JacobiSD

Notations

Traditional name

Jacobi elliptic function sd

Traditional notation

$\mathrm{sd}(z \mid m)$

Mathematica StandardForm notation

`JacobiSD[z, m]`

Primary definition

09.35.02.0001.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case $m = 0$

09.35.03.0001.01

$$\mathrm{sd}(z \mid 0) = \sin(z)$$

09.35.03.0002.01

$$\mathrm{sd}\left(z + \frac{\pi}{2} \,\middle|\, 0\right) = \cos(z)$$

09.35.03.0026.01

$$\mathrm{sd}\left(z + \frac{\pi k}{2} \,\middle|\, 0\right) = \sin\left(z + \frac{\pi k}{2}\right) /; k \in \mathbb{Z}$$

Case $m = 1$

09.35.03.0003.01

$$\mathrm{sd}(z \mid 1) = \sinh(z)$$

09.35.03.0004.01

$$\operatorname{sd}\left(z+\frac{\pi i}{2}\,\middle|\,1\right)=i\cosh(z)$$

09.35.03.0027.01

$$\operatorname{sd}\left(z+\frac{i\pi k}{2}\,\middle|\,1\right)=\sinh\left(z+\frac{i\pi k}{2}\right)\;/;\;k\in\mathbb{Z}$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.35.03.0005.01

$$\operatorname{sd}(0\mid m)=0$$

09.35.03.0006.01

$$\operatorname{sd}(K(m)\mid m)=\frac{1}{\sqrt{1-m}}$$

09.35.03.0007.01

$$\operatorname{sd}(2K(m)\mid m)=0$$

09.35.03.0008.01

$$\operatorname{sd}(3K(m)\mid m)=-\frac{1}{\sqrt{1-m}}$$

09.35.03.0009.01

$$\operatorname{sd}(4K(m)\mid m)=0$$

09.35.03.0010.01

$$\operatorname{sd}(iK(1-m)\mid m)=\frac{i}{\sqrt{m}}$$

09.35.03.0011.01

$$\operatorname{sd}(2iK(1-m)\mid m)=0$$

09.35.03.0012.01

$$\operatorname{sd}(3iK(1-m)\mid m)=-\frac{i}{\sqrt{m}}$$

09.35.03.0013.01

$$\operatorname{sd}(4iK(1-m)\mid m)=0$$

09.35.03.0014.01

$$\operatorname{sd}(K(m)+iK(1-m)\mid m)=\tilde{\infty}$$

09.35.03.0015.01

$$\operatorname{sd}(2K(m)+iK(1-m)\mid m)=-\frac{i}{\sqrt{m}}$$

09.35.03.0016.01

$$\operatorname{sd}(3K(m)+iK(1-m)\mid m)=\tilde{\infty}$$

09.35.03.0017.01

$$\operatorname{sd}(4K(m)+iK(1-m)\mid m)=\frac{i}{\sqrt{m}}$$

09.35.03.0018.01

$$\operatorname{sd}((2r+1)K(m)+(2s+1)\,i\,K(1-m)\mid m)=\tilde{\infty}\;/;\;\{r,s\}\in\mathbb{Z}$$

09.35.03.0019.01

$$\operatorname{sd}(K(m)+2\,i\,K(1-m)\mid m)=-\frac{1}{\sqrt{1-m}}$$

09.35.03.0020.01

$$\operatorname{sd}(2\,K(m)+2\,i\,K(1-m)\mid m)=0$$

09.35.03.0021.01

$$\operatorname{sd}(3\,K(m)+2\,i\,K(1-m)\mid m)=\frac{1}{\sqrt{1-m}}$$

09.35.03.0022.01

$$\operatorname{sd}(4\,K(m)+2\,i\,K(1-m)\mid m)=0$$

Values at half-quarter-period points

09.35.03.0023.01

$$\operatorname{sd}\left(\frac{K(m)}{2}\,\middle|\,m\right)=\frac{1}{\sqrt[4]{1-m}\,\sqrt{1+\sqrt{1-m}}}$$

09.35.03.0024.01

$$\operatorname{sd}\left(\frac{i\,K(1-m)}{2}\,\middle|\,m\right)=\frac{i}{\sqrt[4]{m}\,\sqrt{1+\sqrt{m}}}$$

09.35.03.0025.01

$$\operatorname{sd}\left(\frac{K(m)}{2}+\frac{i\,K(1-m)}{2}\,\middle|\,m\right)=\frac{1}{\sqrt[4]{1-m}\,\sqrt[4]{m}}\,\frac{\sqrt{1+\sqrt{m}}+i\sqrt{1-\sqrt{m}}}{\sqrt{1+\sqrt{1-m}}-i\sqrt{1-\sqrt{1-m}}}$$

General characteristics

Domain and analyticity

$\operatorname{cd}(z\mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.35.04.0001.01

$$(z*m)\longrightarrow\operatorname{sd}(z\mid m)::(\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Parity

$\operatorname{sd}(z\mid m)$ is an odd function with respect to z.

09.35.04.0002.01

$$\operatorname{sd}(-z\mid m)=-\operatorname{sd}(z\mid m)$$

Mirror symmetry

09.35.04.0003.01

$\mathrm{sd}(\bar{z} \mid \bar{m}) = \overline{\mathrm{sd}(z \mid m)}$

Periodicity

$\mathrm{sd}(z \mid m)$ is a doubly periodic function with respect to z with periods $4\, i\, K(1-m)$ and $4\, K(m)$.

09.35.04.0004.01

$\mathrm{sd}(z + 2\, K(m) \mid m) = -\mathrm{sd}(z \mid m)$

09.35.04.0005.01

$\mathrm{sd}(z + 4\, K(m) \mid m) = \mathrm{sd}(z \mid m)$

09.35.04.0006.01

$\mathrm{sd}(z + 2\, i\, K(1-m) \mid m) = -\mathrm{sd}(z \mid m)$

09.35.04.0007.01

$\mathrm{sd}(z + 4\, i\, K(1-m) \mid m) = \mathrm{sd}(z \mid m)$

09.35.04.0008.01

$\mathrm{sd}(z + 2\, K(m) + 2\, i\, K(1-m) \mid m) = \mathrm{sd}(z \mid m)$

09.35.04.0009.01

$\mathrm{sd}(z + 2\, i\, s\, K(1-m) + 2\, r\, K(m) \mid m) = (-1)^{r+s}\, \mathrm{sd}(z \mid m) \;/;\; \{r, s\} \in \mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed m, the function $\mathrm{sd}(z \mid m)$ has an infinite set of singular points:

a) $z = (2\, r + 1)\, K(m) + (2\, s + 1)\, i\, K(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $\frac{(-1)^{r+s-1}\, i}{\sqrt{m}\, \sqrt{1-m}}$;

b) $z = \tilde{\infty}$ is an essential singular point.

09.35.04.0010.01

$\mathcal{Sing}_z(\mathrm{sd}(z \mid m)) = \{\{\{(2\, s + 1)\, i\, K(1-m) + (2\, r + 1)\, K(m), 1\} \;/;\; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$

09.35.04.0011.01

$$\mathrm{res}_z(\mathrm{sd}(z \mid m))\, ((2\, s + 1)\, i\, K(1-m) + (2\, r + 1)\, K(m)) = \frac{(-1)^{r+s-1}\, i}{\sqrt{m}\, \sqrt{1-m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

Branch points

With respect to *m*

For fixed z, the function $\mathrm{sd}(z \mid m)$ is a meromorphic function in m that has no branch points.

09.35.04.0014.01

$\mathcal{BP}_m(\mathrm{sd}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\mathrm{sd}(z \mid m)$ does not have branch points.

09.35.04.0012.01

$\mathcal{BP}_z(\operatorname{sd}(z \mid m)) = \{\}$

Branch cuts

With respect to *m*

For fixed z, the function $\operatorname{sd}(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.35.04.0015.01

$\mathcal{BC}_m(\operatorname{sd}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\operatorname{sd}(z \mid m)$ does not have branch cuts.

