InverseWeierstrassP4

Notations

Traditional name

Generalized inverse Weierstrass elliptic function

Traditional notation

$\wp^{-1}(z_1, z_2; g_2, g_3)$

Mathematica StandardForm notation

InverseWeierstrassP[{z_1, z_2}, {g_2, g_3}]

Primary definition

\[ \wp(w; g_2, g_3) = z_1 /; w = \wp^{-1}(z_1, z_2; g_2, g_3) \]
\[ \int_{z_1}^{z_2} \frac{1}{\sqrt{4 t^3 - g_2 t - g_3}} dt /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3} \]

$\wp^{-1}(z_1, z_2; g_2, g_3)$ is the unique value of $u$ for which $z_1 = \wp(u; g_2, g_3)$ and $z_2 = \wp'(u; g_2, g_3)$. For $\wp^{-1}(z_1, z_2; g_2, g_3)$ to exist, $z_1$ and $z_2$ must be related by $z_2^3 = 4 z_1^3 - g_2 z_1 - g_3$.

General characteristics

Domain and analyticity

$\wp^{-1}(z_1, z_2; g_2, g_3)$ is an analytical function of $z_1, z_2, g_2, g_3$ which is defined in $\mathbb{C}^4$.

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

...
No periodicity

Poles and essential singularities

With respect to \( g_3 \)

The function \( \varphi^{-1}(z_1, z_2; g_2, g_3) \) does not have poles and essential singularities with respect to \( g_3 \).

\[ \text{Sing}_{g_3}(\varphi^{-1}(z_1, z_2; g_2, g_3)) = \{ \} \]

With respect to \( g_2 \)

The function \( \varphi^{-1}(z_1, z_2; g_2, g_3) \) does not have poles and essential singularities with respect to \( g_2 \).

\[ \text{Sing}_{g_2}(\varphi^{-1}(z_1, z_2; g_2, g_3)) = \{ \} \]

With respect to \( z_1 \)

The function \( \varphi^{-1}(z_1, z_2; g_2, g_3) \) does not have poles and essential singularities with respect to \( z_1 \).

\[ \text{Sing}_{z_1}(\varphi^{-1}(z_1, z_2; g_2, g_3)) = \{ \} \]

Branch cuts

Branch cut locations: complicated

Integral representations

On the real axis

Of the direct function

\[ \varphi^{-1}(z_1, z_2; g_2, g_3) = \int_0^{z_1} \frac{1}{\sqrt{4 t^3 - g_2 t - g_3}} \, dt; \, z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3} \]

Differential equations

Ordinary nonlinear differential equations

\[ (4 z_1^3 - g_2 z_1 - g_3) w'(z_1)^2 - 1 = 0; \, w(z_1) = \varphi^{-1}(z_1, z_2; g_2, g_3) \]

Differentiation

Low-order differentiation
\[
\frac{\partial \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1} = \frac{1}{\varphi'(\varphi^{-1}(z_1; g_2, g_3); g_2, g_3)} /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3}
\]

\[
\frac{\partial^2 \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^2} = \frac{1}{4 z_1^3 - g_2 z_1 - g_3} /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3}
\]

\[
\frac{\partial^2 \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^2} = \frac{\varphi''(\varphi^{-1}(z_1; g_2, g_3); g_2, g_3)^2}{2 \varphi'(\varphi^{-1}(z_1; g_2, g_3); g_2, g_3)^3} /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3}
\]

Symbolic differentiation

\[
\frac{\partial^n \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^n} = \frac{\delta_{n-1}}{4 z_1^3 - g_2 z_1 - g_3} + \varphi^{-1}(z_1, z_2; g_2, g_3) \delta_n + \sum_{m=0}^{n-1} \frac{1}{m!} \left( \frac{1}{2} - m \right) \sum_{j=0}^{m-1} (-1)^j \binom{m}{j} (4 z_1^3 - g_2 z_1 - g_3)^{j-m} \sum_{k_1=0}^{m-j} \sum_{k_2=0}^{m-j} (-1)^{m-j+k_1+k_2} \delta_{m-j+k_1+k_2+k_3} (k_1 + k_2 + k_3; k_1, k_2, k_3) 4^k z_1^{k_1} g_2^{k_2} g_3^{k_3} (-3 k_1 - k_2)_{m-1} z_1^{m+k_1+k_2+k_3} /; n \in \mathbb{N} /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3}
\]

Representations through equivalent functions

With inverse function

\[
\varphi(\varphi^{-1}(z_1, z_2; g_2, g_3); g_2, g_3) = z_1 /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3}
\]

\[
\varphi'(\varphi^{-1}(z_1, z_2; g_2, g_3); g_2, g_3) = z_2 /; z_2 = \sqrt[4]{z_1^3 - g_2 z_1 - g_3}
\]

History

– L. Euler (1761)
– J.-L. Lagrange (1769)
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