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Notations

Traditional name

Imaginary unit

Traditional notation

i

Mathematica StandardForm notation

I

Primary definition

02.01.02.0001.01

$$i = \sqrt{-1}$$

Specific values

02.01.03.0001.01

$$i^2 = -1$$

General characteristics

The imaginary unit i is a constant. It is the pure complex number.

Complex characteristics

Real part

02.01.19.0001.01

$$\operatorname{Re}(i) = 0$$

Imaginary part

02.01.19.0002.01

$$\operatorname{Im}(i) = 1$$

Absolute value

02.01.19.0003.01

$$|i| = 1$$

Argument

02.01.19.0004.01

$$\arg(i) = \frac{\pi}{2}$$

Conjugate value

02.01.19.0005.01

$$\bar{i} = -i$$

Signum value

02.01.19.0006.01

$$\operatorname{sgn}(i) = i$$

Differentiation

Low-order differentiation

02.01.20.0001.01

$$\frac{\partial i}{\partial z} = 0$$

Fractional integro-differentiation

02.01.20.0002.01

$$\frac{\partial^\alpha i}{\partial z^\alpha} = \frac{z^{-\alpha} i}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

02.01.21.0001.01

$$\int i dz = iz$$

02.01.21.0002.01

$$\int z^{\alpha-1} i dz = \frac{z^\alpha i}{\alpha}$$

Integral transforms

Fourier exp transforms

02.01.22.0001.01

$$\mathcal{F}_t[i](z) = i \sqrt{2\pi} \delta(z)$$

Inverse Fourier exp transforms

02.01.22.0002.01

$$\mathcal{F}_t^{-1}[i](z) = i \sqrt{2\pi} \delta(z)$$

Fourier cos transforms

02.01.22.0003.01

$$\mathcal{F}_{c_t}[i](z) = i \sqrt{\frac{\pi}{2}} \delta(z)$$

Fourier sin transforms

02.01.22.0004.01

$$\mathcal{F}_{s_t}[i](z) = \sqrt{\frac{2}{\pi}} \frac{i}{z}$$

Laplace transforms

02.01.22.0005.01

$$\mathcal{L}_t[i](z) = \frac{i}{z}$$

Inverse Laplace transforms

02.01.22.0006.01

$$\mathcal{L}_t^{-1}[i](z) = i \delta(z)$$

Representations through more general functions

Through Meijer G

02.01.26.0004.01

$$i = i G_{0,1}^{1,0}(z | 0) + i G_{1,2}^{1,1}\left(z \left| \begin{matrix} 1 \\ 1, 0 \end{matrix} \right.\right)$$

Through other functions

02.01.26.0001.01

$$i = \sqrt{z} \ ; \ z = -1$$

02.01.26.0002.01

$$i = (-1)^a \ ; \ a = \frac{1}{2}$$

02.01.26.0003.01

$$i = (z; z^2 + 1)_2^{-1}$$

Representations through equivalent functions

02.01.27.0001.01

$$e^{\pi i} = -1$$

identity due to L.Euler

02.01.27.0002.01

$$e^{2\pi i} = 1$$

02.01.27.0003.01

$$e^{\pi i k} = (-1)^k ; k \in \mathbb{Z}$$

02.01.27.0004.01

$$i^i = e^{-\frac{\pi}{2}}$$

History

- Joh. Bernoulli (1702)
- H. Kühn (1753)
- L. Euler (1755) used the word "complex" (1777) and first used the letter i for $\sqrt{-1}$
- H. D. Truel (1786)
- C. V. Mourey (1828)
- J. R. Argand (1806)
- C. F. Gauss (1831) introduced the name "imaginary unit"

The constant i is encountered often in mathematics and the natural sciences.

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