KelvinKei2

Notations

Traditional name

Kelvin function of the second kind

Traditional notation

$\mathrm{kei}_\nu(z)$

Mathematica StandardForm notation

`KelvinKei[ν, z]`

Primary definition

03.19.02.0001.01

$$\mathrm{kei}_\nu(z) = -\frac{1}{4} i\, e^{-\frac{3}{4} i \pi \nu} \pi z^{-\nu} \left(\sqrt[4]{-1}\, z\right)^{-\nu} \csc(\pi \nu) \left(\left(\sqrt[4]{-1}\, z\right)^{2\nu} \left(I_{-\nu}\left(\sqrt[4]{-1}\, z\right) - e^{\frac{3 i \pi \nu}{2}} J_{-\nu}\left(\sqrt[4]{-1}\, z\right)\right) - e^{\frac{i \pi \nu}{2}} z^{2\nu} \left(I_\nu\left(\sqrt[4]{-1}\, z\right) - e^{\frac{i \pi \nu}{2}} J_\nu\left(\sqrt[4]{-1}\, z\right)\right)\right) /;\ \nu \notin \mathbb{Z}$$

03.19.02.0002.01

$$\mathrm{kei}_\nu(z) = \lim_{\mu \to \nu} \mathrm{kei}_\mu(z) /;\ \nu \in \mathbb{Z}$$

Specific values

Specialized values

For fixed ν

03.19.03.0001.01

$$\mathrm{kei}_\nu(0) = ¿$$

For fixed z

Explicit rational ν

03.19.03.0002.01

$$\mathrm{kei}_0(z) = \mathrm{kei}(z)$$

03.19.03.0003.01

$$\operatorname{kei}_{-\frac{14}{3}}(z) = \frac{(-1)^{3/4}\pi}{243\,2^{5/6}\sqrt[6]{3}\,z^{8/3}((1+i)z)^{5/3}}\Bigg(144\sqrt[3]{3}\left(9z^2+110i\right)\left(2\left(\sqrt[4]{-1}\,z\right)^{2/3}-\left(-i+\sqrt{3}\right)z^{2/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\sqrt[3]{z}+$$
$$48\sqrt[3]{3}\left(9z^2+110i\right)\left(\left(3-i\sqrt{3}\right)z^{2/3}+2^{2/3}\sqrt{3}\,((1+i)z)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\sqrt[3]{z}-$$
$$\frac{144\sqrt[3]{3}\,z\left(9z^2-110i\right)\left(2^{2/3}(1+i)\sqrt[3]{z}+\left(i+\sqrt{3}\right)\sqrt[3]{(1+i)z}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)}{\sqrt[3]{(1+i)z}}-$$
$$\frac{1}{((1+i)z)^{4/3}}\Big(3\Big(14080\,2^{2/3}\,i\,((1+i)z)^{2/3}\sqrt[3]{z}-4320\,2^{2/3}((1+i)z)^{2/3}z^{7/3}-81\,i\,2^{2/3}((1+i)z)^{2/3}z^{13/3}-$$
$$162(-1)^{2/3}z^5-8640\sqrt[6]{-1}\,z^3+28160(-1)^{2/3}z\Big)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\Big)+$$
$$\frac{1}{((1+i)z)^{4/3}}\Big(3\,i\Big(-14080\,2^{2/3}((1+i)z)^{2/3}\sqrt[3]{z}+4320\,2^{2/3}\,i\,((1+i)z)^{2/3}z^{7/3}+81\,2^{2/3}((1+i)z)^{2/3}z^{13/3}+$$
$$162(-1)^{5/6}z^5-8640\sqrt[3]{-1}\,z^3-28160(-1)^{5/6}z\Big)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\Big)+$$
$$\frac{48\sqrt[3]{3}\,z\left(9iz^2+110\right)\left(2^{2/3}\sqrt{3}\,(-1+i)\sqrt[3]{z}+\left(-3i+\sqrt{3}\right)\sqrt[3]{(1+i)z}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)}{\sqrt[3]{(1+i)z}}-$$
$$\frac{1}{((1+i)z)^{4/3}}\Big(i\sqrt{3}\sqrt[3]{z}\Big(-28160\sqrt[6]{-1}\,z^{2/3}-8640(-1)^{2/3}z^{8/3}+162\sqrt[6]{-1}\,z^{14/3}-81\,2^{2/3}((1+i)z)^{2/3}z^4+$$
$$4320\,2^{2/3}\,i\,((1+i)z)^{2/3}z^2+14080\,2^{2/3}((1+i)z)^{2/3}\Big)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\Big)-$$
$$\frac{1}{((1+i)z)^{4/3}}\Big(i\sqrt{3}\sqrt[3]{z}\Big(-28160(-1)^{5/6}z^{2/3}-8640\sqrt[3]{-1}\,z^{8/3}+162(-1)^{5/6}z^{14/3}-81\,2^{2/3}((1+i)z)^{2/3}z^4-$$
$$4320\,i\,2^{2/3}((1+i)z)^{2/3}z^2+14080\,2^{2/3}((1+i)z)^{2/3}\Big)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\Big)\Bigg)$$