09.35.04.0013.01

$\mathcal{BC}_z(\operatorname{sd}(z \mid m)) = \{\}$

Series representations

Generalized power series

Expansions at *z* == 0

09.35.06.0005.01

$$\operatorname{sd}(z \mid m) \propto z + \frac{1}{6}(2m-1)z^3 + \frac{1}{120}\left(1 - 16m + 16m^2\right)z^5 + \ldots \;/;\; (z \to 0)$$

09.35.06.0001.02

$$\operatorname{sd}(z \mid m) \propto z + \frac{1}{6}(-1+2m)z^3 + \frac{1}{120}\left(1 - 16m + 16m^2\right)z^5 + \frac{\left(-1 + 138m - 408m^2 + 272m^3\right)z^7}{5040} + \frac{\left(1 - 1232m + 9168m^2 - 15872m^3 + 7936m^4\right)z^9}{362880} + O\left(z^{11}\right)$$

09.35.06.0006.01

$$\operatorname{sd}(z \mid m) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)(-1)^{k-j}\operatorname{sn}_{k-j}(m)}{(2k-2j+1)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}q_{r,j}\,z^{2k+1} \;/;\; q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i + i - k)(-1)^i\operatorname{dn}_i(m)\,q_{j,k-i}}{(2i)!} \bigwedge k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\operatorname{cn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge \operatorname{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.35.06.0007.01

$$\operatorname{sd}(z \mid m) \propto z\left(1 + O\left(z^2\right)\right)$$

Expansions at *z* == (2 *r* + 1) *K*(*m*) + (2 *s* + 1) *i* *K*(1 − *m*)

09.35.06.0008.01

$$\operatorname{sd}(z\mid m)\propto\frac{i\,(-1)^{r+s-1}}{\sqrt{m}\,\sqrt{1-m}}\left(\frac{1}{z-z_0}+\frac{1}{6}(1-2m)(z-z_0)+\frac{1}{360}\left(-8m^2+8m+7\right)(z-z_0)^3+\ldots\right)/;$$
$$(z\to z_0)\bigwedge z_0=(2r+1)K(m)+(2s+1)\,i\,K(1-m)\bigwedge r\in\mathbb{Z}\bigwedge s\in\mathbb{Z}$$

09.35.06.0009.01

$$\operatorname{sd}(z\mid m)=\frac{i\,(-1)^{r+s-1}}{\sqrt{m}\,\sqrt{1-m}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)(-1)^{k-j}\operatorname{dn}_{k-j}(m)}{(2k-2j)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}q_{r,j}\,(z-z_0)^{2k-1}\;/;$$
$$z_0=(2r+1)K(m)+(2s+1)\,i\,K(1-m)\bigwedge r\in\mathbb{Z}\bigwedge s\in\mathbb{Z}\bigwedge q_{j,0}=1\bigwedge q_{j,k}=\frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i+i-k)(-1)^i\operatorname{sn}_i(m)\,q_{j,k-i}}{(2i+1)!}\bigwedge$$
$$k\in\mathbb{N}^+\bigwedge\operatorname{sn}_0(m)=1\bigwedge\operatorname{sn}_n(m)=\sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\operatorname{cn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n}\bigwedge\operatorname{cn}_0(m)=1\bigwedge$$
$$\operatorname{cn}_n(m)=\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n+1}\bigwedge\operatorname{dn}_0(m)=1\bigwedge\operatorname{dn}_n(m)=m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.35.06.0010.01

$$\operatorname{sd}(z\mid m)\propto\frac{i\,(-1)^{r+s-1}}{\sqrt{m}\,\sqrt{1-m}\,(z-z_0)}\left(1+O\left((z-z_0)^2\right)\right)/;\ z_0=(2r+1)K(m)+(2s+1)\,i\,K(1-m)\bigwedge r\in\mathbb{Z}\bigwedge s\in\mathbb{Z}$$

Expansions at *m* == 0

09.35.06.0011.01

$$\operatorname{sd}(z\mid m)\propto\sin(z)-\frac{1}{16}(4z\cos(z)-7\sin(z)+\sin(3z))\,m+$$
$$\frac{1}{256}\left(-48z\cos(z)+12z\cos(3z)-\left(8z^2-79\right)\sin(z)-16\sin(3z)+\sin(5z)\right)m^2+\ldots/;(m\to 0)$$

09.35.06.0012.01

$$\operatorname{sd}(z\mid m)\propto\sin(z)-\frac{1}{16}(4z\cos(z)-7\sin(z)+\sin(3z))\,m+$$
$$\frac{1}{256}\left(-48z\cos(z)+12z\cos(3z)-\left(8z^2-79\right)\sin(z)-16\sin(3z)+\sin(5z)\right)m^2+$$
$$\frac{1}{12\,288}\left(8z\left(4z^2-237\right)\cos(z)+756z\cos(3z)-60z\cos(5z)-\right.$$
$$\left.3\left(136z^2-1013\right)\sin(z)+6(6z-11)(6z+11)\sin(3z)+72\sin(5z)-3\sin(7z)\right)m^3+$$
$$\frac{1}{196\,608}\left(4z\left(176z^2-6555\right)\cos(z)-216z\left(4z^2-61\right)\cos(3z)-1740z\cos(5z)+84z\cos(7z)+\right.$$
$$\left.2\left(16z^4-3144z^2+20\,679\right)\sin(z)+3\left(1872z^2-3647\right)\sin(3z)-6\left(100z^2-227\right)\sin(5z)-96\sin(7z)+3\sin(9z)\right)m^4-$$
$$\frac{1}{15\,728\,640}\left(8z\left(16z^4-7820z^2+233\,445\right)\cos(z)+1080z\left(124z^2-1005\right)\cos(3z)-200z\left(100z^2-927\right)\cos(5z)-\right.$$
$$15\,540z\cos(7z)+540z\cos(9z)-360\left(12z^4-1324z^2+8091\right)\sin(z)+45\left(288z^4-12\,360z^2+18\,391\right)\sin(3z)+$$
$$\left.1200\left(85z^2-99\right)\sin(5z)-30\left(196z^2-365\right)\sin(7z)-600\sin(9z)+15\sin(11z)\right)m^5+$$
$$\frac{1}{754\,974\,720}\left(-12z\left(1024z^4-259\,960z^2+6\,783\,615\right)\cos(z)+432z\left(216z^4-20\,670z^2+121\,345\right)\cos(3z)+\right.$$
$$4500z\left(520z^2-2343\right)\cos(5z)-840z\left(196z^2-1401\right)\cos(7z)-72\,900z\cos(9z)+1980z\cos(11z)-$$

