3\, s + 1}{z^2} + \frac{r\, (r + 2\, s - 1)}{z^2\, (a\, z^r - 1)} - \frac{r\, (b_1\, (b_2\, r - r - 2\, s + 1) - (r + 2\, s - 1)\, b_2)}{z^2\, (a\, z^r - 1)}\right) w'(z) + \left(\frac{a\, r\, (a_1\, (s - r\, a_2)\, (s - r\, a_3) + s\, (2\, r + 3\, s + s\, a_3 + a_2\, (s - r\, a_3)))\, z^{r-3}}{a\, z^r - 1} + \frac{r\, s\, (b_1\, (b_2\, r - r - s) - (r + s)\, b_2)}{z^3\, (a\, z^r - 1)} - \frac{s\left(2\, r^2 + 3\, s\, r + s^2\right)}{z^3} - \frac{r\, s\, (r + s)}{z^3\, (a\, z^r - 1)}\right) w(z) = 0\, /;$$

$$w(z) = c_1\, z^s\, {}_3\tilde{F}_2(a_1, a_2, a_3;\, b_1, b_2;\, a\, z^r) + c_2\, z^s \left(G_{3,3}^{2,3}\!\left(a\, z^r \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_1, 1 - b_2 \end{matrix}\right) + G_{3,3}^{2,3}\!\left(a\, z^r \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_2, 1 - b_1 \end{matrix}\right)\right) + c_3\, z^s\, G_{3,3}^{3,3}\!\left(-a\, z^r \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_1, 1 - b_2 \end{matrix}\right)$$

07.27.13.0012.01

$$W_z\!\left(z^s\, {}_3\tilde{F}_2(a_1, a_2, a_3;\, b_1, b_2;\, a\, z^r),\, z^s \left(G_{3,3}^{2,3}\!\left(a\, z^r \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_1, 1 - b_2 \end{matrix}\right) + G_{3,3}^{2,3}\!\left(a\, z^r \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_2, 1 - b_1 \end{matrix}\right)\right),\, z^s\, G_{3,3}^{3,3}\!\left(-a\, z^r \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_1, 1 - b_2 \end{matrix}\right)\right) = a^3\, r^3\, z^{3\, r + 3\, s - 3}\, (-a\, z^r)^{-b_1 - b_2}\, (a\, z^r)^{-b_1 - b_2 - 1}\, (1 - a\, z^r)^{-a_1 - a_2 - a_3 + b_1 + b_2 - 2} \left((a\, z^r)^{b_2}\, (-a\, z^r)^{b_1} + (a\, z^r)^{b_1}\, (-a\, z^r)^{b_2}\right) \Gamma(a_1 - b_1 + 1)\, \Gamma(a_2 - b_1 + 1)\, \Gamma(a_3 - b_1 + 1)\, \Gamma(a_1 - b_2 + 1)\, \Gamma(a_2 - b_2 + 1)\, \Gamma(a_3 - b_2 + 1)$$

07.27.13.0013.01

$$w^{(3)}(z) + \left(\frac{a\, r^z \log(r)\, (a_1 + a_2 + a_3 + 3) - \log(r)\, (b_1 + b_2 + 1)}{a\, r^z - 1} - 3\, (\log(r) + \log(s))\right) w''(z) + \left(2 \log^2(r) + 6 \log(s) \log(r) + 3 \log^2(s) + \frac{1}{a\, r^z - 1} (\log(r)\, (-a\, (\log(r) + 2 \log(s))\, (a_1 + a_2 + a_3 + 3)\, r^z + 2 \log(s) + 2 \log(s)\, b_1 + 2 \log(s)\, b_2 - \log(r)\, b_1\, b_2 + \log(r)\, (a\, ((a_2 + 1)\, (a_3 + 1) + a_1\, (a_2 + a_3 + 1))\, r^z + b_1 + b_2 + 1)))\right) w'(z) + \left(\frac{r^{-z} \left(3\, a\, r^z \log^2(r) \log^2(s)\, s^z + a\, r^z \log^3(r) \log(s)\, s^z\right) s^{-z}}{a \log(r)} - \log(s) \left(3 \log^2(r) + 6 \log(s) \log(r) + \log^2(s)\right) + \frac{1}{a\, r^z - 1} (\log(r)\, (a\, a_1\, (\log(s) - \log(r)\, a_2)\, (\log(s) - \log(r)\, a_3)\, r^z + \log(s)\, (a\, (2 \log(r) + 3 \log(s) + \log(s)\, a_3 + a_2\, (\log(s) - \log(r)\, a_3))\, r^z - (\log(r) + \log(s))\, (b_2 + 1) - b_1\, (-b_2 \log(r) + \log(r) + \log(s)))))\right) w(z) = 0\, /;$$

$$w(z) = c_1\, s^z\, {}_3\tilde{F}_2(a_1, a_2, a_3;\, b_1, b_2;\, a\, r^z) + c_2\, s^z \left(G_{3,3}^{2,3}\!\left(a\, r^z \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_1, 1 - b_2 \end{matrix}\right) + G_{3,3}^{2,3}\!\left(a\, r^z \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_2, 1 - b_1 \end{matrix}\right)\right) + c_3\, s^z\, G_{3,3}^{3,3}\!\left(-a\, r^z \,\middle|\, \begin{matrix} 1 - a_1, 1 - a_2, 1 - a_3 \\ 0, 1 - b_1, 1 - b_2 \end{matrix}\right)$$

07.27.13.0014.01

$$W_z\left(s^z\,{}_3\tilde{F}_2(a_1, a_2, a_3; b_1, b_2; a\,r^z),\; s^z\left(G_{3,3}^{2,3}\!\left(a\,r^z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right) + G_{3,3}^{2,3}\!\left(a\,r^z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_2, 1-b_1 \end{matrix}\right)\right),\; s^z\,G_{3,3}^{3,3}\!\left(-a\,r^z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right)\right) = a^3\,r^{3z}\,(-a\,r^z)^{-b_1-b_2}\,(a\,r^z)^{-b_1-b_2-1}\,(1-a\,r^z)^{-a_1-a_2-a_3+b_1+b_2-2}\left((a\,r^z)^{b_2}\,(-a\,r^z)^{b_1} + (a\,r^z)^{b_1}\,(-a\,r^z)^{b_2}\right)s^{3z}\,\Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(a_1-b_2+1)\,\Gamma(a_2-b_2+1)\,\Gamma(a_3-b_2+1)\log^3(r)$$

Representation of fundamental system solutions near unit

07.27.13.0015.01

$$(1-z)\,z^2\,w^{(3)}(z) + (-(a_1+a_2+a_3+3)\,z+b_1+b_2+1)\,z\,w''(z) + (b_1\,b_2-(a_2\,a_1+a_3\,a_1+a_1+a_2+a_2\,a_3+a_3+1)\,z)\,w'(z) - a_1\,a_2\,a_3\,w(z) = 0 \,/;\, \left(w(z) = c_1\,G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right) + c_2\,G_{5,5}^{2,5}\!\left(z \,\middle|\, \begin{matrix} 0, b_1, 1-a_1, 1-a_2, 1-a_3 \\ 0, b_1, 0, 1-b_1, 1-b_2 \end{matrix}\right) + c_3\,G_{5,5}^{2,5}\!\left(z \,\middle|\, \begin{matrix} 0, b_2, 1-a_1, 1-a_2, 1-a_3 \\ 0, b_2, 0, 1-b_1, 1-b_2 \end{matrix}\right) \bigwedge |z| < 1 \bigwedge b_1+b_2-a_1-a_2-a_3 \notin \mathbb{Z} \bigwedge b_1 \notin \mathbb{Z} \bigwedge b_2 \notin \mathbb{Z} \bigwedge b_1-b_2 \notin \mathbb{Z}\right)$$

07.27.13.0016.01

$$W_z\left(G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right), G_{5,5}^{2,5}\!\left(z \,\middle|\, \begin{matrix} 0, b_1, 1-a_1, 1-a_2, 1-a_3 \\ 0, b_1, 0, 1-b_1, 1-b_2 \end{matrix}\right), G_{5,5}^{2,5}\!\left(z \,\middle|\, \begin{matrix} 0, b_2, 1-a_1, 1-a_2, 1-a_3 \\ 0, b_2, 0, 1-b_1, 1-b_2 \end{matrix}\right)\right) = G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} -a_1, -a_2, -a_3 \\ 0, -b_1, -b_2 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1-1, -a_2-1, -a_3-1, b_2-2 \\ 0, b_2-2, -b_1-1, -b_2-1 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, b_1 \\ 0, b_1, 1-b_1, 1-b_2 \end{matrix}\right) - G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} -a_1-1, -a_2-1, -a_3-1 \\ 0, -b_1-1, -b_2-1 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1, -a_2, -a_3, b_2-1 \\ 0, b_2-1, -b_1, -b_2 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, b_1 \\ 0, b_1, 1-b_1, 1-b_2 \end{matrix}\right) - G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} -a_1, -a_2, -a_3 \\ 0, -b_1, -b_2 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1-1, -a_2-1, -a_3-1, b_1-2 \\ 0, b_1-2, -b_1-1, -b_2-1 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, b_2 \\ 0, b_2, 1-b_1, 1-b_2 \end{matrix}\right) - G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1-1, -a_2-1, -a_3-1, b_2-2 \\ 0, b_2-2, -b_1-1, -b_2-1 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1, -a_2, -a_3, b_1-1 \\ 0, b_1-1, -b_1, -b_2 \end{matrix}\right) + G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} -a_1-1, -a_2-1, -a_3-1 \\ 0, -b_1-1, -b_2-1 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, b_2 \\ 0, b_2, 1-b_1, 1-b_2 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1, -a_2, -a_3, b_1-1 \\ 0, b_1-1, -b_1, -b_2 \end{matrix}\right) + G_{3,3}^{3,0}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1-1, -a_2-1, -a_3-1, b_1-2 \\ 0, b_1-2, -b_1-1, -b_2-1 \end{matrix}\right) G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} -a_1, -a_2, -a_3, b_2-1 \\ 0, b_2-1, -b_1, -b_2 \end{matrix}\right)$$

Representation of fundamental system solutions near infinity

07.27.13.0017.01

$$(1-z)\,z^2\,w^{(3)}(z) + (-(a_1+a_2+a_3+3)\,z+b_1+b_2+1)\,z\,w''(z) + (b_1\,b_2-(a_2\,a_1+a_3\,a_1+a_1+a_2+a_2\,a_3+a_3+1)\,z)\,w'(z) - a_1\,a_2\,a_3\,w(z) = 0 \,/;\, w(z) = c_1\,z^{-a_1}\,{}_3\tilde{F}_2\!\left(a_1, a_1-b_1+1, a_1-b_2+1; a_1-a_2+1, a_1-a_3+1; \frac{1}{z}\right) + c_2\left(G_{3,3}^{3,2}\!\left(\frac{1}{z} \,\middle|\, \begin{matrix} 1, b_1, b_2 \\ a_1, a_2, a_3 \end{matrix}\right) + G_{3,3}^{3,2}\!\left(\frac{1}{z} \,\middle|\, \begin{matrix} 1, b_2, b_1 \\ a_1, a_2, a_3 \end{matrix}\right)\right) + c_3\,G_{3,3}^{3,3}\!\left(-\frac{1}{z} \,\middle|\, \begin{matrix} 1, b_1, b_2 \\ a_1, a_2, a_3 \end{matrix}\right)$$