03.19.03.0004.01

$$\operatorname{kei}_{-\frac{9}{2}}(z) = \frac{(-1)^{3/8}}{2z^{9/2}}e^{-\sqrt[4]{-1}\,z}\sqrt{\frac{\pi}{2}}$$
$$\left(-\sqrt[4]{-1}\,z^4-10z^3+45(-1)^{3/4}z^2+105\,i\,z+105\sqrt[4]{-1}+e^{i\sqrt{2}\,z}\left(z^4+\sqrt{2}\,(5+5i)z^3+45\,i\,z^2+105(-1)^{3/4}z-105\right)\right)$$

03.19.03.0005.01

$$\operatorname{kei}_{-\frac{13}{3}}(z) = \frac{(-1)^{3/4}\,\pi}{162\,\sqrt[3]{6}\,z^{13/3}\,((1+i)\,z)^{2/3}}$$
$$\Bigg(7\,\sqrt[6]{3}\,\left(9\,i\,z^2+80\right)\left(4\,\sqrt{3}\,z^{2/3}+2^{2/3}\left(3\,i+\sqrt{3}\right)i\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}+$$
$$7\,\sqrt[6]{3}\,\left(9\,z^2+80\,i\right)\left(4\,\sqrt{3}\,z^{2/3}+2^{2/3}\left(3+i\,\sqrt{3}\right)((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}+$$
$$\sqrt{3}\,\left(4480\,z^{2/3}+3024\,i\,z^{8/3}-81\,z^{14/3}+81\,(-1)^{5/6}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^4-4480\,(-1)^{5/6}\left(\sqrt[4]{-1}\,z\right)^{2/3}-3024\,(-1)^{5/6}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)$$
$$\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\sqrt{3}\,\Big(-4480\,i\,z^{2/3}-3024\,z^{8/3}+81\,i\,z^{14/3}-81\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^4+$$
$$3024\,\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^2+4480\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}\Big)\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\frac{42\,\sqrt[6]{3}\,z^2\left(9\,z^2-80\,i\right)\left(4\,z^{2/3}+2^{2/3}\left(-i+\sqrt{3}\right)((1+i)\,z)^{2/3}\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{((1+i)\,z)^{4/3}}+$$
$$\frac{42\,i\,\sqrt[6]{3}\,z^{5/3}\left(9\,z^2+80\,i\right)\left(2^{2/3}\left(i+\sqrt{3}\right)\sqrt[3]{z}-(2-2\,i)\,\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{((1+i)\,z)^{2/3}}+$$
$$\frac{1}{((1+i)\,z)^{10/3}}\,z^{11/3}\left(81\,z^4-3024\,i\,z^2-4480\right)\left(2^{2/3}\left(-i+\sqrt{3}\right)\sqrt[3]{z}-(2-2\,i)\,\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\frac{1}{4}\left(-81\,i\,z^4+3024\,z^2+4480\,i\right)\left(4\,z^{2/3}+2^{2/3}\left(i+\sqrt{3}\right)((1+i)\,z)^{2/3}\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)$$

