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InverseJacobiDC

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Notations

Traditional name

Inverse of the Jacobi elliptic function cs

Traditional notation

$\text{dc}^{-1}(z \mid m)$

Mathematica StandardForm notation

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InverseJacobiDC[z, m]
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Primary definition

09.40.02.0001.01

$$z = \text{dc}(w \mid m)\ /;\ w = \text{dc}^{-1}(z \mid m)$$

09.40.02.0002.01

$$\text{dc}^{-1}(z \mid m) = \int_1^z \frac{1}{\sqrt{t^2-1}\,\sqrt{t^2-m}}\,dt\ /;\ z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 - m > 0 \bigwedge m < 1$$

Specific values

Specialized values

For fixed *z*

09.40.03.0001.01

$$\text{dc}^{-1}(z \mid 0) = \sec^{-1}(z)$$

09.40.03.0002.01

$$\text{dc}^{-1}\left(z \,\middle|\, \frac{1}{2}\right) = \sqrt{2}\,\left(K(2) - F\left(\sin^{-1}(z) \,\middle|\, 2\right)\right)\ /;\ z > 1$$

09.40.03.0003.01

$$\text{dc}^{-1}(z \mid 1) = \tilde{\infty}$$

For fixed *m*

09.40.03.0004.01

$$\text{dc}^{-1}(-1 \mid m) = \frac{2}{\sqrt{m}}\,K\left(\frac{1}{m}\right)\ /;\ m > 1$$

09.40.03.0005.01

$$\mathrm{dc}^{-1}\left(-\frac{1}{2} \,\middle|\, m\right) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) + F\left(\frac{\pi}{6} \,\middle|\, \frac{1}{m}\right)\right) /; m > 1$$

09.40.03.0006.01

$$\mathrm{dc}^{-1}(0 \mid m) = \frac{1}{\sqrt{m}} K\left(\frac{1}{m}\right) /; m > 1$$

09.40.03.0007.01

$$\mathrm{dc}^{-1}\left(\frac{1}{2} \,\middle|\, m\right) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - F\left(\frac{\pi}{6} \,\middle|\, \frac{1}{m}\right)\right) /; m > 1$$

09.40.03.0008.01

$$\mathrm{dc}^{-1}(1 \mid m) = 0$$

09.40.03.0009.01

$$\mathrm{dc}^{-1}(i \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - F\left(\sin^{-1}(i) \,\middle|\, \frac{1}{m}\right)\right) /; m > 1$$

09.40.03.0010.01

$$\mathrm{dc}^{-1}(-i \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) + F\left(\sin^{-1}(i) \,\middle|\, \frac{1}{m}\right)\right) /; m > 1$$

Values at infinities

09.40.03.0011.01

$$\mathrm{dc}^{-1}(z \mid \infty) = 0$$

09.40.03.0012.01

$$\mathrm{dc}^{-1}(z \mid -\infty) = 0$$

09.40.03.0013.01

$$\mathrm{dc}^{-1}(\infty \mid m) = K(m)$$

09.40.03.0014.01

$$\mathrm{dc}^{-1}(-\infty \mid m) = -2\, i\, K(1-m) + \frac{2}{\sqrt{m}} K\left(\frac{1}{m}\right) - K(m) /; m > 1$$

General characteristics

Domain and analyticity

$\mathrm{dc}^{-1}(z \mid m)$ is an analytical function of z and m which is defined over $\mathbb{C}^2$.

09.40.04.0001.01

$$(z * m) \longrightarrow \mathrm{dc}^{-1}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

09.40.04.0002.01

$$\mathrm{dc}^{-1}(\bar{z} \mid \bar{m}) = \overline{\mathrm{dc}^{-1}(z \mid m)}$$

Quasi-reflection symmetry

09.40.04.0003.01

$$\operatorname{dc}^{-1}(-z \mid m) = \frac{2}{\sqrt{m}} F\left(\sin^{-1}(z) \,\middle|\, \frac{1}{m}\right) + \operatorname{dc}^{-1}(z \mid m)$$

Poles and essential singularities

With respect to *m*

The function $\operatorname{dc}^{-1}(z \mid m)$ does not have poles and essential singularities with respect to m.

