JacobiDS

Notations

Traditional name

Jacobi elliptic function ds

Traditional notation

$\operatorname{ds}(z \mid m)$

Mathematica StandardForm notation

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JacobiDS[z, m]
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Primary definition

09.30.02.0001.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)}{\operatorname{sn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case *m* = 0

09.30.03.0001.01

$$\operatorname{ds}(z \mid 0) = \csc(z)$$

09.30.03.0002.01

$$\operatorname{ds}\left(z + \frac{\pi}{2} \,\middle|\, 0\right) = \sec(z)$$

09.30.03.0025.01

$$\operatorname{ds}\left(z + \frac{\pi k}{2} \,\middle|\, 0\right) = \csc\left(z + \frac{\pi k}{2}\right) /; k \in \mathbb{Z}$$

Case *m* = 1

09.30.03.0003.01

$$\operatorname{ds}\left(z+\frac{\pi i}{2}\,\middle|\,1\right)=-i\operatorname{sech}(z)$$

09.30.03.0026.01

$$\operatorname{ds}\left(z+\frac{i\pi k}{2}\,\middle|\,1\right)=\operatorname{csch}\left(z+\frac{i\pi k}{2}\right)\;/;\;k\in\mathbb{Z}$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.30.03.0004.01

$$\operatorname{ds}(0\mid m)=\tilde{\infty}$$

09.30.03.0005.01

$$\operatorname{ds}(K(m)\mid m)=\sqrt{1-m}$$

09.30.03.0006.01

$$\operatorname{ds}(2K(m)\mid m)=\tilde{\infty}$$

09.30.03.0007.01

$$\operatorname{ds}(3K(m)\mid m)=-\sqrt{1-m}$$

09.30.03.0008.01

$$\operatorname{ds}(4K(m)\mid m)=\tilde{\infty}$$

09.30.03.0009.01

$$\operatorname{ds}(iK(1-m)\mid m)=-i\sqrt{m}$$

09.30.03.0010.01

$$\operatorname{ds}(2iK(1-m)\mid m)=\tilde{\infty}$$

09.30.03.0011.01

$$\operatorname{ds}(3iK(1-m)\mid m)=i\sqrt{m}$$

09.30.03.0012.01

$$\operatorname{ds}(4iK(1-m)\mid m)=\tilde{\infty}$$

09.30.03.0013.01

$$\operatorname{ds}(K(m)+iK(1-m)\mid m)=0$$

09.30.03.0014.01

$$\operatorname{ds}(2K(m)+iK(1-m)\mid m)=i\sqrt{m}$$

09.30.03.0015.01

$$\operatorname{ds}(3K(m)+iK(1-m)\mid m)=0$$

09.30.03.0016.01

$$\operatorname{ds}(4K(m)+iK(1-m)\mid m)=-i\sqrt{m}$$

09.30.03.0017.01

$$\operatorname{ds}(K(m)+2iK(1-m)\mid m)=-\sqrt{1-m}$$

09.30.03.0018.01

$$\operatorname{ds}(2K(m)+2iK(1-m)\mid m)=\tilde{\infty}$$

09.30.03.0019.01

$$\operatorname{ds}(3K(m)+2iK(1-m)\mid m)=\sqrt{1-m}$$

09.30.03.0020.01

$$\operatorname{ds}(4\,K(m)+2\,i\,K(1-m)\mid m)=\tilde{\infty}$$

09.30.03.0021.01

$$\operatorname{ds}(2\,r\,K(m)+2\,s\,i\,K(1-m)\mid m)=\tilde{\infty}\;/;\;\{r,s\}\in\mathbb{Z}$$

Values at half-quarter-period points

09.30.03.0022.01

$$\operatorname{ds}\left(\frac{K(m)}{2}\,\middle|\,m\right)=\sqrt[4]{1-m}\,\sqrt{1+\sqrt{1-m}}$$

09.30.03.0023.01

$$\operatorname{ds}\left(\frac{i\,K(1-m)}{2}\,\middle|\,m\right)=-i\,\sqrt[4]{m}\,\sqrt{1+\sqrt{m}}$$

09.30.03.0024.01

$$\operatorname{ds}\left(\frac{K(m)}{2}+\frac{i\,K(1-m)}{2}\,\middle|\,m\right)=\sqrt[4]{1-m}\,\sqrt[4]{m}\,\frac{\sqrt{1+\sqrt{1-m}}-i\sqrt{1-\sqrt{1-m}}}{\sqrt{1+\sqrt{m}}+i\sqrt{1-\sqrt{m}}}$$

General characteristics

Domain and analyticity

$\operatorname{ds}(z\mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.30.04.0001.01

$$(z*m)\longrightarrow\operatorname{ds}(z\mid m)::(\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Parity

$\operatorname{ds}(z\mid m)$ is an odd function with respect to z.

09.30.04.0002.01

$$\operatorname{ds}(-z\mid m)=-\operatorname{ds}(z\mid m)$$

Mirror symmetry

09.30.04.0003.01

$$\operatorname{ds}(\bar{z}\mid\bar{m})=\overline{\operatorname{ds}(z\mid m)}$$

Periodicity

$\operatorname{ds}(z\mid m)$ is a doubly periodic function with respect to z with periods $4\,i\,K(1-m)$ and $4\,K(m)$.

09.30.04.0004.01

$$\operatorname{ds}(z+2\,K(m)\mid m)=-\operatorname{ds}(z\mid m)$$

09.30.04.0005.01

$$\operatorname{ds}(z+4\,K(m)\mid m)=\operatorname{ds}(z\mid m)$$

09.30.04.0006.01

$\mathrm{ds}(z + 2\,i\,K(1-m) \mid m) = -\mathrm{ds}(z \mid m)$

09.30.04.0007.01

$\mathrm{ds}(z + 4\,i\,K(1-m) \mid m) = \mathrm{ds}(z \mid m)$

09.30.04.0008.01

$\mathrm{ds}(z + 2\,K(m) + 2\,i\,K(1-m) \mid m) = \mathrm{ds}(z \mid m)$

09.30.04.0009.01

$\mathrm{ds}(z + 2\,i\,s\,K(1-m) + 2\,r\,K(m) \mid m) = (-1)^{r+s}\,\mathrm{ds}(z \mid m) \;/;\; \{r, s\} \in \mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed m, the function ds$(z \mid m)$ has an infinite set of singular points:

a) $z = 2\,r\,K(m) + 2\,s\,i\,K(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $(-1)^{r+s}$;

b) $z = \tilde{\infty}$ is an essential singular point.

09.30.04.0010.01

$\mathcal{Sing}_z(\mathrm{ds}(z \mid m)) = \{\{\{2\,s\,i\,K(1-m) + 2\,r\,K(m), 1\} \;/;\; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$

09.30.04.0011.01

$\mathrm{res}_z(\mathrm{ds}(z \mid m))\,(2\,s\,i\,K(1-m) + 2\,r\,K(m)) = (-1)^{r+s} \;/;\; \{r, s\} \in \mathbb{Z}$

Branch points

With respect to *m*

For fixed z, the function ds$(z \mid m)$ is a meromorphic function in m that has no branch points.

09.30.04.0014.01

$\mathcal{BP}_m(\mathrm{ds}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function ds$(z \mid m)$ does not have branch points.

09.30.04.0012.01

$\mathcal{BP}_z(\mathrm{ds}(z \mid m)) = \{\}$

Branch cuts

With respect to *m*

For fixed z, the function ds$(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.30.04.0015.01

$\mathcal{BC}_m(\mathrm{ds}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function ds$(z \mid m)$ does not have branch cuts.

09.30.04.0013.01

$$\mathcal{BC}_z(\text{ds}(z \mid m)) = \{\}$$

Series representations

Generalized power series

Expansions at $z = 0$

09.30.06.0005.01

$$\text{ds}(z \mid m) \propto \frac{1}{z} + \frac{1}{6}(1-2m)\,z + \frac{1}{360}\left(7+8m-8m^2\right)z^3 + \ldots \;/;\; (z \to 0)$$

09.30.06.0001.02

$$\text{ds}(z \mid m) \propto \frac{1}{z} + \frac{1}{6}(1-2m)\,z + \frac{1}{360}\left(7+8m-8m^2\right)z^3 + \frac{\left(31-78m+48m^2-32m^3\right)z^5}{15\,120} + \frac{\left(127-224m+96m^2+256m^3-128m^4\right)z^7}{604\,800} + \frac{1}{23\,950\,080}\left(\left(511-1294m+1072m^2-1568m^3+1280m^4-512m^5\right)z^9\right) + O\left(z^{11}\right)$$

09.30.06.0006.01

$$\text{ds}(z \mid m) = \sum_{k=0}^{\infty}\sum_{j=0}^{k} \frac{(j+1)(-1)^{k-j}\,\text{dn}_{k-j}(m)}{(2k-2j)!} \sum_{r=0}^{j} \frac{(-1)^r}{r+1}\binom{j}{r} q_{r,j}\, z^{2k-1} \;/;\; q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k}\sum_{i=1}^{k} \frac{(j\,i+i-k)(-1)^i\,\text{sn}_i(m)\,q_{j,k-i}}{(2i+1)!} \bigwedge k \in \mathbb{N}^+ \bigwedge \text{sn}_0(m) = 1 \bigwedge \text{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\text{cn}_j(m)\,\text{dn}_k(m)\,\delta_{j+k-n} \bigwedge \text{cn}_0(m) = 1 \bigwedge \text{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\text{sn}_j(m)\,\text{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \text{dn}_0(m) = 1 \bigwedge \text{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\text{sn}_j(m)\,\text{cn}_k(m)\,\delta_{j+k-n+1}$$

09.30.06.0007.01

$$\text{ds}(z \mid m) \propto \frac{1}{z}\left(1 + O\left(z^2\right)\right)$$

Expansions at $z = 2\,r\,K(m) + 2\,i\,s\,K(1-m)$

09.30.06.0008.01

$$\text{ds}(z \mid m) \propto (-1)^{r+s}\left(\frac{1}{z-z_0} + \frac{1}{6}(1-2m)(z-z_0) + \frac{1}{360}\left(-8m^2+8m+7\right)(z-z_0)^3 + \ldots\right) /;\; (z \to z_0) \bigwedge z_0 = 2\,r\,K(m) + 2\,i\,s\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

