InverseErfc

Notations

Traditional name

Inverse complementary error function

Traditional notation

$\operatorname{erfc}^{-1}(z)$

Mathematica StandardForm notation

`InverseErfc[z]`

Primary definition

06.31.02.0001.01

$z = \operatorname{erfc}(w) \,/;\, w = \operatorname{erfc}^{-1}(z)$

Specific values

Values at fixed points

06.31.03.0001.01

$\operatorname{erfc}^{-1}(0) = \infty$

06.31.03.0002.01

$\operatorname{erfc}^{-1}(1) = 0$

General characteristics

Domain and analyticity

$\operatorname{erfc}^{-1}(z)$ is an analytical function of z which is defined in the whole complex z-plane.

06.31.04.0001.01

$z \longrightarrow \operatorname{erfc}^{-1}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

06.31.06.0003.01

$$\text{erfc}^{-1}(z) \propto \text{erfc}^{-1}(z_0) - \frac{\sqrt{\pi}}{2} e^{\text{erfc}^{-1}(z_0)^2} (z - z_0) + \frac{\pi}{4} e^{2\,\text{erfc}^{-1}(z_0)^2} \text{erfc}^{-1}(z_0) (z - z_0)^2 - \frac{\pi^{3/2}}{24} e^{3\,\text{erfc}^{-1}(z_0)^2} \left(4\,\text{erfc}^{-1}(z_0)^2 + 1\right) (z - z_0)^3 + \frac{\pi^2}{96} e^{4\,\text{erfc}^{-1}(z_0)^2} \text{erfc}^{-1}(z_0) \left(12\,\text{erfc}^{-1}(z_0)^2 + 7\right) (z - z_0)^4 - \frac{\pi^{5/2}}{960} e^{5\,\text{erfc}^{-1}(z_0)^2} \left(8\,\text{erfc}^{-1}(z_0)^2 + 7\right) \left(12\,\text{erfc}^{-1}(z_0)^2 + 1\right) (z - z_0)^5 + \frac{\pi^3}{5760} e^{6\,\text{erfc}^{-1}(z_0)^2} \text{erfc}^{-1}(z_0) \left(480\,\text{erfc}^{-1}(z_0)^4 + 652\,\text{erfc}^{-1}(z_0)^2 + 127\right) (z - z_0)^6 - \frac{\pi^{7/2}}{80\,640} e^{7\,\text{erfc}^{-1}(z_0)^2} \left(5760\,\text{erfc}^{-1}(z_0)^6 + 10\,224\,\text{erfc}^{-1}(z_0)^4 + 3480\,\text{erfc}^{-1}(z_0)^2 + 127\right) (z - z_0)^7 + \frac{\pi^4}{645\,120} e^{8\,\text{erfc}^{-1}(z_0)^2} \text{erfc}^{-1}(z_0) \left(40\,320\,\text{erfc}^{-1}(z_0)^6 + 88\,848\,\text{erfc}^{-1}(z_0)^4 + 44\,808\,\text{erfc}^{-1}(z_0)^2 + 4369\right) (z - z_0)^8 - \frac{\pi^{9/2}}{11\,612\,160} e^{9\,\text{erfc}^{-1}(z_0)^2} \left(645\,120\,\text{erfc}^{-1}(z_0)^8 + 1\,703\,808\,\text{erfc}^{-1}(z_0)^6 + 1\,161\,168\,\text{erfc}^{-1}(z_0)^4 + 204\,328\,\text{erfc}^{-1}(z_0)^2 + 4369\right) (z - z_0)^9 + \frac{\pi^5}{116\,121\,600} e^{10\,\text{erfc}^{-1}(z_0)^2} \text{erfc}^{-1}(z_0) \left(5\,806\,080\,\text{erfc}^{-1}(z_0)^8 + 17\,914\,752\,\text{erfc}^{-1}(z_0)^6 + 15\,561\,936\,\text{erfc}^{-1}(z_0)^4 + 4\,161\,288\,\text{erfc}^{-1}(z_0)^2 + 243\,649\right) (z - z_0)^{10} + \ldots \;/;\; (z \to z_0)$$

06.31.06.0004.01

$$\text{erfc}^{-1}(z) \propto \text{erfc}^{-1}(z_0) (1 + O(z - z_0))$$

Expansions at $z == 1$

06.31.06.0001.02

$$\text{erfc}^{-1}(z) == -\frac{\sqrt{\pi}}{2} \left((z-1) + \frac{\pi}{12} (z-1)^3 + \frac{7\pi^2}{480} (z-1)^5 + \ldots\right) /; (z \to 1)$$