09.40.04.0004.01

$$\mathcal{S}ing_m\left(\operatorname{dc}^{-1}(z \mid m)\right) = \{\}$$

With respect to *z*

The function $\operatorname{dc}^{-1}(z \mid m)$ does not have poles and essential singularities with respect to z.

09.40.04.0005.01

$$\mathcal{S}ing_z\left(\operatorname{dc}^{-1}(z \mid m)\right) = \{\}$$

Branch points

With respect to *m*

For fixed z, the function $\operatorname{dc}^{-1}(z \mid m)$ has two branch points: $m = z^2$, $m = \tilde{\infty}$.

09.40.04.0006.01

$$\mathcal{BP}_m\left(\operatorname{dc}^{-1}(z \mid m)\right) = \left\{z^2, \tilde{\infty}\right\}$$

09.40.04.0007.01

$$\mathcal{R}_m\left(\operatorname{dc}^{-1}(z \mid m), z^2\right) = \log$$

09.40.04.0008.01

$$\mathcal{R}_m\left(\operatorname{dc}^{-1}(z \mid m), \tilde{\infty}\right) = 2$$

With respect to *z*

For fixed m, the function $\operatorname{dc}^{-1}(z \mid m)$ has six branch points: $z = 0$, $z = \pm 1$, $z = \pm\sqrt{m}$, $z = \tilde{\infty}$.

09.40.04.0009.01

$$\mathcal{BP}_z\left(\operatorname{dc}^{-1}(z \mid m)\right) = \left\{0, 1, -1, \sqrt{m}, -\sqrt{m}, \tilde{\infty}\right\}$$

09.40.04.0010.01

$$\mathcal{R}_z\left(\operatorname{dc}^{-1}(z \mid m), 0\right) = \log$$

09.40.04.0011.01

$$\mathcal{R}_z\left(\operatorname{dc}^{-1}(z \mid m), 1\right) = 2$$

09.40.04.0012.01

$$\mathcal{R}_z\left(\operatorname{dc}^{-1}(z \mid m), -1\right) = 2$$

09.40.04.0013.01

$$\mathcal{R}_z\left(\operatorname{dc}^{-1}(z \mid m), \sqrt{m}\right) == 2$$

09.40.04.0014.01

$$\mathcal{R}_z\left(\operatorname{dc}^{-1}(z \mid m), -\sqrt{m}\right) == 2$$

09.40.04.0015.01

$$\mathcal{R}_z\left(\operatorname{dc}^{-1}(z \mid m), \tilde{\infty}\right) == \log$$

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

Expansions at $z == 0$

09.40.06.0001.02

$$\operatorname{dc}^{-1}(z \mid m) \propto \frac{1}{\sqrt{m}}\left(K\left(\frac{1}{m}\right) - z - \frac{(1+m)\,z^3}{6\,m} - \frac{\left(3+2\,m+3\,m^2\right)z^5}{40\,m^2} \ldots\right) /; (z \to 0)$$

09.40.06.0002.01

$$\operatorname{dc}^{-1}(z \mid m) == \frac{1}{\sqrt{m}}\,K\left(\frac{1}{m}\right) - \frac{1}{\sqrt{m}} \sum_{k=0}^{\infty} \frac{m^{-k}\left(\frac{1}{2}\right)_k}{(2\,k+1)\,k!}\,{}_2F_1\left(\frac{1}{2}, -k; \frac{1}{2}-k; m\right) z^{2\,k+1}$$

09.40.06.0007.01

$$\operatorname{dc}^{-1}(z \mid m) \propto \frac{1}{\sqrt{m}}\,K\left(\frac{1}{m}\right)(1+O(z))$$

Expansions at $m == 0$

09.40.06.0003.02

$$\operatorname{dc}^{-1}(z \mid m) \propto \sec^{-1}(z) + \frac{1}{4\,z}\left(z\sec^{-1}(z) + \sqrt{1-\frac{1}{z^2}}\right)m + \frac{3}{64\,z^3}\left(3\sec^{-1}(z)\,z^3 + \left(3\,z^2+2\right)\sqrt{1-\frac{1}{z^2}}\right)m^2 + \ldots /; (m \to 0)$$

09.40.06.0004.01

$$\operatorname{dc}^{-1}(z \mid m) == \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k}{k!}\left(\frac{\sqrt{\pi}\;\Gamma\left(k+\frac{3}{2}\right)}{(2\,k+1)\,k!} - \frac{z^{-2\,k-1}}{2\,k+1}\,{}_2F_1\left(\frac{1}{2}, k+\frac{1}{2}; k+\frac{3}{2}; \frac{1}{z^2}\right)\right)m^k$$

