InverseJacobiSN

Notations

Traditional name

Inverse of the Jacobi elliptic function sn

Traditional notation

$\operatorname{sn}^{-1}(z \mid m)$

Mathematica StandardForm notation

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InverseJacobiSN[z, m]
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Primary definition

09.48.02.0001.01

$$z = \operatorname{sn}(w \mid m) \,/;\, w = \operatorname{sn}^{-1}(z \mid m)$$

09.48.02.0002.01

$$\operatorname{sn}^{-1}(z \mid m) = \int_0^z \frac{1}{\sqrt{1-t^2}\,\sqrt{1-m\,t^2}}\,dt \,/;\, -1 < z < 1 \bigwedge m\,z^2 < 1$$

Specific values

Specialized values

For fixed *z*

09.48.03.0001.01

$$\operatorname{sn}^{-1}(z \mid 0) = \sin^{-1}(z)$$

09.48.03.0002.01

$$\operatorname{sn}^{-1}\left(z \,\middle|\, \frac{1}{2}\right) = F\left(\sin^{-1}(z) \,\middle|\, \frac{1}{2}\right)$$

09.48.03.0003.01

$$\operatorname{sn}^{-1}(z \mid 1) = \tanh^{-1}(z)$$

For fixed *m*

09.48.03.0004.01

$$\operatorname{sn}^{-1}(-1 \mid m) = -K(m)$$

09.48.03.0005.01

$$\mathrm{sn}^{-1}\left(-\frac{1}{2}\,\middle|\,m\right) = -F\left(\frac{\pi}{6}\,\middle|\,m\right)$$

09.48.03.0006.01

$$\mathrm{sn}^{-1}(0 \mid m) = 0$$

09.48.03.0007.01

$$\mathrm{sn}^{-1}\left(\frac{1}{2}\,\middle|\,m\right) = F\left(\frac{\pi}{6}\,\middle|\,m\right)$$

09.48.03.0008.01

$$\mathrm{sn}^{-1}(1 \mid m) = K(m)$$

09.48.03.0009.01

$$\mathrm{sn}^{-1}(i \mid m) = F\left(i \sinh^{-1}(1) \,\middle|\, m\right)$$

09.48.03.0010.01

$$\mathrm{sn}^{-1}(-i \mid m) = -F\left(i \sinh^{-1}(1) \,\middle|\, m\right)$$

Values at infinities

09.48.03.0011.01

$$\mathrm{sn}^{-1}(z \mid \infty) = 0$$

09.48.03.0012.01

$$\mathrm{sn}^{-1}(z \mid -\infty) = 0$$

09.48.03.0013.01

$$\mathrm{sn}^{-1}(\infty \mid m) = K(m) - \frac{1}{\sqrt{m}} K\left(\frac{1}{m}\right) /; m > 1$$

09.48.03.0014.01

$$\mathrm{sn}^{-1}(-\infty \mid m) = \frac{1}{\sqrt{m}} K\left(\frac{1}{m}\right) - K(m) /; m > 1$$

General characteristics

Domain and analyticity

$\mathrm{sn}^{-1}(z \mid m)$ is an analytical function of z and m which is defined over $\mathbb{C}^2$.

09.48.04.0001.01

$$(z * m) \longrightarrow \mathrm{sn}^{-1}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

09.48.04.0002.01

$$\mathrm{ds}^{-1}(\bar{z} \mid \bar{m}) = \overline{\mathrm{ds}^{-1}(z \mid m)}$$

Quasi-reflection symmetry

09.48.04.0003.01

$$\mathrm{sn}^{-1}(-z \mid m) = -\mathrm{sn}^{-1}(z \mid m)$$

Poles and essential singularities

With respect to *m*

The function $\mathrm{sn}^{-1}(z \mid m)$ does not have poles and essential singularities with respect to m.

09.48.04.0004.01

$$\mathcal{Sing}_m\left(\mathrm{sn}^{-1}(z \mid m)\right) = \{\}$$

With respect to *z*

The function $\mathrm{sn}^{-1}(z \mid m)$ does not have poles and essential singularities with respect to z.

