IntegerPart

Notations

Traditional name

Integer part

Traditional notation

$\operatorname{int}(z)$

Mathematica StandardForm notation

`IntegerPart[z]`

Primary definition

04.04.02.0001.01

$\operatorname{int}(x) = n \;/;\; x \in \mathbb{R} \land n \in \mathbb{Z} \land 0 \leq \operatorname{sgn}(x)\,(x - n) < 1 \land x \neq 0$

04.04.02.0002.01

$\operatorname{int}(z) = \operatorname{int}(\operatorname{Re}(z)) + i \operatorname{int}(\operatorname{Im}(z))$

For real z, the function $\operatorname{int}(z)$ is the integer part of z.

Examples: $\operatorname{int}(3.2) = 3$, $\operatorname{int}(3) = 3$, $\operatorname{int}(-0.2) = 0$, $\operatorname{int}(-2.3) = -2$, $\operatorname{int}\left(\frac{2}{3}\right) = 0$, $\operatorname{int}(-\pi) = -3$, $\operatorname{int}\left(-4 - \frac{5}{3} i\right) = -4 - i$, $\operatorname{int}\left(\frac{5}{2}\right) = 2$, $\operatorname{int}\left(\frac{7}{2}\right) = 3$.

Specific values

Specialized values

04.04.03.0001.01

$\operatorname{int}(x) = x \;/;\; x \in \mathbb{Z}$

04.04.03.0002.01

$\operatorname{int}(i\, x) = i\, x \;/;\; x \in \mathbb{Z}$

04.04.03.0003.01

$\operatorname{int}(x + i\, y) = \operatorname{int}(x) + i \operatorname{int}(y) \;/;\; x \in \mathbb{R} \land y \in \mathbb{R}$

Values at fixed points

04.04.03.0004.01

$\operatorname{int}(0) = 0$

04.04.03.0005.01

$\text{int}(1) = 1$

04.04.03.0006.01

$\text{int}(-1) = -1$

04.04.03.0007.01

$\text{int}(i) = i$

04.04.03.0008.01

$\text{int}(-i) = -i$

04.04.03.0009.01

$\text{int}(2) = 2$

04.04.03.0010.01

$\text{int}(-3) = -3$

04.04.03.0011.01

$\text{int}(-\pi) = -3$

04.04.03.0012.01

$$\text{int}\left(-\frac{27}{10}\right) = -2$$

04.04.03.0013.01

$\text{int}(-3.4) = -3$

04.04.03.0014.01

$$\text{int}\left(\frac{23}{10} - i\, e\right) = 2 - 2\, i$$

Values at infinities

04.04.03.0015.01

$\text{int}(\infty) = \infty$

04.04.03.0016.01

$\text{int}(-\infty) = -\infty$

04.04.03.0017.01

$\text{int}(i\, \infty) = i\, \infty$

04.04.03.0018.01

$\text{int}(-i\, \infty) = -i\, \infty$

04.04.03.0019.01

$\text{int}(\tilde{\infty}) = \tilde{\infty}$

General characteristics

Domain and analyticity

int(z) is a nonanalytical function; it is a piecewise constant function which is defined in the whole complex z-plane.

04.04.04.0001.01

$z \longrightarrow \text{int}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

int(z) is an odd function.

04.04.04.0002.01

$$\text{int}(-z) = -\text{int}(z)$$

Mirror symmetry

04.04.04.0003.01

$$\text{int}(\bar{z}) = \overline{\text{int}(z)}$$

Periodicity

No periodicity

Sets of discontinuity

The function int(z) is a piecewise constant function with unit jumps on the lines $\text{Re}(z) = k \lor \text{Im}(z) = l /; k, l \in \mathbb{Z}, k \neq 0, l \neq 0$.

The function int(z) is continuous from the right on the intervals $(k - i\infty, k + i\infty)$, $k \in \mathbb{N}^+$, and from the left on the intervals $(-k - i\infty, -k + i\infty)$, $k \in \mathbb{N}^+$.

The function int(z) is continuous from above on the intervals $(-\infty + i k, \infty + i k)$, $k \in \mathbb{N}^+$, and from below on the intervals $(-\infty - i k, \infty - i k)$, $k \in \mathbb{N}^+$.