$$8\left(32 z^6-32\,700 z^4+2\,706\,615 z^2-15\,734\,205\right)\sin(z)-270\left(5184 z^4-111\,036 z^2+139\,787\right)\sin(3 z)+$$
$$15\left(20\,000 z^4-484\,200 z^2+399\,333\right)\sin(5 z)+5040\left(147 z^2-130\right)\sin(7 z)-$$
$$90\left(324 z^2-535\right)\sin(9 z)-2160\sin(11 z)+45\sin(13 z)\Big) m^6+\frac{1}{84\,557\,168\,640}$$
$$\Big(4 z\left(256 z^6-486\,528 z^4+88\,220\,160 z^2-2\,099\,102\,355\right)\cos(z)+756 z\left(35\,424 z^4-1\,612\,920 z^2+7\,728\,505\right)\cos(3 z)-$$
$$2100 z\left(4000 z^4-204\,200 z^2+628\,377\right)\cos(5 z)-2940 z\left(18\,424 z^2-60\,261\right)\cos(7 z)+204\,120 z\left(12 z^2-73\right)\cos(9 z)+$$
$$734\,580 z\cos(11 z)-16\,380 z\cos(13 z)-14\left(4736 z^6-2\,373\,840 z^4+164\,375\,460 z^2-920\,985\,345\right)\sin(z)+$$
$$63\left(20\,736 z^6-3\,836\,160 z^4+57\,130\,200 z^2-64\,038\,815\right)\sin(3 z)+$$
$$105\left(880\,000 z^4-9\,930\,000 z^2+6\,578\,709\right)\sin(5 z)-105\left(76\,832 z^4-1\,367\,688 z^2+812\,787\right)\sin(7 z)-$$
$$25\,200\left(405 z^2-301\right)\sin(9 z)+6930\left(44 z^2-67\right)\sin(11 z)+17\,640\sin(13 z)-315\sin(15 z)\Big) m^7+$$
$$\frac{1}{1\,352\,914\,698\,240}\Big(84 z\left(512 z^6-456\,736 z^4+66\,969\,680 z^2-1\,488\,442\,935\right)\cos(z)-$$
$$432 z\left(5184 z^6-1\,681\,344 z^4+51\,519\,300 z^2-214\,135\,145\right)\cos(3 z)-$$
$$42\,000 z\left(9800 z^4-225\,500 z^2+543\,171\right)\cos(5 z)+588 z\left(76\,832 z^4-2\,777\,320 z^2+5\,945\,595\right)\cos(7 z)+$$
$$34\,020 z\left(3960 z^2-10\,553\right)\cos(9 z)-9240 z\left(484 z^2-2637\right)\cos(11 z)-999\,180 z\cos(13 z)+$$
$$18\,900 z\cos(15 z)+\left(512 z^8-1\,662\,976 z^6+573\,071\,520 z^4-35\,062\,219\,080 z^2+190\,895\,208\,525\right)\sin(z)+$$
$$63\left(953\,856 z^6-80\,866\,080 z^4+955\,001\,880 z^2-981\,794\,585\right)\sin(3 z)-$$
$$70\left(400\,000 z^6-37\,800\,000 z^4+282\,908\,700 z^2-160\,660\,881\right)\sin(5 z)-$$
$$1680\left(249\,704 z^4-1\,973\,769 z^2+907\,212\right)\sin(7 z)+945\left(23\,328 z^4-341\,928 z^2+164\,329\right)\sin(9 z)+$$
$$5040\left(3509 z^2-2312\right)\sin(11 z)-630\left(676 z^2-971\right)\sin(13 z)-20\,160\sin(15 z)+315\sin(17 z)\Big) m^8-$$
$$\frac{1}{194\,819\,716\,546\,560}\Big(16 z\left(128 z^8-666\,432 z^6+396\,402\,552 z^4-50\,087\,404\,080 z^2+1\,055\,822\,405\,715\right)\cos(z)+$$
$$1944 z\left(528\,768 z^6-76\,241\,088 z^4+1\,809\,037\,860 z^2-6\,760\,275\,865\right)\cos(3 z)-$$
$$4500 z\left(160\,000 z^6-25\,603\,200 z^4+381\,412\,920 z^2-772\,279\,011\right)\cos(5 z)-$$
$$15\,876 z\left(1\,459\,808 z^4-22\,857\,520 z^2+37\,107\,615\right)\cos(7 z)+20\,412 z\left(69\,984 z^4-2\,027\,160 z^2+3\,423\,715\right)\cos(9 z)+$$
$$124\,740 z\left(20\,328 z^2-46\,943\right)\cos(11 z)-98\,280 z\left(676 z^2-3399\right)\cos(13 z)-11\,736\,900 z\cos(15 z)+$$
$$192\,780 z\cos(17 z)-18\left(12\,032 z^8-17\,713\,920 z^6+4\,803\,020\,880 z^4-268\,044\,548\,940 z^2+1\,425\,876\,118\,065\right)\sin(z)+$$
$$162\left(186\,624 z^8-98\,775\,936 z^6+5\,487\,138\,720 z^4-55\,014\,259\,860 z^2+52\,863\,102\,635\right)\sin(3 z)+$$
$$1890\left(7\,200\,000 z^6-298\,400\,000 z^4+1\,710\,777\,900 z^2-865\,438\,233\right)\sin(5 z)-$$
$$126\left(15\,059\,072 z^6-977\,303\,040 z^4+4\,946\,017\,860 z^2-1\,893\,369\,285\right)\sin(7 z)-$$
$$68\,040\left(174\,960 z^4-1\,098\,306 z^2+396\,593\right)\sin(9 z)+945\left(468\,512 z^4-6\,002\,568 z^2+2\,483\,103\right)\sin(11 z)+$$
$$136\,080\left(1859 z^2-1121\right)\sin(13 z)-5670\left(900 z^2-1237\right)\sin(15 z)-204\,120\sin(17 z)+2835\sin(19 z)\Big) m^9+$$
$$\frac{1}{15\,585\,577\,323\,724\,800}\Big(-80 z\left(6656 z^8-15\,268\,032 z^6+6\,985\,982\,808 z^4-789\,796\,360\,080 z^2+15\,954\,035\,719\,545\right)$$
$$\cos(z)+4860 z\left(41\,472 z^8-33\,903\,360 z^6+3\,132\,700\,704 z^4-61\,972\,017\,240 z^2+213\,083\,709\,055\right)\cos(3 z)+$$
$$18\,000 z\left(11\,800\,000 z^6-809\,445\,000 z^4+9\,069\,298\,350 z^2-16\,109\,960\,643\right)\cos(5 z)-$$
$$17\,640 z\left(2\,151\,296 z^6-230\,496\,000 z^4+2\,268\,001\,260 z^2-3\,019\,923\,495\right)\cos(7 z)-$$
$$510\,300 z\left(909\,792 z^4-11\,072\,592 z^2+13\,834\,049\right)\cos(9 z)+$$
$$41\,580 z\left(468\,512 z^4-11\,620\,840 z^2+16\,583\,175\right)\cos(11 z)+245\,700 z\left(95\,992 z^2-199\,353\right)\cos(13 z)-$$
$$5\,103\,000 z\left(100 z^2-473\right)\cos(15 z)-74\,220\,300 z\cos(17 z)+1\,077\,300 z\cos(19 z)-$$
$$\left(4096 z^{10}-32\,509\,440 z^8+31\,120\,266\,240 z^6-7\,123\,026\,708\,000 z^4+370\,045\,682\,269\,200 z^2-1\,930\,805\,954\,624\,925\right)$$
$$\sin(z)-8505\left(995\,328 z^8-228\,407\,040 z^6+9\,631\,175\,040 z^4-85\,379\,407\,560 z^2+77\,680\,098\,395\right)\sin(3 z)+$$
$$225\left(40\,000\,000 z^8-10\,152\,800\,000 z^6+266\,928\,480\,000 z^4-1\,265\,178\,448\,800 z^2+584\,519\,733\,567\right)\sin(5 z)+$$

$$315\left(1867324928\,z^6-51409827840\,z^4+194009059440\,z^2-64674151155\right)\sin(7\,z)-$$
$$5670\left(7558272\,z^6-384212160\,z^4+1502509500\,z^2-441741125\right)\sin(9\,z)-$$
$$75600\left(1991176\,z^4-10623921\,z^2+3227163\right)\sin(11\,z)+$$
$$4725\left(913952\,z^4-10606440\,z^2+3922749\right)\sin(13\,z)+1134000\left(1665\,z^2-938\right)\sin(15\,z)-$$
$$28350\left(1156\,z^2-1535\right)\sin(17\,z)-1134000\sin(19\,z)+14175\sin(21\,z)\Big)\,m^{10}+O\!\left(m^{11}\right)$$

09.35.06.0013.01

$$\operatorname{sd}(z\mid m)\propto\sin(z)\,(1+O(m))$$

Expansions at $m == 1$

09.35.06.0014.01

$$\operatorname{sd}(z\mid m)\propto\sinh(z)+\frac{1}{16}\,(4\,z\cosh(z)-7\sinh(z)+\sinh(3\,z))\,(m-1)+$$
$$\frac{1}{256}\left(-48\,z\cosh(z)+12\,z\cosh(3\,z)+2\left(4\,z^2-15\cosh(2\,z)+\cosh(4\,z)+32\right)\sinh(z)\right)(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.35.06.0015.01

$$\operatorname{sc}(z\mid m)\propto\sinh(z)-\frac{1}{8}\cosh(z)\,(\sinh(2\,z)-2\,z)\,(m-1)+$$
$$\frac{1}{256}\left(-24\,z\cosh(z)-12\,z\cosh(3\,z)+\left(8\,z^2+7\right)\sinh(z)+8\sinh(3\,z)+\sinh(5\,z)\right)(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.35.06.0016.01

$$\operatorname{sd}(z\mid m)\propto\sinh(z)\,(1+O(m-1))$$

q-series

09.35.06.0002.01

$$\operatorname{sd}(z\mid m)==\frac{2\,\pi}{\sqrt{m}\,\sqrt{1-m}\,K(m)}\sum_{k=0}^{\infty}\frac{(-1)^k\,q(m)^{k+1/2}}{q(m)^{2k+1}+1}\sin\!\left(\frac{(2\,k+1)\,\pi\,z}{2\,K(m)}\right)$$