09.48.04.0005.01

$$\mathcal{Sing}_z\left(\mathrm{sn}^{-1}(z \mid m)\right) = \{\}$$

Branch points

With respect to *m*

For fixed z, the function $\mathrm{sn}^{-1}(z \mid m)$ has two branch points: $m = \frac{1}{z^2}$, $m = \tilde{\infty}$.

09.48.04.0006.01

$$\mathcal{BP}_m\left(\mathrm{sn}^{-1}(z \mid m)\right) = \left\{\frac{1}{z^2}, \tilde{\infty}\right\}$$

09.48.04.0007.01

$$\mathcal{R}_m\left(\mathrm{sn}^{-1}(z \mid m), \frac{1}{z^2}\right) = \log$$

09.48.04.0008.01

$$\mathcal{R}_m\left(\mathrm{sn}^{-1}(z \mid m), \tilde{\infty}\right) = \log$$

With respect to *z*

For fixed m, the function $\mathrm{sn}^{-1}(z \mid m)$ has five branch points: $z = \pm 1$, $z = \pm \frac{1}{\sqrt{m}}$, $z = \tilde{\infty}$.

09.48.04.0009.01

$$\mathcal{BP}_z\left(\mathrm{sn}^{-1}(z \mid m)\right) = \left\{1, -1, \frac{1}{\sqrt{m}}, -\frac{1}{\sqrt{m}}, \tilde{\infty}\right\}$$

09.48.04.0010.01

$$\mathcal{R}_z\left(\mathrm{sn}^{-1}(z \mid m), 1\right) = 2$$

09.48.04.0011.01

$$\mathcal{R}_z\left(\mathrm{sn}^{-1}(z \mid m), -1\right) = 2$$

09.48.04.0012.01

$$\mathcal{R}_z\left(\mathrm{sn}^{-1}(z \mid m), \frac{1}{\sqrt{m}}\right) = 2$$

09.48.04.0013.01

$$\mathcal{R}_z\left(\text{sn}^{-1}(z \mid m), -\frac{1}{\sqrt{m}}\right) == 2$$

09.48.04.0014.01

$$\mathcal{R}_z\left(\text{sn}^{-1}(z \mid m), \tilde{\infty}\right) == \log$$

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

Expansions at $z == 0$

09.48.06.0001.02

$$\text{sn}^{-1}(z \mid m) \propto z + \frac{1+m}{6} z^3 + \frac{3+2m+3m^2}{40} z^5 + \ldots \text{ /; } (z \to 0)$$

09.48.06.0002.01

$$\text{sn}^{-1}(z \mid m) == \sum_{k=0}^{\infty} \frac{m^k \left(\frac{1}{2}\right)_k}{(2k+1)\,k!} \, {}_2F_1\left(\frac{1}{2}, -k; \frac{1}{2}-k; \frac{1}{m}\right) z^{2k+1}$$

09.48.06.0007.01

$$\text{sn}^{-1}(z \mid m) \propto z\left(1 + O\left(z^2\right)\right)$$

Expansions at $m == 0$

09.48.06.0003.02

$$\text{sn}^{-1}(z \mid m) \propto \sin^{-1}(z) - \frac{1}{4}\left(z\sqrt{1-z^2} - \sin^{-1}(z)\right) m - \frac{3}{64}\left(z\left(2z^2+3\right)\sqrt{1-z^2} - 3\sin^{-1}(z)\right) m^2 + \ldots \text{ /; } (m \to 0)$$

09.48.06.0004.01

$$\text{sn}^{-1}(z \mid m) == \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k z^{2k+1}}{(2k+1)\,k!} \, {}_2F_1\left(\frac{1}{2}, k+\frac{1}{2}; k+\frac{3}{2}; z^2\right) m^k \text{ /; } |m| < 1$$

09.48.06.0008.01

$$\text{sn}^{-1}(z \mid m) = \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k^2}{(k!)^2} \left(\sin^{-1}(z) - \frac{\sqrt{1-z^2}}{2z} \sum_{j=1}^{k} \frac{(j-1)!\, z^{2j}}{\left(\frac{1}{2}\right)_j}\right) m^k \text{ /; } |m| < 1$$

09.48.06.0005.01

$$\text{sn}^{-1}(z \mid m) == \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_j \left(\frac{1}{2}\right)_k m^k z^{2j+2k+1}}{(2j+2k+1)\, j!\, k!}$$