09.30.06.0009.01

$$\operatorname{ds}(z\mid m)=(-1)^{r+s}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)\,(-1)^{k-j}\,\operatorname{dn}_{k-j}(m)}{(2k-2j)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}q_{r,j}\,(z-z_0)^{2k-1}\,/;$$

$$z_0=2\,r\,K(m)+2\,i\,s\,K(1-m)\bigwedge r\in\mathbb{Z}\bigwedge s\in\mathbb{Z}\bigwedge q_{j,0}=1\bigwedge q_{j,k}=\frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i+i-k)\,(-1)^i\,\operatorname{sn}_i(m)\,q_{j,k-i}}{(2i+1)!}\bigwedge$$

$$k\in\mathbb{N}^+\bigwedge\operatorname{sn}_0(m)=1\bigwedge\operatorname{sn}_n(m)=\sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\operatorname{cn}_j(m)\,\operatorname{dn}_k(m)\,\delta_{j+k-n}\bigwedge\operatorname{cn}_0(m)=1\bigwedge$$

$$\operatorname{cn}_n(m)=\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\,\operatorname{dn}_k(m)\,\delta_{j+k-n+1}\bigwedge\operatorname{dn}_0(m)=1\bigwedge\operatorname{dn}_n(m)=m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\,\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.30.06.0010.01

$$\operatorname{ds}(z\mid m)\propto\frac{(-1)^{r+s}}{z-z_0}\left(1+O\left((z-z_0)^2\right)\right)/;\,z_0=2\,r\,K(m)+2\,i\,s\,K(1-m)\land r\in\mathbb{Z}\land s\in\mathbb{Z}$$

Expansions at $m == 0$

09.30.06.0011.01

$$\operatorname{ds}(z\mid m)\propto\csc(z)+\frac{1}{16}\,(4\,z\cos(z)-7\sin(z)+\sin(3\,z))\csc^2(z)\,m+$$
$$\frac{1}{512}\left(24\,z^2+12\sin(2\,z)\,z-4\sin(4\,z)\,z+8\left(z^2+4\right)\cos(2\,z)-3\cos(4\,z)-29\right)\csc^3(z)\,m^2+\ldots\,/;\,(m\to 0)$$

09.30.06.0012.01

$$\operatorname{ds}(z\mid m)\propto\csc(z)+\frac{1}{16}\,(4\,z\cos(z)-7\sin(z)+\sin(3\,z))\csc^2(z)\,m+$$
$$\frac{1}{512}\left(24\,z^2+12\sin(2\,z)\,z-4\sin(4\,z)\,z+8\left(z^2+4\right)\cos(2\,z)-3\cos(4\,z)-29\right)\csc^3(z)\,m^2\;+$$
$$\frac{1}{49152}\left(16\,z\left(46\,z^2+15\right)\cos(z)+16\,z\left(2\,z^2-21\right)\cos(3\,z)+96\,z\cos(5\,z)+\right.$$
$$\left.6\left(16\cos(2\,z)\,z^2+8\cos(4\,z)\,z^2+168\,z^2+257\cos(2\,z)-18\cos(4\,z)-\cos(6\,z)-238\right)\sin(z)\right)\csc^4(z)\,m^3+\frac{1}{1572864}$$
$$\left(3680\,z^4+9480\,z^2+12\left(632\,z^2+407\right)\sin(2\,z)\,z+12\left(40\,z^2-307\right)\sin(4\,z)\,z-4\left(8\,z^2-195\right)\sin(6\,z)\,z+36\sin(8\,z)\,z+\right.$$
$$\left.\left(2432\,z^4-9768\,z^2+30447\right)\cos(2\,z)+2\left(16\,z^4+300\,z^2-4233\right)\cos(4\,z)-39\left(8\,z^2-7\right)\cos(6\,z)+60\cos(8\,z)-22314\right)$$
$$\csc^5(z)\,m^4+\frac{1}{251658240}\left(8\,z\left(26912\,z^4+54080\,z^2+28635\right)\cos(z)+48\,z\left(632\,z^4-8350\,z^2-8325\right)\cos(3\,z)+\right.$$
$$16\,z\left(8\,z^4-2170\,z^2+12615\right)\cos(5\,z)+180\,z\left(16\,z^2-149\right)\cos(7\,z)-4500\,z\cos(9\,z)+$$
$$280\left(1388\,z^4+5679\,z^2-15162\right)\sin(z)+30\left(6512\,z^4-19620\,z^2+72503\right)\sin(3\,z)+15\left(96\,z^4+3704\,z^2-31017\right)$$
$$\left.\sin(5\,z)+10\left(16\,z^4-1320\,z^2+39\right)\sin(7\,z)-30\left(36\,z^2-161\right)\sin(9\,z)+15\sin(11\,z)\right)\csc^6(z)\,m^5+$$
$$\frac{1}{24159191040}\left(3014144\,z^6+16319520\,z^4+56145600\,z^2+24\left(485744\,z^4+1269960\,z^2+770145\right)\sin(2\,z)\,z+\right.$$
$$84\left(22592\,z^4-160320\,z^2-205725\right)\sin(4\,z)\,z+36\left(288\,z^4-34920\,z^2+166925\right)\sin(6\,z)\,z-$$
$$24\left(16\,z^4-1680\,z^2+7305\right)\sin(8\,z)\,z+180\left(72\,z^2-1381\right)\sin(10\,z)\,z-900\sin(12\,z)\,z+$$
$$2\left(1349504\,z^6-3975840\,z^4-38433240\,z^2+116155305\right)\cos(2\,z)+$$
$$4\left(46208\,z^6-2084160\,z^4+5587560\,z^2-23539905\right)\cos(4\,z)+$$
$$\left(256\,z^6-20160\,z^4-1808640\,z^2+15633045\right)\cos(6\,z)-30\left(368\,z^4-2736\,z^2-19017\right)\cos(8\,z)+$$

$$225\left(432 z^2-967\right)\cos(10 z)-1620\cos(12 z)-154135350\Big)\csc^7(z)\,m^6+$$

$$\frac{1}{5411658792960}\Big(8 z\left(33244544 z^6+170063712 z^4+342812400 z^2+160989885\right)\cos(z)+$$
$$12 z\left(5176064 z^6-94961216 z^4-332047520 z^2-204001455\right)\cos(3 z)+$$
$$8 z\left(278912 z^6-27447840 z^4+145308660 z^2+187870725\right)\cos(5 z)+$$
$$4 z\left(256 z^6-366912 z^4+17686200 z^2-76415535\right)\cos(7 z)+$$
$$84 z\left(896 z^4+143240 z^2-784815\right)\cos(9 z)-5040 z\left(630 z^2-5647\right)\cos(11 z)+$$
$$258300 z\cos(13 z)+189\left(3207168 z^6+20784960 z^4+85868640 z^2-253271975\right)\sin(z)+$$
$$28\left(16336768 z^6+1462800 z^4-300501630 z^2+1026824535\right)\sin(3 z)+$$
$$14\left(2404096 z^6-57351120 z^4+136471140 z^2-678730635\right)\sin(5 z)+$$
$$7\left(4352 z^6-835680 z^4-6135120 z^2+172064925\right)\sin(7 z)+7\left(256 z^6+115680 z^4-6263280 z^2+14958765\right)$$
$$\sin(9 z)-630\left(432 z^4-22276 z^2+35169\right)\sin(11 z)+2520\left(25 z^2-109\right)\sin(13 z)-315\sin(15 z)\Big)\csc^8(z)\,m^7+$$

$$\frac{1}{173173081374720}\Big(1196803584 z^8+11636626176 z^6+54896637600 z^4+205016064120 z^2+$$
$$28\left(225682432 z^6+1385952768 z^4+3229458120 z^2+1284454305\right)\sin(2 z)\,z+$$
$$4\left(464093696 z^6-3559129056 z^4-17629256400 z^2-9048398625\right)\sin(4 z)\,z+$$
$$72\left(959872 z^6-47697552 z^4+230563200 z^2+193412625\right)\sin(6 z)\,z+$$
$$12\left(3328 z^6-273056 z^4-23444120 z^2+42941325\right)\sin(8 z)\,z-8\left(128 z^6+544656 z^4-59191440 z^2+252167265\right)$$
$$\sin(10 z)\,z+756\left(864 z^4-95200 z^2+578185\right)\sin(12 z)\,z-420\left(1000 z^2-16371\right)\sin(14 z)\,z+8820\sin(16 z)\,z+$$
$$2\left(636233728 z^8-1429788416 z^6-27307576800 z^4-154793887920 z^2+499901171175\right)\cos(2 z)+$$
$$4\left(42446336 z^8-2032549120 z^6-2792512800 z^4+31520509650 z^2-123914582505\right)\cos(4 z)+$$
$$63\left(53248 z^8-10261760 z^6+168837600 z^4-321859800 z^2+2156834375\right)\cos(6 z)+$$
$$2\left(256 z^8-151424 z^6+158348400 z^4-1186709580 z^2-6231780135\right)\cos(8 z)-$$
$$21\left(2816 z^6+3608800 z^4-66159480 z^2+99713535\right)\cos(10 z)+1260\left(8640 z^4-195390 z^2+251237\right)\cos(12 z)-$$
$$1260\left(2300 z^2-4459\right)\cos(14 z)+16380\cos(16 z)-625789226790\Big)\csc^9(z)\,m^8+\frac{1}{49873847435919360}$$