04.04.04.0004.01

$$\mathcal{DS}_z(\text{int}(z)) = \{\{\{(k - i\infty, k + i\infty), -1\} /; k \in \mathbb{N}^+\}, \{\{(-k - i\infty, -k + i\infty), 1\} /; k \in \mathbb{N}^+\}, \{\{(i k - \infty, i k + \infty), -i\} /; k \in \mathbb{N}^+\}, \{\{(-i k - \infty, -i k + \infty), i\} /; k \in \mathbb{N}^+\}\}$$

04.04.04.0005.01

$$\lim_{\epsilon \to +0} \text{int}(z + \epsilon) = \text{int}(z) /; \text{Re}(z) \in \mathbb{N}^+$$

04.04.04.0006.01

$$\lim_{\epsilon \to +0} \text{int}(z - \epsilon) = \text{int}(z) /; -\text{Re}(z) \in \mathbb{N}^+$$

04.04.04.0007.01

$$\lim_{\epsilon \to +0} \text{int}(z + \epsilon) = \text{int}(z) + 1 /; -\text{Re}(z) \in \mathbb{N}^+$$

04.04.04.0008.01

$$\lim_{\epsilon \to +0} \text{int}(z - \epsilon) = \text{int}(z) - 1 /; \text{Re}(z) \in \mathbb{N}^+$$

04.04.04.0009.01

$$\lim_{\epsilon \to +0} \text{int}(z + i\epsilon) = \text{int}(z) /; \text{Im}(z) \in \mathbb{N}^+$$

04.04.04.0010.01

$$\lim_{\epsilon \to +0} \text{int}(z - i\epsilon) = \text{int}(z) /; -\text{Im}(z) \in \mathbb{N}^+$$

04.04.04.0011.01

$$\lim_{\epsilon \to +0} \text{int}(z + i\epsilon) = \text{int}(z) + i /; -\text{Im}(z) \in \mathbb{N}^+$$

04.04.04.0012.01

$$\lim_{\epsilon \to +0} \operatorname{int}(z - i\,\epsilon) = \operatorname{int}(z) - i \;/;\; \operatorname{Im}(z) \in \mathbb{N}^+$$

Series representations

Exponential Fourier series

04.04.06.0001.01

$$\operatorname{int}(x) = x + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2\,\pi\,k\,x)}{k} - \theta(x) + \frac{1}{2} \;/;\; x \in \mathbb{R} \bigwedge x \notin \mathbb{Z}$$

Other series representations

04.04.06.0002.01

$$\operatorname{int}\left(\frac{m}{n}\right) = \frac{m}{n} - \frac{1}{2} \operatorname{sgn}\left(\frac{m}{n}\right) + \frac{1}{2\,n} \sum_{k=1}^{n-1} \sin\left(\frac{2\,\pi\,k\,m}{n}\right) \cot\left(\frac{\pi\,k}{n}\right) \;/;\; m \in \mathbb{Z} \bigwedge n \in \mathbb{Z} \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigwedge n > 1$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

04.04.16.0001.01

$$\operatorname{int}(-z) = -\operatorname{int}(z)$$

04.04.16.0002.01

$$\operatorname{int}(i\,z) = i \operatorname{int}(z)$$

04.04.16.0003.01

$$\operatorname{int}(-i\,z) = -i \operatorname{int}(z)$$

04.04.16.0004.01

$$\operatorname{int}(n + z) = \operatorname{int}(z) + n - \theta(n + z) + \theta(z) \;/;\; n \in \mathbb{Z} \bigwedge z \notin \mathbb{Z}$$

Argument involving related functions

04.04.16.0005.01

$$\operatorname{int}(\operatorname{int}(z)) = \operatorname{int}(z)$$

04.04.16.0006.01

$$\operatorname{int}(z - \operatorname{int}(z)) = 0$$

04.04.16.0007.01

$$\operatorname{int}(\operatorname{frac}(z)) = 0$$

04.04.16.0008.01

$$\operatorname{int}(\lfloor z \rceil) = \lfloor z \rceil$$

04.04.16.0009.01

$$\operatorname{int}(\lfloor z \rfloor) = \lfloor z \rfloor$$

04.04.16.0010.01

$$\operatorname{int}(\lceil z \rceil) = \lceil z \rceil$$

04.04.16.0013.01

$$\operatorname{int}(\operatorname{quotient}(m, n)) = \left\lfloor \frac{m}{n} \right\rfloor$$

Addition formulas

04.04.16.0011.01

$$\operatorname{int}(n + z) = \operatorname{int}(z) + n - \theta(n + z) + \theta(z) \;/;\; n \in \mathbb{Z} \bigwedge z \notin \mathbb{Z}$$