09.48.06.0006.01

$$\operatorname{sn}^{-1}(z \mid m) == z\, F_{1\times 0\times 0}^{1\times 1\times 1}\left(\begin{matrix} \frac{1}{2};\ \frac{1}{2};\ \frac{1}{2}; \\ \frac{3}{2};;; \end{matrix}\ m z^2, z^2\right)$$

09.48.06.0009.01

$$\operatorname{sn}^{-1}(z \mid m) \propto \sin^{-1}(z)\,(1 + O(m))$$

Integral representations

On the real axis

Of the direct function

09.48.07.0001.01

$$\operatorname{sn}^{-1}(z \mid m) == \int_0^z \frac{1}{\sqrt{1-t^2}\,\sqrt{1-m t^2}}\, dt \,/;\, -1 < z < 1 \bigwedge m z^2 < 1$$

09.48.07.0002.01

$$\operatorname{sn}^{-1}(z \mid m) = \frac{\sqrt{1-m z^2}\ \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right)}{\sqrt{1-z^2}} \int_0^z \frac{1}{\sqrt{1-t^2}\,\sqrt{1-m t^2}}\, dt \,/;$$

$$\neg\, \exists_{\tau,\{\tau\in\mathbb{R},0<\tau<1\}} \left(\operatorname{Im}\left(1-\tau^2 z^2\right) = 0 \bigwedge 1-\tau^2 z^2 < 0 \bigwedge \operatorname{Im}\left(1-m\tau^2 z^2\right) = 0 \bigwedge 1-m\tau^2 z^2 < 0\right)$$

09.48.07.0003.01

$$\operatorname{sn}^{-1}(z \mid m) = \operatorname{sn}^{-1}(z_0 \mid m) + \frac{\sqrt{1-m z^2}\ \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right)}{\sqrt{1-z^2}} \int_{z_0}^z \frac{1}{\sqrt{1-t^2}\,\sqrt{1-m t^2}}\, dt \,/;$$

$$\neg\, \exists_{\tau,\{\tau\in\mathbb{R},0<\tau<1\}} \left(\operatorname{Im}\left(1-(\tau(z-z_0)+z_0)^2\right) = 0 \bigwedge 1-(\tau(z-z_0)+z_0)^2 < 0 \bigwedge \operatorname{Im}\left(1-m(\tau(z-z_0)+z_0)^2\right) = 0 \bigwedge 1-m(\tau(z-z_0)+z_0)^2 < 0\right)$$

Differential equations

Ordinary nonlinear differential equations

09.48.13.0001.01

$$w''(z) + \left(2 m z^2 - m - 1\right) z\, w'(z)^3 == 0 \,/;\, w(z) == \operatorname{sn}^{-1}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.48.16.0001.01

$$\operatorname{sn}^{-1}(-z \mid m) == -\operatorname{sn}^{-1}(z \mid m)$$

Products, sums, and powers of the direct function

Sums of the direct function

09.48.16.0002.01

$$\operatorname{sn}^{-1}(z_1 \mid m) + \operatorname{sn}^{-1}(z_2 \mid m) = \operatorname{sn}^{-1}\left(\frac{\sqrt{(1-z_2^2)(1-m z_2^2)}\, z_1 + \sqrt{(1-z_1^2)(1-m z_1^2)}\, z_2}{1 - m z_1^2 z_2^2} \,\middle|\, m \right) /;$$
$$z_1 \in \mathbb{R} \bigwedge z_2 \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge -\frac{1}{m} < z_1 < \frac{1}{m} \bigwedge -\frac{1}{m} < z_2 < \frac{1}{m}$$

Identities

Functional identities

09.48.17.0001.01

$$\left(m z_1^2 z_2^2 - 1\right)^2 \operatorname{sn}(w(z_1) + w(z_2) \mid m)^4 - 2\left(\left(\left(m\left(z_1^2 + z_2^2\right) - 2(m+1)\right) z_2^2 + 1\right) z_1^2 + z_2^2\right) \operatorname{sn}(w(z_1) + w(z_2) \mid m)^2 + \left(z_1^2 - z_2^2\right)^2 = 0 /;$$
$$w(z) = \operatorname{sn}^{-1}(z \mid m)$$