03.19.03.0006.01

$$\begin{aligned}\operatorname{kei}_{-\frac{11}{3}}(z) = -\frac{\pi}{324\, 2^{5/6} \sqrt[6]{3}\, z^{11/3} ((1+i)z)^{5/3}} \Bigg(& 40\sqrt{3}\, ((1+i)z)^{2/3} \left(-64(-1)^{2/3} z^{2/3} + 18\sqrt[6]{-1}\, z^{8/3} - 9\, 2^{2/3} ((1+i)z)^{2/3} z^2 + 32\, 2^{2/3}\, i\, ((1+i)z)^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) \\ & \sqrt[3]{z} + 40\sqrt{3} \left(64(-1)^{5/6} z^{2/3} + 18\sqrt[3]{-1}\, z^{8/3} - 32\, 2^{2/3} ((1+i)z)^{2/3} + \frac{9((1+i)z)^{8/3}}{\sqrt[3]{2}}\right) ((1+i)z)^{2/3} \\ & \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) \sqrt[3]{z} + 18\sqrt[3]{3}\, (9z^2 + 160 i) \left(2\left(\sqrt[4]{-1}\, z\right)^{2/3} - \left(-i + \sqrt{3}\right) z^{2/3}\right) \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) z^{7/3} + \\ & 9\sqrt[3]{3}\, ((1+i)z)^{5/3} (9z^2 - 160 i) \left(2^{2/3} (1+i) \sqrt[3]{z} + \left(i + \sqrt{3}\right) \sqrt[3]{(1+i)z}\right) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) z + \\ & 3\sqrt[3]{3}\, ((1+i)z)^{5/3} (9z^2 - 160 i) \left(2^{2/3} \sqrt{3}\, (1+i) \sqrt[3]{z} + \left(-3 - i\sqrt{3}\right) \sqrt[3]{(1+i)z}\right) \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) z - \\ & 120((1+i)z)^{2/3} \left(-32\, i\, 2^{2/3} ((1+i)z)^{2/3} \sqrt[3]{z} + 9\, 2^{2/3} ((1+i)z)^{2/3} z^{7/3} + 18\sqrt[6]{-1}\, z^3 - 64(-1)^{2/3} z\right) \\ & \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + 120\, i\, ((1+i)z)^{2/3} \\ & \left(32\, 2^{2/3}\, i\, ((1+i)z)^{2/3} \sqrt[3]{z} + 9\, 2^{2/3} ((1+i)z)^{2/3} z^{7/3} + 18(-1)^{5/6} z^3 - 64\sqrt[3]{-1}\, z\right) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + \\ & \frac{6\sqrt[3]{3}\, z^3 (9z^2 + 160 i) \left(2^{2/3} \sqrt{3}\, (1+i) \sqrt[3]{z} + \left(3 - i\sqrt{3}\right) \sqrt[3]{(1+i)z}\right) \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right)}{\sqrt[3]{(1+i)z}} \Bigg)\end{aligned}$$

03.19.03.0007.01

$$\operatorname{kei}_{-\frac{7}{2}}(z) = \frac{(-1)^{7/8}}{2 z^{7/2}} e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \left(\sqrt[4]{-1}\, z^3 + 6z^2 - 15(-1)^{3/4} z - 15 i + e^{i\sqrt{2}\, z} \left(z^3 + 6\sqrt[4]{-1}\, z^2 + 15\, i\, z + 15(-1)^{3/4}\right)\right)$$

03.19.03.0008.01

$$\operatorname{kei}_{-\frac{10}{3}}(z) = -\frac{\pi z^{10/3}}{54\, 2^{2/3}\sqrt[3]{3}} \Bigg(16\sqrt{3}\left(-9 i z^2 - 14\right)\left(\frac{1}{\left(\sqrt[4]{-1}\, z\right)^{20/3}} + \frac{\sqrt[3]{-1}}{z^{20/3}}\right) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$16\sqrt[6]{-1}\sqrt{3}\left(9 z^2 + 14 i\right)\left(\frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\, z\right)^{20/3}} + \frac{1}{z^{20/3}}\right)\operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \frac{1}{z^{20/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}}$$

$$3\sqrt[6]{3}\,((1+i) z)^{2/3}\left(112 i z^{2/3} - 9 z^{8/3} + 112\sqrt[3]{-1}\left(\sqrt[4]{-1}\, z\right)^{2/3} + 9\sqrt[3]{-1}\left(\sqrt[4]{-1}\, z\right)^{8/3}\right)\operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) -$$

$$\frac{1}{z^{20/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} 3\sqrt[6]{3}\,((1+i) z)^{2/3}\left(112 i z^{2/3} + 9 z^{8/3} - 112(-1)^{2/3}\left(\sqrt[4]{-1}\, z\right)^{2/3} + 9(-1)^{2/3}\left(\sqrt[4]{-1}\, z\right)^{8/3}\right)$$

$$\operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 16\left(-9 i z^2 - 14\right)\left(\frac{\sqrt[3]{-1}}{z^{20/3}} - \frac{1}{\left(\sqrt[4]{-1}\, z\right)^{20/3}}\right)\operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$\frac{16\left(14 i z^{2/3} + 9 z^{8/3} + 9\sqrt[6]{-1}\left(\sqrt[4]{-1}\, z\right)^{2/3} z^2 + 14(-1)^{2/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)}{z^{20/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} +$$