Multiple arguments

04.04.16.0012.01

$$\operatorname{int}(n\, z) = \operatorname{int}(z)\, n + n \operatorname{sgn}(\chi_{\mathbb{Z}}(z) + \theta(z)) - \operatorname{sgn}(\chi_{\mathbb{Z}}(n\, z) + \theta(z)) + \sum_{k=0}^{n-1} k\, \theta\left(z \bmod 1 - \frac{k}{n}\right)\left(1 - \theta\left(z \bmod 1 - \frac{k+1}{n}\right)\right) - n + 1 \;/;$$
$$n \in \mathbb{N} \land z \in \mathbb{R}$$

Complex characteristics

Real part

04.04.19.0001.01

$$\operatorname{Re}(\operatorname{int}(x + i\, y)) = \operatorname{int}(x)$$

04.04.19.0006.01

$$\operatorname{Re}(\operatorname{int}(z)) = \operatorname{int}(\operatorname{Re}(z))$$

Imaginary part

04.04.19.0002.01

$$\operatorname{Im}(\operatorname{int}(x + i\, y)) = \operatorname{int}(y)$$

04.04.19.0007.01

$$\operatorname{Im}(\operatorname{int}(z)) = \operatorname{int}(\operatorname{Im}(z))$$

Absolute value

04.04.19.0003.01

$$|\operatorname{int}(x + i\, y)| = \sqrt{\operatorname{int}(x)^2 + \operatorname{int}(y)^2}$$

04.04.19.0008.01

$$|\operatorname{int}(z)| = \sqrt{\operatorname{int}(\operatorname{Im}(z))^2 + \operatorname{int}(\operatorname{Re}(z))^2}$$

Argument

04.04.19.0004.01

$$\arg(\operatorname{int}(x + i\, y)) = \tan^{-1}(\operatorname{int}(x), \operatorname{int}(y))$$

04.04.19.0009.01

$$\arg(\operatorname{int}(z)) = \tan^{-1}(\operatorname{int}(\operatorname{Re}(z)), \operatorname{int}(\operatorname{Im}(z)))$$

Conjugate value

04.04.19.0005.01

$$\overline{\operatorname{int}(x + i\, y)} = \operatorname{int}(x) - i \operatorname{int}(y)$$

04.04.19.0010.01

$$\overline{\operatorname{int}(z)} = \operatorname{int}(\operatorname{Re}(z)) - i \operatorname{int}(\operatorname{Im}(z))$$

Signum value

04.04.19.0011.01

$$\operatorname{sgn}(\operatorname{int}(x + i\, y)) = \frac{\operatorname{int}(x) + i \operatorname{int}(y)}{\sqrt{\operatorname{int}(x)^2 + \operatorname{int}(y)^2}}$$

04.04.19.0012.01

$$\operatorname{sgn}(\operatorname{int}(z)) = \frac{\operatorname{int}(z)}{|\operatorname{int}(z)|}$$

Differentiation

Low-order differentiation

04.04.20.0001.01

$$\frac{\partial \operatorname{int}(z)}{\partial z} = 0$$

In a distributional sense for $x \in \mathbb{R}$.

04.04.20.0002.01

$$\frac{\partial \operatorname{int}(x)}{\partial x} = \sum_{k=-\infty}^{\infty} \delta_{k,0}\, \delta(x - k)$$

Fractional integro-differentiation

04.04.20.0003.01

$$\frac{\partial^{\alpha} \operatorname{int}(z)}{\partial z^{\alpha}} = \frac{\operatorname{int}(z)\, z^{-\alpha}}{\Gamma(1 - \alpha)}$$

Integration

Indefinite integration

Involving only one direct function

04.04.21.0001.01

$$\int \operatorname{int}(z)\, dz = z \operatorname{int}(z)$$

Involving one direct function and elementary functions

Involving power function

04.04.21.0002.01

$$\int z^{\alpha-1}\,\mathrm{int}(z)\,dz = \frac{z^\alpha\,\mathrm{int}(z)}{\alpha}$$

04.04.21.0003.01

$$\int \frac{\mathrm{int}(z)}{z}\,dz = \log(z)\,\mathrm{int}(z)$$

Definite integration

For the direct function itself

In the following formulas $a \in \mathbb{R}$.