$$\Big(8 z\left(17769803264 z^8+154333234944 z^6+658896390432 z^4+1407530896380 z^2+362130876765\right)\cos(z)+4 z$$
$$\left(11114481664 z^8-217986748416 z^6-1797855073056 z^4-5056519328280 z^2-1382731177275\right)\cos(3 z)+$$
$$20 z\left(179849216 z^8-17460979200 z^6+71947897056 z^4+564845891400 z^2+161241132465\right)\cos(5 z)+$$
$$8 z\left(5036288 z^8-1686287232 z^6+60858081648 z^4-313633139400 z^2-15268961295\right)\cos(7 z)+$$
$$8 z\left(256 z^8-1169280 z^6-963679248 z^4+33854058000 z^2-99675132795\right)\cos(9 z)+$$
$$36 z\left(9728 z^6+66681888 z^4-2916648840 z^2+10776893715\right)\cos(11 z)-$$
$$11340 z\left(23328 z^4-1053928 z^2+5114407\right)\cos(13 z)+56700 z\left(3400 z^2-23959\right)\cos(15 z)-4524660 z\cos(17 z)+$$
$$63\left(5765827584 z^8+64166102528 z^6+333730895520 z^4+1325343713760 z^2-4307074148655\right)\sin(z)+$$
$$216\left(1654030208 z^8+3788596000 z^6-26569322920 z^4-230872244295 z^2+831876702090\right)\sin(3 z)+$$
$$360\left(149785536 z^8-3223529792 z^6-7078597260 z^4+44068988880 z^2-209000751309\right)\sin(5 z)+$$
$$9\left(120776192 z^8-10973310208 z^6+131103658560 z^4-134250966360 z^2+1915278069285\right)\sin(7 z)+$$
$$9\left(12800 z^8-59381504 z^6+10897393920 z^4-90497521800 z^2-110680748955\right)\sin(9 z)+$$
$$36\left(128 z^8+2929920 z^6-606712680 z^4+7685227620 z^2-9452706795\right)\sin(11 z)-$$
$$189\left(62208 z^6-12659840 z^4+187016760 z^2-212694795\right)\sin(13 z)+$$
$$945\left(20000 z^4-800040 z^2+997737\right)\sin(15 z)-5670\left(196 z^2-839\right)\sin(17 z)+2835\sin(19 z)\Big)\csc^{10}(z)\,m^9+$$

$$\frac{1}{7979815589747097600}\Big(3127485374464 z^{10}+43993064586240 z^8+329797755302400 z^6+$$

$$1\,513\,901\,172\,168\,000\,z^4 + 5\,479\,777\,375\,934\,400\,z^2 + 240\left(83\,452\,515\,584\,z^8 + 827\,660\,584\,512\,z^6 + 3\,867\,171\,581\,448\,z^4 + 9\,326\,318\,248\,800\,z^2 + 1\,174\,089\,492\,135\right)\sin(2z)\,z + 120\left(69\,804\,095\,744\,z^8 - 411\,013\,092\,096\,z^6 - 5\,327\,507\,238\,480\,z^4 - 18\,940\,300\,243\,200\,z^2 - 1\,802\,961\,717\,165\right)\sin(4z)\,z + 40\left(18\,228\,084\,992\,z^8 - 790\,535\,470\,464\,z^6 + 1\,646\,629\,919\,424\,z^4 + 25\,904\,678\,506\,560\,z^2 - 1\,148\,910\,594\,615\right)\sin(6z)\,z + 360\left(22\,940\,672\,z^8 - 3\,575\,028\,736\,z^6 + 110\,596\,799\,712\,z^4 - 658\,808\,191\,440\,z^2 + 477\,932\,160\,195\right)\sin(8z)\,z + 120\left(4352\,z^8 + 37\,633\,920\,z^6 - 14\,004\,944\,352\,z^4 + 342\,040\,028\,400\,z^2 - 979\,532\,361\,045\right)\sin(10z)\,z - 20\left(512\,z^8 + 53\,217\,792\,z^6 - 18\,912\,731\,040\,z^4 + 539\,471\,197\,440\,z^2 - 1\,819\,814\,251\,815\right)\sin(12z)\,z + 540\left(186\,624\,z^6 - 62\,952\,736\,z^4 + 1\,785\,363\,720\,z^2 - 7\,673\,444\,625\right)\sin(14z)\,z - 18\,900\left(20\,000\,z^4 - 1\,596\,816\,z^2 + 7\,072\,143\right)\sin(16z)\,z + 18\,900\left(2744\,z^2 - 41\,535\right)\sin(18z)\,z - 510\,300\sin(20z)\,z + \left(3\,714\,757\,763\,072\,z^{10} - 723\,491\,573\,760\,z^8 - 262\,397\,077\,770\,240\,z^6 - 1\,943\,491\,925\,642\,400\,z^4 - 8\,702\,764\,170\,067\,800\,z^2 + 30\,758\,459\,515\,640\,325\right)\cos(2z) + 2\left(365\,160\,251\,392\,z^{10} - 18\,369\,497\,514\,240\,z^8 - 81\,320\,502\,170\,880\,z^6 + 133\,554\,317\,929\,200\,z^4 + 2\,091\,791\,621\,455\,200\,z^2 - 8\,657\,487\,490\,731\,975\right)\cos(4z) + \left(37\,339\,713\,536\,z^{10} - 6\,398\,908\,139\,520\,z^8 + 87\,398\,181\,884\,160\,z^6 + 235\,747\,482\,415\,200\,z^4 - 978\,366\,651\,759\,000\,z^2 + 6\,218\,861\,152\,229\,775\right)\cos(6z) + 32\left(7\,556\,864\,z^{10} - 4\,114\,395\,360\,z^8 + 240\,986\,274\,480\,z^6 - 1\,951\,658\,197\,650\,z^4 - 1\,928\,846\,922\,975\,z^2 - 37\,158\,703\,329\,825\right)\cos(8z) + \left(4096\,z^{10} - 8\,202\,240\,z^8 + 158\,795\,884\,800\,z^6 - 13\,173\,126\,876\,000\,z^4 + 103\,379\,598\,264\,600\,z^2 + 26\,012\,863\,464\,525\right)\cos(10z) - 90\left(11\,008\,z^8 + 346\,123\,008\,z^6 - 28\,798\,124\,880\,z^4 + 295\,641\,672\,480\,z^2 - 323\,929\,341\,225\right)\cos(12z) + 4725\left(622\,080\,z^6 - 47\,770\,336\,z^4 + 557\,168\,808\,z^2 - 594\,087\,585\right)\cos(14z) - 4725\left(1\,120\,000\,z^4 - 18\,727\,968\,z^2 + 17\,561\,337\right)\cos(16z) + 2\,778\,300\left(124\,z^2 - 221\right)\cos(18z) - 963\,900\cos(20z) - 18\,525\,543\,027\,901\,875\Big)\csc^{11}(z)\,m^{10} + O\!\left(m^{11}\right)$$

09.30.06.0013.01

$$\operatorname{ds}(z \mid m) \propto \csc(z)\,(1 + O(m))$$

Expansions at *m* == 1

09.30.06.0014.01

$$\operatorname{ds}(z \mid m) \propto \operatorname{csch}(z) - \frac{1}{16}\,(4z\cosh(z) - 7\sinh(z) + \sinh(3z))\,\operatorname{csch}^2(z)\,(m-1) + \frac{1}{512}\left(24z^2 + 12\sinh(2z)\,z - 4\sinh(4z)\,z + 8\left(z^2 - 4\right)\cosh(2z) + 3\cosh(4z) + 29\right)\operatorname{csch}^3(z)\,(m-1)^2 + \ldots \;/;\; (m \to 1)$$

09.30.06.0015.01

$$\operatorname{ds}(z \mid m) \propto \operatorname{csch}(z) - \frac{1}{16}\,(4z\cosh(z) - 7\sinh(z) + \sinh(3z))\,\operatorname{csch}^2(z)\,(m-1) + \frac{1}{512}\left(24z^2 + 12\sinh(2z)\,z - 4\sinh(4z)\,z + 8\left(z^2 - 4\right)\cosh(2z) + 3\cosh(4z) + 29\right)\operatorname{csch}^3(z)\,(m-1)^2 - \frac{1}{49\,152}\left(16z\left(46z^2 - 15\right)\cosh(z) + 16z\left(2z^2 + 21\right)\cosh(3z) - 96z\cosh(5z) + 3\left(320z^2 + 733\right)\sinh(z) + 3\left(8z^2 - 275\right)\sinh(3z) + 3\left(8z^2 + 17\right)\sinh(5z) + 3\sinh(7z)\right)\operatorname{csch}^4(z)\,(m-1)^3 + \frac{1}{1\,572\,864}\left(2\left(1840z^4 - 4740z^2 - 11\,157\right) + \left(2432z^4 + 9768z^2 + 30\,447\right)\cosh(2z) + 2\left(16z^4 - 300z^2 - 4233\right)\cosh(4z) + 39\left(8z^2 + 7\right)\cosh(6z) + 60\cosh(8z) + 12z\left(632z^2 - 407\right)\sinh(2z) + 12z\left(40z^2 + 307\right)\sinh(4z) - 4z\left(8z^2 + 195\right)\sinh(6z) - 36z\sinh(8z)\right)\operatorname{csch}^5(z)\,(m-1)^4 - \frac{1}{251\,658\,240}\left(8z\left(26\,912z^4 - 54\,080z^2 + 28\,635\right)\cosh(z) + 48z\left(632z^4 + 8350z^2 - 8325\right)\cosh(3z) + \right.$$