06.31.06.0005.01

$$\text{erfc}^{-1}(z) == -\frac{\sqrt{\pi}}{2} \left((z-1) + \frac{\pi}{12} (z-1)^3 + \frac{7\pi^2}{480} (z-1)^5\right) + O\left((z-1)^7\right)$$

06.31.06.0006.01

$$\text{erfc}^{-1}(z) \propto -\frac{1}{2} \sqrt{\pi}\, (z-1) \left(1 + \frac{\pi}{12} (z-1)^2 + \frac{7\pi^2}{480} (z-1)^4 + \frac{127\pi^3}{40\,320} (z-1)^6 + \frac{4369\pi^4}{5\,806\,080} (z-1)^8\right) + O\left((z-1)^{11}\right)$$

06.31.06.0002.01

$$\operatorname{erfc}^{-1}(z) == -\sum_{k=0}^{\infty} \frac{c_k}{2k+1} \left(\frac{\sqrt{\pi}}{2} (z-1) \right)^{2k+1} /; c_0 == 1 \bigwedge c_k == \sum_{m=0}^{k-1} \frac{c_m c_{k-1-m}}{(m+1)(2m+1)}$$

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06.31.06.0007.01

$$\operatorname{erfc}^{-1}(z) == -\frac{\sqrt{\pi}}{2} (z-1) + O\left((z-1)^3\right)$$

Asymptotic series expansions

06.31.06.0008.01

$$\operatorname{erfc}^{-1}(z) \propto \frac{1}{\sqrt{2}} \sqrt{\log\left(\frac{2}{\pi z^2}\right) - \log\left(\log\left(\frac{2}{\pi z^2}\right)\right)} /; (z \to 0)$$

Differential equations

Ordinary nonlinear differential equations

06.31.13.0001.01

$$w''(z) - 2 w(z) w'(z)^2 == 0 /; w(z) == \operatorname{erfc}^{-1}(z)$$

Differentiation

Low-order differentiation

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$$\frac{\partial \operatorname{erfc}^{-1}(z)}{\partial z} == -\frac{\sqrt{\pi}}{2} e^{\operatorname{erfc}^{-1}(z)^2}$$

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$$\frac{\partial^2 \operatorname{erfc}^{-1}(z)}{\partial z^2} == -\frac{\pi}{2} e^{2 \operatorname{erfc}^{-1}(z)^2} \operatorname{erfc}^{-1}(z)$$

Symbolic differentiation

06.31.20.0003.01

$$\frac{\partial^n \operatorname{erfc}^{-1}(z)}{\partial z^n} == \operatorname{erfc}^{-1}(z) \delta_n - \frac{\pi^{n/2}}{2^n} e^{n \operatorname{erfc}^{-1}(z)^2} \sum_{j_2=0}^{n} \cdots \sum_{j_n=0}^{n} \delta_{\sum_{i=2}^{n} (i-1) j_i, n-1} (-1)^{\sum_{i=2}^{n} j_i} \left(n + \sum_{i=2}^{n} j_i - 1\right)!$$
$$\prod_{i=2}^{n} \frac{1}{j_i!} \left(\frac{(-1)^{i-1} 2^{i-1} e^{\operatorname{erfc}^{-1}(z)^2} \sqrt{\pi} \operatorname{erfc}^{-1}(z)^{1-i}}{i!} \right)^{j_i} {}_2\tilde{F}_2\left(\frac{1}{2}, 1; 1 - \frac{i}{2}, \frac{3-i}{2}; -\operatorname{erfc}^{-1}(z)^2\right)^{j_i} /; n \in \mathbb{N}$$

Integration

Indefinite integration

Involving only one direct function

06.31.21.0001.01

$$\int \operatorname{erfc}^{-1}(z)\, dz = \frac{1}{\sqrt{\pi}} e^{-\operatorname{erfc}^{-1}(z)^2}$$

Representations through more general functions

Through other functions

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$$\operatorname{erfc}^{-1}(z) = \operatorname{erf}^{-1}(\infty, -z)$$

Representations through equivalent functions

With inverse function

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$$\operatorname{erfc}\left(\operatorname{erfc}^{-1}(z)\right) = z$$

06.31.27.0003.01

$$\operatorname{erfc}\left(\operatorname{erf}^{-1}(1-z)\right) = z$$

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$$\operatorname{erfc}\left(\operatorname{erf}^{-1}(\infty, -z)\right) = z$$

With related functions

06.31.27.0002.01

$$\operatorname{erfc}^{-1}(z) = \operatorname{erf}^{-1}(1-z)$$

Zeros

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$$\operatorname{erfc}^{-1}(0) = \infty$$

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$$\operatorname{erfc}^{-1}(z) = 0 \;/;\; z = 1$$

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