Other series representations

09.35.06.0003.01

$$\operatorname{sd}(z\mid m)==-\frac{\pi}{2\sqrt{m}\,\sqrt{1-m}\,K(1-m)}\sum_{k=-\infty}^{\infty}(-1)^k\operatorname{sech}\!\left(\pi\,\frac{K(m)}{K(1-m)}\left(k+\frac{1}{2}+\frac{z}{2\,K(m)}\right)\right)$$

09.35.06.0004.01

$$\operatorname{sd}(z\mid m)\propto\frac{(-1)^{r+s-1}}{\sqrt{m}\,\sqrt{1-m}\,(z-i\,(2\,s+1)\,K(1-m)-(2\,r+1)\,K(m))}+O(1)\;/;$$
$$(z\to(2\,s+1)\,i\,K(1-m)+(2\,r+1)\,K(m))\bigwedge\{r,s\}\in\mathbb{Z}$$

Product representations

09.35.08.0001.01

$$\operatorname{sd}(z\mid m)==\frac{2\sqrt[4]{q(m)}}{\sqrt[4]{m}\,\sqrt[4]{1-m}}\sin\!\left(\frac{\pi\,z}{2\,K(m)}\right)\prod_{k=1}^{\infty}\frac{1-2\,q(m)^{2k}\cos\!\left(\frac{\pi z}{K(m)}\right)+q(m)^{4k}}{1+2\,q(m)^{2k-1}\cos\!\left(\frac{\pi z}{K(m)}\right)+q(m)^{4k-2}}$$

Differential equations

Ordinary nonlinear differential equations

09.35.13.0001.01

$$w''(z) + w(z)\left(2\,m\,(1-m)\,w(z)^2 - 2\,m + 1\right) = 0\ /;\ w(z) = \mathrm{sd}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.35.16.0001.01

$$\mathrm{sd}(i\,z \mid m) = i\,\mathrm{sd}(z \mid 1-m)$$

09.35.16.0002.01

$$\mathrm{sd}(z \mid 1-m) = -i\,\mathrm{sd}(i\,z \mid m)$$

09.35.16.0003.01

$$\mathrm{sd}(i\,z \mid 1-m) = i\,\mathrm{sd}(z \mid m)$$

09.35.16.0007.01

$$\mathrm{sd}(x + i\,y \mid m) = \frac{\mathrm{dn}(y \mid 1-m)\,\mathrm{sn}(x \mid m) + i\,\mathrm{cn}(x \mid m)\,\mathrm{cn}(y \mid 1-m)\,\mathrm{dn}(x \mid m)\,\mathrm{sn}(y \mid 1-m)}{\mathrm{dn}(x \mid m)\,\mathrm{cn}(y \mid 1-m)\,\mathrm{dn}(y \mid 1-m) - i\,m\,\mathrm{sn}(x \mid m)\,\mathrm{cn}(x \mid m)\,\mathrm{sn}(y \mid 1-m)}\ /;\ \{x, y\} \in \mathbb{R}$$

09.35.16.0008.01

$$\mathrm{sd}\left(\sqrt{1-m}\ z \,\middle|\, \frac{m}{m-1}\right) = \sqrt{1-m}\ \mathrm{sn}(z \mid m)$$

09.35.16.0009.01

$$\mathrm{sd}\left(\sqrt{m}\ z \,\middle|\, \frac{1}{m}\right) = \sqrt{m}\ \mathrm{sc}(z \mid m)$$

09.35.16.0010.01

$$\mathrm{sd}\left(i\,\sqrt{1-m}\ z \,\middle|\, \frac{1}{1-m}\right) = i\,\sqrt{1-m}\ \mathrm{sn}(z \mid m)$$

09.35.16.0011.01

$$\mathrm{sd}\left(i\,\sqrt{m}\ z \,\middle|\, \frac{m-1}{m}\right) = i\,\sqrt{m}\ \mathrm{sc}(z \mid m)$$

Landen's transformation:

09.35.16.0012.01

$$\mathrm{sd}\left(\left(1+\sqrt{1-m}\right) z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{\left(1+\sqrt{1-m}\right)\mathrm{sn}(z \mid m)\,\mathrm{cn}(z \mid m)}{1-\left(1-\sqrt{1-m}\right)\mathrm{sn}(z \mid m)^2}$$

Gauss' transformation:

09.35.16.0013.01

$$\operatorname{sd}\left(\left(1+\sqrt{m}\right) z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{\left(1+\sqrt{m}\right)\operatorname{sn}(z\mid m)}{1-\sqrt{m}\,\operatorname{sn}(z\mid m)^2}$$

n th degree transformations:

09.35.16.0014.01

$$\operatorname{sd}\left(\frac{z}{M} \,\middle|\, l\right) = \frac{1}{M}\operatorname{sd}(z\mid m)\prod_{r=1}^{\frac{n-1}{2}} \frac{1-\operatorname{ns}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2}{1-m\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2\operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n+1}{2}\in\mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n-1}{2}} \operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

09.35.16.0015.01

$$\operatorname{sd}\left(\frac{z}{M}+\frac{K(m)}{nM} \,\middle|\, l\right) = \frac{\operatorname{dn}(z\mid m)}{\sqrt{1-l}}\prod_{r=1}^{\frac{n}{2}} \frac{1-\operatorname{ns}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2}{1-m\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2\operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n}{2}\in\mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n}{2}} \operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

Argument involving half-periods

09.35.16.0004.01

$$\operatorname{sd}(z+K(m)\mid m) = \frac{1}{\sqrt{1-m}}\operatorname{cn}(z\mid m)$$

09.35.16.0020.01

$$\operatorname{sd}(z-K(m)\mid m) = -\frac{1}{\sqrt{1-m}}\operatorname{cn}(z\mid m)$$

09.35.16.0021.01

$$\operatorname{sd}(z+3K(m)\mid m) = -\frac{1}{\sqrt{1-m}}\operatorname{cn}(z\mid m)$$

09.35.16.0022.01

$$\operatorname{sd}(z+(2r+1)K(m)\mid m) = \frac{(-1)^r}{\sqrt{1-m}}\operatorname{cn}(z\mid m) \;/;\; r\in\mathbb{Z}$$

09.35.16.0005.01

$$\operatorname{sd}(z+iK(1-m)\mid m) = \frac{i}{\sqrt{m}}\operatorname{nc}(z\mid m)$$

09.35.16.0023.01

$$\operatorname{sd}(z-iK(1-m)\mid m) = -\frac{i}{\sqrt{m}}\operatorname{nc}(z\mid m)$$

09.35.16.0024.01

$$\mathrm{sd}(z+3\,i\,K(1-m)\mid m)=-\frac{i}{\sqrt{m}}\,\mathrm{nc}(z\mid m)$$

09.35.16.0025.01

$$\mathrm{sd}(z+(2\,s+1)\,i\,K(1-m)\mid m)=\frac{(-1)^s\,i}{\sqrt{m}}\,\mathrm{nc}(z\mid m)\;/;\;s\in\mathbb{Z}$$

09.35.16.0006.01

$$\mathrm{sd}(z+i\,K(1-m)+K(m)\mid m)=-\frac{i\,\mathrm{ds}(z\mid m)}{\sqrt{m}\,\sqrt{1-m}}$$

09.35.16.0026.01

$$\mathrm{sd}(z+K(m)-i\,K(1-m)\mid m)=\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)$$

09.35.16.0027.01

$$\mathrm{sd}(z-K(m)+i\,K(1-m)\mid m)=\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)$$

09.35.16.0028.01

$$\mathrm{sd}(z-K(m)-i\,K(1-m)\mid m)=-\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)$$

09.35.16.0029.01

$$\mathrm{sd}(z+3\,K(m)+i\,K(1-m)\mid m)=\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)$$