$$\frac{1}{z^{20/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} 3^{2/3}((1+i) z)^{2/3}\left(-112 i z^{2/3} + 9 z^{8/3} + 112\sqrt[3]{-1}\left(\sqrt[4]{-1}\, z\right)^{2/3} + 9\sqrt[3]{-1}\left(\sqrt[4]{-1}\, z\right)^{8/3}\right)$$

$$\operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \frac{1}{z^{20/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}}$$

$$3^{2/3}((1+i) z)^{2/3}\left(112 i z^{2/3} + 9 z^{8/3} + 9\sqrt[6]{-1}\left(\sqrt[4]{-1}\, z\right)^{2/3} z^2 + 112(-1)^{2/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\Bigg)$$

03.19.03.0009.01

$$\operatorname{kei}_{-\frac{8}{3}}(z) = -\frac{(-1)^{3/4}\,\pi}{54\,2^{5/6}\,\sqrt[6]{3}\,z^{8/3}\,((1+i)\,z)^{5/3}} \Bigg(-30\,\sqrt[3]{3}\,\left(2^{2/3}\,\sqrt{3}\,z^2\,((1+i)\,z)^{2/3} + \left(-3 - i\,\sqrt{3}\right)z^{8/3}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} - 90\,\sqrt[3]{3}\,\left(\left(i + \sqrt{3}\right)z^{2/3} + 2^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^{7/3} - 90\,\sqrt[3]{3}\,\left(\left(-i + \sqrt{3}\right)z^{2/3} - 2\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^{7/3} + 3\,((1+i)\,z)^{2/3}\left(-40\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\,\sqrt[3]{z} + 9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^{7/3} + 18\,\sqrt[6]{-1}\,z^3 - 80\,(-1)^{2/3}\,z\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 3\,((1+i)\,z)^{4/3}\left(40\,2^{2/3}\,i\,\sqrt[3]{z} + 9\,2^{2/3}\,z^{7/3} + 9\,\sqrt[3]{-1}\,((1+i)\,z)^{4/3}\,z + (-1)^{5/6}\,(40 + 40\,i)\,\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 30\,\sqrt[3]{3}\,\left(2^{2/3}\,\sqrt{3}\,z^{7/3}\,((1+i)\,z)^{2/3} + \left(3 - i\,\sqrt{3}\right)z^3\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \frac{1}{((1+i)\,z)^{4/3}}\,2\,\sqrt{3}\,z^{7/3}\left(-80\,\sqrt[6]{-1}\,z^{2/3} - 18\,(-1)^{2/3}\,z^{8/3} + 40\,2^{2/3}\,((1+i)\,z)^{2/3} + \frac{9\,((1+i)\,z)^{8/3}}{\sqrt[3]{2}}\right)\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \frac{1}{((1+i)\,z)^{4/3}}\,2\,\sqrt{3}\,z^{7/3}\left(80\,(-1)^{5/6}\,z^{2/3} + 18\,\sqrt[3]{-1}\,z^{8/3} - 40\,2^{2/3}\,((1+i)\,z)^{2/3} + \frac{9\,((1+i)\,z)^{8/3}}{\sqrt[3]{2}}\right)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)$$

03.19.03.0010.01

$$\operatorname{kei}_{-\frac{5}{2}}(z) = \frac{(-1)^{3/8}}{2\,z^{5/2}}\,e^{-\sqrt[4]{-1}\,z}\,\sqrt{\frac{\pi}{2}}\,\left(\sqrt[4]{-1}\,z^2 + 3\,z - 3\,(-1)^{3/4} - e^{i\,\sqrt{2}\,z}\left(z^2 + 3\,\sqrt[4]{-1}\,z + 3\,i\right)\right)$$

03.19.03.0011.01

$$\operatorname{kei}_{-\frac{7}{3}}(z) = \frac{\sqrt[4]{-1}\,\pi\, z^{7/3}}{18\, 2^{2/3}\sqrt[3]{3}} \left(\frac{24\sqrt[6]{3}\, i \left(z^{2/3} - \sqrt[6]{-1}\left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3}}{z^{14/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} + \right.$$

$$\frac{8\, 3^{2/3}\left(z^{2/3} + (-1)^{5/6}\left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3}}{z^{14/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$\frac{24\sqrt[6]{3}\left(z^{2/3} - (-1)^{5/6}\left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3}}{z^{14/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$\frac{8\, i\, 3^{2/3}\left(z^{2/3} + \sqrt[6]{-1}\left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3}}{z^{14/3}\left(\sqrt[4]{-1}\, z\right)^{2/3}} +$$