Differentiation

Low-order differentiation

With respect to z

09.48.20.0001.02

$$\frac{\partial \operatorname{sn}^{-1}(z \mid m)}{\partial z} = \frac{\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{1 - z^2}$$

09.48.20.0002.02

$$\frac{\partial \operatorname{sn}^{-1}(z \mid m)}{\partial z} = \frac{1}{\sqrt{1-z^2}\,\sqrt{1-m z^2}} /; -1 < z < 1 \bigwedge m z^2 < 1$$

09.48.20.0003.02

$$\frac{\partial^2 \operatorname{sn}^{-1}(z \mid m)}{\partial z^2} = \frac{z\left(m\left(2 z^2 - 1\right) - 1\right) \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{\left(z^2 - 1\right)^2 \left(m z^2 - 1\right)}$$

09.48.20.0011.01

$$\frac{\partial^2 \operatorname{sn}^{-1}(z \mid m)}{\partial z^2} = \frac{\sqrt{1 - m z^2}\, \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{\sqrt{1-z^2}} \, \frac{\partial \frac{1}{\sqrt{1-z^2}\,\sqrt{1-m z^2}}}{\partial z}$$

With respect to m

09.48.20.0012.01

$$\frac{\partial \operatorname{sn}^{-1}(z \mid m)}{\partial m} = \frac{E\left(\operatorname{am}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right) \mid m\right) + (m-1) \operatorname{sn}^{-1}(z \mid m) - m z \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{2(1-m) m}$$

09.48.20.0004.01

$$\frac{\partial \operatorname{sn}^{-1}(z \mid m)}{\partial m} = \frac{E\left(\sin^{-1}(z) \mid m\right) - (1-m) F\left(\sin^{-1}(z) \mid m\right) - m z \operatorname{cd}\left(F\left(\sin^{-1}(z) \mid m\right) \mid m\right)}{2(1-m) m}$$

09.48.20.0005.01

$$\frac{\partial \operatorname{sn}^{-1}(z \mid m)}{\partial m} = \frac{1}{2(m-1)m}\left(\frac{m\sqrt{1-z^2}\, z}{\sqrt{1-m z^2}} - E\left(\sin^{-1}(z) \mid m\right) - (m-1) F\left(\sin^{-1}(z) \mid m\right)\right) /; -1 < z < 1 \bigwedge m z^2 < 1$$

09.48.20.0006.01

$$\frac{\partial^2 \operatorname{sn}^{-1}(z \mid m)}{\partial m^2} = \frac{1}{4(m-1)^2 m^2}\left(3 \operatorname{sn}^{-1}(z \mid m)(m-1)^2 + F\left(\operatorname{am}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right) \mid m\right)(m-1) + (4m-2) E\left(\operatorname{am}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right) \mid m\right) + \frac{\left(m z \left(m\left((2-4m) z^2 + 3\right) - 1\right)\right) \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{m z^2 - 1}\right)$$

09.48.20.0013.01

$$\frac{\partial^3 \operatorname{sn}^{-1}(z \mid m)}{\partial m^3} = -\frac{1}{8(m-1)^3 m^3}\left((23(m-1)m + 8) E\left(\operatorname{am}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right) \mid m\right) + (m-1)(11m-7) F\left(\operatorname{am}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right) \mid m\right) + \frac{1}{\left(1-m z^2\right)^{7/2}}\left(15(m-1)^3 \operatorname{sn}^{-1}(z \mid m)\left(1-m z^2\right)^{7/2} + m z \left((m-1) \operatorname{cn}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)\left(m z^2 - 1\right)^3 + \sqrt{1-m z^2}\,\left(1-m z^2\right) \left(m\left(-m(m(23m-24)+9) z^4 + (5m(7m-6)+11) z^2 - 15m + 12\right) - 5\right) \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)\right)\right)\right)$$