07.27.13.0018.01

$$W_z\left(z^{-a_1}\,{}_3\tilde{F}_2\left(a_1,a_1-b_1+1,a_1-b_2+1;a_1-a_2+1,a_1-a_3+1;\frac{1}{z}\right),\right.$$
$$G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}1,b_1,b_2\\a_1,a_2,a_3\end{matrix}\right)+G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}1,b_2,b_1\\a_1,a_2,a_3\end{matrix}\right),\left.G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}1,b_1,b_2\\a_1,a_2,a_3\end{matrix}\right)\right)=$$
$$z^{-a_1-6}\left(\left(G_{4,4}^{3,4}\left(-\frac{1}{z}\middle|\begin{matrix}-2,-1,b_1-2,b_2-2\\a_1-2,a_2-2,a_3-2,0\end{matrix}\right)-2\,z\,G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)\right)\left(-{}_3\tilde{F}_2\left(a_1,a_1-b_1+1,a_1-b_2+1;\right.\right.\right.$$
$$\left.a_1-a_2+1,a_1-a_3+1;\frac{1}{z}\right)\left(G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)+G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}-1,b_2-1,b_1-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)\right)-$$
$$\left(G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}1,b_1,b_2\\a_1,a_2,a_3\end{matrix}\right)+G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}1,b_2,b_1\\a_1,a_2,a_3\end{matrix}\right)\right)a_1\left(-z\,{}_3\tilde{F}_2\left(a_1,a_1-b_1+1,a_1-b_2+1;a_1-a_2+1,a_1-a_3+1;\right.\right.$$
$$\left.\left.\left.\frac{1}{z}\right)-{}_3\tilde{F}_2\left(a_1+1,a_1-b_1+2,a_1-b_2+2;a_1-a_2+2,a_1-a_3+2;\frac{1}{z}\right)(a_1-b_1+1)(a_1-b_2+1)\right)\right)-$$
$$\left(2\,z\,G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)+2\,z\,G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}-1,b_2-1,b_1-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)+G_{4,4}^{3,3}\left(\frac{1}{z}\middle|\begin{matrix}-2,-1,b_1-2,b_2-2\\a_1-2,a_2-2,a_3-2,0\end{matrix}\right)+\right.$$
$$\left.G_{4,4}^{3,3}\left(\frac{1}{z}\middle|\begin{matrix}-2,-1,b_2-2,b_1-2\\a_1-2,a_2-2,a_3-2,0\end{matrix}\right)\right)$$
$$\left({}_3\tilde{F}_2\left(a_1,a_1-b_1+1,a_1-b_2+1;a_1-a_2+1,a_1-a_3+1;\frac{1}{z}\right)G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)-\right.$$
$$G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}1,b_1,b_2\\a_1,a_2,a_3\end{matrix}\right)a_1\left(-z\,{}_3\tilde{F}_2\left(a_1,a_1-b_1+1,a_1-b_2+1;a_1-a_2+1,a_1-a_3+1;\frac{1}{z}\right)-\right.$$
$$\left.\left.{}_3\tilde{F}_2\left(a_1+1,a_1-b_1+2,a_1-b_2+2;a_1-a_2+2,a_1-a_3+2;\frac{1}{z}\right)(a_1-b_1+1)(a_1-b_2+1)\right)\right)+$$
$$\left(G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}1,b_1,b_2\\a_1,a_2,a_3\end{matrix}\right)G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)+G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}1,b_2,b_1\\a_1,a_2,a_3\end{matrix}\right)G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)+\right.$$
$$\left.\left(G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}-1,b_1-1,b_2-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)+G_{3,3}^{3,2}\left(\frac{1}{z}\middle|\begin{matrix}-1,b_2-1,b_1-1\\a_1-1,a_2-1,a_3-1\end{matrix}\right)\right)G_{3,3}^{3,3}\left(-\frac{1}{z}\middle|\begin{matrix}1,b_1,b_2\\a_1,a_2,a_3\end{matrix}\right)\right)$$
$$a_1\left({}_3\tilde{F}_2\left(a_1,a_1-b_1+1,a_1-b_2+1;a_1-a_2+1,a_1-a_3+1;\frac{1}{z}\right)(a_1+1)\,z^2+\right.$$
$$2\,{}_3\tilde{F}_2\left(a_1+1,a_1-b_1+2,a_1-b_2+2;a_1-a_2+2,a_1-a_3+2;\frac{1}{z}\right)(a_1-b_1+1)(a_1-b_2+1)\,z+$$
$$2\,{}_3\tilde{F}_2\left(a_1+1,a_1-b_1+2,a_1-b_2+2;a_1-a_2+2,a_1-a_3+2;\frac{1}{z}\right)a_1\,(a_1-b_1+1)(a_1-b_2+1)\,z+$$
$${}_3\tilde{F}_2\left(a_1+2,a_1-b_1+3,a_1-b_2+3;a_1-a_2+3,a_1-a_3+3;\frac{1}{z}\right)$$
$$\left.\left.(a_1+1)(a_1-b_1+1)(a_1-b_1+2)(a_1-b_2+1)(a_1-b_2+2)\right)\right)$$

07.27.13.0019.01

$$(1-z)\,z^2\,w^{(3)}(z)\;+(-(a_1+a_2+a_3+3)\,z+b_1+b_2+1)\,z\,w''(z)+ (b_1\,b_2-(a_2\,a_1+a_3\,a_1+a_1+a_2+a_2\,a_3+a_3+1)\,z)\,w'(z)-a_1\,a_2\,a_3\,w(z)=0\;/;$$
$$w(z)=c_1\,z^{-a_1}\,{}_3\tilde{F}_2\!\left(a_1+1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)+ c_2\,z^{-a_2}\,{}_3\tilde{F}_2\!\left(a_2+1,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right)+c_3\,z^{-a_3} {}_3\tilde{F}_2\!\left(a_3+1,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)\bigwedge a_1-a_2\notin\mathbb{Z}\bigwedge a_1-a_3\notin\mathbb{Z}\bigwedge a_2-a_3\notin\mathbb{Z}$$

07.27.13.0020.01

$$W_z\!\left(z^{-a_1}\,{}_3\tilde{F}_2\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right), z^{-a_2}\,{}_3\tilde{F}_2\!\left(a_2,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right), z^{-a_3}\,{}_3\tilde{F}_2\!\left(a_3,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)\right)=$$
$$z^{-a_1-a_2-a_3-6}\left(a_1\left({}_3\tilde{F}_2\!\left(a_2,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right)\right.\right.$$
$$a_3\left(-z\,{}_3\tilde{F}_2\!\left(a_3,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)-\right.$$
$$\left.{}_3\tilde{F}_2\!\left(a_3+1,\,a_3-b_1+2,\,a_3-b_2+2;\,-a_1+a_3+2,\,-a_2+a_3+2;\,\frac{1}{z}\right)(a_3-b_1+1)\,(a_3-b_2+1)\right)-$$
$${}_3\tilde{F}_2\!\left(a_3,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)a_2$$
$$\left(-z\,{}_3\tilde{F}_2\!\left(a_2,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right)-\right.$$
$$\left.\left.{}_3\tilde{F}_2\!\left(a_2+1,\,a_2-b_1+2,\,a_2-b_2+2;\,-a_1+a_2+2,\,a_2-a_3+2;\,\frac{1}{z}\right)(a_2-b_1+1)\,(a_2-b_2+1)\right)\right)$$
$$\left({}_3\tilde{F}_2\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)(a_1+1)\,z^2+\right.$$
$$2\,{}_3\tilde{F}_2\!\left(a_1+1,\,a_1-b_1+2,\,a_1-b_2+2;\,a_1-a_2+2,\,a_1-a_3+2;\,\frac{1}{z}\right)(a_1-b_1+1)\,(a_1-b_2+1)\,z+$$
$$2\,{}_3\tilde{F}_2\!\left(a_1+1,\,a_1-b_1+2,\,a_1-b_2+2;\,a_1-a_2+2,\,a_1-a_3+2;\,\frac{1}{z}\right)a_1\,(a_1-b_1+1)\,(a_1-b_2+1)\,z+$$
$${}_3\tilde{F}_2\!\left(a_1+2,\,a_1-b_1+3,\,a_1-b_2+3;\,a_1-a_2+3,\,a_1-a_3+3;\,\frac{1}{z}\right)(a_1+1)\,(a_1-b_1+1)\,(a_1-b_1+2)$$
$$\left.(a_1-b_2+1)\,(a_1-b_2+2)\right)-a_2\left({}_3\tilde{F}_2\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)\right.$$
$$a_3\left(-z\,{}_3\tilde{F}_2\!\left(a_3,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)-\right.$$
$$\left.{}_3\tilde{F}_2\!\left(a_3+1,\,a_3-b_1+2,\,a_3-b_2+2;\,-a_1+a_3+2,\,-a_2+a_3+2;\,\frac{1}{z}\right)(a_3-b_1+1)\,(a_3-b_2+1)\right)-$$
$${}_3\tilde{F}_2\!\left(a_3,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)a_1$$

$$\left(-z\, {}_3\tilde{F}_2\left(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1; a_1 - a_2 + 1, a_1 - a_3 + 1; \frac{1}{z}\right) - {}_3\tilde{F}_2\left(a_1 + 1, a_1 - b_1 + 2, a_1 - b_2 + 2; a_1 - a_2 + 2, a_1 - a_3 + 2; \frac{1}{z}\right)(a_1 - b_1 + 1)(a_1 - b_2 + 1)\right)\right)$$

$$\left({}_3\tilde{F}_2\left(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1; -a_1 + a_2 + 1, a_2 - a_3 + 1; \frac{1}{z}\right)(a_2 + 1) z^2 + 2\, {}_3\tilde{F}_2\left(a_2 + 1, a_2 - b_1 + 2, a_2 - b_2 + 2; -a_1 + a_2 + 2, a_2 - a_3 + 2; \frac{1}{z}\right)(a_2 - b_1 + 1)(a_2 - b_2 + 1) z + 2\, {}_3\tilde{F}_2\left(a_2 + 1, a_2 - b_1 + 2, a_2 - b_2 + 2; -a_1 + a_2 + 2, a_2 - a_3 + 2; \frac{1}{z}\right) a_2 (a_2 - b_1 + 1)(a_2 - b_2 + 1) z + {}_3\tilde{F}_2\left(a_2 + 2, a_2 - b_1 + 3, a_2 - b_2 + 3; -a_1 + a_2 + 3, a_2 - a_3 + 3; \frac{1}{z}\right)(a_2 + 1)(a_2 - b_1 + 1)(a_2 - b_1 + 2)(a_2 - b_2 + 1)(a_2 - b_2 + 2)\right) +$$

$$a_3 \left({}_3\tilde{F}_2\left(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1; a_1 - a_2 + 1, a_1 - a_3 + 1; \frac{1}{z}\right) a_2 \left(-z\, {}_3\tilde{F}_2\left(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1; -a_1 + a_2 + 1, a_2 - a_3 + 1; \frac{1}{z}\right) - {}_3\tilde{F}_2\left(a_2 + 1, a_2 - b_1 + 2, a_2 - b_2 + 2; -a_1 + a_2 + 2, a_2 - a_3 + 2; \frac{1}{z}\right)(a_2 - b_1 + 1)(a_2 - b_2 + 1)\right) - {}_3\tilde{F}_2\left(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1; -a_1 + a_2 + 1, a_2 - a_3 + 1; \frac{1}{z}\right) a_1 \left(-z\, {}_3\tilde{F}_2\left(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1; a_1 - a_2 + 1, a_1 - a_3 + 1; \frac{1}{z}\right) - {}_3\tilde{F}_2\left(a_1 + 1, a_1 - b_1 + 2, a_1 - b_2 + 2; a_1 - a_2 + 2, a_1 - a_3 + 2; \frac{1}{z}\right)(a_1 - b_1 + 1)(a_1 - b_2 + 1)\right)\right)$$

$$\left({}_3\tilde{F}_2\left(a_3, a_3 - b_1 + 1, a_3 - b_2 + 1; -a_1 + a_3 + 1, -a_2 + a_3 + 1; \frac{1}{z}\right)(a_3 + 1) z^2 + 2\, {}_3\tilde{F}_2\left(a_3 + 1, a_3 - b_1 + 2, a_3 - b_2 + 2; -a_1 + a_3 + 2, -a_2 + a_3 + 2; \frac{1}{z}\right)(a_3 - b_1 + 1)(a_3 - b_2 + 1) z + 2\, {}_3\tilde{F}_2\left(a_3 + 1, a_3 - b_1 + 2, a_3 - b_2 + 2; -a_1 + a_3 + 2, -a_2 + a_3 + 2; \frac{1}{z}\right) a_3 (a_3 - b_1 + 1)(a_3 - b_2 + 1) z + {}_3\tilde{F}_2\left(a_3 + 2, a_3 - b_1 + 3, a_3 - b_2 + 3; -a_1 + a_3 + 3, -a_2 + a_3 + 3; \frac{1}{z}\right)(a_3 + 1)(a_3 - b_1 + 1)(a_3 - b_1 + 2)(a_3 - b_2 + 1)(a_3 - b_2 + 2)\right)\right)$$