09.35.16.0030.01

$$\mathrm{sd}(z+(4\,s+1)\,i\,K(1-m)+(4\,r+1)\,K(m)\mid m)=-\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)\;/;\;\{r,\,s\}\in\mathbb{Z}$$

09.35.16.0031.01

$$\mathrm{sd}(z+(4\,s+1)\,i\,K(1-m)-(4\,r+1)\,K(m)\mid m)=\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)\;/;\;\{r,\,s\}\in\mathbb{Z}$$

09.35.16.0032.01

$$\mathrm{sd}(z+(4\,s-1)\,i\,K(1-m)+(4\,r+1)\,K(m)\mid m)=\frac{i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)\;/;\;\{r,\,s\}\in\mathbb{Z}$$

09.35.16.0033.01

$$\mathrm{sd}(z+(2\,s+1)\,i\,K(1-m)+(2\,r+1)\,K(m)\mid m)=\frac{(-1)^{r+s-1}\,i}{\sqrt{m}\,\sqrt{1-m}}\,\mathrm{ds}(z\mid m)\;/;\;\{r,\,s\}\in\mathbb{Z}$$

Argument involving inverse Jacobi functions

09.35.16.0034.01

$$\mathrm{sd}\left(\mathrm{cd}^{-1}(z\mid m)\,\middle|\,m\right)^2=\frac{1-z^2}{1-m}$$

09.35.16.0035.01

$$\mathrm{sd}\left(\mathrm{cn}^{-1}(z\mid m)\,\middle|\,m\right)^2=\frac{1-z^2}{m\,z^2-m+1}$$

09.35.16.0036.01

$$\operatorname{sd}\left(\operatorname{cs}^{-1}(z \mid m) \mid m\right)^2 = \frac{1}{z^2 - m + 1}$$

09.35.16.0037.01

$$\operatorname{sd}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)^2 = \frac{1 - z^2}{(m-1)\, z^2}$$

09.35.16.0038.01

$$\operatorname{sd}\left(\operatorname{dn}^{-1}(z \mid m) \mid m\right)^2 = \frac{1 - z^2}{m\, z^2}$$

09.35.16.0039.01

$$\operatorname{sd}\left(\operatorname{ds}^{-1}(z \mid m) \mid m\right) = \frac{1}{z}$$

09.35.16.0040.01

$$\operatorname{sd}\left(\operatorname{nc}^{-1}(z \mid m) \mid m\right)^2 = \frac{1 - z^2}{(m-1)\, z^2 - m}$$

09.35.16.0041.01

$$\operatorname{sd}\left(\operatorname{nd}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2 - 1}{m}$$

09.35.16.0042.01

$$\operatorname{sd}\left(\operatorname{ns}^{-1}(z \mid m) \mid m\right)^2 = \frac{1}{z^2 - m}$$

09.35.16.0043.01

$$\operatorname{sd}\left(\operatorname{sc}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2}{1 - (m-1)\, z^2}$$

09.35.16.0044.01

$$\operatorname{sd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2}{1 - m\, z^2}$$

Addition formulas

09.35.16.0016.01

$$\operatorname{sd}(u + v \mid m) = \frac{\operatorname{cn}(v \mid m)\operatorname{dn}(v \mid m)\operatorname{sn}(u \mid m) + \operatorname{cn}(u \mid m)\operatorname{dn}(u \mid m)\operatorname{sn}(v \mid m)}{\operatorname{dn}(u \mid m)\operatorname{dn}(v \mid m) - m\operatorname{sn}(u \mid m)\operatorname{cn}(u \mid m)\operatorname{sn}(v \mid m)\operatorname{cn}(v \mid m)}$$

09.35.16.0017.01

$$\operatorname{sd}(u + v \mid m)\operatorname{sd}(u - v \mid m) = \frac{\operatorname{sn}(u \mid m)^2 - \operatorname{sn}(v \mid m)^2}{\operatorname{dn}(v \mid m)^2 - m\operatorname{cn}(v \mid m)^2 \operatorname{sn}(u \mid m)^2}$$

Half-angle formulas

09.35.16.0018.01

$$\operatorname{sd}\left(\frac{z}{2} \,\middle|\, m\right)^2 = \frac{1 - \operatorname{cn}(z \mid m)}{1 - m + \operatorname{dn}(z \mid m) + m\operatorname{cn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.35.16.0019.01

$$\operatorname{sd}(2\,z \mid m) = \frac{2\operatorname{sn}(z \mid m)\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{dn}(z \mid m)^2 - m\operatorname{sn}(z \mid m)^2\operatorname{cn}(z \mid m)^2}$$

Identities

Functional identities

09.35.17.0001.01

$$\left((m-1)\,m\,w(z)^4 - 1\right)^2 w(2\,z)^2 - 4\,w(z)^2\left((m-1)\,m\,w(z)^4 + (2\,m-1)\,w(z)^2 + 1\right) = 0 \;/;\; w(z) = \operatorname{sd}(z \mid m)$$

Complex characteristics

Real part

09.35.19.0001.01

$$\operatorname{Re}(\operatorname{sd}(x + i\,y \mid m)) = \frac{\operatorname{cn}(y \mid 1-m)\operatorname{dn}(x \mid m)\operatorname{sn}(x \mid m)\left(\operatorname{dn}(y \mid 1-m)^2 - m\operatorname{cn}(x \mid m)^2\operatorname{sn}(y \mid 1-m)^2\right)}{\operatorname{cn}(y \mid 1-m)^2\operatorname{dn}(x \mid m)^2\operatorname{dn}(y \mid 1-m)^2 + m^2\operatorname{cn}(x \mid m)^2\operatorname{sn}(x \mid m)^2\operatorname{sn}(y \mid 1-m)^2} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Imaginary part

09.35.19.0002.01

$$\operatorname{Im}(\operatorname{sd}(x + i\,y \mid m)) = \frac{\operatorname{cn}(x \mid m)\operatorname{dn}(y \mid 1-m)\left(\operatorname{cn}(y \mid 1-m)^2\operatorname{dn}(x \mid m)^2 + m\operatorname{sn}(x \mid m)^2\right)\operatorname{sn}(y \mid 1-m)}{\operatorname{cn}(y \mid 1-m)^2\operatorname{dn}(x \mid m)^2\operatorname{dn}(y \mid 1-m)^2 + m^2\operatorname{cn}(x \mid m)^2\operatorname{sn}(x \mid m)^2\operatorname{sn}(y \mid 1-m)^2} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Absolute value

09.35.19.0003.01

$$|\operatorname{sd}(x + i\,y \mid m)| = \sqrt{\frac{\operatorname{dn}(y \mid 1-m)^2\operatorname{sn}(x \mid m)^2 + \operatorname{cn}(x \mid m)^2\operatorname{cn}(y \mid 1-m)^2\operatorname{dn}(x \mid m)^2\operatorname{sn}(y \mid 1-m)^2}{\operatorname{cn}(y \mid 1-m)^2\operatorname{dn}(x \mid m)^2\operatorname{dn}(y \mid 1-m)^2 + m^2\operatorname{cn}(x \mid m)^2\operatorname{sn}(x \mid m)^2\operatorname{sn}(y \mid 1-m)^2}} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Argument

09.35.19.0004.01

$$\arg(\operatorname{sd}(x + i\,y \mid m)) = \tan^{-1}\Big(\operatorname{cn}(y \mid 1-m)\operatorname{dn}(x \mid m)\operatorname{sn}(x \mid m)\left(\operatorname{dn}(y \mid 1-m)^2 - m\operatorname{cn}(x \mid m)^2\operatorname{sn}(y \mid 1-m)^2\right),$$
$$\operatorname{cn}(x \mid m)\operatorname{dn}(y \mid 1-m)\left(\operatorname{cn}(y \mid 1-m)^2\operatorname{dn}(x \mid m)^2 + m\operatorname{sn}(x \mid m)^2\right)\operatorname{sn}(y \mid 1-m)\Big) \;/;\; \{x, y, m\} \in \mathbb{R}$$