Symbolic differentiation

With respect to *z*

09.48.20.0014.01

$$\frac{\partial^n \operatorname{sn}^{-1}(z \mid m)}{\partial z^n} = \operatorname{sn}^{-1}(z \mid m)\,\delta_n + \frac{\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{1-z^2} \sum_{j=0}^{n-1} \frac{(1-n)_{2(n-j)-2}}{(n-j-1)!\,(2z)^{n-2j-1}} \sum_{k=0}^{j} \binom{j}{k} \left(\frac{1}{2}\right)_k \left(\frac{1}{2}\right)_{j-k} m^{j-k} \left(1-z^2\right)^{-k} \left(1-m z^2\right)^{k-j} /; n \in \mathbb{N}$$

09.48.20.0015.01

$$\frac{\partial^n \operatorname{sn}^{-1}(z \mid m)}{\partial z^n} = \operatorname{sn}^{-1}(z \mid m)\,\delta_n + \frac{\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{1-z^2} \sum_{j=0}^{n-1} \frac{2^{2j-n+1} m^j z^{2j-n+1} \left(1-m z^2\right)^{-j} \left(\frac{1}{2}\right)_j (1-n)_{2(n-j)-2}}{(n-j-1)!}\, {}_2F_1\left(\frac{1}{2}, -j; \frac{1}{2} - j; \frac{1-m z^2}{m\left(1-z^2\right)}\right) /; n \in \mathbb{N}$$

09.48.20.0016.01

$$\frac{\partial^n \operatorname{sn}^{-1}(z \mid m)}{\partial z^n} = \operatorname{sn}^{-1}(z \mid m)\,\delta_n + \frac{\sqrt{1-m z^2}\, \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{\sqrt{1-z^2}} \frac{\partial^{n-1} \frac{1}{\sqrt{1-z^2}\,\sqrt{1-m z^2}}}{\partial z^{n-1}} /; n \in \mathbb{N}^+$$

09.48.20.0007.01

$$\frac{\partial^n \operatorname{sn}^{-1}(z \mid m)}{\partial z^n} = -\frac{2^{n-1}\,\pi\,(-z)^n\,(n-1)!\,\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{z - z^3} \sum_{j=0}^{n-1} \frac{m^{n-j-1}\,(1-z^2)^{-j}\,(1-m z^2)^{j-n+1}}{j!\,(n-j-1)!\,\Gamma\left(\frac{1}{2}-j\right)\Gamma\left(j-n+\frac{3}{2}\right)} \, {}_2F_1\left(\frac{1-j}{2}, -\frac{j}{2}; \frac{1}{2}-j; 1-\frac{1}{z^2}\right) {}_2F_1\left(\frac{j-n+2}{2}, \frac{j-n+1}{2}; j-n+\frac{3}{2}; 1-\frac{1}{m z^2}\right) /; n \in \mathbb{N}^+$$

With respect to *m*

09.48.20.0008.02

$$\frac{\partial^n \operatorname{sn}^{-1}(z \mid m)}{\partial m^n} = \frac{z^{2n+1}\left(\frac{1}{2}\right)_n \sqrt{1-m z^2}\,\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{(2n+1)\sqrt{1-z^2}} F_1\left(n+\frac{1}{2}; \frac{1}{2}, n+\frac{1}{2}; n+\frac{3}{2}; z^2, m z^2\right) /; n \in \mathbb{N}$$

09.48.20.0017.01

$$\frac{\partial^n \operatorname{sn}^{-1}(z \mid m)}{\partial m^n} = \frac{\sqrt{1-m z^2}\,\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)}{\sqrt{1-z^2}} \frac{\partial^n F\left(\sin^{-1}(z) \mid m\right)}{\partial m^n} /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.48.20.0009.01

$$\frac{\partial^\alpha \operatorname{sn}^{-1}(z \mid m)}{\partial z^\alpha} = z^{1-\alpha} \sqrt{\pi}\, \tilde{F}_{2\times 0\times 0}^{2\times 1\times 1}\left(\begin{matrix} \frac{1}{2}, 1; \frac{1}{2}; \frac{1}{2}; \\ \frac{3-\alpha}{2}, 1-\frac{\alpha}{2};;; \end{matrix}\; z^2, m z^2\right) /; -1 < z < 1 \bigwedge m z^2 < 1$$