07.27.13.0021.01

$$(1 - z) z^2 w^{(3)}(z) + (-(a_1 + a_2 + a_3 + 3) z + b_1 + b_2 + 1) z\, w''(z) + (b_1 b_2 - (a_2 a_1 + a_3 a_1 + a_1 + a_2 + a_2 a_3 + a_3 + 1) z) w'(z) - a_1 a_2 a_3 w(z) = 0 /; w(z) = c_1 z^{-a_1}\, {}_3F_2\left(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1; a_1 - a_2 + 1, a_1 - a_3 + 1; \frac{1}{z}\right) + c_2 z^{-a_2}\, {}_3F_2\left(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1; -a_1 + a_2 + 1, a_2 - a_3 + 1; \frac{1}{z}\right) + c_3 z^{-a_3}\, {}_3F_2\left(a_3, a_3 - b_1 + 1, a_3 - b_2 + 1; -a_1 + a_3 + 1, -a_2 + a_3 + 1; \frac{1}{z}\right) \bigwedge a_1 - a_2 \notin \mathbb{Z} \bigwedge a_1 - a_3 \notin \mathbb{Z} \bigwedge a_2 - a_3 \notin \mathbb{Z}$$

07.27.13.0022.01

$$W_z\left(z^{-a_1}\,{}_3F_2\left(a_1+1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right),\right.$$
$$z^{-a_2}\,{}_3F_2\left(a_2+1,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right),$$
$$\left.z^{-a_3}\,{}_3F_2\left(a_3+1,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)\right)=$$
$$z^{-a_1-a_2-a_3-6}\left(\left({}_3F_2\left(a_1+1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)\right.\right.$$
$$\left(\left({}_3F_2\left(a_2+2,\,a_2-b_1+2,\,a_2-b_2+2;\,-a_1+a_2+2,\,a_2-a_3+2;\,\frac{1}{z}\right)(a_2+1)\,(a_2-b_1+1)\,(a_2-b_2+1)\right)\Big/\right.$$
$$((a_1-a_2-1)\,(a_2-a_3+1))-z\,{}_3F_2\left(a_2+1,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right)a_2\Big)-$$
$${}_3F_2\left(a_2+1,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right)$$
$$\left(-z\,{}_3F_2\left(a_1+1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)a_1-\left({}_3F_2\left(a_1+2,\,a_1-b_1+2,\,a_1-b_2+\right.\right.\right.$$
$$\left.\left.\left.2;\,a_1-a_2+2,\,a_1-a_3+2;\,\frac{1}{z}\right)(a_1+1)\,(a_1-b_1+1)\,(a_1-b_2+1)\right)\Big/((a_1-a_2+1)\,(a_1-a_3+1))\right)\Big)$$
$$(a_3+1)\left({}_3F_2\left(a_3+1,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)a_3\,z^2+\right.$$
$$\left(2\,z\,{}_3F_2\left(a_3+2,\,a_3-b_1+2,\,a_3-b_2+2;\,-a_1+a_3+2,\,-a_2+a_3+2;\,\frac{1}{z}\right)(a_3-b_1+1)\,(a_3-b_2+1)\right)\Big/$$
$$((-a_1+a_3+1)\,(-a_2+a_3+1))+\left(2\,z\,{}_3F_2\left(a_3+2,\,a_3-b_1+2,\,a_3-b_2+2;\,-a_1+a_3+2,\,-a_2+a_3+2;\,\frac{1}{z}\right)\right.$$
$$\left.a_3\,(a_3-b_1+1)\,(a_3-b_2+1)\right)\Big/((-a_1+a_3+1)\,(-a_2+a_3+1))+$$
$$\left({}_3F_2\left(a_3+3,\,a_3-b_1+3,\,a_3-b_2+3;\,-a_1+a_3+3,\,-a_2+a_3+3;\,\frac{1}{z}\right)(a_3+2)\,(a_3-b_1+1)\,(a_3-b_1+2)\right.$$
$$\left.\left.(a_3-b_2+1)\,(a_3-b_2+2)\right)\Big/((-a_1+a_3+1)\,(-a_1+a_3+2)\,(-a_2+a_3+1)\,(-a_2+a_3+2))\right)-$$
$$(a_2+1)\left({}_3F_2\left(a_1+1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)\right.$$
$$\left(-z\,{}_3F_2\left(a_3+1,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)a_3-\right.$$
$$\left({}_3F_2\left(a_3+2,\,a_3-b_1+2,\,a_3-b_2+2;\,-a_1+a_3+2,\,-a_2+a_3+2;\,\frac{1}{z}\right)(a_3+1)\,(a_3-b_1+1)\,(a_3-b_2+1)\right)\Big/$$
$$\left.((-a_1+a_3+1)\,(-a_2+a_3+1))\right)-{}_3F_2\left(a_3+1,\,a_3-b_1+1,\,a_3-b_2+1;\,-a_1+a_3+1,\,-a_2+a_3+1;\,\frac{1}{z}\right)$$
$$\left(-z\,{}_3F_2\left(a_1+1,\,a_1-b_1+1,\,a_1-b_2+1;\,a_1-a_2+1,\,a_1-a_3+1;\,\frac{1}{z}\right)a_1-\right.$$
$$\left({}_3F_2\left(a_1+2,\,a_1-b_1+2,\,a_1-b_2+2;\,a_1-a_2+2,\,a_1-a_3+2;\,\frac{1}{z}\right)(a_1+1)\,(a_1-b_1+1)\,(a_1-b_2+1)\right)\Big/$$
$$\left.\left.((a_1-a_2+1)\,(a_1-a_3+1))\right)\right)\left({}_3F_2\left(a_2+1,\,a_2-b_1+1,\,a_2-b_2+1;\,-a_1+a_2+1,\,a_2-a_3+1;\,\frac{1}{z}\right)a_2\,z^2+\right.$$

$$\left(2 z\, {}_3F_2\left(a_2+2, a_2-b_1+2, a_2-b_2+2; -a_1+a_2+2, a_2-a_3+2; \frac{1}{z}\right)(a_2-b_1+1)(a_2-b_2+1)\right) \Big/ \\ ((-a_1+a_2+1)(a_2-a_3+1)) + \left(2 z\, {}_3F_2\left(a_2+2, a_2-b_1+2, a_2-b_2+2; -a_1+a_2+2, a_2-a_3+2; \frac{1}{z}\right) \\ a_2 (a_2-b_1+1)(a_2-b_2+1)\right) \Big/ ((-a_1+a_2+1)(a_2-a_3+1)) + \\ \left({}_3F_2\left(a_2+3, a_2-b_1+3, a_2-b_2+3; -a_1+a_2+3, a_2-a_3+3; \frac{1}{z}\right)(a_2+2)(a_2-b_1+1)(a_2-b_1+2) \\ (a_2-b_2+1)(a_2-b_2+2)\right) \Big/ ((-a_1+a_2+1)(-a_1+a_2+2)(a_2-a_3+1)(a_2-a_3+2))\Bigg) + \\ (a_1+1)\left({}_3F_2\left(a_1+1, a_1-b_1+1, a_1-b_2+1; a_1-a_2+1, a_1-a_3+1; \frac{1}{z}\right) a_1 z^2 + \\ \left(2 z\, {}_3F_2\left(a_1+2, a_1-b_1+2, a_1-b_2+2; a_1-a_2+2, a_1-a_3+2; \frac{1}{z}\right)(a_1-b_1+1)(a_1-b_2+1)\right) \Big/ \\ ((a_1-a_2+1)(a_1-a_3+1)) + \left(2 z\, {}_3F_2\left(a_1+2, a_1-b_1+2, a_1-b_2+2; a_1-a_2+2, a_1-a_3+2; \frac{1}{z}\right) \\ a_1 (a_1-b_1+1)(a_1-b_2+1)\right) \Big/ ((a_1-a_2+1)(a_1-a_3+1)) + \\ \left({}_3F_2\left(a_1+3, a_1-b_1+3, a_1-b_2+3; a_1-a_2+3, a_1-a_3+3; \frac{1}{z}\right)(a_1+2)(a_1-b_1+1)(a_1-b_1+2) \\ (a_1-b_2+1)(a_1-b_2+2)\right) \Big/ ((a_1-a_2+1)(a_1-a_2+2)(a_1-a_3+1)(a_1-a_3+2))\Bigg) \\ \left({}_3F_2\left(a_2+1, a_2-b_1+1, a_2-b_2+1; -a_1+a_2+1, a_2-a_3+1; \frac{1}{z}\right) \right. \\ \left(-z\, {}_3F_2\left(a_3+1, a_3-b_1+1, a_3-b_2+1; -a_1+a_3+1, -a_2+a_3+1; \frac{1}{z}\right) a_3 - \right. \\ \left({}_3F_2\left(a_3+2, a_3-b_1+2, a_3-b_2+2; -a_1+a_3+2, -a_2+a_3+2; \frac{1}{z}\right)(a_3+1)(a_3-b_1+1)(a_3-b_2+1)\right) \Big/ \\ ((-a_1+a_3+1)(-a_2+a_3+1))\Bigg) - {}_3F_2\left(a_3+1, a_3-b_1+1, a_3-b_2+1; -a_1+a_3+1, -a_2+a_3+1; \frac{1}{z}\right) \\ \left(\left({}_3F_2\left(a_2+2, a_2-b_1+2, a_2-b_2+2; -a_1+a_2+2, a_2-a_3+2; \frac{1}{z}\right)(a_2+1)(a_2-b_1+1)(a_2-b_2+1)\right) \Big/ \right. \\ \left.\left. ((a_1-a_2-1)(a_2-a_3+1)) - z\, {}_3F_2\left(a_2+1, a_2-b_1+1, a_2-b_2+1; -a_1+a_2+1, a_2-a_3+1; \frac{1}{z}\right) a_2\right)\right)\Bigg)$$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