Conjugate value

09.35.19.0005.01

$$\overline{\operatorname{sd}(x + i\,y \mid m)} = \frac{\operatorname{dn}(y \mid 1-m)\operatorname{sn}(x \mid m) - i\operatorname{cn}(x \mid m)\operatorname{cn}(y \mid 1-m)\operatorname{dn}(x \mid m)\operatorname{sn}(y \mid 1-m)}{\operatorname{dn}(x \mid m)\operatorname{cn}(y \mid 1-m)\operatorname{dn}(y \mid 1-m) + i\,m\operatorname{sn}(x \mid m)\operatorname{cn}(x \mid m)\operatorname{sn}(y \mid 1-m)} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.35.20.0001.01

$$\frac{\partial \operatorname{sd}(z \mid m)}{\partial z} = \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m)$$

09.35.20.0002.01

$$\frac{\partial^2 \operatorname{sd}(z \mid m)}{\partial z^2} = \left(m \operatorname{cd}(z \mid m)^2 + (m-1) \operatorname{nd}(z \mid m)^2\right) \operatorname{sd}(z \mid m)$$

With respect to *m*

09.35.20.0003.01

$$\frac{\partial \operatorname{sd}(z \mid m)}{\partial m} = \frac{\operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m) ((1-m) z - E(\operatorname{am}(z \mid m) \mid m) + m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m))}{2 m (1-m)}$$

09.35.20.0004.01

$$\frac{\partial^2 \operatorname{sd}(z \mid m)}{\partial m^2} = \frac{1}{4 (m-1)^2 m^2} \Big(m \big(\big(((m-1) z + E(\operatorname{am}(z \mid m) \mid m))^2 - (m+1) \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) + m \operatorname{dn}(z \mid m)^2 \operatorname{sc}(z \mid m)^2\big) \operatorname{sd}(z \mid m) + m \operatorname{dc}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) \operatorname{sn}(z \mid m)\big) \operatorname{cd}(z \mid m)^2 + \operatorname{nd}(z \mid m) \Big(z \operatorname{cn}(z \mid m) \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m) m^3 - 2 z m^2 - z \operatorname{cn}(z \mid m) \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m) m^2 + E(\operatorname{am}(z \mid m) \mid m) \operatorname{cn}(z \mid m) \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m) m^2 + 4 z m - 3 E(\operatorname{am}(z \mid m) \mid m) m - F(\operatorname{am}(z \mid m) \mid m) m - ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{dc}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m) m - \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m) \sqrt{1 - m \operatorname{sn}(z \mid m)^2}\, m - 2 z + E(\operatorname{am}(z \mid m) \mid m) + F(\operatorname{am}(z \mid m) \mid m) + \operatorname{dn}(z \mid m) \left(2 \operatorname{sc}(z \mid m) m^2 - \operatorname{cn}(z \mid m) \operatorname{sc}(z \mid m)^2 \operatorname{sn}(z \mid m) m^2 + ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \sqrt{1 - m \operatorname{sn}(z \mid m)^2}\right)\Big) \operatorname{cd}(z \mid m) + (m-1) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{nd}(z \mid m)^2 ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)) \operatorname{sd}(z \mid m)\Big)$$

Symbolic differentiation

With respect to *z*

09.35.20.0007.01

$$\frac{\partial^n \operatorname{sd}(z \mid m)}{\partial z^n} = \operatorname{sd}(z \mid m) \delta_n + \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \operatorname{cd}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \operatorname{nd}(z \mid m)}{\partial z^{-j+n-1}} \text{ /; } n \in \mathbb{N}$$

09.35.20.0005.01

$$\frac{\partial^n \operatorname{sd}(z \mid m)}{\partial z^n} = \frac{2^{1-n} \pi^{n+1}}{\sqrt{m} \sqrt{1-m} K(m)^{n+1}} \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)^n q(m)^{k+\frac{1}{2}}}{q(m)^{2k+1} + 1} \sin\left(\frac{\pi n}{2} + \frac{(2k+1) \pi z}{2 K(m)}\right) \text{ /; } n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.35.20.0006.01

$$\frac{\partial^\alpha \mathrm{sd}(z \mid m)}{\partial z^\alpha} = \frac{2^{\alpha-1} \pi^{5/2} z^{1-\alpha}}{\sqrt{1-m}\,\sqrt{m}\,K(m)^2} \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)\, q(m)^{k+\frac{1}{2}}}{q(m)^{2k+1}+1}\, {}_1\tilde{F}_2\left(1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16\, K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.35.21.0001.01

$$\int \mathrm{sd}(z \mid m)\, dz = -\frac{\sin^{-1}\left(\sqrt{m}\, \mathrm{cd}(z \mid m)\right) \sqrt{1-m\, \mathrm{cd}(z \mid m)^2}\, \mathrm{dn}(z \mid m)}{(1-m)\sqrt{m}}$$

Representations through equivalent functions

With inverse function

09.35.27.0001.01

$$\mathrm{sd}\left(\mathrm{sd}^{-1}(z \mid m) \mid m\right) = z$$

With related functions

Involving am

09.35.27.0026.01

$$\mathrm{sd}(z \mid m)^2 = \frac{1}{\csc^2(\mathrm{am}(z \mid m)) - m}$$

Involving one other Jacobi elliptic function

Involving cd

09.35.27.0004.01

$$\mathrm{sd}(z \mid m)^2 = \frac{1-\mathrm{cd}(z \mid m)^2}{1-m}$$

Involving cn

09.35.27.0005.01

$$\mathrm{sd}(z \mid m)^2 = \frac{1-\mathrm{cn}(z \mid m)^2}{m\, \mathrm{cn}(z \mid m)^2 - m + 1}$$

Involving cs

09.35.27.0007.01

$$\operatorname{sd}(z \mid m)^2 = \frac{1}{\operatorname{cs}(z \mid m)^2 - m + 1}$$

Involving dc

09.35.27.0009.01

$$\operatorname{sd}(z \mid m)^2 = \frac{1 - \operatorname{dc}(z \mid m)^2}{(m-1)\operatorname{dc}(z \mid m)^2}$$

Involving dn

09.35.27.0012.01

$$\operatorname{sd}(z \mid m)^2 = \frac{1 - \operatorname{dn}(z \mid m)^2}{m \operatorname{dn}(z \mid m)^2}$$

Involving ds

09.35.27.0013.01

$$\operatorname{sd}(z \mid m) = \frac{1}{\operatorname{ds}(z \mid m)}$$

Involving nc

09.35.27.0014.01

$$\operatorname{sd}(z \mid m)^2 = \frac{1 - \operatorname{nc}(z \mid m)^2}{(m-1)\operatorname{nc}(z \mid m)^2 - m}$$

Involving nd

09.35.27.0017.01

$$\operatorname{sd}(z \mid m)^2 = \frac{\operatorname{nd}(z \mid m)^2 - 1}{m}$$

Involving ns

09.35.27.0018.01

$$\operatorname{sd}(z \mid m)^2 = \frac{1}{\operatorname{ns}(z \mid m)^2 - m}$$

Involving sc

09.35.27.0019.01

$$\operatorname{sd}(z \mid m)^2 = \frac{\operatorname{sc}(z \mid m)^2}{1 - (m-1)\operatorname{sc}(z \mid m)^2}$$

Involving sn

09.35.27.0020.01

$$\operatorname{sd}(z \mid m)^2 = \frac{\operatorname{sn}(z \mid m)^2}{1 - m \operatorname{sn}(z \mid m)^2}$$

Involving two other Jacobi elliptic functions

Involving cd and cs

09.35.27.0002.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

Involving cd and sc

09.35.27.0003.01

$$\operatorname{sd}(z \mid m) = \operatorname{sc}(z \mid m) \operatorname{cd}(z \mid m)$$

Involving cs and dc

09.35.27.0006.01

$$\operatorname{sd}(z \mid m) = \frac{1}{\operatorname{cs}(z \mid m) \operatorname{dc}(z \mid m)}$$

Involving dc and sc

09.35.27.0008.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{sc}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

Involving dn and ns

09.35.27.0010.01

$$\operatorname{sd}(z \mid m) = \frac{1}{\operatorname{ns}(z \mid m) \operatorname{dn}(z \mid m)}$$

09.35.27.0027.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{ns}(z \mid m)^2 - m}$$

Involving dn and sn

09.35.27.0011.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{sn}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