$$16\,z\left(8\,z^4+2170\,z^2+12\,615\right)\cosh(5\,z)-180\,z\left(16\,z^2+149\right)\cosh(7\,z)-4500\,z\cosh(9\,z)+$$
$$280\left(1388\,z^4-5679\,z^2-15\,162\right)\sinh(z)+30\left(6512\,z^4+19\,620\,z^2+72\,503\right)\sinh(3\,z)+$$
$$15\left(96\,z^4-3704\,z^2-31\,017\right)\sinh(5\,z)+10\left(16\,z^4+1320\,z^2+39\right)\sinh(7\,z)+30\left(36\,z^2+161\right)\sinh(9\,z)+15\sinh(11\,z)\Big)$$
$$\operatorname{csch}^6(z)\,(m-1)^5+\frac{1}{24\,159\,191\,040}\Big(2\left(1\,507\,072\,z^6-8\,159\,760\,z^4+28\,072\,800\,z^2+77\,067\,675\right)+$$
$$2\left(1\,349\,504\,z^6+3\,975\,840\,z^4-38\,433\,240\,z^2-116\,155\,305\right)\cosh(2\,z)+$$
$$4\left(46\,208\,z^6+2\,084\,160\,z^4+5\,587\,560\,z^2+23\,539\,905\right)\cosh(4\,z)+$$
$$\left(256\,z^6+20\,160\,z^4-1\,808\,640\,z^2-15\,633\,045\right)\cosh(6\,z)+30\left(368\,z^4+2736\,z^2-19\,017\right)\cosh(8\,z)+$$
$$225\left(432\,z^2+967\right)\cosh(10\,z)+1620\cosh(12\,z)+24\,z\left(485\,744\,z^4-1\,269\,960\,z^2+770\,145\right)\sinh(2\,z)+$$
$$84\,z\left(22\,592\,z^4+160\,320\,z^2-205\,725\right)\sinh(4\,z)+36\,z\left(288\,z^4+34\,920\,z^2+166\,925\right)\sinh(6\,z)-$$
$$24\,z\left(16\,z^4+1680\,z^2+7305\right)\sinh(8\,z)-180\,z\left(72\,z^2+1381\right)\sinh(10\,z)-900\,z\sinh(12\,z)\Big)\operatorname{csch}^7(z)\,(m-1)^6-$$
$$\frac{1}{5\,411\,658\,792\,960}\Big(8\,z\left(33\,244\,544\,z^6-170\,063\,712\,z^4+342\,812\,400\,z^2-160\,989\,885\right)\cosh(z)+$$
$$12\,z\left(5\,176\,064\,z^6+94\,961\,216\,z^4-332\,047\,520\,z^2+204\,001\,455\right)\cosh(3\,z)+$$
$$8\,z\left(278\,912\,z^6+27\,447\,840\,z^4+145\,308\,660\,z^2-187\,870\,725\right)\cosh(5\,z)+$$
$$4\,z\left(256\,z^6+366\,912\,z^4+17\,686\,200\,z^2+76\,415\,535\right)\cosh(7\,z)-$$
$$84\,z\left(896\,z^4-143\,240\,z^2-784\,815\right)\cosh(9\,z)-5040\,z\left(630\,z^2+5647\right)\cosh(11\,z)-$$
$$258\,300\,z\cosh(13\,z)+189\left(3\,207\,168\,z^6-20\,784\,960\,z^4+85\,868\,640\,z^2+253\,271\,975\right)\sinh(z)+$$
$$28\left(16\,336\,768\,z^6-1\,462\,800\,z^4-300\,501\,630\,z^2-1\,026\,824\,535\right)\sinh(3\,z)+$$
$$14\left(2\,404\,096\,z^6+57\,351\,120\,z^4+136\,471\,140\,z^2+678\,730\,635\right)\sinh(5\,z)+$$
$$7\left(4352\,z^6+835\,680\,z^4-6\,135\,120\,z^2-172\,064\,925\right)\sinh(7\,z)+$$
$$7\left(256\,z^6-115\,680\,z^4-6\,263\,280\,z^2-14\,958\,765\right)\sinh(9\,z)+630\left(432\,z^4+22\,276\,z^2+35\,169\right)\sinh(11\,z)+$$
$$2520\left(25\,z^2+109\right)\sinh(13\,z)+315\sinh(15\,z)\Big)\operatorname{csch}^8(z)\,(m-1)^7+$$
$$\frac{1}{173\,173\,081\,374\,720}\Big(18\left(66\,489\,088\,z^8-646\,479\,232\,z^6+3\,049\,813\,200\,z^4-11\,389\,781\,340\,z^2-34\,766\,068\,155\right)+$$
$$2\left(636\,233\,728\,z^8+1\,429\,788\,416\,z^6-27\,307\,576\,800\,z^4+154\,793\,887\,920\,z^2+499\,901\,171\,175\right)\cosh(2\,z)+$$
$$4\left(42\,446\,336\,z^8+2\,032\,549\,120\,z^6-2\,792\,512\,800\,z^4-31\,520\,509\,650\,z^2-123\,914\,582\,505\right)\cosh(4\,z)+$$
$$63\left(53\,248\,z^8+10\,261\,760\,z^6+168\,837\,600\,z^4+321\,859\,800\,z^2+2\,156\,834\,375\right)\cosh(6\,z)+$$
$$2\left(256\,z^8+151\,424\,z^6+158\,348\,400\,z^4+1\,186\,709\,580\,z^2-6\,231\,780\,135\right)\cosh(8\,z)+$$
$$21\left(2816\,z^6-3\,608\,800\,z^4-66\,159\,480\,z^2-99\,713\,535\right)\cosh(10\,z)+$$
$$1260\left(8640\,z^4+195\,390\,z^2+251\,237\right)\cosh(12\,z)+1260\left(2300\,z^2+4459\right)\cosh(14\,z)+$$
$$16\,380\cosh(16\,z)+28\,z\left(225\,682\,432\,z^6-1\,385\,952\,768\,z^4+3\,229\,458\,120\,z^2-1\,284\,454\,305\right)\sinh(2\,z)+$$
$$4\,z\left(464\,093\,696\,z^6+3\,559\,129\,056\,z^4-17\,629\,256\,400\,z^2+9\,048\,398\,625\right)\sinh(4\,z)+$$
$$72\,z\left(959\,872\,z^6+47\,697\,552\,z^4+230\,563\,200\,z^2-193\,412\,625\right)\sinh(6\,z)+$$
$$12\,z\left(3328\,z^6+273\,056\,z^4-23\,444\,120\,z^2-42\,941\,325\right)\sinh(8\,z)-$$
$$8\,z\left(128\,z^6-544\,656\,z^4-59\,191\,440\,z^2-252\,167\,265\right)\sinh(10\,z)-756\,z\left(864\,z^4+95\,200\,z^2+578\,185\right)\sinh(12\,z)-$$
$$420\,z\left(1000\,z^2+16\,371\right)\sinh(14\,z)-8820\,z\sinh(16\,z)\Big)\operatorname{csch}^9(z)\,(m-1)^8-\frac{1}{49\,873\,847\,435\,919\,360}$$
$$\Big(8\,z\left(17\,769\,803\,264\,z^8-154\,333\,234\,944\,z^6+658\,896\,390\,432\,z^4-1\,407\,530\,896\,380\,z^2+362\,130\,876\,765\right)\cosh(z)+$$
$$4\,z\left(11\,114\,481\,664\,z^8+217\,986\,748\,416\,z^6-1\,797\,855\,073\,056\,z^4+5\,056\,519\,328\,280\,z^2-1\,382\,731\,177\,275\right)$$
$$\cosh(3\,z)+20\,z\left(179\,849\,216\,z^8+17\,460\,979\,200\,z^6+71\,947\,897\,056\,z^4-564\,845\,891\,400\,z^2+161\,241\,132\,465\right)$$
$$\cosh(5\,z)+8\,z\left(5\,036\,288\,z^8+1\,686\,287\,232\,z^6+60\,858\,081\,648\,z^4+313\,633\,139\,400\,z^2-15\,268\,961\,295\right)\cosh(7\,z)+$$
$$8\,z\left(256\,z^8+1\,169\,280\,z^6-963\,679\,248\,z^4-33\,854\,058\,000\,z^2-99\,675\,132\,795\right)\cosh(9\,z)-$$

$$36\,z\left(9728\,z^6-66\,681\,888\,z^4-2\,916\,648\,840\,z^2-10\,776\,893\,715\right)\cosh(11\,z)-11\,340\,z\left(23\,328\,z^4+1\,053\,928\,z^2+5\,114\,407\right)\cosh(13\,z)-56\,700\,z\left(3400\,z^2+23\,959\right)\cosh(15\,z)-4\,524\,660\,z\cosh(17\,z)+63\left(5\,765\,827\,584\,z^8-64\,166\,102\,528\,z^6+333\,730\,895\,520\,z^4-1\,325\,343\,713\,760\,z^2-4\,307\,074\,148\,655\right)\sinh(z)+216\left(1\,654\,030\,208\,z^8-3\,788\,596\,000\,z^6-26\,569\,322\,920\,z^4+230\,872\,244\,295\,z^2+831\,876\,702\,090\right)\sinh(3\,z)+360\left(149\,785\,536\,z^8+3\,223\,529\,792\,z^6-7\,078\,597\,260\,z^4-44\,068\,988\,880\,z^2-209\,000\,751\,309\right)\sinh(5\,z)+9\left(120\,776\,192\,z^8+10\,973\,310\,208\,z^6+131\,103\,658\,560\,z^4+134\,250\,966\,360\,z^2+1\,915\,278\,069\,285\right)\sinh(7\,z)+9\left(12\,800\,z^8+59\,381\,504\,z^6+10\,897\,393\,920\,z^4+90\,497\,521\,800\,z^2-110\,680\,748\,955\right)\sinh(9\,z)+36\left(128\,z^8-2\,929\,920\,z^6-606\,712\,680\,z^4-7\,685\,227\,620\,z^2-9\,452\,706\,795\right)\sinh(11\,z)+189\left(62\,208\,z^6+12\,659\,840\,z^4+187\,016\,760\,z^2+212\,694\,795\right)\sinh(13\,z)+945\left(20\,000\,z^4+800\,040\,z^2+997\,737\right)\sinh(15\,z)+5670\left(196\,z^2+839\right)\sinh(17\,z)+2835\sinh(19\,z)\Big)$$