07.27.16.0001.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; c\,z)\,{}_3F_2(\alpha_1, \alpha_2, \alpha_3; \beta_1, \beta_2; d\,z) = \sum_{k=0}^{\infty} c_k\, z^k \;/;$$

$$c_k = \frac{d^k \prod_{j=1}^{3} (\alpha_j)_k}{k!\, \prod_{j=1}^{2} (\beta_j)_k}\, {}_6F_5\left(-k, 1-k-\beta_1, 1-k-\beta_2, a_1, a_2, a_3; 1-k-\alpha_1, 1-k-\alpha_2, 1-k-\alpha_3, b_1, b_2; \frac{c}{d}\right) \bigvee$$

$$c_k = \frac{c^k \prod_{j=1}^{3} (a_j)_k}{k!\, \prod_{j=1}^{2} (b_j)_k}\, {}_6F_5\left(-k, 1-k-b_1, 1-k-b_2, \alpha_1, \alpha_2, \alpha_3; 1-k-a_1, 1-k-a_2, 1-k-a_3, \beta_1, \beta_2; \frac{d}{c}\right)$$

07.27.16.0002.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; c\,z)\,{}_3F_2(\alpha_1, \alpha_2, \alpha_3; \beta_1, \beta_2; d\,z) = \sum_{k=0}^{\infty} \sum_{m=0}^{k} \frac{\left(\prod_{j=1}^{3} (a_j)_m\, c^m\right) \left(\prod_{j=1}^{3} (\alpha_j)_{k-m}\, d^{k-m}\right) z^k}{\left(\prod_{j=1}^{2} (b_j)_m\, m!\right) \prod_{j=1}^{2} (\beta_j)_{k-m}\, (k-m)!}$$

07.27.16.0003.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; c\,z)\,{}_3F_2(\alpha_1, \alpha_2, \alpha_3; \beta_1, \beta_2; d\,z) = F_{0;2;2}^{0:3;3}\left(\begin{matrix} : a_1, a_2, a_3; \alpha_1, \alpha_2, \alpha_3; \\ : b_1, b_2; \beta_1, \beta_2; \end{matrix}\; c\,z, d\,z\right)$$

Identities

Recurrence identities

Consecutive neighbors

07.27.17.0001.01

$${}_3F_2(a, a_2, a_3; b_1, b_2; z) = (B_1 + C_1 z)\,{}_3F_2(a+1, a_2, a_3; b_1, b_2; z) + (B_2 + C_2 z)\,{}_3F_2(a+2, a_2, a_3; b_1, b_2; z) + (B_3 + C_3 z)\,{}_3F_2(a+3, a_2, a_3; b_1, b_2; z) \;/;$$

$$B_1 = \frac{b_1 b_2 + (a+1)(3a - 2b_1 - 2b_2 + 4)}{(a-b_1+1)(a-b_2+1)} \bigwedge C_1 = \frac{(-a+a_2-1)(a-a_3+1)}{(a-b_1+1)(a-b_2+1)} \bigwedge B_2 = \frac{(a+1)(-3a+b_1+b_2-5)}{(a-b_1+1)(a-b_2+1)} \bigwedge$$

$$C_2 = \frac{(a+1)(2a-a_2-a_3+3)}{(a-b_1+1)(a-b_2+1)} \bigwedge B_3 = \frac{(a+1)(a+2)}{(a-b_1+1)(a-b_2+1)} \bigwedge C_3 = -\frac{(a+1)(a+2)}{(a-b_1+1)(a-b_2+1)}$$

07.27.17.0002.01

$${}_3F_2(a, a_2, a_3; b_1, b_2; z) = \frac{B_1 + C_1 z}{z-1}\,{}_3F_2(a-1, a_2, a_3; b_1, b_2; z) + \frac{B_2 + C_2 z}{z-1}\,{}_3F_2(a-2, a_2, a_3; b_1, b_2; z) + \frac{B_3}{z-1}\,{}_3F_2(a-3, a_2, a_3; b_1, b_2; z) \;/;$$

$$B_1 = \frac{b_1 + b_2 - 3a + 4}{a-1} \bigwedge C_1 = \frac{2a - a_2 - a_3 - 3}{a-1} \bigwedge B_2 = \frac{(3a - 2b_1 - 2b_2 - 5)(a-2) + b_1 b_2}{(a-1)(a-2)} \bigwedge$$

$$C_2 = \frac{(a-2)(a_2 + a_3 - a + 2) - a_2 a_3}{(a-1)(a-2)} \bigwedge B_3 = -\frac{(a-b_1-2)(a-b_2-2)}{(a-1)(a-2)}$$

07.27.17.0003.01

$${}_3F_2(a_1,\,a_2,\,a_3;\,b,\,b_2;\,z) = \frac{B_1+C_1\,z}{z-1}\,{}_3F_2(a_1,\,a_2,\,a_3;\,b+1,\,b_2;\,z) + \frac{B_2+C_2\,z}{z-1}\,{}_3F_2(a_1,\,a_2,\,a_3;\,b+2,\,b_2;\,z) + \frac{C_3\,z}{z-1}\,{}_3F_2(a_1,\,a_2,\,a_3;\,b+3,\,b_2;\,z)\;/;$$
$$B_1 = \frac{b_2-2\,b-2}{b} \bigwedge C_1 = \frac{3\,b-a_1-a_2-a_3+3}{b} \bigwedge B_2 = \frac{b-b_2+2}{b} \bigwedge$$
$$C_2 = \frac{(2\,b+3)\,(a_1+a_2+a_3)-3\,b^2-9\,b-a_1\,a_2-a_1\,a_3-a_2\,a_3-7}{b\,(b+1)} \bigwedge C_3 = \frac{(b-a_1+2)\,(b-a_2+2)\,(b-a_3+2)}{b\,(b+1)\,(b+2)}$$

07.27.17.0004.01

$${}_3F_2(a_1,\,a_2,\,a_3;\,b,\,b_2;\,z) = \frac{B_1+C_1\,z}{z}\,{}_3F_2(a_1,\,a_2,\,a_3;\,b-1,\,b_2;\,z) + \frac{B_2+C_2\,z}{z}\,{}_3F_2(a_1,\,a_2,\,a_3;\,b-2,\,b_2;\,z) +$$
$$\frac{B_3+C_3\,z}{z}\,{}_3F_2(a_1,\,a_2,\,a_3;\,b-3,\,b_2;\,z)\;/;\; B_1 = \frac{(b-1)\,(b-2)\,(b-b_2-1)}{(1-b+a_1)\,(1-b+a_2)\,(1-b+a_3)} \bigwedge$$
$$C_1 = \left((b-1)\left((2\,b-3)\,(a_1+a_2+a_3)-3\,b^2+9\,b-a_1\,a_2-a_1\,a_3-a_2\,a_3-7\right)\right) / ((1-b+a_1)\,(1-b+a_2)\,(1-b+a_3)) \bigwedge$$
$$B_2 = \frac{(b-1)\,(b-2)\,(b_2-2\,b+4)}{(1-b+a_1)\,(1-b+a_2)\,(1-b+a_3)} \bigwedge C_2 = \frac{(b-1)\,(b-2)\,(3\,b-a_1-a_2-a_3-6)}{(1-b+a_1)\,(1-b+a_2)\,(1-b+a_3)} \bigwedge$$
$$B_3 = \frac{(b-1)\,(b-2)\,(b-3)}{(1-b+a_1)\,(1-b+a_2)\,(1-b+a_3)} \bigwedge C_3 = -\frac{(b-1)\,(b-2)\,(b-3)}{(1-b+a_1)\,(1-b+a_2)\,(1-b+a_3)}$$

Distant neighbors with respect to *q*

07.27.17.0005.01

$${}_3F_2(a_1,\,a_2,\,a_3;\,b_1,\,b_2;\,z) = \frac{\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_3)} \sum_{k=0}^{\infty} \mathcal{E}_k^{(2)}(\{a_1,\,a_2,\,a_3\},\,\{b_1,\,b_2\})\,{}_2\tilde{F}_1(a_1,\,a_2;\,k-a_3+b_1+b_2;\,z)$$

Functional identities

Relations between contiguous functions

07.27.17.0006.01

$$b\,{}_3F_2(a,\,b+1,\,a_3;\,b_1,\,b_2;\,z) - a\,{}_3F_2(a+1,\,b,\,a_3;\,b_1,\,b_2;\,z) + (a-b)\,{}_3F_2(a,\,b,\,a_3;\,b_1,\,b_2;\,z) = 0$$

07.27.17.0007.01

$$b\,{}_3F_2(a,\,a_2,\,a_3;\,b,\,b_2;\,z) - a\,{}_3F_2(a+1,\,a_2,\,a_3;\,b+1,\,b_2;\,z) + (a-b)\,{}_3F_2(a,\,a_2,\,a_3;\,b+1,\,b_2;\,z) = 0$$

07.27.17.0008.01

$$(b-c)\,{}_3F_2(a_1,\,a_2,\,a_3;\,b+1,\,c+1;\,z) + c\,{}_3F_2(a_1,\,a_2,\,a_3;\,b+1,\,c;\,z) - b\,{}_3F_2(a_1,\,a_2,\,a_3;\,b,\,c+1;\,z) = 0$$

07.27.17.0009.01

$$(a-b)\,c\,{}_3F_2(a,\,b,\,c+1;\,b_1,\,b_2;\,z) - b\,(a-c)\,{}_3F_2(a,\,b+1,\,c;\,b_1,\,b_2;\,z) + a\,(b-c)\,{}_3F_2(a+1,\,b,\,c;\,b_1,\,b_2;\,z) = 0$$

07.27.17.0010.01

$$a\,(b-c)\,{}_3F_2(a+1,\,a_2,\,a_3;\,b+1,\,c+1;\,z) - c\,(b-a)\,{}_3F_2(a,\,a_2,\,a_3;\,b+1,\,c;\,z) + (c-a)\,b\,{}_3F_2(a,\,a_2,\,a_3;\,b,\,c+1;\,z) = 0$$

07.27.17.0011.01

$$(a-b)\,c\,{}_3F_2(a,\,b,\,a_3;\,c,\,b_2;\,z) - a\,(c-b)\,{}_3F_2(a+1,\,b,\,a_3;\,c+1,\,b_2;\,z) + (c-a)\,b\,{}_3F_2(a,\,b+1,\,a_3;\,c+1,\,b_2;\,z) = 0$$

07.27.17.0012.01

$$b\,c\,z\,{}_3F_2(a+1,\,b+1,\,c+1;\,d+1,\,e+1;\,z) + d\,e\,({}_3F_2(a,\,b,\,c;\,d,\,e;\,z) - {}_3F_2(a+1,\,b,\,c;\,d,\,e;\,z)) = 0$$

07.27.17.0013.01

$$d\,(d+1)\,e\,({}_3F_2(a,b,c;d,e;z)-{}_3F_2(a,b,c;d+1,e;z))-a\,b\,c\,z\,{}_3F_2(a+1,b+1,c+1;d+2,e+1;z)=0$$

07.27.17.0014.01

$$(b-a)\,c\,z\,{}_3F_2(a+1,b+1,c+1;d+1,e+1;z)+d\,e\,({}_3F_2(a,b+1,c;d,e;z)-{}_3F_2(a+1,b,c;d,e;z))=0$$

07.27.17.0015.01

$$(d-a)\,b\,c\,z\,{}_3F_2(a+1,b+1,c+1;d+2,e+1;z)+(d+1)\,d\,e\,({}_3F_2(a,b,c;d,e;z)-{}_3F_2(a+1,b,c;d+1,e;z))=0$$

07.27.17.0016.01

$$a\,c\,z\,{}_3F_2(a+1,b+1,c+1;d+1,e+1;z)+ e\,(-a\,{}_3F_2(a+1,b+1,c;d+1,e;z)-(d-a)\,{}_3F_2(a,b+1,c;d+1,e;z)+d\,{}_3F_2(a,b,c;d,e;z))=0$$