09.40.06.0008.01

$$\operatorname{dc}^{-1}(z \mid m) = \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k^2}{(k!)^2}\left(\sec^{-1}(z) + \frac{1}{2}\,z\sqrt{1-\frac{1}{z^2}}\,\sum_{j=1}^{k} \frac{(j-1)!\,z^{-2\,j}}{\left(\frac{1}{2}\right)_j}\right)m^k /; |m| < 1$$

09.40.06.0005.01

$$\operatorname{dc}^{-1}(z \mid m) == K(m) - \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{\left(\frac{1}{2}\right)_j \left(\frac{1}{2}\right)_k m^k z^{-2j-2k-1}}{j!\, k!\, (2j + 2k + 1)}$$

09.40.06.0006.01

$$\operatorname{dc}^{-1}(z \mid m) == K(m) - \frac{1}{z} F_{1\times 0\times 0}^{1\times 1\times 1}\left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \\ \frac{3}{2};;; \end{matrix} \frac{m}{z^2}, \frac{1}{z^2}\right)$$

09.40.06.0009.01

$$\operatorname{dc}^{-1}(z \mid m) \propto \sec^{-1}(z)\,(1 + O(m))$$

Integral representations

On the real axis

Of the direct function

09.40.07.0001.01

$$\operatorname{dc}^{-1}(z \mid m) == \int_1^z \frac{1}{\sqrt{t^2 - 1}\, \sqrt{t^2 - m}}\, dt \,/;\, z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 - m > 0 \bigwedge m < 1$$

09.40.07.0002.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{\sqrt{z^2 - m}\, \operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{\sqrt{z^2 - 1}} \int_1^z \frac{1}{\sqrt{t^2 - 1}\, \sqrt{t^2 - m}}\, dt \,/;$$

$$\neg \exists_{\tau, \{\tau \in \mathbb{R}, 0 < \tau < 1\}} \left(\operatorname{Im}\left(((z-1)\,\tau + 1)^2 - 1\right) = 0 \bigwedge ((z-1)\,\tau + 1)^2 - 1 < 0 \bigwedge \operatorname{Im}\left(((z-1)\,\tau + 1)^2 - m\right) = 0 \bigwedge ((z-1)\,\tau + 1)^2 - m < 0\right)$$

09.40.07.0003.01

$$\operatorname{dc}^{-1}(z \mid m) = \operatorname{dc}^{-1}(z_0 \mid m) + \frac{\sqrt{z^2 - m}\, \operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{\sqrt{z^2 - 1}} \int_{z_0}^z \frac{1}{\sqrt{t^2 - 1}\, \sqrt{t^2 - m}}\, dt \,/;$$

$$\neg \exists_{\tau, \{\tau \in \mathbb{R}, 0 < \tau < 1\}} \left(\operatorname{Im}\left((\tau\,(z - z_0) + z_0)^2 - 1\right) = 0 \bigwedge (\tau\,(z - z_0) + z_0)^2 - 1 < 0 \bigwedge \operatorname{Im}\left((\tau\,(z - z_0) + z_0)^2 - m\right) = 0 \bigwedge (\tau\,(z - z_0) + z_0)^2 - m < 0\right)$$

09.40.07.0004.01

$$\operatorname{dc}^{-1}(z \mid m) = K(m) - \frac{\sqrt{z^2 - m}\, \operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{\sqrt{z^2 - 1}} \int_z^{\infty} \frac{1}{\sqrt{t^2 - 1}\, \sqrt{t^2 - m}}\, dt \,/;$$

$$\neg \exists_{\tau, \{\tau \in \mathbb{R}, 0 < \tau < 1\}} \left(\operatorname{Im}\left(\tau^2 z^2 - 1\right) = 0 \bigwedge \tau^2 z^2 - 1 < 0 \bigwedge \operatorname{Im}\left(\tau^2 z^2 - m\right) = 0 \bigwedge \tau^2 z^2 - m < 0\right)$$