04.04.21.0004.01

$$\int_0^n \mathrm{int}(t)\,dt = \frac{n\,(n-1)}{2} \;/;\; n \in \mathbb{N}$$

04.04.21.0005.01

$$\int_0^a \mathrm{int}(t)\,dt = \frac{1}{2}\,(2\,a - \mathrm{int}(a) - 1)\,\mathrm{int}(a)$$

04.04.21.0006.01

$$\int_0^a t^{\alpha-1}\,\mathrm{int}(t)\,dt = \frac{\mathrm{int}(a)\,a^\alpha - \zeta(-\alpha) + \zeta(-\alpha,\,\mathrm{int}(a)+1)}{\alpha}$$

04.04.21.0007.01

$$\int_a^\infty t^{\alpha-1}\,\mathrm{int}(t)\,dt = -\frac{1}{\alpha}\,(\mathrm{int}(a)\,a^\alpha + \zeta(-\alpha,\,\mathrm{int}(a)+1)) \;/;\; \mathrm{Re}(\alpha) < -1$$

04.04.21.0008.01

$$\int_1^\infty t^{\alpha-1}\,\mathrm{int}(t)\,dt = -\frac{\zeta(-\alpha)}{\alpha} \;/;\; \mathrm{Re}(\alpha) < -1$$

04.04.21.0009.01

$$\int_0^\infty t^{\alpha-1}\,\mathrm{int}(t)\,dt = -\frac{\zeta(-\alpha)}{\alpha} \;/;\; \mathrm{Re}(\alpha) < -1$$

04.04.21.0010.01

$$\int_{-a}^a \mathrm{int}(t)\,dt = 0$$

Integral transforms

Fourier exp transforms

04.04.22.0001.01

$$\mathcal{F}_t[\mathrm{int}(t)]\,(z) = -\frac{i}{\sqrt{2\,\pi}\;z} + \frac{i}{\sqrt{2\,\pi}} \sum_{k=1}^\infty \frac{\delta(2\,k\,\pi - z) - \delta(2\,\pi\,k + z)}{k} - i\,\sqrt{2\,\pi}\;\delta'(z)$$

Fourier cos transforms

04.04.22.0002.01

$$\mathcal{F}c_t[\operatorname{int}(t)](z) = -\frac{1}{\sqrt{2\pi}\, z}\cot\left(\frac{z}{2}\right) - \sqrt{\frac{\pi}{2}}\,\delta(z)$$

Fourier sin transforms

04.04.22.0003.01

$$\mathcal{F}s_t[\operatorname{int}(t)](z) = -\frac{1}{\sqrt{2\pi}\, z} - \sqrt{2\pi}\,\delta'(z) + \frac{1}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{\delta(2k\pi - z) - \delta(2\pi k + z)}{k}$$

Laplace transforms

04.04.22.0004.01

$$\mathcal{L}_t[\operatorname{int}(t)](z) = \frac{1}{(e^z - 1)\, z} \;/;\; \operatorname{Re}(z) > 0$$

Mellin transforms

04.04.22.0005.01

$$\mathcal{M}_t[\operatorname{int}(t)](z) = -\frac{\zeta(-z)}{z} \;/;\; \operatorname{Re}(z) < -1$$

Representations through equivalent functions

With related functions

With Floor

For real arguments

04.04.27.0009.01

$$\operatorname{int}(x) = \lfloor x \rfloor \;/;\; x \in \mathbb{R} \land x > 0 \lor x \in \mathbb{Z}$$

04.04.27.0010.01

$$\operatorname{int}(x) = \lfloor x \rfloor + 1 \;/;\; x \in \mathbb{R} \bigwedge x < 0 \bigwedge x \notin \mathbb{Z}$$

04.04.27.0011.01

$$\operatorname{int}(x) = \lfloor x \rfloor + 1 - \operatorname{sgn}(\chi_{\mathbb{Z}}(x) + \theta(x)) \;/;\; x \in \mathbb{R}$$

For complex arguments

04.04.27.0012.01

$$\operatorname{int}(z) = \lfloor z \rfloor \;/;\; \operatorname{Re}(z) \geq 0 \land \operatorname{Im}(z) \geq 0 \lor z \in \mathbb{Z} \lor i\, z \in \mathbb{Z}$$

04.04.27.0013.01

$$\operatorname{int}(z) = \lfloor z \rfloor + 1 \;/;\; z \in \mathbb{R} \bigwedge z < 0 \bigwedge z \notin \mathbb{Z} \bigvee \operatorname{Re}(z) < 0 \land \operatorname{Im}(z) > 0$$

