**InverseErf2**

**Notations**

**Traditional name**

Inverse of the generalized error function

**Traditional notation**

\[ \text{erf}^{-1}(z_1, z_2) \]

**Mathematica StandardForm notation**

\[ \text{InverseErf}[z_1, z_2] \]

**Primary definition**

\[ z_2 = \text{erf}(z_1, w) /; w = \text{erf}^{-1}(z_1, z_2) \]

**Specific values**

**Specialized values**

\[ \text{erf}^{-1}(0, z) = \text{erf}^{-1}(z) \]

**Values at fixed points**

\[ \text{erf}^{-1}(0, 0) = 0 \]

\[ \text{erf}^{-1}(0, 1) = \infty \]

\[ \text{erf}^{-1}(1, 0) = 1 \]

**Values at infinities**

\[ \text{erf}^{-1}(\infty, z) = \text{erfc}^{-1}(-z) \]

**General characteristics**
Domain and analyticity

\[ \text{erf}^{-1}(z_1, z_2) \text{ is an analytical function of } z_1 \text{ and } z_2 \text{ which is defined in } \mathbb{C}^2. \]

\[ (z_1 + z_2) \rightarrow \text{erf}^{-1}(z_1, z_2) : (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C} \]

Symmetries and periodicities

Symmetry
No symmetry

Periodicity
No periodicity

Series representations

Generalized power series

Expansions at \( z_1 = 0 \)

\[ \text{erf}^{-1}(z_1, z_2) \propto \text{erf}^{-1}(z_2) + e^{\text{erf}^{-1}(z_2)^2} \left( z_1 + e^{\text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2) \frac{z_1^2}{3} + \left( 4 e^{2 \text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2)^2 + e^{2 \text{erf}^{-1}(z_2)^2} - 1 \right) \frac{z_1^3}{3} + \right. \]

\[ \left. e^{\text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2) \left( 12 e^{2 \text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2)^2 + 7 e^{2 \text{erf}^{-1}(z_2)^2} - 4 \right) \frac{z_1^4}{6} + \left( 96 e^{4 \text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2)^4 - 
\right. \right. \]

\[ \left. \left. 40 e^{2 \text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2)^2 + 92 e^{2 \text{erf}^{-1}(z_2)^2} \text{erf}^{-1}(z_2)^2 - 10 e^{2 \text{erf}^{-1}(z_2)^2} + 7 e^{2 \text{erf}^{-1}(z_2)^2} + 3 \right) \frac{z_1^5}{30} + \ldots \right) / (z_1 \to 0) \]

Expansions at \( z_2 = 0 \)

\[ \text{erf}^{-1}(z_1, z_2) \propto z_1 + \left. \frac{1}{2} e^2 \sqrt{\pi} z_2 + \frac{\pi z_1}{4} e^{\frac{z_1^2}{2}} z_2 + \frac{\pi^{3/2}(1 + 4 z_1^2)}{24} e^{\frac{z_1^2}{2}} z_2^3 + \right. \]

\[ \left. \frac{\pi^2 z_1 (7 + 12 z_1^2)}{96} e^{\frac{z_1^2}{2}} z_2^4 + \frac{\pi^{3/2}(7 + 8 z_1^2)(1 + 12 z_1^2)}{960} e^{\frac{z_1^2}{2}} z_2^5 + \frac{\pi^3 z_1 (127 + 652 z_1^2 + 480 z_1^4)}{5760} e^{\frac{z_1^2}{2}} z_2^6 + \right. \]

\[ \left. \frac{\pi^{7/2}(127 + 3480 z_1^2 + 10224 z_1^4 + 5760 z_1^6)}{80640} e^{\frac{z_1^2}{2}} z_2^7 + \frac{\pi^4 z_1 (4369 + 44808 z_1^2 + 88848 z_1^4 + 40320 z_1^6)}{645120} e^{\frac{z_1^2}{2}} z_2^8 + \ldots / (z_2 \to 0) \right) \]

Differential equations
Ordinary nonlinear differential equations

\[ w''(z_2) - 2w(z_2) w'(z_2)^2 = 0 \; / \; w(z_2) = \text{erf}^{-1}(z_1, z_2) \]

Differentiation

Low-order differentiation

With respect to \( z_1 \)

\[ \frac{\partial \text{erf}^{-1}(z_1, z_2)}{\partial z_1} = e^{\text{erf}^{-1}(z_1, z_2)^2 - z_1^2} \]

\[ \frac{\partial^2 \text{erf}^{-1}(z_1, z_2)}{\partial z_1^2} = e^{\text{erf}^{-1}(z_1, z_2)^2 - z_1^2} \left( 2 e^{\text{erf}^{-1}(z_1, z_2)^2 - z_1^2} \text{erf}^{-1}(z_1, z_2) - 2 z_1 \right) \]

With respect to \( z_2 \)

\[ \frac{\partial \text{erf}^{-1}(z_1, z_2)}{\partial z_2} = \frac{\sqrt{\pi}}{2} e^{\text{erf}^{-1}(z_1, z_2)^2} \]

\[ \frac{\partial^2 \text{erf}^{-1}(z_1, z_2)}{\partial z_2^2} = \frac{\pi}{2} e^{2 \text{erf}^{-1}(z_1, z_2)^2} \text{erf}^{-1}(z_1, z_2) \]

Symbolic differentiation

With respect to \( z_2 \)

\[ \frac{\partial^n \text{erf}^{-1}(z_1, z_2)}{\partial z_2^n} = \text{erf}^{-1}(z_1, z_2) \delta_n + \frac{\pi^{n/2}}{2^n} e^{n \text{erf}^{-1}(z_1, z_2)^2} \sum_{j=0}^{n} \sum_{j=0}^{n} \Delta_{2n-(j+1)j-n-1} \left( \frac{(-1)^n}{j!} \right) \left( \sum_{i=2}^{n} \frac{2^{i-1} e^{\text{erf}^{-1}(z_1, z_2)^2} \sqrt{\pi} \text{erf}^{-1}(z_1, z_2)^{1-i}}{i!} \right)^h \]

\[ \left( \text{erf}^{-1}(z_1, z_2)^2 \right) \left( \text{erf}^{-1}(z_1, z_2)^2 \right)^{1-h} \left( \frac{1}{2} \right)^{1-h} \left( 1 - \frac{1}{2} \right)^{1-h} \left( \frac{3 - i}{2} \right)^{1-h} \left( \frac{-\text{erf}^{-1}(z_1, z_2)^2}{2} \right)^{1-h} \]

Integration

Indefinite integration

Involving one direct function with respect to \( z_2 \)

\[ \int \text{erf}^{-1}(z_1, z_2) \, dz_2 = -\frac{1}{\sqrt{\pi}} e^{-\text{erf}^{-1}(z_1, z_2)^2} \]
Representations through equivalent functions

With inverse function

\[ \text{erf}(z_1, \text{erf}^{-1}(z_1, z_2)) = z_2 \]

Zeros

\[ \text{erf}^{-1}(z_1, z_2) = 0 \big/; z_1 = 0 \wedge z_2 = 0 \]
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