Differential equations

Ordinary nonlinear differential equations

09.40.13.0001.01

$$w''(z) + \left(2\,z^2 - m - 1\right) z\, w'(z)^3 = 0 \;/;\; w(z) = \operatorname{dc}^{-1}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.40.16.0001.01

$$\operatorname{dc}^{-1}(-z \mid m) = \frac{2}{\sqrt{m}} F\left(\sin^{-1}(z) \,\middle|\, \frac{1}{m}\right) + \operatorname{dc}^{-1}(z \mid m)$$

Identities

Functional identities

09.40.17.0001.01

$$\left(\left(z_2^2 - 1\right) z_1^2 - z_2^2 + m\right) \operatorname{dc}(w(z_1) + w(z_2) \mid m)^2 - 2\,(m-1)\, z_1\, z_2 \operatorname{dc}(w(z_1) + w(z_2) \mid m) + \left(z_2^2 - 1\right) m + z_1^2 \left(m - z_2^2\right) = 0 \;/;\; w(z) = \operatorname{dc}^{-1}(z \mid m)$$

Differentiation

Low-order differentiation

With respect to *z*

09.40.20.0001.02

$$\frac{\partial \operatorname{dc}^{-1}(z \mid m)}{\partial z} = \frac{\operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \,\middle|\, m\right)}{z^2 - 1}$$

09.40.20.0002.01

$$\frac{\partial \operatorname{dc}^{-1}(z \mid m)}{\partial z} = \frac{1}{\sqrt{z^2 - 1}\,\sqrt{z^2 - m}} \;/;\; z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 - m > 0 \bigwedge m < 1$$

09.40.20.0003.02

$$\frac{\partial^2 \operatorname{dc}^{-1}(z \mid m)}{\partial z^2} = -\frac{z\left(-2\,z^2 + m + 1\right) \operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \,\middle|\, m\right)}{\left(m - z^2\right)\left(z^2 - 1\right)^2}$$

09.40.20.0011.01

$$\frac{\partial^2 \operatorname{dc}^{-1}(z \mid m)}{\partial z^2} = -\frac{\sqrt{m - z^2}\, \operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \,\middle|\, m\right)}{\sqrt{1 - z^2}} \frac{\partial \frac{1}{\sqrt{1-z^2}\,\sqrt{m-z^2}}}{\partial z}$$

With respect to *m*

09.40.20.0004.01

$$\frac{\partial \operatorname{dc}^{-1}(z \mid m)}{\partial m} = \frac{E\left(\operatorname{am}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \mid m\right) - (1-m)\operatorname{dc}^{-1}(z \mid m)}{2(1-m)\,m}$$

09.40.20.0005.01

$$\frac{\partial \operatorname{dc}^{-1}(z \mid m)}{\partial m} = \frac{1}{2\,m\,(1-m)} \left(\frac{\sqrt{1-z^2}\; z}{\sqrt{m-z^2}} + \sqrt{m} \left(E\left(\frac{1}{m}\right) - E\left(\sin^{-1}(z) \,\middle|\, \frac{1}{m}\right)\right)\right) /;\ z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 - m > 0 \bigwedge m < 1$$

09.40.20.0006.02

$$\frac{\partial^2 \operatorname{dc}^{-1}(z \mid m)}{\partial m^2} = \frac{1}{4\,(m-1)^2\, m^2} \left(\frac{m \operatorname{sc}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \left(\frac{(1-m) z^2}{z^2-m}\right)^{3/2}}{z^2} + (4\,m-2)\, E\left(\operatorname{am}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \mid m\right) + (m-1)\, F\left(\operatorname{am}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \mid m\right) + 3\,(m-1)^2 \operatorname{dc}^{-1}(z \mid m)\right)$$

09.40.20.0012.01

$$\frac{\partial^3 \operatorname{dc}^{-1}(z \mid m)}{\partial m^3} = \frac{1}{8\,(m-1)^3\, m^3\, z^2\left(m-z^2\right)} \left(-m\left(11\,m^2 - \left(8\,z^2+7\right) m + 4\,z^2\right) \operatorname{sc}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \left(\frac{(m-1)\, z^2}{m-z^2}\right)^{3/2} + z^2\left(m-z^2\right)\left((-23\,(m-1)\,m-8)\, E\left(\operatorname{am}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \mid m\right) - (m-1)\,(11\,m-7)\, F\left(\operatorname{am}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) \mid m\right)\right) - 15\,(m-1)^3\, z^2\left(m-z^2\right) \operatorname{dc}^{-1}(z \mid m)\right)$$