Involving nd and ns

09.35.27.0015.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{nd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.35.27.0028.01

$$\mathrm{sd}(z \mid m) = \frac{(\mathrm{nd}(z \mid m) - 1)\,(\mathrm{nd}(z \mid m) + 1)\,\mathrm{ns}(z \mid m)}{m\,\mathrm{nd}(z \mid m)}$$

Involving nd and sn

09.35.27.0016.01

$$\mathrm{sd}(z \mid m) = \mathrm{sn}(z \mid m)\,\mathrm{nd}(z \mid m)$$

Involving three other Jacobi elliptic functions

09.35.27.0029.01

$$\mathrm{sd}(z \mid m) = -\frac{(\mathrm{cn}(z \mid m) - 1)\,(\mathrm{cn}(z \mid m) + 1)\,\mathrm{cs}(z \mid m)}{\mathrm{cn}(z \mid m)^2\,\mathrm{dc}(z \mid m)}$$

09.35.27.0030.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,(\mathrm{dc}(z \mid m) - \mathrm{dn}(z \mid m))\,(\mathrm{dc}(z \mid m) + \mathrm{dn}(z \mid m))}{\mathrm{dc}(z \mid m)\,\mathrm{dn}(z \mid m)^2}$$

09.35.27.0031.01

$$\mathrm{sd}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{dn}(z \mid m)}{\mathrm{cs}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 - m + 1\right)\mathrm{nc}(z \mid m)}$$

09.35.27.0032.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}{\mathrm{dc}(z \mid m)}$$

09.35.27.0033.01

$$\mathrm{sd}(z \mid m) = -\frac{\mathrm{dn}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cs}(z \mid m)\left(m\,\mathrm{nc}(z \mid m)^2 - \mathrm{nc}(z \mid m)^2 - m\right)}$$

09.35.27.0034.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}{\mathrm{cs}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 - m + 1\right)\mathrm{nd}(z \mid m)}$$

09.35.27.0035.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.35.27.0036.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)\,\mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.35.27.0037.01

$$\mathrm{sd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m)^2}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.35.27.0038.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2}{\operatorname{cs}(z \mid m)^2 + 1}$$

09.35.27.0039.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)\operatorname{nd}(z \mid m)^2}{\operatorname{nc}(z \mid m)^2}$$

09.35.27.0040.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{nc}(z \mid m)(\operatorname{nd}(z \mid m) - 1)(\operatorname{nd}(z \mid m) + 1)}{m\operatorname{nd}(z \mid m)}$$

09.35.27.0041.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)(\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m) - 1)(\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m) + 1)}{\operatorname{dc}(z \mid m)^2}$$

09.35.27.0042.01

$$\operatorname{sd}(z \mid m) = -\frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{dn}(z \mid m)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right)\operatorname{ns}(z \mid m)}$$

09.35.27.0043.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)^2}{\left(m\operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m\right)\operatorname{ns}(z \mid m)}$$

09.35.27.0044.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{-\operatorname{cs}(z \mid m)^2 + m - 1}$$

09.35.27.0045.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m)}$$

09.35.27.0046.01

$$\operatorname{sd}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{ns}(z \mid m)}{m}$$

09.35.27.0047.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{ns}(z \mid m)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right)\operatorname{nd}(z \mid m)}$$

09.35.27.0048.01

$$\operatorname{sd}(z \mid m) = -(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)$$

09.35.27.0049.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{cs}(z \mid m)^2 + 1}$$

09.35.27.0050.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)^2}$$

09.35.27.0051.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) - 1) (\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) + 1) \operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.35.27.0052.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.35.27.0053.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)^2 (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.35.27.0054.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.35.27.0055.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{cd}(z \mid m) \left(\operatorname{ns}(z \mid m)^2 - m\right)}$$

09.35.27.0056.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) (\operatorname{cn}(z \mid m) - 1) (\operatorname{cn}(z \mid m) + 1)}{\operatorname{cn}(z \mid m)^2 \operatorname{sc}(z \mid m)}$$

09.35.27.0057.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m)}{(m - 1) \operatorname{sc}(z \mid m)}$$

09.35.27.0058.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) (\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{sc}(z \mid m)}$$

09.35.27.0059.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) (\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1)}{\operatorname{dn}(z \mid m) \left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{sc}(z \mid m)}$$

09.35.27.0060.01

$$\operatorname{sd}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1) \operatorname{nc}(z \mid m)}{m \operatorname{dn}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.35.27.0061.01

$$\operatorname{sd}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - 1) (\operatorname{cn}(z \mid m) + 1) \operatorname{nd}(z \mid m)}{\operatorname{cn}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.35.27.0062.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}{m \operatorname{sc}(z \mid m)}$$

09.35.27.0063.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}{\left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right) \operatorname{sc}(z \mid m)}$$

09.35.27.0064.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1)\,(\operatorname{ns}(z \mid m) + 1)\,\operatorname{sc}(z \mid m)}$$

09.35.27.0065.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\left(m\operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)\operatorname{sc}(z \mid m)}{m\operatorname{nd}(z \mid m)}$$

09.35.27.0066.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{nd}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}{(m-1)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0067.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}$$

09.35.27.0068.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{cn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}$$

09.35.27.0069.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.35.27.0070.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)^2\operatorname{sc}(z \mid m)}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.35.27.0071.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2\operatorname{sc}(z \mid m)^2}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.35.27.0072.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m)^2}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.35.27.0073.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m)}{m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1}$$

09.35.27.0074.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{ns}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.35.27.0075.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nc}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.35.27.0076.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nd}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.35.27.0077.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{cd}(z \mid m) - 1)\,(\operatorname{cd}(z \mid m) + 1)\operatorname{cn}(z \mid m)}{(m-1)\operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0078.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\,(\operatorname{dc}(z \mid m) - 1)\,(\operatorname{dc}(z \mid m) + 1)}{(m-1)\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0079.01

$$\operatorname{sd}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right)\operatorname{sn}(z \mid m)}{(m-1)\operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m)}$$

09.35.27.0080.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\left(m\operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m\right)}$$

09.35.27.0081.01

$$\operatorname{sd}(z \mid m) = -\frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right)\operatorname{nd}(z \mid m)}$$

09.35.27.0082.01

$$\operatorname{sd}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right)\operatorname{sn}(z \mid m)}{(m-1)\operatorname{dc}(z \mid m)^2\operatorname{nd}(z \mid m)}$$

09.35.27.0083.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)^2\operatorname{sn}(z \mid m)}{\left(m\operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m\right)\operatorname{nd}(z \mid m)}$$

09.35.27.0084.01

$$\operatorname{sd}(z \mid m) = -\frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.35.27.0085.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\,(\operatorname{sn}(z \mid m) - 1)\,(\operatorname{sn}(z \mid m) + 1)}$$

09.35.27.0086.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{sn}(z \mid m)^2}{\operatorname{dc}(z \mid m)\,(\operatorname{sn}(z \mid m) - 1)\,(\operatorname{sn}(z \mid m) + 1)}$$

09.35.27.0087.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)^2}{\operatorname{sc}(z \mid m)\,(\operatorname{sn}(z \mid m) - 1)\,(\operatorname{sn}(z \mid m) + 1)}$$

09.35.27.0088.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\left(m\operatorname{sn}(z \mid m)^2 - 1\right)}$$

Involving four other Jacobi elliptic functions

09.35.27.0089.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\,(\operatorname{dn}(z \mid m) + m\,\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))}{m\,\operatorname{cs}(z \mid m)}$$

09.35.27.0090.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{cn}(z \mid m) - \operatorname{dc}(z \mid m)\,\operatorname{nd}(z \mid m))}{\operatorname{cn}(z \mid m)\,\operatorname{dc}(z \mid m)}$$

09.35.27.0091.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\left(\operatorname{dc}(z \mid m)^2\,\operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)\right)}{\operatorname{dc}(z \mid m)\,\operatorname{dn}(z \mid m)}$$

09.35.27.0092.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\,(\operatorname{ds}(z \mid m)\,\operatorname{nc}(z \mid m)\,\operatorname{nd}(z \mid m) - \operatorname{cs}(z \mid m))}{\operatorname{nc}(z \mid m)}$$