$$\operatorname{csch}^{10}(z)\,(m-1)^9+\frac{1}{7\,979\,815\,589\,747\,097\,600}\Big(3\,127\,485\,374\,464\,z^{10}-43\,993\,064\,586\,240\,z^8+329\,797\,755\,302\,400\,z^6-1\,513\,901\,172\,168\,000\,z^4+5\,479\,777\,375\,934\,400\,z^2+240\left(83\,452\,515\,584\,z^8-827\,660\,584\,512\,z^6+3\,867\,171\,581\,448\,z^4-9\,326\,318\,248\,800\,z^2+1\,174\,089\,492\,135\right)\sinh(2\,z)\,z+120\left(69\,804\,095\,744\,z^8+411\,013\,092\,096\,z^6-5\,327\,507\,238\,480\,z^4+18\,940\,300\,243\,200\,z^2-1\,802\,961\,717\,165\right)\sinh(4\,z)\,z+40\left(18\,228\,084\,992\,z^8+790\,535\,470\,464\,z^6+1\,646\,629\,919\,424\,z^4-25\,904\,678\,506\,560\,z^2-1\,148\,910\,594\,615\right)\sinh(6\,z)\,z+360\left(22\,940\,672\,z^8+3\,575\,028\,736\,z^6+110\,596\,799\,712\,z^4+658\,808\,191\,440\,z^2+477\,932\,160\,195\right)\sinh(8\,z)\,z+120\left(4352\,z^8-37\,633\,920\,z^6-14\,004\,944\,352\,z^4-342\,040\,028\,400\,z^2-979\,532\,361\,045\right)\sinh(10\,z)\,z-20\left(512\,z^8-53\,217\,792\,z^6-18\,912\,731\,040\,z^4-539\,471\,197\,440\,z^2-1\,819\,814\,251\,815\right)\sinh(12\,z)\,z-540\left(186\,624\,z^6+62\,952\,736\,z^4+1\,785\,363\,720\,z^2+7\,673\,444\,625\right)\sinh(14\,z)\,z-18\,900\left(20\,000\,z^4+1\,596\,816\,z^2+7\,072\,143\right)\sinh(16\,z)\,z-18\,900\left(2744\,z^2+41\,535\right)\sinh(18\,z)\,z-510\,300\sinh(20\,z)\,z+\left(3\,714\,757\,763\,072\,z^{10}+723\,491\,573\,760\,z^8-262\,397\,077\,770\,240\,z^6+1\,943\,491\,925\,642\,400\,z^4-8\,702\,764\,170\,067\,800\,z^2-30\,758\,459\,515\,640\,325\right)\cosh(2\,z)+2\left(365\,160\,251\,392\,z^{10}+18\,369\,497\,514\,240\,z^8-81\,320\,502\,170\,880\,z^6-133\,554\,317\,929\,200\,z^4+2\,091\,791\,621\,455\,200\,z^2+8\,657\,487\,490\,731\,975\right)\cosh(4\,z)+\left(37\,339\,713\,536\,z^{10}+6\,398\,908\,139\,520\,z^8+87\,398\,181\,884\,160\,z^6-235\,747\,482\,415\,200\,z^4-978\,366\,651\,759\,000\,z^2-6\,218\,861\,152\,229\,775\right)\cosh(6\,z)+32\left(7\,556\,864\,z^{10}+4\,114\,395\,360\,z^8+240\,986\,274\,480\,z^6+1\,951\,658\,197\,650\,z^4-1\,928\,846\,922\,975\,z^2+37\,158\,703\,329\,825\right)\cosh(8\,z)+\left(4096\,z^{10}+8\,202\,240\,z^8+158\,795\,884\,800\,z^6+13\,173\,126\,876\,000\,z^4+103\,379\,598\,264\,600\,z^2-26\,012\,863\,464\,525\right)\cosh(10\,z)+90\left(11\,008\,z^8-346\,123\,008\,z^6-28\,798\,124\,880\,z^4-295\,641\,672\,480\,z^2-323\,929\,341\,225\right)\cosh(12\,z)+4725\left(622\,080\,z^6+47\,770\,336\,z^4+557\,168\,808\,z^2+594\,087\,585\right)\cosh(14\,z)+4725\left(1\,120\,000\,z^4+18\,727\,968\,z^2+17\,561\,337\right)\cosh(16\,z)+2\,778\,300\left(124\,z^2+221\right)\cosh(18\,z)+963\,900\cosh(20\,z)+18\,525\,543\,027\,901\,875\Big)\operatorname{csch}^{11}(z)\,(m-1)^{10}+O\left((m-1)^{11}\right)$$

09.30.06.0016.01

$$\operatorname{ds}(z\mid m)\propto\operatorname{csch}(z)\,(1+O(m-1))$$

q-series

09.30.06.0002.01

$$\operatorname{ds}(z\mid m)=\frac{\pi}{2\,K(m)}\csc\left(\frac{\pi\,z}{2\,K(m)}\right)-\frac{2\,\pi}{K(m)}\sum_{k=0}^{\infty}\frac{q(m)^{2k+1}}{q(m)^{2k+1}+1}\sin\left(\frac{(2\,k+1)\,\pi\,z}{2\,K(m)}\right)$$

Other series representations

09.30.06.0003.01

$$\operatorname{ds}(z \mid m) = \frac{\pi}{2 K(1-m)} \sum_{k=-\infty}^{\infty} (-1)^k \operatorname{csch}\left(\pi \frac{K(m)}{K(1-m)} \left(k + \frac{z}{2 K(m)}\right)\right)$$

09.30.06.0004.01

$$\operatorname{ds}(z \mid m) \propto \frac{(-1)^{r+s}}{z - 2 s i K(1-m) - 2 r K(m)} + O(1) \;/;\; (z \to 2 s i K(1-m) + 2 r K(m)) \bigwedge \{r, s\} \in \mathbb{Z}$$

Product representations

09.30.08.0001.01

$$\operatorname{ds}(z \mid m) = \sqrt[4]{1-m}\, \frac{\sqrt[4]{m}}{2 \sqrt[4]{q(m)}} \csc\left(\frac{\pi z}{2 K(m)}\right) \prod_{k=1}^{\infty} \frac{1 + 2 q(m)^{2k-1} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k-2}}{1 - 2 q(m)^{2k} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k}}$$

Differential equations

Ordinary nonlinear differential equations

09.30.13.0001.01

$$w''(z) - w(z) \left(2 w(z)^2 + 2 m - 1\right) = 0 \;/;\; w(z) = \operatorname{ds}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.30.16.0001.01

$$\operatorname{ds}(i z \mid m) = -i \operatorname{ds}(z \mid 1-m)$$

09.30.16.0002.01

$$\operatorname{ds}(z \mid 1-m) = i \operatorname{ds}(i z \mid m)$$

09.30.16.0003.01

$$\operatorname{ds}(i z \mid 1-m) = -i \operatorname{ds}(z \mid m)$$

09.30.16.0007.01

$$\operatorname{ds}(x + i y \mid m) = (\operatorname{dn}(x \mid m) \operatorname{cn}(y \mid 1-m) \operatorname{dn}(y \mid 1-m) - i m \operatorname{sn}(x \mid m) \operatorname{cn}(x \mid m) \operatorname{sn}(y \mid 1-m)) / (\operatorname{dn}(y \mid 1-m) \operatorname{sn}(x \mid m) + i \operatorname{cn}(x \mid m) \operatorname{cn}(y \mid 1-m) \operatorname{dn}(x \mid m) \operatorname{sn}(y \mid 1-m)) \;/;\; \{x, y\} \in \mathbb{R}$$

09.30.16.0008.01

$$\operatorname{ds}\left(\sqrt{1-m}\, z \,\middle|\, \frac{m}{m-1}\right) = \frac{\operatorname{ns}(z \mid m)}{\sqrt{1-m}}$$

09.30.16.0009.01

$$\operatorname{ds}\left(\sqrt{m}\, z \,\middle|\, \frac{1}{m}\right) = \frac{\operatorname{cs}(z \mid m)}{\sqrt{m}}$$

09.30.16.0010.01

$$\operatorname{ds}\left(i \sqrt{1-m}\, z \,\middle|\, \frac{1}{1-m}\right) = -\frac{i \operatorname{ns}(z \mid m)}{\sqrt{1-m}}$$

09.30.16.0011.01

$$\operatorname{ds}\left(i\sqrt{m}\,z \,\middle|\, \frac{m-1}{m}\right) = -\frac{i\operatorname{cs}(z\mid m)}{\sqrt{m}}$$

Landen's transformation:

09.30.16.0012.01

$$\operatorname{ds}\left(\left(1+\sqrt{1-m}\right)z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{1-\left(1-\sqrt{1-m}\right)\operatorname{sn}(z\mid m)^2}{\left(1+\sqrt{1-m}\right)\operatorname{sn}(z\mid m)\operatorname{cn}(z\mid m)}$$

Gauss' transformation:

09.30.16.0013.01

$$\operatorname{ds}\left(\left(1+\sqrt{m}\right)z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{1-\sqrt{m}\operatorname{sn}(z\mid m)^2}{\left(1+\sqrt{m}\right)\operatorname{sn}(z\mid m)}$$

n th degree transformations:

09.30.16.0014.01

$$\operatorname{ds}\left(\frac{z}{M} \,\middle|\, l\right) = M\operatorname{ds}(z\mid m)\prod_{r=1}^{\frac{n-1}{2}} \frac{1-m\operatorname{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2}{1-\operatorname{ns}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n+1}{2}\in\mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n-1}{2}} \operatorname{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2}$$

09.30.16.0015.01

$$\operatorname{ds}\left(\frac{z}{M}+\frac{K(m)}{nM} \,\middle|\, l\right) = \frac{\sqrt{1-l}}{\operatorname{dn}(z\mid m)}\prod_{r=1}^{\frac{n}{2}} \frac{1-m\operatorname{sn}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2}{1-\operatorname{ns}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n}{2}\in\mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n}{2}} \operatorname{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2}$$

Argument involving half-periods

09.30.16.0004.01

$$\operatorname{ds}(z+K(m)\mid m) = \sqrt{1-m}\operatorname{nc}(z\mid m)$$

09.30.16.0020.01

$$\operatorname{ds}(z-K(m)\mid m) = -\sqrt{1-m}\operatorname{nc}(z\mid m)$$

09.30.16.0021.01

$$\operatorname{ds}(z+3K(m)\mid m) = -\sqrt{1-m}\operatorname{nc}(z\mid m)$$

09.30.16.0022.01

$$\operatorname{ds}(z+(2r+1)K(m)\mid m) = (-1)^r\sqrt{1-m}\operatorname{nc}(z\mid m) \;/;\; r\in\mathbb{Z}$$

09.30.16.0005.01

$$\operatorname{ds}(z + i\,K(1-m) \mid m) == -i\,\sqrt{m}\,\operatorname{cn}(z \mid m)$$

09.30.16.0023.01

$$\operatorname{ds}(z - i\,K(1-m) \mid m) = i\,\sqrt{m}\,\operatorname{cn}(z \mid m)$$

09.30.16.0024.01

$$\operatorname{ds}(z + 3\,i\,K(1-m) \mid m) = i\,\sqrt{m}\,\operatorname{cn}(z \mid m)\;/;\; s \in \mathbb{Z}$$

09.30.16.0025.01

$$\operatorname{ds}(z + (2\,s+1)\,i\,K(1-m) \mid m) = (-1)^{s-1}\,i\,\sqrt{m}\,\operatorname{cn}(z \mid m)\;/;\; s \in \mathbb{Z}$$