Symbolic differentiation

With respect to *z*

09.40.20.0013.01

$$\frac{\partial^n \operatorname{dc}^{-1}(z \mid m)}{\partial z^n} = \delta_n \operatorname{dc}^{-1}(z \mid m) - \frac{\operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{z^2-1} \sum_{j=0}^{n-1} \frac{(-1)^{j-1}\,(1-n)_{2(n-j)-2}}{(n-j-1)!\,(2\,z)^{n-2\,j-1}} \sum_{k=0}^{j} \binom{j}{k} \left(\frac{1}{2}\right)_k \left(\frac{1}{2}\right)_{j-k} \left(z^2-1\right)^{-k} \left(z^2-m\right)^{k-j} /;\ n \in \mathbb{N}$$

09.40.20.0014.01

$$\frac{\partial^n \operatorname{dc}^{-1}(z \mid m)}{\partial z^n} = \delta_n \operatorname{dc}^{-1}(z \mid m) - \frac{\operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{z^2-1} \sum_{j=0}^{n-1} \frac{(-1)^{j-1}\, 2^{2\,j-n+1}\, z^{2\,j-n+1} \left(z^2-m\right)^{-j} \left(\frac{1}{2}\right)_j (1-n)_{2(n-j)-2}}{(n-j-1)!} {}_2F_1\left(\frac{1}{2}, -j;\ \frac{1}{2}-j;\ \frac{z^2-m}{z^2-1}\right) /;\ n \in \mathbb{N}$$

09.40.20.0015.01

$$\frac{\partial^n \operatorname{dc}^{-1}(z \mid m)}{\partial z^n} = \operatorname{dc}^{-1}(z \mid m)\, \delta_n + \frac{\sqrt{z^2-m}\; \operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{\sqrt{z^2-1}} \frac{\partial^{n-1} \frac{1}{\sqrt{z^2-1}\,\sqrt{z^2-m}}}{\partial z^{n-1}} /;\ n \in \mathbb{N}^+$$

09.40.20.0007.01

$$\frac{\partial^n \mathrm{dc}^{-1}(z \mid m)}{\partial z^n} = \frac{2^{n-1} \pi z^{n-1} (n-1)! \, \mathrm{sn}\left(\mathrm{dc}^{-1}(z \mid m) \mid m\right)}{z^2-1} \sum_{j=0}^{n-1} \frac{\left(z^2-1\right)^{-j} \left(z^2-m\right)^{j-n+1}}{j! \, (n-j-1)! \, \Gamma\left(\frac{1}{2}-j\right) \Gamma\left(j-n+\frac{3}{2}\right)} \, {}_2F_1\left(\frac{1-j}{2}, -\frac{j}{2}; \frac{1}{2}-j; 1-\frac{1}{z^2}\right) {}_2F_1\left(\frac{j-n+2}{2}, \frac{j-n+1}{2}; j-n+\frac{3}{2}; 1-\frac{m}{z^2}\right) /; n \in \mathbb{N}^+$$

With respect to m

09.40.20.0008.02

$$\frac{\partial^n \mathrm{dc}^{-1}(z \mid m)}{\partial m^n} = \frac{(-1)^{n-1} \sqrt{\pi} \, z^{-2n-1}}{(2n+1) \Gamma\left(\frac{1}{2}-n\right)} F_1\left(n+\frac{1}{2}; \frac{1}{2}, n+\frac{1}{2}; n+\frac{3}{2}; \frac{1}{z^2}, \frac{m}{z^2}\right) + \frac{\pi m^{-n}}{2} \, {}_2\tilde{F}_1\left(\frac{1}{2}, \frac{1}{2}; 1-n; m\right) /; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

09.40.20.0009.01

$$\frac{\partial^\alpha \mathrm{dc}^{-1}(z \mid m)}{\partial z^\alpha} = \frac{z^{-\alpha}}{\sqrt{m} \, \Gamma(1-\alpha)} K\left(\frac{1}{m}\right) - \frac{z^{1-\alpha} \sqrt{\pi}}{\sqrt{m}} \tilde{F}_{2\times 0\times 0}^{2\times 1\times 1}\left(\begin{matrix} \frac{1}{2}, 1; \frac{1}{2}; \frac{1}{2}; \\ \frac{3-\alpha}{2}, 1-\frac{\alpha}{2};;; \end{matrix} \; z^2, \frac{z^2}{m}\right) /; z > 1 \land m < 1$$