07.27.17.0017.01

$${}_3F_2(a,b,c;d,e;z)-\frac{a\,b\,(d-c)\,(e-c)}{d\,e\,(a-c)\,(b-c)}\,{}_3F_2(a+1,b+1,c;d+1,e+1;z)- \frac{a\,c\,(d-b)\,(e-b)}{d\,e\,(a-b)\,(c-b)}\,{}_3F_2(a+1,b,c+1;d+1,e+1;z)-\frac{b\,c\,(d-a)\,(e-a)}{d\,e\,(b-a)\,(c-a)}\,{}_3F_2(a,b+1,c+1;d+1,e+1;z)=0$$

07.27.17.0018.01

$$(a+(a_2+a_3-d-e)\,z)\,{}_3F_2(a,a_2,a_3;d,e;z)+\frac{(d-a)\,(d-a_2)\,(d-a_3)\,z}{d\,(d-e)}\,{}_3F_2(a,a_2,a_3;d+1,e;z)+ \frac{(e-a)\,(e-a_2)\,(e-a_3)\,z}{e\,(e-d)}\,{}_3F_2(a,a_2,a_3;d,e+1;z)=a\,(1-z)\,{}_3F_2(a+1,a_2,a_3;d,e;z)$$

Relations of special kind

07.27.17.0019.01

$${}_3F_2(a_1,a_2,a_3;-c,c+1;z)+{}_3F_2(a_1,a_2,a_3;c,1-c;z)=2\,{}_3F_2(a_1,a_2,a_3;c+1,1-c;z)$$

07.27.17.0020.01

$${}_3F_2(a,a_2,a_3;-a,a+1;z)-2\,{}_3F_2(a,a_2,a_3;1-a,a+1;z)=-{}_2F_1(a_2,a_3;1-a;z)$$

07.27.17.0021.01

$${}_3F_2(-a,a_2,a_3;1-a,b_2;z)+{}_3F_2(a,a_2,a_3;a+1,b_2;z)=2\,{}_4F_3(a,-a,a_2,a_3;a+1,1-a,b_2;z)$$

07.27.17.0022.01

$${}_3F_2(-a,a+1,a_3;b_1,b_2;z)+{}_3F_2(a,1-a,a_3;b_1,b_2;z)=2\,{}_3F_2(a,-a,a_3;b_1,b_2;z)$$

Division on even and odd parts and generalization

07.27.17.0023.01

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;z)=A^+(z)+A^-(z)\;/;\;A^+(z)=\frac{1}{2}\,({}_3F_2(a_1,a_2,a_3;b_1,b_2;z)+{}_3F_2(a_1,a_2,a_3;b_1,b_2;-z))\bigwedge A^-(z)=\frac{1}{2}\,({}_3F_2(a_1,a_2,a_3;b_1,b_2;z)-{}_3F_2(a_1,a_2,a_3;b_1,b_2;-z))$$

07.27.17.0024.01

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;z)=A^+(z)+A^-(z)\;/;$$
$$A^+(z)={}_6F_5\left(\frac{a_1}{2},\frac{a_2}{2},\frac{a_3}{2},\frac{a_1+1}{2},\frac{a_2+1}{2},\frac{a_3+1}{2};\frac{1}{2},\frac{b_1}{2},\frac{b_2}{2},\frac{b_1+1}{2},\frac{b_2+1}{2};z^2\right)\bigwedge$$
$$A^-(z)=\frac{z\prod_{j=1}^3 a_j}{b_1\,b_2}\,{}_6F_5\left(\frac{a_1+1}{2},\frac{a_2+1}{2},\frac{a_3+1}{2},\frac{a_1+2}{2},\frac{a_2+2}{2},\frac{a_3+2}{2};\frac{3}{2},\frac{b_1+1}{2},\frac{b_2+1}{2},\frac{b_1+2}{2},\frac{b_2+2}{2};z^2\right)$$

07.27.17.0025.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{n-1} \frac{z^k \prod_{j=1}^{3} (a_j)_k}{k!\,(b_1)_k\,(b_2)_k}\, {}_{3n+1}F_{3n}\Big(1, \frac{a_1+k}{n}, \ldots, \frac{a_1+k+n-1}{n}, \frac{a_2+k}{n}, \ldots, \frac{a_2+k+n-1}{n}, \frac{a_3+k}{n}, \ldots, \frac{a_3+k+n-1}{n}; \frac{k+1}{n}, \ldots, \frac{k+n}{n}, \frac{b_1+k}{n}, \ldots, \frac{b_1+k+n-1}{n}, \frac{b_2+k}{n}, \ldots, \frac{b_2+k+n-1}{n}; z^n\Big) $$

General cases

07.27.17.0026.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \frac{\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)} \Bigg(\frac{\Gamma(a_1)\,\Gamma(a_2-a_1)\,\Gamma(a_3-a_1)}{\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)} (-z)^{-a_1}\, {}_3F_2\Big(a_1, a_1-b_1+1, a_1-b_2+1; a_1-a_2+1, a_1-a_3+1; \frac{1}{z}\Big) + \frac{\Gamma(a_2)\,\Gamma(a_1-a_2)\,\Gamma(a_3-a_2)}{\Gamma(b_1-a_2)\,\Gamma(b_2-a_2)} (-z)^{-a_2}\, {}_3F_2\Big(a_2, a_2-b_1+1, a_2-b_2+1; -a_1+a_2+1, a_2-a_3+1; \frac{1}{z}\Big) + \frac{\Gamma(a_3)\,\Gamma(a_1-a_3)\,\Gamma(a_2-a_3)}{\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)} (-z)^{-a_3}\, {}_3F_2\Big(a_3, a_3-b_1+1, a_3-b_2+1; -a_1+a_3+1, -a_2+a_3+1; \frac{1}{z}\Big)\Bigg) /; a_1-a_2 \notin \mathbb{Z} \bigwedge a_1-a_3 \notin \mathbb{Z} \bigwedge a_2-a_3 \notin \mathbb{Z} \bigwedge z \notin (0, 1) $$

07.27.17.0027.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; w\,z) = (1-z)^{-a_1} \sum_{k=0}^{\infty} \frac{(a_1)_k}{k!}\, {}_3F_2(-k, a_2, a_3; b_1, b_2; w) \left(\frac{z}{z-1}\right)^k $$

07.27.17.0028.01

$$ {}_3F_2(a, b, c; a-b+1, a-c+1; z) = (1-z)^{-a}\, {}_3F_2\Big(a-b-c+1, \frac{a}{2}, \frac{a+1}{2}; a-b+1, a-c+1; -\frac{4z}{(1-z)^2}\Big) /; z \notin (1, \infty) $$

07.27.17.0029.01

$$ {}_3F_2(a, b, c; a+1, e; z) = \frac{1}{a-e+1} \big(a\, {}_2F_1(b, c; e; z) - (e-1)\, {}_3F_2(a, b, c; a+1, e-1; z)\big) $$

For fixed a_1, a_2, b_1, z

07.27.17.0030.01

$$ {}_3F_2\Big(a, b, a+\frac{1}{2}; d, 2a+b-d+1; z\Big) = \left(\frac{2}{z}\left(1-\sqrt{1-z}\right)\right)^{2a} {}_3F_2\Big(2a, d-b, 2a-d+1; d, 2a+b-d+1; 1-\frac{2}{z}\left(1-\sqrt{1-z}\right)\Big) $$

07.27.17.0031.01

$$ {}_3F_2\Big(a, b, 1-a; \frac{a+b+1}{2}, \frac{b-a}{2}+1; z\Big) = (1-4z)^{-b}\, {}_3F_2\Big(\frac{b}{3}, \frac{b+1}{3}, \frac{b+2}{3}; \frac{a+b+1}{2}, \frac{b-a}{2}+1; -\frac{27z}{(1-4z)^3}\Big) $$

07.27.17.0032.01

$$ {}_3F_2\Big(a, b, a+b-\frac{1}{2}; 2a, 2b; z\Big) = \left(1-\frac{z}{4}\right)^{\frac{1}{2}-a-b} {}_3F_2\Big(\frac{2a+2b-1}{6}, \frac{2a+2b+1}{6}, \frac{2a+2b+3}{6}; a+\frac{1}{2}, b+\frac{1}{2}; \frac{27z^2}{(4-z)^3}\Big) $$

07.27.17.0033.01

$$ {}_3F_2\Big(a, b, a+b-\frac{1}{2}; 2a, 2b; z\Big) = \left(1-\frac{z}{4}\right)^{\frac{1}{2}-a-b} {}_3F_2\Big(\frac{2a+2b-1}{6}, \frac{2a+2b+1}{6}, \frac{2a+2b+3}{6}; a+\frac{1}{2}, b+\frac{1}{2}; \frac{27z^2}{(4-z)^3}\Big) $$

Functional identities for $z = 1$

07.27.17.0034.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = \frac{\Gamma(b_1)\,\Gamma(b_1+b_2-a_1-a_2-a_3)}{\Gamma(b_1-a_1)\,\Gamma(b_1+b_2-a_2-a_3)}\, {}_3F_2(a_1, b_2-a_2, b_2-a_3; b_2, -a_2-a_3+b_1+b_2; 1) \,/;\, \mathrm{Re}(b_1+b_2-a_1-a_2-a_3) > 0 \bigwedge \mathrm{Re}(b_1-a_1) > 0 $$

07.27.17.0035.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = \frac{\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_1+b_2-a_1-a_2-a_3)}{\Gamma(a_1)\,\Gamma(b_1+b_2-a_1-a_2)\,\Gamma(b_1+b_2-a_1-a_3)}\, {}_3F_2(b_1-a_1, b_2-a_1, b_1+b_2-a_1-a_2-a_3; b_1+b_2-a_1-a_2, b_1+b_2-a_1-a_3; 1) \,/;\, \mathrm{Re}(b_1+b_2-a_1-a_2-a_3) > 0 \bigwedge \mathrm{Re}(a_1) > 0 $$

07.27.17.0036.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = \frac{\Gamma(1-a_2)\,\Gamma(a_3-a_1)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1-a_2+1)\,\Gamma(a_3)\,\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)}\, {}_3F_2(a_1, a_1-b_1+1, a_1-b_2+1; a_1-a_2+1, a_1-a_3+1; 1) + \frac{\Gamma(1-a_2)\,\Gamma(a_1-a_3)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_3-a_2+1)\,\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)}\, {}_3F_2(a_3, a_3-b_1+1, a_3-b_2+1; a_3-a_1+1, a_3-a_2+1; 1) \,/;\, \mathrm{Re}(b_1+b_2-a_1-a_2-a_3) > 0 $$

07.27.17.0037.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = \frac{\Gamma(a_1+a_2-b_1)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_1+b_2-a_1-a_2-a_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(b_2-a_3)\,\Gamma(b_1+b_2-a_1-a_2)}\, {}_3F_2(b_1-a_1, b_1-a_2, b_1+b_2-a_1-a_2-a_3; b_1-a_1-a_2+1, b_1+b_2-a_1-a_2; 1) + \frac{\Gamma(b_1-a_1-a_2)\,\Gamma(b_1)}{\Gamma(b_1-a_1)\,\Gamma(b_1-a_2)}\, {}_3F_2(a_1, a_2, b_2-a_3; b_2, a_1+a_2-b_1+1; 1) \,/;\, \mathrm{Re}(b_1+b_2-a_1-a_2-a_3) > 0 \bigwedge \mathrm{Re}(a_3-b_1+1) > 0 $$

07.27.17.0038.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = \frac{\Gamma(a_1-b_1+1)\,\Gamma(a_3-b_1+1)}{\Gamma(1-b_1)\,\Gamma(a_1+a_3-b_1+1)}\, {}_3F_2(a_1, a_3, b_2-a_2; a_1+a_3-b_1+1, b_2; 1) + \frac{\Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(2-b_1)\,\Gamma(1-b_1+b_2)}\, {}_3F_2(a_1-b_1+1, a_2-b_1+1, a_3-b_1+1; 2-b_1, 1-b_1+b_2; 1) \,/;\, \mathrm{Re}(b_1+b_2-a_1-a_2-a_3) > 0 \bigwedge \mathrm{Re}(a_2-b_1+1) > 0 $$

