I

Notations

Traditional name

Imaginary unit

Traditional notation

i

Mathematica StandardForm notation

I

Primary definition

02.01.02.0001.01

$i = \sqrt{-1}$

Specific values

02.01.03.0001.01

$i^2 = -1$

General characteristics

The imaginary unit i is a constant. It is the pure complex niumber.

Complex characteristics

Real part

02.01.19.0001.01

$\mathrm{Re}(i) = 0$

Imaginary part

02.01.19.0002.01

$\mathrm{Im}(i) = 1$

Absolute value

02.01.19.0003.01

$$|i| = 1$$

Argument

02.01.19.0004.01

$$\arg(i) = \frac{\pi}{2}$$

Conjugate value

02.01.19.0005.01

$$\bar{i} = -i$$

Signum value

02.01.19.0006.01

$$\operatorname{sgn}(i) = i$$

Differentiation

Low-order differentiation

02.01.20.0001.01

$$\frac{\partial i}{\partial z} = 0$$

Fractional integro-differentiation

02.01.20.0002.01

$$\frac{\partial^\alpha i}{\partial z^\alpha} = \frac{z^{-\alpha}\, i}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

02.01.21.0001.01

$$\int i\, dz = i\, z$$

02.01.21.0002.01

$$\int z^{\alpha-1}\, i\, dz = \frac{z^\alpha\, i}{\alpha}$$

Integral transforms

Fourier exp transforms

02.01.22.0001.01

$$\mathcal{F}_t[i](z) = i\sqrt{2\pi}\,\delta(z)$$

Inverse Fourier exp transforms

02.01.22.0002.01

$$\mathcal{F}_t^{-1}[i](z) = i\sqrt{2\pi}\,\delta(z)$$

Fourier cos transforms

02.01.22.0003.01

$$\mathcal{F}c_t[i](z) = i\sqrt{\frac{\pi}{2}}\,\delta(z)$$

Fourier sin transforms

02.01.22.0004.01

$$\mathcal{F}s_t[i](z) = \sqrt{\frac{2}{\pi}}\,\frac{i}{z}$$

Laplace transforms

02.01.22.0005.01

$$\mathcal{L}_t[i](z) = \frac{i}{z}$$

Inverse Laplace transforms

02.01.22.0006.01

$$\mathcal{L}_t^{-1}[i](z) = i\,\delta(z)$$

Representations through more general functions

Through Meijer *G*

02.01.26.0004.01

$$i = i\,G_{0,1}^{1,0}(z \mid 0) + i\,G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ 1,\, 0 \end{matrix}\right)$$

Through other functions

02.01.26.0001.01

$$i = \sqrt{z} \;/;\; z = -1$$

02.01.26.0002.01

$$i = (-1)^a \;/;\; a = \frac{1}{2}$$

02.01.26.0003.01

$$i = \left(z;\, z^2 + 1\right)_2^{-1}$$

Representations through equivalent functions

02.01.27.0001.01

$$e^{\pi i} == -1$$

identity due to L.Euler

02.01.27.0002.01

$$e^{2 \pi i} == 1$$

02.01.27.0003.01

$$e^{\pi i k} == (-1)^k \text{ /; } k \in \mathbb{Z}$$

02.01.27.0004.01

$$i^i == e^{-\frac{\pi}{2}}$$

History

– Joh. Bernoulli (1702)
– H. Kühn (1753)
– L. Euler (1755) used the word "complex" (1777) and first used the letter i for $\sqrt{-1}$
– H. D. Truel (1786)
– C. V. Mourey (1828)
– J. R. Argand (1806)
– C. F. Gauss (1831) introduced the name "imaginary unit"

The constant i is encountered often in mathematics and the natural sciences.

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