09.30.16.0006.01

$$\operatorname{ds}(z + K(m) + i\,K(1-m) \mid m) == i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0026.01

$$\operatorname{ds}(z - i\,K(1-m) + K(m) \mid m) = -i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0027.01

$$\operatorname{ds}(z + i\,K(1-m) - K(m) \mid m) = -i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0028.01

$$\operatorname{ds}(z - i\,K(1-m) - K(m) \mid m) = i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0029.01

$$\operatorname{ds}(z + i\,K(1-m) + 3\,K(m) \mid m) = -i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0030.01

$$\operatorname{ds}(z + (4\,s+1)\,i\,K(1-m) + (4\,r+1)\,K(m) \mid m) = i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0031.01

$$\operatorname{ds}(z + (4\,s+1)\,i\,K(1-m) + (4\,r-1)\,K(m) \mid m) = -i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0032.01

$$\operatorname{ds}(z + (4\,s-1)\,i\,K(1-m) + (4\,r+1)\,K(m) \mid m) = -i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0033.01

$$\operatorname{ds}(z + (4\,s-1)\,i\,K(1-m) + (4\,r-1)\,K(m) \mid m) = i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

09.30.16.0034.01

$$\operatorname{ds}(z + (2\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = (-1)^{r+s}\,i\,\sqrt{m}\,\sqrt{1-m}\,\operatorname{sd}(z \mid m)$$

Argument involving inverse Jacobi functions

09.30.16.0035.01

$$\operatorname{ds}\left(\operatorname{cd}^{-1}(z \mid m) \mid m\right)^2 = \frac{1-m}{1-z^2}$$

09.30.16.0036.01

$$\operatorname{ds}\left(\operatorname{cn}^{-1}(z \mid m) \mid m\right)^2 = \frac{m\,z^2 - m + 1}{1-z^2}$$

09.30.16.0037.01

$$\operatorname{ds}\left(\operatorname{cs}^{-1}(z \mid m) \mid m\right)^2 = z^2 - m + 1$$

09.30.16.0038.01

$$\operatorname{ds}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)^2 = \frac{(1-m)\,z^2}{z^2-1}$$

09.30.16.0039.01

$$\operatorname{ds}\left(\operatorname{dn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\, z^2}{1 - z^2}$$

09.30.16.0040.01

$$\operatorname{ds}\left(\operatorname{nc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{(m-1)\, z^2 - m}{1 - z^2}$$

09.30.16.0041.01

$$\operatorname{ds}\left(\operatorname{nd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m}{z^2 - 1}$$

09.30.16.0042.01

$$\operatorname{ds}\left(\operatorname{ns}^{-1}(z \mid m) \,\middle|\, m\right)^2 = z^2 - m$$

09.30.16.0043.01

$$\operatorname{ds}\left(\operatorname{sc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1 - (m-1)\, z^2}{z^2}$$

09.30.16.0044.01

$$\operatorname{ds}\left(\operatorname{sd}^{-1}(z \mid m) \,\middle|\, m\right) = \frac{1}{z}$$

09.30.16.0045.01

$$\operatorname{ds}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1 - m\, z^2}{z^2}$$

Addition formulas

09.30.16.0016.01

$$\operatorname{ds}(u + v \mid m) = \frac{\operatorname{dn}(u \mid m)\, \operatorname{dn}(v \mid m) - m\, \operatorname{sn}(u \mid m)\, \operatorname{cn}(u \mid m)\, \operatorname{sn}(v \mid m)\, \operatorname{cn}(v \mid m)}{\operatorname{cn}(v \mid m)\, \operatorname{dn}(v \mid m)\, \operatorname{sn}(u \mid m) + \operatorname{cn}(u \mid m)\, \operatorname{dn}(u \mid m)\, \operatorname{sn}(v \mid m)}$$

09.30.16.0017.01

$$\operatorname{ds}(u + v \mid m)\, \operatorname{ds}(u - v \mid m) = \frac{\operatorname{dn}(v \mid m)^2 - m\, \operatorname{cn}(v \mid m)^2\, \operatorname{sn}(u \mid m)^2}{\operatorname{sn}(u \mid m)^2 - \operatorname{sn}(v \mid m)^2}$$

Half-angle formulas

09.30.16.0018.01

$$\operatorname{ds}\left(\frac{z}{2} \,\middle|\, m\right)^2 = \frac{1 - m + \operatorname{dn}(z \mid m) + m\, \operatorname{cn}(z \mid m)}{1 - \operatorname{cn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.30.16.0019.01

$$\operatorname{ds}(2\, z \mid m) = \frac{\operatorname{dn}(z \mid m)^2 - m\, \operatorname{sn}(z \mid m)^2\, \operatorname{cn}(z \mid m)^2}{2\, \operatorname{sn}(z \mid m)\, \operatorname{cn}(z \mid m)\, \operatorname{dn}(z \mid m)}$$

Identities

Functional identities

09.30.17.0001.01

$$4\,w(z)^2\left(w(z)^2+m-1\right)\left(w(z)^2+m\right)w(2\,z)^2-\left(w(z)^4-m^2+m\right)^2 = 0\;/;\;w(z) = \mathrm{ds}(z\mid m)$$

Complex characteristics

Real part

09.30.19.0001.01

$$\mathrm{Re}(\mathrm{ds}(x+i\,y\mid m)) = \left(\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)^2\,\mathrm{sn}(x\mid m)-m\,\mathrm{cn}(x\mid m)^2\,\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)^2\right)\Big/ \left(\mathrm{dn}(y\mid 1-m)^2\,\mathrm{sn}(x\mid m)^2+\mathrm{cn}(x\mid m)^2\,\mathrm{cn}(y\mid 1-m)^2\,\mathrm{dn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2\right)\;/;\;\{x,y,m\}\in\mathbb{R}$$

Imaginary part

09.30.19.0002.01

$$\mathrm{Im}(\mathrm{ds}(x+i\,y\mid m)) = -\frac{\mathrm{cn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)\left(\mathrm{cn}(y\mid 1-m)^2\,\mathrm{dn}(x\mid m)^2+m\,\mathrm{sn}(x\mid m)^2\right)\mathrm{sn}(y\mid 1-m)}{\mathrm{dn}(y\mid 1-m)^2\,\mathrm{sn}(x\mid m)^2+\mathrm{cn}(x\mid m)^2\,\mathrm{cn}(y\mid 1-m)^2\,\mathrm{dn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Absolute value

09.30.19.0003.01

$$|\mathrm{ds}(x+i\,y\mid m)| = \sqrt{\frac{\mathrm{cn}(y\mid 1-m)^2\,\mathrm{dn}(x\mid m)^2\,\mathrm{dn}(y\mid 1-m)^2+m^2\,\mathrm{cn}(x\mid m)^2\,\mathrm{sn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2}{\mathrm{dn}(y\mid 1-m)^2\,\mathrm{sn}(x\mid m)^2+\mathrm{cn}(x\mid m)^2\,\mathrm{cn}(y\mid 1-m)^2\,\mathrm{dn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2}}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Argument

09.30.19.0004.01

$$\arg(\mathrm{ds}(x+i\,y\mid m)) = \tan^{-1}\left(\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)^2\,\mathrm{sn}(x\mid m)-m\,\mathrm{cn}(x\mid m)^2\,\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)^2,\; -\left(\mathrm{cn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)\left(\mathrm{cn}(y\mid 1-m)^2\,\mathrm{dn}(x\mid m)^2+m\,\mathrm{sn}(x\mid m)^2\right)\mathrm{sn}(y\mid 1-m)\right)\right)\;/;\;\{x,y,m\}\in\mathbb{R}$$

Conjugate value

09.30.19.0005.01

$$\overline{\mathrm{ds}(x+i\,y\mid m)} = \frac{\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)+i\,m\,\mathrm{cn}(x\mid m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)}{\mathrm{sn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)-i\,\mathrm{cn}(x\mid m)\,\mathrm{dn}(x\mid m)\,\mathrm{cn}(y\mid 1-m)\,\mathrm{sn}(y\mid 1-m)}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.30.20.0001.01

$$\frac{\partial\,\mathrm{ds}(z\mid m)}{\partial z} = -\mathrm{cs}(z\mid m)\,\mathrm{ns}(z\mid m)$$

09.30.20.0002.01

$$\frac{\partial^2 \operatorname{ds}(z \mid m)}{\partial z^2} == \operatorname{ds}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 + \operatorname{ns}(z \mid m)^2\right)$$

With respect to *m*

09.30.20.0003.01

$$\frac{\partial \operatorname{ds}(z \mid m)}{\partial m} == \frac{1}{2\, m\,(1-m)} (\operatorname{cs}(z \mid m) \operatorname{ns}(z \mid m) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)))$$

09.30.20.0004.01

$$\frac{\partial^2 \operatorname{ds}(z \mid m)}{\partial m^2} == \frac{1}{4\,(m-1)^2\, m^2} \Bigg(\operatorname{ds}(z \mid m) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{cs}(z \mid m)^2 +$$
$$2\,(m-1) \operatorname{ns}(z \mid m) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)) \operatorname{cs}(z \mid m) +$$
$$2\, m \operatorname{ns}(z \mid m) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)) \operatorname{cs}(z \mid m) +$$
$$(1-m)\, m \operatorname{ns}(z \mid m) \Bigg(2\, z + \frac{E(\operatorname{am}(z \mid m) \mid m) - F(\operatorname{am}(z \mid m) \mid m)}{m} - 2 \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m) +$$
$$\frac{1}{m-1} (m \operatorname{cn}(z \mid m) \operatorname{sc}(z \mid m) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)) \operatorname{sn}(z \mid m)) -$$
$$\frac{1}{m-1} (\operatorname{dc}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))) +$$
$$\frac{1}{(m-1)\, m} \left(((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{dn}(z \mid m) - m \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m)) \sqrt{1 - m \operatorname{sn}(z \mid m)^2}\,\right)\Bigg) \operatorname{cs}(z \mid m) + \operatorname{ds}(z \mid m)$$
$$\operatorname{ns}(z \mid m)^2 ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)) ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))\Bigg)$$

Symbolic differentiation

With respect to *z*

09.30.20.0007.01

$$\frac{\partial^n \operatorname{ds}(z \mid m)}{\partial z^n} = \operatorname{ds}(z \mid m)\, \delta_n - \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \operatorname{cs}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \operatorname{ns}(z \mid m)}{\partial z^{-j+n-1}} \,/;\, n \in \mathbb{N}$$