07.27.17.0039.01

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; 1) = (\Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(b_2))\,/\,(\Gamma(1-b_1)\,\Gamma(a_1+a_2-b_1+1)\,\Gamma(a_1+a_3-b_1+1)\,\Gamma(b_2-a_1))\; {}_3F_2(a_1, a_1-b_1+1, a_1+a_2+a_3-b_1-b_2+1; a_1+a_2-b_1+1, a_1+a_3-b_1+1; 1) + (\Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(b_1)\,\Gamma(b_2))\,/\,(\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(2-b_1)\,\Gamma(1-b_1+b_2))\; {}_3F_2(a_1-b_1+1, a_2-b_1+1, a_3-b_1+1; 2-b_1, -b_1+b_2+1; 1) \,/;\, \mathrm{Re}(b_1+b_2-a_1-a_2-a_3) > 0 \bigwedge \mathrm{Re}(b_2-a_1) > 0 $$

07.27.17.0040.01

$$ {}_3F_2(1, b, c; d, e; 1) + \frac{b\,c\,(b+c-d-e+2)}{d\,e\,(b\,c-(d-1)\,(e-1))}\, {}_3F_2(2, b+1, c+1; d+1, e+1; 1) = \frac{(1-d)\,(e-1)}{b\,c-(d-1)\,(e-1)} \,/;\, \mathrm{Re}(b+c-d-e) < -2 $$

07.27.17.0041.01

$$ {}_3F_2(-n, b, c; d, e; 1) = \frac{(d-b)_n\,(e-b)_n}{(d)_n\,(e)_n}\, {}_3F_2(-n, b, b+c-d-e-n+1; b-d-n+1, b-e-n+1; 1) \,/;\, n \in \mathbb{N} $$

07.27.17.0042.01

$${}_3F_2(-n,\, b,\, c;\, d,\, e;\, 1) = \frac{(b)_n\, (d+e-b-c)_n}{(d)_n\, (e)_n}\, {}_3F_2(-n,\, d-b,\, e-b;\, 1-b-n,\, d+e-b-c;\, 1) \,/;\, n \in \mathbb{N}$$

07.27.17.0043.01

$${}_3F_2(-n,\, b,\, c;\, d,\, e;\, 1) = \frac{(d+e-b-c)_n}{(d)_n}\, {}_3F_2(-n,\, e-b,\, e-c;\, e,\, d+e-b-c;\, 1) \,/;\, n \in \mathbb{N}$$

07.27.17.0044.01

$${}_3F_2(-n,\, b,\, c;\, d,\, e;\, 1) = \frac{(-1)^n\, (b)_n\, (c)_n}{(d)_n\, (e)_n}\, {}_3F_2(-n,\, 1-d-n,\, 1-e-n;\, 1-b-n,\, 1-c-n;\, 1) \,/;\, n \in \mathbb{N}$$

07.27.17.0045.01

$${}_3F_2(-n,\, b,\, c;\, d,\, e;\, 1) = \frac{(-1)^n\, (e-b)_n\, (e-c)_n}{(d)_n\, (e)_n}\, {}_3F_2(-n,\, 1-e-n,\, b+c-d-e-n+1;\, b-e-n+1,\, c-e-n+1;\, 1) \,/;\, n \in \mathbb{N}$$

07.27.17.0046.01

$${}_3F_2(-n,\, b,\, c;\, d,\, e;\, 1) = \frac{(d-b)_n}{(d)_n}\, {}_3F_2(-n,\, b,\, e-c;\, e,\, b-d-n+1;\, 1) \,/;\, n \in \mathbb{N}$$

07.27.17.0047.01

$${}_3F_2(-n,\, b,\, c;\, d,\, e;\, 1) = \frac{(d-b)_n\, (c)_n}{(d)_n\, (e)_n}\, {}_3F_2(-n,\, e-c,\, 1-d-n;\, 1-c-n,\, b-d-n+1;\, 1) \,/;\, n \in \mathbb{N}$$

07.27.17.0048.01

$${}_3F_2(-n,\, 1,\, c;\, d,\, c-m;\, 1) = \frac{(d-1)\, (c-m-1)}{(d+n-1)\, (c-1)}\, {}_3F_2(-m,\, 1,\, 2-d;\, 2-d-n,\, 2-c;\, 1) \,/;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

07.27.17.0049.01

$${}_3F_2(-n,\, 1,\, 1;\, l,\, m;\, 1) = \frac{l-1}{l+n-1}\, {}_3F_2(-n,\, m-1,\, 1;\, m,\, 2-l-n;\, 1) \,/;\, m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} \bigwedge l-2 \in \mathbb{Z} \bigwedge (l-2 \geq 0 \bigvee l \leq -n)$$

07.27.17.0050.01

$${}_3F_2(-n,\, -n,\, 1;\, -2\, n,\, m;\, 1) = \frac{n!^2}{(2\, n)!}\, {}_3F_2(-n,\, -n,\, m-1;\, 1,\, m;\, 1) \,/;\, m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}$$

07.27.17.0051.01

$${}_3F_2\left(-\frac{n}{2},\, -\frac{1}{2}\, (n-1),\, -m;\, 1,\, -m-n;\, 1\right) = 2^{-n}\, {}_3F_2(-n,\, -n,\, m+1;\, 1,\, -m-n;\, -1) \,/;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

07.27.17.0052.01

$${}_3F_2\left(-\frac{n}{2},\, -\frac{n-1}{2},\, -m;\, 1,\, -m-n;\, 1\right) = \frac{2^n}{n!} \left(\frac{1}{2}\right)_n {}_3F_2\left(-n,\, -\frac{n}{2},\, -\frac{n-1}{2};\, \frac{1}{2}-n,\, -m-n;\, 1\right) \,/;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

Functional identities for $z = -1$

07.27.17.0053.01

$${}_3F_2(a,\, b,\, c;\, a-b+1,\, a-c+1;\, -1) = \frac{\Gamma(a-b+1)\, \Gamma(a-c+1)}{\Gamma(a+1)\, \Gamma(a-b-c+1)}\, {}_3F_2\left(\frac{1}{2},\, b,\, c;\, \frac{a}{2}+1,\, \frac{a+1}{2};\, 1\right) \,/;\, \mathrm{Re}(a-b-c) > -1$$

07.27.17.0054.01

$${}_3F_2(-n,\, -n,\, -n;\, 1,\, 1;\, -1) = 2^n\, {}_3F_2\left(-\frac{n}{2},\, -\frac{n-1}{2},\, n+1;\, 1,\, 1;\, 1\right) \,/;\, n \in \mathbb{N}$$

Differentiation

Low-order differentiation

With respect to a_1

07.27.20.0001.01

$$ {}_3F_2^{(\{1,0,0\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k \psi(k+a_1) z^k}{(b_1)_k (b_2)_k k!} - \psi(a_1) {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) /; |z| < 1 $$

07.27.20.0002.01

$$ {}_3F_2^{(\{1,0,0\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \frac{z a_2 a_3}{b_1 b_2} F_{3\times0\times1}^{3\times1\times2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1; 1; 1, a_1; \\ 2, b_1+1, b_2+1;; a_1+1; \end{matrix} z, z \right) $$

With respect to a_2

07.27.20.0003.01

$$ {}_3F_2^{(\{0,1,0\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k \psi(k+a_2) z^k}{(b_1)_k (b_2)_k k!} - \psi(a_2) {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) /; |z| < 1 $$

07.27.20.0004.01

$$ {}_3F_2^{(\{0,1,0\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \frac{z a_1 a_3}{b_1 b_2} F_{3\times0\times1}^{3\times1\times2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1; 1; 1, a_2; \\ 2, b_1+1, b_2+1;; a_2+1; \end{matrix} z, z \right) $$

With respect to a_3

07.27.20.0005.01

$$ {}_3F_2^{(\{0,0,1\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k \psi(k+a_3) z^k}{(b_1)_k (b_2)_k k!} - \psi(a_3) {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) /; |z| < 1 $$

07.27.20.0006.01

$$ {}_3F_2^{(\{0,0,1\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \frac{z a_1 a_2}{b_1 b_2} F_{3\times0\times1}^{3\times1\times2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1; 1; 1, a_3; \\ 2, b_1+1, b_2+1;; a_3+1; \end{matrix} z, z \right) $$

With respect to b_1

07.27.20.0007.01

$$ {}_3F_2^{(\{0,0,0\},\{1,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \psi(b_1) {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) - \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k \psi(k+b_1) z^k}{(b_1)_k (b_2)_k k!} /; |z| < 1 $$

07.27.20.0008.01

$$ {}_3F_2^{(\{0,0,0\},\{1,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = -\frac{z \prod_{j=1}^{3} a_j}{b_1^2 b_2} F_{3\times0\times1}^{3\times1\times2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1; 1; 1, b_1; \\ 2, b_1+1, b_2+1;; b_1+1; \end{matrix} z, z \right) $$

With respect to b_2

07.27.20.0009.01

$$ {}_3F_2^{(\{0,0,0\},\{0,1\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \psi(b_2) {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) - \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k \psi(k+b_2) z^k}{(b_1)_k (b_2)_k k!} /; |z| < 1 $$

07.27.20.0010.01

$$ {}_3F_2^{(\{0,0,0\},\{0,1\},0)}(a_1, a_2, a_3; b_1, b_2; z) = -\frac{z \prod_{j=1}^{3} a_j}{b_2^2 \, b_1} F_{3\times 0\times 1}^{3\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1; 1; 1, b_2; \\ 2, b_1+1, b_2+1;; b_2+1; \end{matrix} z, z\right) $$

With respect to element of parameters ||| With respect to element of parameters

07.27.20.0011.01

$$ \frac{\partial\, {}_3F_2(a, a_2, a_3; a+1, b_2; z)}{\partial a} = \frac{z\, a_2\, a_3}{b_2\, (a+1)^2}\, {}_4F_3(a+1, a+1, a_2+1, a_3+1; a+2, a+2, b_2+1; z) $$

07.27.20.0012.01

$$ \frac{\partial\, {}_3F_2(a+1, a_2, a_3; a, b_2; z)}{\partial a} = -\frac{z\, a_2\, a_3}{a^2\, b_2}\, {}_2F_1(a_2+1, a_3+1; b_2+1; z) $$

With respect to z

07.27.20.0013.01

$$ \frac{\partial\, {}_3F_2(a_1, a_2, a_3; b_1, b_2; z)}{\partial z} = \frac{a_1\, a_2\, a_3}{b_1\, b_2}\, {}_3F_2(a_1+1, a_2+1, a_3+1; b_1+1, b_2+1; z) $$

07.27.20.0014.01

$$ \frac{\partial^2\, {}_3F_2(a_1, a_2, a_3; b_1, b_2; z)}{\partial z^2} = \frac{a_1\,(a_1+1)\, a_2\,(a_2+1)\, a_3\,(a_3+1)}{b_1\,(b_1+1)\, b_2\,(b_2+1)}\, {}_3F_2(a_1+2, a_2+2, a_3+2; b_1+2, b_2+2; z) $$

Symbolic differentiation

With respect to a_1

07.27.20.0015.01

$$ {}_3F_2^{(\{n,0,0\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_2)_k\,(a_3)_k}{(b_1)_k\,(b_2)_k\, k!} \frac{\partial^n (a_1)_k}{\partial a_1^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+ $$