With respect to m

09.40.20.0010.01

$$\frac{\partial^\alpha \mathrm{dc}^{-1}(z \mid m)}{\partial m^\alpha} = \frac{\pi m^{-\alpha}}{2} \, {}_2\tilde{F}_1\left(\frac{1}{2}, \frac{1}{2}; 1-\alpha; m\right) - \frac{m^{-\alpha} \sqrt{\pi}}{2z} \tilde{F}_{1\times 0\times 1}^{1\times 1\times 2}\left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}, 1; \\ \frac{3}{2};; 1-\alpha; \end{matrix} \; \frac{1}{z^2}, \frac{m}{z^2}\right) /; z < 0 \land m < 0 \lor z > 1 \land m < 1$$

Integration

Indefinite integration

Involving only one direct function

09.40.21.0001.01

$$\int \mathrm{dc}^{-1}(z \mid m) \, dz = \mathrm{dc}^{-1}(z \mid m) \, z - \log\left(\mathrm{nc}\left(\mathrm{dc}^{-1}(z \mid m) \mid m\right) + \mathrm{sc}\left(\mathrm{dc}^{-1}(z \mid m) \mid m\right)\right)$$

Involving only one direct function with respect to m

09.40.21.0002.01

$$\int \mathrm{dc}^{-1}(z \mid m) \, dm = 2\sqrt{m} \left(E\left(\frac{1}{m}\right) - E\left(\sin^{-1}(z) \,\middle|\, \frac{1}{m}\right)\right) /; -1 < z < 1 \land m > 1 \lor z > 1 \land m < 1$$

Representations through more general functions

Through hypergeometric functions of two variables

09.40.26.0001.01

$$\mathrm{dc}^{-1}(z \mid m) = K(m) - \frac{1}{z} F_{1\times 0\times 0}^{1\times 1\times 1}\left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \\ \frac{3}{2};;; \end{matrix} \frac{m}{z^2}, \frac{1}{z^2}\right)$$

Through other functions

Involving some hypergeometric-type functions

09.40.26.0002.01

$$\mathrm{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - z\, F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z^2, \frac{z^2}{m}\right)\right) /; z > 1 \land m < 1$$

Representations through equivalent functions

With inverse function

09.40.27.0001.01

$$\mathrm{dc}\left(\mathrm{dc}^{-1}(z \mid m) \,\middle|\, m\right) = z$$

With related functions

Involving cd^{-1}

09.40.27.0002.01

$$\mathrm{dc}^{-1}(z \mid m) = \mathrm{cd}^{-1}\left(\frac{1}{z} \,\middle|\, m\right)$$

Involving cn^{-1}

09.40.27.0003.01

$$\mathrm{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - \mathrm{cn}^{-1}\left(\sqrt{1 - z^2} \,\middle|\, \frac{1}{m}\right)\right) /; 0 < z < 1 \land m > 1$$

Involving cs^{-1}

09.40.27.0004.01

$$\mathrm{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - i\, \mathrm{cs}^{-1}\left(\frac{i}{z} \,\middle|\, 1 - \frac{1}{m}\right)\right) /; z < 1 \land m > 1$$

Involving dn^{-1}

09.40.27.0005.01

$$\mathrm{dc}^{-1}(z \mid m) = 2\, i\, K(1 - m) - i\, \mathrm{dn}^{-1}(z \mid 1 - m)$$

Involving ds^{-1}

09.40.27.0006.01

$$\mathrm{dc}^{-1}(z \mid m) = K(m) + \frac{i}{\sqrt{1 - m}}\, \mathrm{ds}^{-1}\left(-\frac{i\, z}{\sqrt{1 - m}} \,\middle|\, \frac{1}{1 - m}\right) /; 0 < m < 1$$

Involving nc^{-1}

09.40.27.0007.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{1-m}} \operatorname{nc}^{-1}\left(z \,\middle|\, \frac{m}{m-1}\right) /; z > 0 \land m > 1$$