With respect to *m*

09.48.20.0010.01

$$\frac{\partial^\alpha \operatorname{sn}^{-1}(z \mid m)}{\partial m^\alpha} = \frac{m^{-\alpha} \sqrt{\pi}\, z}{2} \tilde{F}_{1\times 0\times 1}^{1\times 1\times 2}\left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}, 1; \\ \frac{3}{2};; 1-\alpha; \end{matrix}\; z^2, m z^2\right) /; -1 < z < 1 \bigwedge m z^2 < 1$$

Integration

Indefinite integration

Involving only one direct function

09.48.21.0001.01

$$\int \operatorname{sn}^{-1}(z \mid m)\, dz = \operatorname{sn}^{-1}(z \mid m)\, z - \frac{\log\left(\operatorname{dn}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right) - \sqrt{m}\,\operatorname{cn}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)\right)}{\sqrt{m}}$$

Involving only one direct function with respect to *m*

09.48.21.0002.01

$$\int \operatorname{sn}^{-1}(z \mid m)\, dm = 2\left(\frac{\sqrt{1-z^2}\,\sqrt{1-m z^2} - 1}{z} + E\left(\sin^{-1}(z) \mid m\right) + (m-1)\, F\left(\sin^{-1}(z) \mid m\right)\right) /; -1 < z < 1 \land m < 1$$

Representations through more general functions

Through hypergeometric functions of two variables

09.48.26.0001.01

$$\mathrm{sn}^{-1}(z \mid m) = z\, F_{1\times 0\times 0}^{1\times 1\times 1}\left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \\ \frac{3}{2};;; \end{matrix} m z^2, z^2\right)$$

Through other functions

Involving some hypergeometric-type functions

09.48.26.0002.01

$$\mathrm{sn}^{-1}(z \mid m) = z\, F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z^2, m z^2\right) /; -1 < z < 1 \bigwedge m z^2 < 1$$

Representations through equivalent functions

With inverse function

09.48.27.0001.01

$$\mathrm{sn}\left(\mathrm{sn}^{-1}(z \mid m) \,\middle|\, m\right) = z$$

With related functions

Involving cd^{-1}

09.48.27.0002.01

$$\mathrm{sn}^{-1}(z \mid m) = K(m) - \mathrm{cd}^{-1}(z \mid m) /; z \in \mathbb{R} \land m \in \mathbb{R}$$

Involving cn^{-1}

09.48.27.0003.01

$$\mathrm{sn}^{-1}(z \mid m) = K(m) - \frac{1}{\sqrt{1-m}}\, \mathrm{cn}^{-1}\left(z \,\middle|\, \frac{m}{m-1}\right) /; -1 < z < 1 \land m < 1$$

09.48.27.0004.01

$$\mathrm{sn}^{-1}(z \mid m) = \mathrm{cn}^{-1}\left(\sqrt{1-z^2} \,\middle|\, m\right) /; 0 < z < 1 \land m \in \mathbb{R}$$

Involving cs^{-1}

09.48.27.0005.01

$$\mathrm{sn}^{-1}(z \mid m) = K(m) + \frac{i}{\sqrt{m}}\, \mathrm{dn}^{-1}\left(z \,\middle|\, \frac{m-1}{m}\right) /; z < 0 \land m > 1$$

Involving dc^{-1}

09.48.27.0006.01

$$\mathrm{sn}^{-1}(z \mid m) = K(m) - \frac{1}{\sqrt{m}}\, \mathrm{dc}^{-1}\left(z \,\middle|\, \frac{1}{m}\right) /; z > 1 \land m > 1$$

Involving dn^{-1}

09.48.27.0007.01

$$\text{sn}^{-1}(z \mid m) = K(m) + \frac{i}{\sqrt{m}} \text{dn}^{-1}\left(z \,\middle|\, \frac{m-1}{m}\right) /;\ z < 0 \wedge m > 1$$

09.48.27.0008.01

$$\text{sn}^{-1}(z \mid m) = \text{dn}^{-1}\left(\sqrt{1 - m z^2} \,\middle|\, m\right) /;\ z > 1 \wedge m < 0$$

Involving ds^{-1}

09.48.27.0009.01

$$\text{sn}^{-1}(z \mid m) = \frac{1}{\sqrt{1-m}} \text{ds}^{-1}\left(\frac{1}{\sqrt{1-m}\ z} \,\middle|\, \frac{m}{m-1}\right) /;\ z > 0 \wedge m > 1$$