09.30.20.0005.01

$$\frac{\partial^n \operatorname{ds}(z \mid m)}{\partial z^n} == (-1)^n\, n!\, z^{-n-1} + z^{-n-1} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \left(2^{2k-1} - 1\right) B_{2k}}{k\,(2k-n-1)!} \left(\frac{\pi z}{2\, K(m)}\right)^{2k} -$$
$$\frac{2^{1-n}\, \pi^{n+1}}{K(m)^{n+1}} \sum_{k=0}^{\infty} \frac{(2k+1)^n\, q(m)^{2k+1}}{q(m)^{2k+1} + 1} \sin\left(\frac{\pi n}{2} + \frac{(2k+1)\, \pi z}{2\, K(m)}\right) /;\, n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.30.20.0006.01

$$\frac{\partial^{\alpha} \operatorname{ds}(z \mid m)}{\partial z^{\alpha}} = \mathcal{FC}_{\exp}^{(\alpha)}(z, -1)\, z^{-\alpha-1} + 2 \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \left(2^{2k-1}-1\right) B_{2k}}{\Gamma(2k-\alpha)\,(2k+1)} \left(\frac{\pi}{2 K(m)}\right)^{2k} z^{2k-\alpha-1} - \frac{2^{\alpha-1} \pi^{5/2} z^{1-\alpha}}{K(m)^2} \sum_{k=0}^{\infty} \frac{(2k+1)\, q(m)^{2k+1}}{q(m)^{2k+1}+1}\, {}_1\tilde{F}_2\!\left(1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.30.21.0001.01

$$\int \operatorname{ds}(z \mid m)\, dz = \log\left(\frac{1-\operatorname{cn}(z \mid m)}{\operatorname{sn}(z \mid m)}\right)$$

Representations through equivalent functions

With inverse function

09.30.27.0001.01

$$\operatorname{ds}\left(\operatorname{ds}^{-1}(z \mid m) \mid m\right) = z$$

With related functions

Involving am

09.30.27.0026.01

$$\operatorname{ds}(z \mid m)^2 = \csc^2(\operatorname{am}(z \mid m)) - m$$

Involving one other Jacobi elliptic function

Involving cd

09.30.27.0004.01

$$\operatorname{ds}(z \mid m)^2 = \frac{1-m}{1-\operatorname{cd}(z \mid m)^2}$$

Involving cn

09.30.27.0005.01

$$\operatorname{ds}(z \mid m)^2 = \frac{m \operatorname{cn}(z \mid m)^2 - m + 1}{1-\operatorname{cn}(z \mid m)^2}$$

Involving cs

09.30.27.0007.01

$$\operatorname{ds}(z \mid m)^2 = \operatorname{cs}(z \mid m)^2 - m + 1$$

Involving dc

09.30.27.0009.01

$$\mathrm{ds}(z \mid m)^2 == \frac{(1-m)\,\mathrm{dc}(z \mid m)^2}{\mathrm{dc}(z \mid m)^2 - 1}$$

Involving dn

09.30.27.0012.01

$$\mathrm{ds}(z \mid m)^2 == \frac{m\,\mathrm{dn}(z \mid m)^2}{1 - \mathrm{dn}(z \mid m)^2}$$

Involving nc

09.30.27.0013.01

$$\mathrm{ds}(z \mid m)^2 == \frac{(m-1)\,\mathrm{nc}(z \mid m)^2 - m}{1 - \mathrm{nc}(z \mid m)^2}$$

Involving nd

09.30.27.0016.01

$$\mathrm{ds}(z \mid m)^2 == \frac{m}{\mathrm{nd}(z \mid m)^2 - 1}$$

Involving ns

09.30.27.0017.01

$$\mathrm{ds}(z \mid m)^2 == \mathrm{ns}(z \mid m)^2 - m$$

Involving sc

09.30.27.0018.01

$$\mathrm{ds}(z \mid m)^2 == \frac{1 - (m-1)\,\mathrm{sc}(z \mid m)^2}{\mathrm{sc}(z \mid m)^2}$$

Involving sd

09.30.27.0019.01

$$\mathrm{ds}(z \mid m) == \frac{1}{\mathrm{sd}(z \mid m)}$$

Involving sn

09.30.27.0020.01

$$\mathrm{ds}(z \mid m)^2 == \frac{1 - m\,\mathrm{sn}(z \mid m)^2}{\mathrm{sn}(z \mid m)^2}$$

Involving two other Jacobi elliptic functions

Involving cd and cs

09.30.27.0002.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{cs}(z \mid m)}{\mathrm{cd}(z \mid m)}$$

Involving cd and sc

09.30.27.0003.01

$$\mathrm{ds}(z \mid m) = \frac{1}{\mathrm{cd}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

Involving cs and dc

09.30.27.0006.01

$$\mathrm{ds}(z \mid m) = \mathrm{dc}(z \mid m)\, \mathrm{cs}(z \mid m)$$

Involving dc and sc

09.30.27.0008.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{dc}(z \mid m)}{\mathrm{sc}(z \mid m)}$$

Involving dn and ns

09.30.27.0010.01

$$\mathrm{ds}(z \mid m) = \mathrm{dn}(z \mid m)\, \mathrm{ns}(z \mid m)$$

Involving dn and sn

09.30.27.0011.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{dn}(z \mid m)}{\mathrm{sn}(z \mid m)}$$

09.30.27.0027.01

$$\mathrm{ds}(z \mid m) = -\frac{m\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)}{(\mathrm{dn}(z \mid m) - 1)\, (\mathrm{dn}(z \mid m) + 1)}$$

Involving nd and ns

09.30.27.0014.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{ns}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

Involving nd and sn

09.30.27.0015.01

$$\operatorname{ds}(z \mid m) = \frac{1}{\operatorname{nd}(z \mid m) \operatorname{sn}(z \mid m)}$$

Involving three other Jacobi elliptic functions

09.30.27.0028.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)^2 \operatorname{dc}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1) \operatorname{cs}(z \mid m)}$$

09.30.27.0029.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

09.30.27.0030.01

$$\operatorname{ds}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m) \operatorname{nc}(z \mid m)}$$

09.30.27.0031.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m)}{\operatorname{cs}(z \mid m) (\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}$$

09.30.27.0032.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{cs}(z \mid m) (\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}$$

09.30.27.0033.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m) \operatorname{nd}(z \mid m)^2}$$

09.30.27.0034.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m) \operatorname{nd}(z \mid m)}$$

09.30.27.0035.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1) \operatorname{ns}(z \mid m)}$$

09.30.27.0036.01

$$\operatorname{ds}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

09.30.27.0037.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{nc}(z \mid m)}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1) \operatorname{ns}(z \mid m)}$$

09.30.27.0038.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)^2}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1) \operatorname{ns}(z \mid m)}$$

09.30.27.0039.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m) (\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.30.27.0040.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\,(\operatorname{ns}(z \mid m) - 1)\,(\operatorname{ns}(z \mid m) + 1)}{\operatorname{cd}(z \mid m)\,\operatorname{ns}(z \mid m)}$$

09.30.27.0041.01

$$\operatorname{ds}(z \mid m) = \frac{m\,\operatorname{cn}(z \mid m)\,\operatorname{dn}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{sc}(z \mid m)}$$

09.30.27.0042.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)^2\,\operatorname{sc}(z \mid m)}{\operatorname{cd}(z \mid m)\,(\operatorname{cn}(z \mid m) - 1)\,(\operatorname{cn}(z \mid m) + 1)}$$

09.30.27.0043.01

$$\operatorname{ds}(z \mid m) = \frac{(\operatorname{ns}(z \mid m) - 1)\,(\operatorname{ns}(z \mid m) + 1)\,\operatorname{sc}(z \mid m)}{\operatorname{cd}(z \mid m)}$$

09.30.27.0044.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{ns}(z \mid m)\,\operatorname{sc}(z \mid m)^2}$$

09.30.27.0045.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{sc}(z \mid m)}$$

09.30.27.0046.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nc}(z \mid m)\,\operatorname{sc}(z \mid m)}$$

09.30.27.0047.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nd}(z \mid m)^2\,\operatorname{sc}(z \mid m)}$$

09.30.27.0048.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m)}$$

09.30.27.0049.01

$$\operatorname{ds}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)^2}$$

09.30.27.0050.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{nc}(z \mid m)^2\,\operatorname{sd}(z \mid m)}{(\operatorname{nc}(z \mid m) - 1)\,(\operatorname{nc}(z \mid m) + 1)\,\operatorname{nd}(z \mid m)^2}$$

09.30.27.0051.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m)^2\,\operatorname{sd}(z \mid m)}{(\operatorname{dc}(z \mid m)\,\operatorname{nd}(z \mid m) - 1)\,(\operatorname{dc}(z \mid m)\,\operatorname{nd}(z \mid m) + 1)}$$

09.30.27.0052.01

$$\operatorname{ds}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)^2\,\operatorname{sc}(z \mid m)^2}$$

09.30.27.0053.01

$$\mathrm{ds}(z \mid m) = \left(\mathrm{cs}(z \mid m)^2 + 1\right) \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)$$

09.30.27.0054.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)}{(\mathrm{dc}(z \mid m) - \mathrm{dn}(z \mid m))\, (\mathrm{dc}(z \mid m) + \mathrm{dn}(z \mid m))}$$

09.30.27.0055.01

$$\mathrm{ds}(z \mid m) = -\frac{\mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)}{(\mathrm{cd}(z \mid m)\, \mathrm{dn}(z \mid m) - 1)\, (\mathrm{cd}(z \mid m)\, \mathrm{dn}(z \mid m) + 1)}$$

09.30.27.0056.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\, \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)\, (\mathrm{nc}(z \mid m) - 1)\, (\mathrm{nc}(z \mid m) + 1)}$$

09.30.27.0057.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\, \mathrm{nc}(z \mid m)\, \mathrm{sn}(z \mid m)}{(\mathrm{nc}(z \mid m) - 1)\, (\mathrm{nc}(z \mid m) + 1)}$$

09.30.27.0058.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\, \mathrm{nc}(z \mid m)^2\, \mathrm{sn}(z \mid m)}{(\mathrm{nc}(z \mid m) - 1)\, (\mathrm{nc}(z \mid m) + 1)}$$

09.30.27.0059.01

$$\mathrm{ds}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right) \mathrm{sn}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