With respect to a_2

07.27.20.0016.01

$$ {}_3F_2^{(\{0,n,0\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k\,(a_3)_k}{(b_1)_k\,(b_2)_k\, k!} \frac{\partial^n (a_2)_k}{\partial a_2^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+ $$

With respect to a_3

07.27.20.0017.01

$$ {}_3F_2^{(\{0,0,n\},\{0,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k\,(a_2)_k}{(b_1)_k\,(b_2)_k\, k!} \frac{\partial^n (a_3)_k}{\partial a_3^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+ $$

With respect to b_1

07.27.20.0018.01

$$ {}_3F_2^{(\{0,0,0\},\{n,0\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k\,(a_2)_k\,(a_3)_k}{(b_2)_k\, k!} \frac{\partial^n \frac{1}{(b_1)_k}}{\partial b_1^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+ $$

With respect to b_2

07.27.20.0019.01

$$ {}_3F_2^{(\{0,0,0\},\{0,n\},0)}(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k}{(b_1)_k\, k!} \frac{\partial^n \frac{1}{(b_2)_k}}{\partial b_2^n} z^k \text{ /; } |z| < 1 \bigwedge n \in \mathbb{N}^+ $$

With respect to element of parameters ||| With respect to element of parameters

07.27.20.0030.01

$$ \frac{\partial^n {}_3F_2(a, a_2, a_3; a+1, b_2; z)}{\partial a^n} = \frac{(-1)^{n-1}\, n!\, z\, a_2\, a_3}{(a+1)^{n+1}\, b_2} {}_{n+3}F_{n+2}(a+1, \ldots, a+1, a_2+1, a_3+1; a+2, \ldots, a+2, b_2+1; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0031.01

$$ \frac{\partial^n {}_3F_2(a+1, a_2, a_3; a, b_2; z)}{\partial a^n} = $$

$$ \frac{(-1)^n\, n!}{a^{n+1}} \left(a\, {}_2F_1(a_2, a_3; b_2; z) + \frac{z}{b_2}\, {}_2F_1(a_2+1, a_3+1; b_2+1; z)\, (a_2\, a_3) \right) + \frac{(-1)^{n-1}\, n!}{a^n}\, {}_2F_1(a_2, a_3; b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

With respect to z

07.27.20.0020.01

$$ \frac{\partial^n {}_3F_2(a_1, a_2, a_3; b_1, b_2; z)}{\partial z^n} = \frac{\prod_{j=1}^{3} (a_j)_n}{\prod_{j=1}^{2} (b_j)_n} {}_3F_2(n+a_1, n+a_2, n+a_3; n+b_1, n+b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0021.01

$$ \frac{\partial^n {}_3F_2(a_1, a_2, a_3; b_1, b_2; z)}{\partial z^n} = z^{-n} \prod_{j=1}^{2} \Gamma(b_j)\, {}_4\tilde{F}_3(1, a_1, a_2, a_3; 1-n, b_1, b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0022.01

$$ \frac{\partial^n (z^\alpha\, {}_3F_2(a_1, a_2, a_3; b_1, b_2; z))}{\partial z^n} = (-1)^n (-\alpha)_n\, z^{\alpha-n}\, {}_4F_3(\alpha+1, a_1, a_2, a_3; 1-n+\alpha, b_1, b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0023.01

$$ \frac{\partial^n \left(z^{a+n-1}\, {}_3F_2(a, a_2, a_3; b_1, b_2; z) \right)}{\partial z^n} = (a)_n\, z^{a-1}\, {}_3F_2(a+n, a_2, a_3; b_1, b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0024.01

$$ \frac{\partial^n \left(z^{c-1}\, {}_3F_2(a_1, a_2, a_3; c, b_2; z) \right)}{\partial z^n} = (c-n)_n\, z^{c-n-1}\, {}_3F_2(a_1, a_2, a_3; c-n, b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0025.01

$$ \frac{\partial^n \left(z^n\, {}_3F_2\left(-n, a_2, a_3; \frac{1}{2}, b_2; z\right) \right)}{\partial z^n} = n!\, {}_4F_3\left(-n, n+1, a_2, a_3; \frac{1}{2}, 1, b_2; z\right) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0026.01

$$ \frac{\partial^n (z^\alpha\, {}_3F_2(-n, a_2, a_3; b_1, b_2; z))}{\partial z^n} = (-1)^n (-\alpha)_n\, z^{\alpha-n}\, {}_4F_3(-n, \alpha+1, a_2, a_3; -n+\alpha+1, b_1, b_2; z) \text{ /; } n \in \mathbb{N}^+ $$

07.27.20.0027.01

$$\frac{\partial^n\left(z^\alpha\,{}_3F_2\left(-\frac{n}{3},\frac{1-n}{3},\frac{2-n}{3};b_1,b_2;z^m\right)\right)}{\partial z^n}=(-1)^n\,(-\alpha)_n\,z^{\alpha-n}\,{}_{m+3}F_{m+2}\left(-\frac{n}{3},\frac{1-n}{3},\frac{2-n}{3},\frac{\alpha+1}{m},\frac{\alpha+2}{m},\ldots,\frac{\alpha+m}{m};\frac{\alpha-n+1}{m},\frac{\alpha-n+2}{m},\ldots,\frac{\alpha-n+m}{m},b_1,b_2;z^m\right)/;m\in\mathbb{N}^+\bigwedge n\in\mathbb{N}^+$$

07.27.20.0028.01

$$\frac{\partial^n\left(e^{-z}\,{}_3F_2(-n,a_2,a_3;b_1,b_2;z)\right)}{\partial z^n}=(-1)^n\,e^{-z}\sum_{k=0}^{n}\frac{(-n)_k\,z^k}{k!\,(b_1)_k\,(b_2)_k}\,{}_4F_2(-n,k-n,k+a_2,k+a_3;k+b_1,k+b_2;z)\,/;\,n\in\mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

07.27.20.0029.01

$$\frac{\partial^\alpha\,{}_3F_2(a_1,a_2,a_3;b_1,b_2;z)}{\partial z^\alpha}=z^{-\alpha}\prod_{j=1}^{2}\Gamma(b_j)\,{}_4\tilde{F}_3(1,a_1,a_2,a_3;1-\alpha,b_1,b_2;z)$$

Integration

Indefinite integration

Involving only one direct function

07.27.21.0001.01

$$\int{}_3F_2(a_1,a_2,a_3;b_1,b_2;c\,z)\,dz=\frac{(b_1-1)\,(b_2-1)}{c\,(a_1-1)\,(a_2-1)\,(a_3-1)}\,{}_3F_2(a_1-1,a_2-1,a_3-1;b_1-1,b_2-1;c\,z)$$

07.27.21.0002.01

$$\int{}_3F_2(a_1,a_2,a_3;b_1,b_2;z)\,dz=\frac{(b_1-1)\,(b_2-1)}{(a_1-1)\,(a_2-1)\,(a_3-1)}\,{}_3F_2(a_1-1,a_2-1,a_3-1;b_1-1,b_2-1;z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear arguments

07.27.21.0003.01

$$\int z^{\alpha-1}\,{}_3F_2(a_1,a_2,a_3;b_1,b_2;z)\,dz=\frac{z^\alpha}{\alpha}\,{}_4F_3(\alpha,a_1,a_2,a_3;\alpha+1,b_1,b_2;z)$$

Power arguments

07.27.21.0004.01

$$\int z^{\alpha-1}\,{}_3F_2(a_1, a_2, a_3; b_1, b_2; c\,z^r)\,dz = \frac{z^\alpha}{\alpha}\,{}_4F_3\left(\frac{\alpha}{r}, a_1, a_2, a_3; \frac{\alpha}{r}+1, b_1, b_2; c\,z^r\right)$$

Definite integration

For the direct function itself

07.27.21.0005.01

$$\int_0^\infty t^{\alpha-1}\,{}_3F_2(a_1, a_2, a_3; b_1, b_2; -t)\,dt = \frac{\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(\alpha)\,\Gamma(a_1-\alpha)\,\Gamma(a_2-\alpha)\,\Gamma(a_3-\alpha)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-\alpha)\,\Gamma(b_2-\alpha)} \;/;\; 0 < \mathrm{Re}(\alpha) < \min(\mathrm{Re}(a_1), \mathrm{Re}(a_2), \mathrm{Re}(a_3))$$

Involving the direct function

07.27.21.0006.01

$$\int_0^\infty t^{\alpha-1}\,e^{-c\,t}\,{}_3F_2(a_1, a_2, a_3; b_1, b_2; -t)\,dt = \frac{\Gamma(\alpha)\,\Gamma(a_1-\alpha)\,\Gamma(a_2-\alpha)\,\Gamma(a_3-\alpha)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-\alpha)\,\Gamma(b_2-\alpha)}\,{}_3F_3(\alpha, \alpha-b_1+1, \alpha-b_2+1; \alpha-a_1+1, \alpha-a_2+1, \alpha-a_3+1; c) + \frac{\Gamma(\alpha-a_1)\,\Gamma(a_2-a_1)\,\Gamma(a_3-a_1)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)}\,c^{a_1-\alpha}\,{}_3F_3(a_1, a_1-b_1+1, a_1-b_2+1; -\alpha+a_1+1, a_1-a_2+1, a_1-a_3+1; c) + \frac{\Gamma(\alpha-a_2)\,\Gamma(a_1-a_2)\,\Gamma(a_3-a_2)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_3)\,\Gamma(b_1-a_2)\,\Gamma(b_2-a_2)}\,c^{a_2-\alpha}\,{}_3F_3(a_2, a_2-b_1+1, a_2-b_2+1; -\alpha+a_2+1, -a_1+a_2+1, a_2-a_3+1; c) + \frac{\Gamma(\alpha-a_3)\,\Gamma(a_1-a_3)\,\Gamma(a_2-a_3)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)}\,c^{a_3-\alpha}\,{}_3F_3(a_3, a_3-b_1+1, a_3-b_2+1; -\alpha+a_3+1, -a_1+a_3+1, -a_2+a_3+1; c) \;/;\; \mathrm{Re}(\alpha) > 0 \land \mathrm{Re}(c) > 0$$

Integral transforms

Laplace transforms

07.27.22.0001.01

$$\mathcal{L}_t[{}_3F_2(a_1, a_2, a_3; b_1, b_2; -t)](z) = \frac{\pi\csc(\pi a_1)\,\Gamma(a_2-a_1)\,\Gamma(a_3-a_1)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)}\,z^{a_1-1}\,{}_2F_2(a_1-b_1+1, a_1-b_2+1; a_1-a_2+1, a_1-a_3+1; z) + \frac{\pi\csc(\pi a_2)\,\Gamma(a_1-a_2)\,\Gamma(a_3-a_2)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-a_2)\,\Gamma(b_2-a_2)}\,z^{a_2-1}\,{}_2F_2(a_2-b_1+1, a_2-b_2+1; -a_1+a_2+1, a_2-a_3+1; z) + \frac{\pi\csc(\pi a_3)\,\Gamma(a_1-a_3)\,\Gamma(a_2-a_3)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)}\,z^{a_3-1}\,{}_2F_2(a_3-b_1+1, a_3-b_2+1; -a_1+a_3+1, -a_2+a_3+1; z) + \frac{(b_1-1)\,(b_2-1)}{(a_1-1)\,(a_2-1)\,(a_3-1)}\,{}_3F_3(1, 2-b_1, 2-b_2; 2-a_1, 2-a_2, 2-a_3; z) \;/;\; \mathrm{Re}(z) > 0$$