Involving nc^{-1}

09.48.27.0010.01

$$\text{sn}^{-1}(z \mid m) = K(m) + \frac{i}{\sqrt{1-m}} \text{nc}^{-1}\left(z \,\middle|\, \frac{1}{1-m}\right) /;\ -1 < z < 1 \wedge m < 1$$

Involving nd^{-1}

09.48.27.0011.01

$$\text{sn}^{-1}(z \mid m) = K(m) + i\, \text{nd}^{-1}(z \mid 1-m) /;\ z \in \mathbb{R} \wedge m \in \mathbb{R}$$

Involving ns^{-1}

09.48.27.0012.01

$$\text{sn}^{-1}(z \mid m) = \text{ns}^{-1}\left(\frac{1}{z} \,\middle|\, m\right) /;\ -1 < z < 0 \wedge m < 0 \vee z > 0 \wedge m < 0$$

Involving sc^{-1}

09.48.27.0013.01

$$\text{sn}^{-1}(z \mid m) = -i\, \text{sc}^{-1}(i\, z \mid 1-m)$$

Involving sd^{-1}

09.48.27.0014.01

$$\text{sn}^{-1}(z \mid m) = -\frac{i}{\sqrt{1-m}} \text{sd}^{-1}\left(\sqrt{m-1}\ z \,\middle|\, \frac{1}{1-m}\right) /;\ -1 < z < 1 \wedge m < 1$$

Involving elliptic integrals

09.48.27.0015.02

$$\text{sn}^{-1}(z \mid m) = F\left(\sin^{-1}(z) \,\middle|\, m\right) /;\ |z| < 1$$

09.48.27.0018.01

$$\text{sn}^{-1}(z \mid m) = \frac{\sqrt{1 - m z^2}\ \text{cd}\left(\text{sn}^{-1}(z \mid m) \,\middle|\, m\right)}{\sqrt{1 - z^2}} F\left(\sin^{-1}(z) \,\middle|\, m\right) /;$$

$$\neg \exists_{\tau, \{\tau \in \mathbb{R}, 0 < \tau < 1\}} \left(\text{Im}\left(1 - \tau^2 z^2\right) = 0 \bigwedge 1 - \tau^2 z^2 < 0 \bigwedge \text{Im}\left(1 - m \tau^2 z^2\right) = 0 \bigwedge 1 - m \tau^2 z^2 < 0\right)$$

09.48.27.0019.01

$$\operatorname{sn}^{-1}(z \mid m) = \operatorname{sn}^{-1}(z_0 \mid m) + \frac{\sqrt{1 - m z^2}\ \operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right)}{\sqrt{1 - z^2}} \left(F\left(\sin^{-1}(z) \,\middle|\, m\right) - F\left(\sin^{-1}(z_0) \,\middle|\, m\right)\right) /;$$
$$\neg\, \exists_{\tau,\{\tau \in \mathbb{R}, 0 < \tau < 1\}} \left(\operatorname{Im}\left(1 - (\tau (z - z_0) + z_0)^2\right) = 0 \bigwedge 1 - (\tau (z - z_0) + z_0)^2 < 0 \bigwedge \operatorname{Im}\left(1 - m (\tau (z - z_0) + z_0)^2\right) = 0 \bigwedge 1 - m (\tau (z - z_0) + z_0)^2 < 0\right)$$

Involving other related functions

09.48.27.0016.01

$$\operatorname{sn}^{-1}(z \mid m) == -z \operatorname{elog}\left(1, \sqrt{a + b + 1}\,; a, b\right) /; \{a, b\} == \left\{-z^2 (m + 1), m z^4\right\} \bigwedge |z| < 1$$

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$$\operatorname{sn}^{-1}(z \mid m) == -\frac{\sqrt{z_2^2}}{z_2} \operatorname{elog}(z_1, z_2; a, b) /; \{a, b, z_1\} == \left\{-m - 1, m, \frac{1}{z^2}\right\} \bigwedge z_1^3 + a z_1^2 + b z_1 - z_2^2 == 0 \bigwedge z > 0 \bigwedge m < 1$$

History

– N. H. Abel (1826)
– A. G. Greenhill (1892)
– L. M. Milne–Thompson (1948)

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