04.04.27.0014.01

$$\operatorname{int}(z) = \lfloor z \rfloor + i \;/;\; i\, z \in \mathbb{R} \bigwedge i\, z > 0 \bigwedge i\, z \notin \mathbb{Z} \bigvee \operatorname{Re}(z) > 0 \land \operatorname{Im}(z) < 0$$

04.04.27.0015.01

$$\operatorname{int}(z) = \lfloor z \rfloor + 1 + i \;/;\; \operatorname{Re}(z) < 0 \land \operatorname{Im}(z) < 0$$

04.04.27.0001.01

$$\operatorname{int}(z) = \lfloor z \rfloor + 1 + i - \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Re}(z)) + \theta(\operatorname{Re}(z))) - i \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Im}(z)) + \theta(\operatorname{Im}(z)))$$

With Round

For real arguments

04.04.27.0016.01

$$\operatorname{int}(x) = \left\lfloor x - \frac{1}{2} \right\rceil /; x \in \mathbb{R} \bigwedge x \geq 0 \bigwedge \frac{x+1}{2} \notin \mathbb{Z}$$

04.04.27.0017.01

$$\operatorname{int}(x) = \left\lfloor x - \frac{1}{2} \right\rceil + 1 /; x \in \mathbb{R} \land x < 0 \bigvee \frac{x+1}{2} \in \mathbb{Z}$$

04.04.27.0018.01

$$\operatorname{int}(x) = \left\lfloor x - \frac{1}{2} \right\rceil + 1 + \chi_{\mathbb{Z}}\left(\frac{x+1}{2}\right) - \operatorname{sgn}(\chi_{\mathbb{Z}}(x) + \theta(x)) /; x \in \mathbb{R}$$

For complex arguments

04.04.27.0004.01

$$\operatorname{int}(z) = \left\lfloor z - \frac{1+i}{2} \right\rceil + 1 + i + \chi_{\mathbb{Z}}\left(\frac{\operatorname{Re}(z)+1}{2}\right) + i\, \chi_{\mathbb{Z}}\left(\frac{\operatorname{Im}(z)+1}{2}\right) - \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Re}(z)) + \theta(\operatorname{Re}(z))) - i \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Im}(z)) + \theta(\operatorname{Im}(z)))$$

With Ceiling

For real arguments

04.04.27.0019.01

$$\operatorname{int}(x) = \lceil x \rceil - 1 /; x \in \mathbb{R} \bigwedge x > 0 \bigwedge x \notin \mathbb{Z}$$

04.04.27.0020.01

$$\operatorname{int}(x) = \lceil x \rceil /; x \in \mathbb{R} \land x \leq 0 \lor x \in \mathbb{Z}$$

04.04.27.0021.01

$$\operatorname{int}(x) = \lceil x \rceil + \operatorname{sgn}(\chi_{\mathbb{Z}}(-x) + \theta(-x)) - 1 /; x \in \mathbb{R}$$

For complex arguments

04.04.27.0003.01

$$\operatorname{int}(z) = \lceil z \rceil - \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Re}(z)) + \theta(\operatorname{Re}(z))) + i\,(-\operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Im}(z)) + \theta(\operatorname{Im}(z))) - \theta(-\chi_{\mathbb{Z}}(\operatorname{Im}(z))) + \theta(\chi_{\mathbb{Z}}(\operatorname{Re}(z)) - 1) + 1)$$

04.04.27.0002.01

$$\operatorname{int}(z) = -\lceil -z \rceil + 1 + i - \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Re}(z)) + \theta(\operatorname{Re}(z))) - i \operatorname{sgn}(\chi_{\mathbb{Z}}(\operatorname{Im}(z)) + \theta(\operatorname{Im}(z)))$$

With FractionalPart

04.04.27.0005.01

$$\operatorname{int}(z) = z - \operatorname{frac}(z)$$

With Mod

For real arguments

04.04.27.0022.01

$$\mathrm{int}(x) = x - x \bmod 1 \;/;\; x \in \mathbb{R} \land x > 0 \lor x \in \mathbb{Z}$$

04.04.27.0023.01

$$\mathrm{int}(x) = x + 1 - x \bmod 1 \;/;\; x \in \mathbb{R} \bigwedge x < 0 \bigwedge x \notin \mathbb{Z}$$