09.30.27.0060.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{nc}(z \mid m)^2\, \mathrm{sn}(z \mid m)}{(\mathrm{nc}(z \mid m) - 1)\, (\mathrm{nc}(z \mid m) + 1)\, \mathrm{nd}(z \mid m)}$$

09.30.27.0061.01

$$\mathrm{ds}(z \mid m) = -\frac{\mathrm{nd}(z \mid m)\, \mathrm{sn}(z \mid m)}{(\mathrm{cd}(z \mid m) - \mathrm{nd}(z \mid m))\, (\mathrm{cd}(z \mid m) + \mathrm{nd}(z \mid m))}$$

09.30.27.0062.01

$$\mathrm{ds}(z \mid m) = \frac{\mathrm{dn}(z \mid m) \left(\mathrm{sc}(z \mid m)^2 + 1\right) \mathrm{sn}(z \mid m)}{\mathrm{sc}(z \mid m)^2}$$

09.30.27.0063.01

$$\mathrm{ds}(z \mid m) = \frac{\left(\mathrm{sc}(z \mid m)^2 + 1\right) \mathrm{sn}(z \mid m)}{\mathrm{nd}(z \mid m)\, \mathrm{sc}(z \mid m)^2}$$

09.30.27.0064.01

$$\mathrm{ds}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\, (\mathrm{sn}(z \mid m) - 1)\, (\mathrm{sn}(z \mid m) + 1)}{\mathrm{cs}(z \mid m)\, \mathrm{sn}(z \mid m)^2}$$

09.30.27.0065.01

$$\mathrm{ds}(z \mid m) = -\frac{\mathrm{sc}(z \mid m)\, (\mathrm{sn}(z \mid m) - 1)\, (\mathrm{sn}(z \mid m) + 1)}{\mathrm{cd}(z \mid m)\, \mathrm{sn}(z \mid m)^2}$$

Involving four other Jacobi elliptic functions

09.30.27.0066.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{dc}(z \mid m)}{\operatorname{cn}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{ns}(z \mid m)}$$

09.30.27.0067.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)}{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{ns}(z \mid m)}$$

09.30.27.0068.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{ns}(z \mid m)}$$

09.30.27.0069.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)}$$

09.30.27.0070.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m)}{\operatorname{cd}(z \mid m)}$$

09.30.27.0071.01

$$\operatorname{ds}(z \mid m) = \frac{m\operatorname{cn}(z \mid m) - m\operatorname{nc}(z \mid m) + \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.30.27.0072.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{cd}(z \mid m)(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.30.27.0073.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.30.27.0074.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.30.27.0075.01

$$\operatorname{ds}(z \mid m) = \operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))$$

09.30.27.0076.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{nc}(z \mid m)}$$

09.30.27.0077.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cd}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{nd}(z \mid m)^2}$$

09.30.27.0078.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{nd}(z \mid m)}$$

09.30.27.0079.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.30.27.0080.01

$$\text{ds}(z \mid m) = \frac{\text{dn}(z \mid m)\,\text{sc}(z \mid m)}{\text{ns}(z \mid m)\,\text{sc}(z \mid m) - \text{cn}(z \mid m)}$$

09.30.27.0081.01

$$\text{ds}(z \mid m) = \frac{(\text{cs}(z \mid m) + \text{sc}(z \mid m))\,\text{sd}(z \mid m)}{\text{nd}(z \mid m)^2\,\text{sc}(z \mid m)}$$

09.30.27.0082.01

$$\text{ds}(z \mid m) = \frac{\text{cs}(z \mid m) + \text{dc}(z \mid m)\,\text{sd}(z \mid m)}{\text{dc}(z \mid m)\,\text{nd}(z \mid m)^2}$$

09.30.27.0083.01

$$\text{ds}(z \mid m) = \frac{\text{cs}(z \mid m)\,\text{nd}(z \mid m) + \text{nc}(z \mid m)\,\text{sd}(z \mid m)}{\text{nc}(z \mid m)\,\text{nd}(z \mid m)^2}$$

09.30.27.0084.01

$$\text{ds}(z \mid m) = \frac{\text{dn}(z \mid m)^2\,(\text{cd}(z \mid m) + \text{sc}(z \mid m)\,\text{sd}(z \mid m))}{\text{sc}(z \mid m)}$$

09.30.27.0085.01

$$\text{ds}(z \mid m) = \frac{\text{cd}(z \mid m) + \text{sc}(z \mid m)\,\text{sd}(z \mid m)}{\text{nd}(z \mid m)^2\,\text{sc}(z \mid m)}$$

09.30.27.0086.01

$$\text{ds}(z \mid m) = \frac{\text{cn}(z \mid m)\,\text{nd}(z \mid m) + \text{sc}(z \mid m)\,\text{sd}(z \mid m)}{\text{nd}(z \mid m)^2\,\text{sc}(z \mid m)}$$

09.30.27.0087.01

$$\text{ds}(z \mid m) = -\frac{-\text{cn}(z \mid m) + m\,\text{dn}(z \mid m)\,\text{sc}(z \mid m)\,\text{sd}(z \mid m) - \text{dn}(z \mid m)\,\text{sc}(z \mid m)\,\text{sd}(z \mid m)}{\text{dn}(z \mid m)\,\text{sc}(z \mid m)}$$

09.30.27.0088.01

$$\text{ds}(z \mid m) = \frac{\text{dc}(z \mid m)\,(\text{ns}(z \mid m) - \text{sn}(z \mid m))}{\text{cn}(z \mid m)}$$

09.30.27.0089.01

$$\text{ds}(z \mid m) = \frac{\text{ns}(z \mid m) - \text{sn}(z \mid m)}{\text{cd}(z \mid m)^2\,\text{dn}(z \mid m)}$$

09.30.27.0090.01

$$\text{ds}(z \mid m) = \frac{\text{dc}(z \mid m)^2\,(\text{ns}(z \mid m) - \text{sn}(z \mid m))}{\text{dn}(z \mid m)}$$

09.30.27.0091.01

$$\text{ds}(z \mid m) = \frac{\text{nc}(z \mid m)\,(\text{ns}(z \mid m) - \text{sn}(z \mid m))}{\text{cd}(z \mid m)}$$

09.30.27.0092.01

$$\text{ds}(z \mid m) = \frac{\text{dc}(z \mid m)\,(\text{ns}(z \mid m) - \text{sn}(z \mid m))}{\text{cs}(z \mid m)\,\text{sn}(z \mid m)}$$

09.30.27.0093.01

$$\operatorname{ds}(z \mid m) = -\frac{(m-1)\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{cn}(z \mid m)}$$

09.30.27.0094.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)(\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.30.27.0095.01

$$\operatorname{ds}(z \mid m) = -\frac{m\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.30.27.0096.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)^2\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.30.27.0097.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m)^2\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)^2\operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.30.27.0098.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cd}(z \mid m)}$$

09.30.27.0099.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.30.27.0100.01

$$\operatorname{ds}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.30.27.0101.01

$$\operatorname{ds}(z \mid m) = \operatorname{dn}(z \mid m)(\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m))$$

09.30.27.0102.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cs}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.30.27.0103.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m))}{\operatorname{dc}(z \mid m)}$$

09.30.27.0104.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cs}(z \mid m) + \operatorname{nc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.30.27.0105.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m) + \operatorname{nd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)^2}$$

09.30.27.0106.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.30.27.0107.01

$$\operatorname{ds}(z \mid m) = -\frac{\operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.30.27.0108.01

$$\operatorname{ds}(z \mid m) = -\frac{-\operatorname{cn}(z \mid m) + m\operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.30.27.0109.01

$$\operatorname{ds}(z \mid m) = -\frac{-\operatorname{cn}(z \mid m) + m\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.30.27.0110.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.30.27.0111.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.30.27.0112.01

$$\operatorname{ds}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)}$$

Involving Weierstrass functions

09.30.27.0021.01

$$\operatorname{ds}(z \mid m) = \frac{1}{\sqrt{e_1 - e_3}} \frac{\sigma_2\left(\frac{z}{\sqrt{e_1-e_3}};\, g_2,\, g_3\right)}{\sigma\left(\frac{z}{\sqrt{e_1-e_3}};\, g_2,\, g_3\right)} \;/;$$

$$\{\omega_1,\, \omega_2,\, \omega_3\} = \{\omega_1(g_2,\, g_3),\, -\omega_1(g_2,\, g_3) - \omega_3(g_2,\, g_3),\, \omega_3(g_2,\, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n;\, g_2,\, g_3) \bigwedge n \in \{1,\, 2,\, 3\}$$

09.30.27.0022.01

$$\operatorname{ds}(z \mid m)^2 = \frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}};\, g_2,\, g_3\right) - e_2}{e_1 - e_3} \;/;$$

$$\{\omega_1,\, \omega_2,\, \omega_3\} = \{\omega_1(g_2,\, g_3),\, -\omega_1(g_2,\, g_3) - \omega_3(g_2,\, g_3),\, \omega_3(g_2,\, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n;\, g_2,\, g_3) \bigwedge n \in \{1,\, 2,\, 3\}$$

Involving theta functions

09.30.27.0023.01

$$\operatorname{ds}(z \mid m) = (m\,(1-m))^{1/4}\, \frac{\vartheta_3\left(\frac{\pi z}{2 K(m)},\, q(m)\right)}{\vartheta_1\left(\frac{\pi z}{2 K(m)},\, q(m)\right)}$$

09.30.27.0024.01

$$\operatorname{ds}(z \mid m) = \frac{\pi}{2\,K(m)}\,\frac{\vartheta_1'(0,\,q(m))}{\vartheta_3(0,\,q(m))}\,\frac{\vartheta_3\left(\frac{z}{\vartheta_3(0,q(m))^2},\,q(m)\right)}{\vartheta_1\left(\frac{z}{\vartheta_3(0,q(m))^2},\,q(m)\right)}$$

09.30.27.0025.01

$$\operatorname{ds}(z \mid m) = \frac{\vartheta_d(z \mid m)}{\vartheta_s(z \mid m)}$$

Zeros

09.30.30.0001.01

$$\operatorname{ds}((2\,r+1)\,K(m) + (2\,s+1)\,i\,K(1-m) \mid m) = 0 \;/;\; \{r,\,s\} \in \mathbb{Z}$$

History

– C. G. J. Jacobi (1827)
– N. H. Abel (1827)
– J. Glaisher (1882) introduced the notation ds

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