Involving nd^{-1}

09.40.27.0008.01

$$\operatorname{dc}^{-1}(z \mid m) = -\frac{i}{\sqrt{m}} \operatorname{nd}^{-1}\left(z \,\middle|\, 1 - \frac{1}{m}\right) /; z > 0 \land m > 0$$

Involving ns^{-1}

09.40.27.0009.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - \operatorname{ns}^{-1}\left(\frac{1}{z} \,\middle|\, \frac{1}{m}\right)\right) /; z < 1 \land m > 1$$

Involving sc^{-1}

09.40.27.0010.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) + i \operatorname{sc}^{-1}\left(i\, z \,\middle|\, \frac{m-1}{m}\right)\right) /; m > 1$$

Involving sd^{-1}

09.40.27.0011.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} K\left(\frac{1}{m}\right) - \frac{1}{\sqrt{1-m}} \operatorname{sd}^{-1}\left(\frac{z\sqrt{1-m}}{\sqrt{m}} \,\middle|\, \frac{m}{m-1}\right) /; -1 < z < 1 \land m > 1$$

Involving sn^{-1}

09.40.27.0012.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - \operatorname{sn}^{-1}\left(z \,\middle|\, \frac{1}{m}\right)\right) /; -1 < z < 1 \land m > 1 \lor z > 1 \land m < 1$$

Involving elliptic integrals

09.40.27.0013.01

$$\operatorname{dc}^{-1}(z \mid m) = \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - F\left(\sin^{-1}(z) \,\middle|\, \frac{1}{m}\right)\right) /; -1 < z < 1 \land m > 1 \lor z > 1 \land m < 1$$

09.40.27.0015.01

$$\operatorname{dc}^{-1}(z \mid m) = \operatorname{dc}^{-1}(z_0 \mid m) + \frac{\operatorname{sn}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)}{\sqrt{z^2-1}} \sqrt{\frac{z^2-m}{z^2}} \sqrt{z^2} \left(\frac{\sqrt{1-z^2}}{\sqrt{z^2-1}\sqrt{z^2-m}} \sqrt{\frac{m-z^2}{m}} F\left(\sin^{-1}(z) \,\middle|\, \frac{1}{m}\right) - \frac{\sqrt{1-z_0^2}}{\sqrt{z_0^2-1}\sqrt{z_0^2-m}} \sqrt{\frac{m-z_0^2}{m}} F\left(\sin^{-1}(z_0) \,\middle|\, \frac{1}{m}\right)\right) /;$$

$$\neg \exists_{\tau,\{\tau\in\mathbb{R},0<\tau<1\}} \left(\operatorname{Im}\left((\tau(z-z_0)+z_0)^2-1\right) = 0 \bigwedge (\tau(z-z_0)+z_0)^2 - 1 < 0 \bigwedge \operatorname{Im}\left((\tau(z-z_0)+z_0)^2-m\right) = 0 \bigwedge (\tau(z-z_0)+z_0)^2 - m < 0\right)$$

09.40.27.0016.01

$$\text{dc}^{-1}(z \mid m) = K(m) - \frac{\text{sn}\left(\text{dc}^{-1}(z \mid m) \,\middle|\, m\right)}{\sqrt{z^2 - 1}} \sqrt{\frac{z^2 - m}{z^2}} \sqrt{z^2}\; F\left(\csc^{-1}(z) \,\middle|\, m\right) /;$$

$$\neg\, \exists_{\tau,\{\tau \in \mathbb{R}, 0 < \tau < 1\}} \left(\text{Im}\left(\tau^2 z^2 - 1\right) = 0 \bigwedge \tau^2 z^2 - 1 < 0 \bigwedge \text{Im}\left(\tau^2 z^2 - m\right) = 0 \bigwedge \tau^2 z^2 - m < 0\right)$$

Involving other related functions

09.40.27.0014.01

$$\text{dc}^{-1}(z \mid m) == K(m) + \frac{\sqrt{z_2^2}}{z_2} \text{elog}(z_1, z_2; a, b) /; \{a, b, z_1\} == \left\{-m - 1, m, z^2\right\} \bigwedge z_1^3 + a z_1^2 + b z_1 - z_2^2 == 0 \bigwedge z > 1 \bigwedge m < 1$$

History

– N. H. Abel (1826)

– A. G. Greenhill (1892)

– L. M. Milne–Thompson (1948)

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