09.35.27.0093.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\,\operatorname{ds}(z \mid m)\,\operatorname{nd}(z \mid m)^2 - \operatorname{cs}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.35.27.0094.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\,\operatorname{cs}(z \mid m) - \operatorname{ns}(z \mid m)}{\operatorname{cn}(z \mid m)\,\operatorname{dc}(z \mid m)}$$

09.35.27.0095.01

$$\operatorname{sd}(z \mid m) = \frac{m\,\operatorname{cd}(z \mid m) - \operatorname{dn}(z \mid m)\,\operatorname{nc}(z \mid m)}{(m-1)\,\operatorname{nc}(z \mid m)\,\operatorname{ns}(z \mid m)}$$

09.35.27.0096.01

$$\operatorname{sd}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\,\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.35.27.0097.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{dc}(z \mid m)\,\operatorname{nc}(z \mid m) - \operatorname{dn}(z \mid m))\,\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)^2}$$

09.35.27.0098.01

$$\operatorname{sd}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\,\operatorname{nd}(z \mid m)\,\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.35.27.0099.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{cd}(z \mid m)\,\operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m))\,\operatorname{ns}(z \mid m)}{m\,\operatorname{cd}(z \mid m)}$$

09.35.27.0100.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{dc}(z \mid m)\,\operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m))\,\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.35.27.0101.01

$$\operatorname{sd}(z \mid m) = \frac{\left(\operatorname{dc}(z \mid m)^2\,\operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)\right)\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)^2}$$

09.35.27.0102.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cd}(z \mid m))\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.35.27.0103.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)}$$

09.35.27.0104.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m)}$$

09.35.27.0105.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{cn}(z \mid m)}{(m-1)\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0106.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m)^2 - \operatorname{dc}(z \mid m)}{\operatorname{dn}(z \mid m)^2\operatorname{sc}(z \mid m)}$$

09.35.27.0107.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0108.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0109.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0110.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) - \operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{(m-1)\operatorname{sc}(z \mid m)}$$

09.35.27.0111.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{cn}(z \mid m) - \operatorname{nd}(z \mid m)}{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0112.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{sc}(z \mid m)}$$

09.35.27.0113.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{m\operatorname{sc}(z \mid m)}$$

09.35.27.0114.01

$$\operatorname{sd}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{nd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.35.27.0115.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\,(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)) \operatorname{sc}(z \mid m)}$$

09.35.27.0116.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{nd}(z \mid m) - 1)\,(\operatorname{nd}(z \mid m) + 1)}{(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)) \operatorname{sc}(z \mid m)}$$

09.35.27.0117.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\,(\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m))}{\operatorname{sc}(z \mid m)}$$

09.35.27.0118.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.35.27.0119.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)^2}{\operatorname{sc}(z \mid m)}$$

09.35.27.0120.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{dc}(z \mid m) + m \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)) \operatorname{sc}(z \mid m)}{m}$$

09.35.27.0121.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.35.27.0122.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)^2}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.35.27.0123.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m)^2 \operatorname{sc}(z \mid m)}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.35.27.0124.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{nc}(z \mid m)}$$

09.35.27.0125.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{nd}(z \mid m)}$$

09.35.27.0126.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{nc}(z \mid m) \operatorname{ns}(z \mid m) - m \operatorname{sc}(z \mid m)}$$

09.35.27.0127.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{cs}(z \mid m) - m \operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.35.27.0128.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m)\,(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.35.27.0129.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m)^2 - \operatorname{ds}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{dn}(z \mid m)^2\operatorname{sc}(z \mid m)}$$

09.35.27.0130.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cn}(z \mid m)}{(m-1)\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0131.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\,(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cn}(z \mid m))}{\operatorname{sc}(z \mid m)}$$

09.35.27.0132.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2\operatorname{sc}(z \mid m) - \operatorname{cd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.35.27.0133.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0134.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\,(\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cn}(z \mid m))}{\operatorname{sc}(z \mid m)}$$

09.35.27.0135.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.35.27.0136.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{ns}(z \mid m) - m\operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m) + \operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0137.01

$$\operatorname{sd}(z \mid m) = \frac{-\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)^2 - \operatorname{cn}(z \mid m) + \operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m)}{(m-1)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0138.01

$$\operatorname{sd}(z \mid m) = \frac{-\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)^2 - \operatorname{dn}(z \mid m) + \operatorname{cd}(z \mid m)\operatorname{nc}(z \mid m)}{(m-1)\operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0139.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{dc}(z \mid m)\,(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m))}$$

09.35.27.0140.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m))}{(m-1)\operatorname{sn}(z \mid m)}$$

09.35.27.0141.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0142.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)^2 \operatorname{nc}(z \mid m) - \operatorname{cn}(z \mid m)}{m \operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0143.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m)}{m \operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0144.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0145.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)\operatorname{sn}(z \mid m)}{-m \operatorname{cd}(z \mid m) + m \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.35.27.0146.01

$$\operatorname{sd}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)(\operatorname{cs}(z \mid m) - m \operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.35.27.0147.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m))}$$

09.35.27.0148.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{cd}(z \mid m)(\operatorname{ns}(z \mid m) - m \operatorname{sn}(z \mid m))}$$

09.35.27.0149.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0150.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cs}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0151.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{sn}(z \mid m) - \operatorname{cn}(z \mid m)}{(m-1)\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0152.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{sn}(z \mid m)\operatorname{ds}(z \mid m)^2 - \operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m)\operatorname{ds}(z \mid m) + m \operatorname{sn}(z \mid m)}{(m-1)\operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m)}$$

09.35.27.0153.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{cn}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}{(m-1)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0154.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.35.27.0155.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.35.27.0156.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{-\operatorname{cn}(z \mid m) + m\operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.35.27.0157.01

$$\operatorname{sd}(z \mid m) = \frac{\left(\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{cn}(z \mid m)\right)\operatorname{ns}(z \mid m)}{m\operatorname{cd}(z \mid m) + \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.35.27.0158.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m)}{(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{sc}(z \mid m)}$$

09.35.27.0159.01

$$\operatorname{sd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0160.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.35.27.0161.01

$$\operatorname{sd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\left(\operatorname{ns}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{ns}(z \mid m) + \operatorname{sn}(z \mid m)\right)}{m\operatorname{cd}(z \mid m) + \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.35.27.0162.01

$$\operatorname{sd}(z \mid m) = \frac{-\operatorname{cn}(z \mid m) + \operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}{(m-1)\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

Involving Weierstrass functions

09.35.27.0021.01

$$\operatorname{sd}(z \mid m) == \sqrt{e_1 - e_3}\,\frac{\sigma\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)}{\sigma_2\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)} \,/;$$

$$\{\omega_1, \omega_2, \omega_3\} == \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m == \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n == \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.35.27.0022.01

$$\operatorname{sd}(z \mid m)^2 == \frac{e_1 - e_3}{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_2} \,/;$$

$$\{\omega_1, \omega_2, \omega_3\} == \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m == \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n == \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

Involving theta functions

09.35.27.0023.02

$$\operatorname{sd}(z \mid m) = \frac{1}{\sqrt[4]{m}\,\sqrt[4]{1-m}}\,\frac{1}{\vartheta_3\left(\frac{\pi z}{2K(m)},\, q(m)\right)}\,\vartheta_1\left(\frac{\pi z}{2K(m)},\, q(m)\right)$$

09.35.27.0024.01

$$\operatorname{sd}(z \mid m) = \frac{2K(m)}{\pi}\,\frac{\vartheta_3(0,\, q(m))}{\vartheta_1'(0,\, q(m))}\,\frac{\vartheta_1\left(\frac{z}{\vartheta_3(0,q(m))^2},\, q(m)\right)}{\vartheta_3\left(\frac{z}{\vartheta_3(0,q(m))^2},\, q(m)\right)}$$

09.35.27.0025.01

$$\operatorname{sd}(z \mid m) = \frac{\vartheta_s(z \mid m)}{\vartheta_d(z \mid m)}$$

Zeros

09.35.30.0001.01

$$\operatorname{sd}(2\,r\,K(m) + 2\,s\,i\,K(1-m) \mid m) = 0 \;/;\; \{r,\, s\} \in \mathbb{Z}$$

History

– C. G. J. Jacobi (1827)

– N. H. Abel (1827)

– J. Glaisher (1882) introduced the notation sd

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