04.04.27.0024.01

$$\mathrm{int}(x) = x + 1 - x \bmod 1 - \mathrm{sgn}(\chi_{\mathbb{Z}}(x) + \theta(x)) \;/;\; x \in \mathbb{R}$$

For complex arguments

04.04.27.0025.01

$$\mathrm{int}(z) = z - z \bmod 1 \;/;\; \mathrm{Re}(z) \geq 0 \land \mathrm{Im}(z) \geq 0 \lor z \in \mathbb{Z} \lor i\, z \in \mathbb{Z}$$

04.04.27.0026.01

$$\mathrm{int}(z) = z - z \bmod 1 + 1 \;/;\; z \in \mathbb{R} \bigwedge z < 0 \bigwedge z \notin \mathbb{Z} \bigvee \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) > 0$$

04.04.27.0027.01

$$\mathrm{int}(z) = z - z \bmod 1 + i \;/;\; i\, z \in \mathbb{R} \bigwedge i\, z > 0 \bigwedge i\, z \notin \mathbb{Z} \bigvee \mathrm{Re}(z) > 0 \land \mathrm{Im}(z) < 0$$

04.04.27.0028.01

$$\mathrm{int}(z) = z - z \bmod 1 + 1 + i \;/;\; \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) < 0$$

04.04.27.0006.01

$$\mathrm{int}(z) = z + 1 + i - z \bmod 1 - i\, \mathrm{sgn}(\chi_{\mathbb{Z}}(\mathrm{Im}(z)) + \theta(\mathrm{Im}(z))) - \mathrm{sgn}(\chi_{\mathbb{Z}}(\mathrm{Re}(z)) + \theta(\mathrm{Re}(z)))$$

With Quotient

For real arguments

04.04.27.0029.01

$$\mathrm{int}(x) = \mathrm{quotient}(x,\, 1) \;/;\; x \in \mathbb{R} \land x > 0 \lor x \in \mathbb{Z}$$

04.04.27.0030.01

$$\mathrm{int}(x) = \mathrm{quotient}(x,\, 1) + 1 \;/;\; x \in \mathbb{R} \bigwedge x < 0 \bigwedge x \notin \mathbb{Z}$$

04.04.27.0031.01

$$\mathrm{int}(x) = \mathrm{quotient}(x,\, 1) + 1 - \mathrm{sgn}(\chi_{\mathbb{Z}}(x) + \theta(x)) \;/;\; x \in \mathbb{R}$$

For complex arguments

04.04.27.0032.01

$$\mathrm{int}(z) = \mathrm{quotient}(z,\, 1) \;/;\; \mathrm{Re}(z) \geq 0 \land \mathrm{Im}(z) \geq 0 \lor z \in \mathbb{Z} \lor i\, z \in \mathbb{Z}$$

04.04.27.0033.01

$$\mathrm{int}(z) = \mathrm{quotient}(z,\, 1) + 1 \;/;\; z \in \mathbb{R} \bigwedge z < 0 \bigwedge z \notin \mathbb{Z} \bigvee \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) > 0$$

04.04.27.0034.01

$$\mathrm{int}(z) = \mathrm{quotient}(z,\, 1) + i \;/;\; i\, z \in \mathbb{R} \bigwedge i\, z > 0 \bigwedge i\, z \notin \mathbb{Z} \bigvee \mathrm{Re}(z) > 0 \land \mathrm{Im}(z) < 0$$

04.04.27.0035.01

$$\mathrm{int}(z) = \mathrm{quotient}(z,\, 1) + 1 + i \;/;\; \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) < 0$$

04.04.27.0007.01

$$\mathrm{int}(z) = \mathrm{quotient}(z,\, 1) + 1 + i - i\, \mathrm{sgn}(\chi_{\mathbb{Z}}(\mathrm{Im}(z)) + \theta(\mathrm{Im}(z))) - \mathrm{sgn}(\chi_{\mathbb{Z}}(\mathrm{Re}(z)) + \theta(\mathrm{Re}(z)))$$

With elementary functions

04.04.27.0008.01

$$\operatorname{int}(z) == z + \frac{\tan^{-1}(\cot(\pi z))}{\pi} - \operatorname{sgn}(\theta(z)) + \frac{1}{2} /; z \in \mathbb{R} \bigwedge z \notin \mathbb{Z}$$

Zeros

04.04.30.0001.01

$$\operatorname{int}(z) == 0 /; |\operatorname{Re}(z)| < 1 \land |\operatorname{Im}(z)| < 1$$

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