Summation

Infinite summation

07.27.23.0001.01

$$\sum_{k=0}^{\infty} \frac{(a_1)_k}{k!} \, {}_3F_2(-k, a_2, a_3; b_1, b_2; w) \, z^k == \left(\frac{1}{1-z}\right)^{a_1} {}_3F_2\left(a_1, a_2, a_3; b_1, b_2; \frac{z\,w}{z-1}\right)$$

Operations

Limit operation

07.27.25.0001.01

$$\lim_{z\to 1} (1-z)^{-\psi_2} \, {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) == \frac{\Gamma(-\psi_2)\,\Gamma(b_1)\,\Gamma(b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)} \;/;\; \psi_2 == b_1 + b_2 - a_1 - a_2 - a_3 \bigwedge \operatorname{Re}(\psi_2) < 0$$

07.27.25.0002.01

$$\lim_{b_1\to -n} \frac{{}_3F_2(a_1, a_2, a_3; b_1, b_2; z)}{\Gamma(b_1)} == z^{n+1} \, (a_1)_{n+1} \, (a_2)_{n+1} \, (a_3)_{n+1} \, {}_3\tilde{F}_2(n+a_1+1, n+a_2+1, n+a_3+1; n+2, n+b_2+1; z) \;/;\; n \in \mathbb{N}$$

07.27.25.0003.01

$$\lim_{a\to\infty} {}_3F_2\left(a, a_2, a_3; b_1, b_2; \frac{z}{a}\right) == {}_2F_2(a_2, a_3; b_1, b_2; z)$$

07.27.25.0004.01

$$\lim_{b\to\infty} \lim_{a\to\infty} {}_3F_2\left(a, b, a_3; b_1, b_2; \frac{z}{a\,b}\right) == {}_1F_2(a_3; b_1, b_2; z)$$

07.27.25.0005.01

$$\lim_{a\to\infty} {}_3F_2\left(a, a_2, a_3; \frac{a}{z}, b_2; 1\right) == {}_2F_1(a_2, a_3; b_2; z) \;/;\; \operatorname{Re}\left(\frac{a\,(1-z)}{z} - a_2 - a_3 + b_2\right) > 0$$

07.27.25.0006.01

$$\lim_{a\to n} \frac{1}{a^2\,\Gamma(1-a)} \, {}_3F_2(a, a, a; a+1, a+1; 1) == (-1)^{n-1} \, S_n^{(2)} \;/;\; n \in \mathbb{N}$$

07.27.25.0007.01

$$\lim_{z\to 1} {}_3F_2(1-m, 2, 2; 1, 1; z) == (-1)^{m-1} \, \Gamma(m) \, \mathcal{S}_3^{(m)} \;/;\; m \in \mathbb{N}^+$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

07.27.26.0001.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) == \Gamma(b_1)\,\Gamma(b_2)\, {}_3\tilde{F}_2(a_1, a_2, a_3; b_1, b_2; z)$$

Involving ${}_pF_q$

07.27.26.0002.01

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) == {}_pF_q\left(a_1, \ldots, a_p; b_1, \ldots, b_q; z\right) \;/;\; p == 3 \bigwedge q == 2$$

07.27.26.0003.01

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;z)={}_4F_3(a_1,a_2,a_3,a_4;b_1,b_2,a_4;z)$$

Through Meijer *G*

Classical cases for the direct function itself

07.27.26.0004.01

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;z)=\frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)}G_{3,3}^{1,3}\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)$$

07.27.26.0005.01

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;z)=\frac{\Gamma(b_1)\Gamma(b_2)}{\pi\sin(\psi_2\pi)\sin(\pi(b_1-b_2))\prod_{k=1}^{3}\Gamma(a_k)}\left(\prod_{k=1}^{3}\sin(\pi(b_1-a_k))\,G_{3,3}^{2,3}\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)-\prod_{k=1}^{3}\sin(\pi(b_2-a_k))\,G_{3,3}^{2,3}\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_2,1-b_1\end{matrix}\right)\right)-\frac{\pi\Gamma(b_1)\Gamma(b_2)}{\sin(\psi_2\pi)\prod_{k=1}^{3}\Gamma(a_k)}\left((1-z)^{\psi_2}(z-1)^{-\psi_2}G_{3,3}^{0,3}\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)+G_{3,3}^{3,0}\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3\\0,1-b_1,1-b_2\end{matrix}\right)\right)/;\ \psi_2=b_1+b_2-a_1-a_2-a_3\land z\notin(-1,0)\land\psi_2\notin\mathbb{Z}$$

Classical cases involving algebraic functions

07.27.26.0006.01

$$(1-z)^{2b+1}\,{}_3F_2(a,2a-2,b;a-1,2a-b-1;z)=\frac{2\Gamma(2a-b-1)}{\Gamma(2a-2b-2)\Gamma(-b-1)}G_{3,3}^{1,3}\left(-z\,\middle|\,\begin{matrix}b+2,4-2a+2b,\frac{3}{2}-a+b\\0,\frac{5}{2}-a+b,2-2a+b\end{matrix}\right)$$

07.27.26.0007.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^a{}_3F_2\left(a,b,c;a-b+1,a-c+1;2z-2\sqrt{z+1}\sqrt{z}+1\right)=\frac{\Gamma(a-b+1)\Gamma(a-c+1)}{2\sqrt{\pi}\,\Gamma(a)\Gamma(a-b-c+1)}G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-\frac{a}{2},\frac{a}{2}-b+1,\frac{a}{2}-c+1\\0,\frac{1}{2},\frac{a}{2}-b-c+1\end{matrix}\right)/;\ z\notin(-1,0)$$

07.27.26.0008.01

$$\left(\sqrt{z+1}-1\right)^a{}_3F_2\left(a,b,c;a-b+1,a-c+1;\frac{2\left(1-\sqrt{z+1}\right)}{z}+1\right)=\frac{\Gamma(a-b+1)\Gamma(a-c+1)}{2\sqrt{\pi}\,\Gamma(a)\Gamma(a-b-c+1)}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{a}{2}+1,\frac{a+1}{2},b+c\\a,b,c\end{matrix}\right)$$

07.27.26.0009.01

$$(z+1)^{-2a}{}_3F_2\left(a,a+\frac{1}{2},b;c,2a+b-c+1;\frac{4z}{(z+1)^2}\right)=\frac{\Gamma(c)\Gamma(2a+b-c+1)}{\Gamma(2a)\Gamma(2a-c+1)\Gamma(c-b)}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}1-2a,c-2a,b-c+1\\0,1-c,c-2a-b\end{matrix}\right)/;\ |z|<1$$

07.27.26.0010.01

$$(1-z)(z+1)^{-2a}{}_3F_2\left(a,a+\frac{1}{2},b;c,2a+b-c;\frac{4z}{(z+1)^2}\right)=\frac{2\Gamma(c)\Gamma(2a+b-c)}{\Gamma(2a)\Gamma(2a-c)\Gamma(c-b)}G_{4,4}^{1,4}\left(z\,\middle|\,\begin{matrix}2-2a,\frac{1}{2}-a,1-2a+c,b-c+1\\0,\frac{3}{2}-a,1-c,1-2a-b+c\end{matrix}\right)/;\ |z|<1$$

07.27.26.0011.01

$$(z+1)^{-2a}\,{}_3F_2\left(a, a+\frac{1}{2}, b-a+\frac{1}{2}; b, a+\frac{3}{2}; \frac{4z}{(z+1)^2}\right) = \frac{(2a+1)\,\Gamma(b)}{4\,\Gamma(2a-1)\,\Gamma(2a-b+1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} \frac{3}{2}-a, b-2a, 1-2a \\ 0, -a-\frac{1}{2}, 1-b \end{array}\right) /; |z| < 1$$

07.27.26.0012.01

$$(z+4)^{-3a}\,{}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; b, 3a-b+\frac{3}{2}; \frac{27z^2}{(z+4)^3}\right) = \frac{\Gamma(b)\,\Gamma\left(3a-b+\frac{3}{2}\right)}{2\pi\,\Gamma(3a)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} 1-3a, \frac{3}{2}-b, b-3a \\ 0, 2-2b, 2b-6a-1 \end{array}\right) /; z \notin (-1, 0)$$

07.27.26.0013.01

$$(8-z)(z+4)^{-3a}\,{}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; b, 3a-b+\frac{1}{2}; \frac{27z^2}{(z+4)^3}\right) = \frac{3\,\Gamma(b)\,\Gamma\left(3a-b+\frac{1}{2}\right)}{2\pi\,\Gamma(3a)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} 2-3a, \frac{3}{2}-b, \frac{2}{3}-2a, 1-3a+b \\ 0, \frac{5}{3}-2a, 2-2b, 1-6a+2b \end{array}\right) /;$$

$$|z| > 1 \land \mathrm{Im}(z) > 0 \lor |z| > 1 \land \mathrm{Re}(z) < 0 \lor |z| < 1 \land z \notin (-1, 0)$$

07.27.26.0014.01

$$(4z+1)^{-3a}\,{}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; b, 3a-b+\frac{3}{2}; \frac{27z}{(4z+1)^3}\right) = \frac{\Gamma\left(3a-b+\frac{3}{2}\right)\Gamma(b)}{2\pi\,\Gamma(3a)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1-3a, 3a-2b+2, 2b-3a-1 \\ 0, 1-b, b-3a-\frac{1}{2} \end{array}\right) /; |z| > 1$$

07.27.26.0015.01

$$(8z-1)(4z+1)^{-3a}\,{}_3F_2\left(a, a+\frac{1}{3}, a+\frac{2}{3}; b, 3a-b+\frac{1}{2}; \frac{27z}{(4z+1)^3}\right) = \frac{3\,\Gamma\left(3a-b+\frac{1}{2}\right)\Gamma(b)}{2\pi\,\Gamma(3a)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} 2-3a, 2b-3a, \frac{1}{3}-a, 3a-2b+1 \\ 0, b-3a+\frac{1}{2}, \frac{4}{3}-a, 1-b \end{array}\right) /; |z| > 1$$

Classical cases involving unit step θ

07.27.26.0016.01

$$\theta(1-|z|)\,{}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \Gamma(1-a_1)\,\Gamma(1-a_2)\,\Gamma(1-a_3)\,\Gamma(b_1)\,\Gamma(b_2)\, G_{3,3}^{1,0}\left(z \,\middle|\, \begin{array}{c} 1-a_1, 1-a_2, 1-a_3 \\ 0, 1-b_1, 1-b_2 \end{array}\right)$$

Generalized cases involving algebraic functions

07.27.26.0017.01

$$\left(\sqrt{z^2+1}-z\right)^a\,{}_3F_2\left(a, b, c; a-b+1, a-c+1; 2z^2-2\sqrt{z^2+1}\,z+1\right) = \frac{\Gamma(a-b+1)\,\Gamma(a-c+1)}{2\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-b-c+1)}\, G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-\frac{a}{2}, \frac{a}{2}-b+1, \frac{a}{2}-c+1 \\ 0, \frac{1}{2}, \frac{a}{2}-b-c+1 \end{array}\right) /; \mathrm{Re}(z) > 0$$

Through other functions

Involving some hypergeometric-type functions

07.27.26.0018.01

$$ {}_3F_2\left(a,\, b,\, a+\frac{1}{2};\, c,\, c+\frac{1}{2};\, z\right) = F_1\left(2\,a;\, b,\, b;\, 2\,c;\, \sqrt{z}\,,\, -\sqrt{z}\,\right) $$

Theorems

The number of closed pathes from the origin

The number of closed paths from the origin of length $2\,n$ in a three–dimensional cubic lattice is $\frac{4^n\,(1/2)_n}{n!}\,{}_3F_2\left(\frac{1}{2},\, -n,\, -n;\, 1,\, 1;\, 4\right)$.

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