$$\sqrt{3}\left(-9\, i\, z^2 - 16\right) \left(\frac{1}{\left(\sqrt[4]{-1}\, z\right)^{14/3}} + \frac{(-1)^{5/6}}{z^{14/3}} \right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) -$$

$$\frac{1}{z^5}\sqrt[6]{-1}\,\sqrt{3}\left(16\, i\sqrt[3]{z} + 9\, z^{7/3} + 9\sqrt[3]{-1}\left(\sqrt[4]{-1}\, z\right)^{4/3} z - 16\sqrt[12]{-1}\sqrt[3]{\sqrt[4]{-1}\, z}\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$

$$\left(-9\, i\, z^2 - 16\right) \left(\frac{(-1)^{5/6}}{z^{14/3}} - \frac{1}{\left(\sqrt[4]{-1}\, z\right)^{14/3}} \right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$

$$\left. \sqrt[6]{-1}\left(-9\, z^2 - 16\, i\right) \left(\frac{(-1)^{5/6}}{\left(\sqrt[4]{-1}\, z\right)^{14/3}} + \frac{1}{z^{14/3}} \right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \right)$$

03.19.03.0012.01

$$\operatorname{kei}_{-\frac{5}{3}}(z) = \frac{(1+i)\pi}{12\, 6^{5/6} z^{11/3}} \Bigg(4\, 6^{2/3} z \left(\sqrt[3]{z} + (-1)^{7/12} \sqrt[3]{\sqrt[4]{-1}\, z}\right) \operatorname{Ai}'\!\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3} + 9\, 2^{2/3} z^3 \left(\sqrt[3]{z} + \frac{\sqrt[6]{-1}\, z}{\left(\sqrt[4]{-1}\, z\right)^{2/3}}\right)$$

$$\operatorname{Ai}\!\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 9\, i\, z^3 \left(2^{2/3} \sqrt[3]{z} + \sqrt[3]{-1}\, (1+i) \sqrt[3]{(1+i) z}\right) \operatorname{Ai}\!\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$4\, 3^{2/3} i\, z^{4/3} \left(\left(i + \sqrt{3}\right) z^{2/3} + 2^{2/3} ((1+i) z)^{2/3}\right) \operatorname{Ai}'\!\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 3\, 2^{2/3} \sqrt{3}\, z^3 \left(\sqrt[3]{z} + (-1)^{11/12} \sqrt[3]{\sqrt[4]{-1}\, z}\right)$$

$$\operatorname{Bi}\!\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - (3 - 3 i) \sqrt[3]{-1}\, \sqrt[6]{2}\, \sqrt{3}\, z^3 \left((-1)^{5/12} \sqrt[3]{z} + \sqrt[3]{\sqrt[4]{-1}\, z}\right) \operatorname{Bi}\!\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$4 \sqrt[6]{3}\, z^{4/3} \left(\left(1 - i \sqrt{3}\right) z^{2/3} + 2^{2/3} i\, ((1+i) z)^{2/3}\right) \operatorname{Bi}'\!\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$\sqrt[12]{-1}\, \sqrt{2}\, \sqrt[6]{3}\, (4 - 4 i) z^{4/3} \left(z^{2/3} + \sqrt[6]{-1} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\!\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\Bigg)$$

03.19.03.0013.01

$$\operatorname{kei}_{-\frac{3}{2}}(z) = -\frac{(-1)^{3/8}}{2 z^{3/2}} e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \left(i + e^{i \sqrt{2}\, z} \left(i z + (-1)^{3/4}\right) + (-1)^{3/4} z\right)$$

03.19.03.0014.01

$$\operatorname{kei}_{-\frac{3}{2}}(z) = \frac{\sqrt{\pi}}{4 z^{3/2}}$$

$$e^{-\frac{z}{\sqrt{2}}} \left(\left(z + \sqrt{2}\right) \cos\!\left(\frac{1}{8} \left(4 \sqrt{2}\, z + \pi\right)\right) + \left(\sqrt{2}\, z + 1\right) \cos\!\left(\frac{1}{8} \left(\pi - 4 \sqrt{2}\, z\right)\right) + z \sin\!\left(\frac{1}{8} \left(4 \sqrt{2}\, z + \pi\right)\right) + \sin\!\left(\frac{1}{8} \left(\pi - 4 \sqrt{2}\, z\right)\right)\right)$$

03.19.03.0015.01

$$\operatorname{kei}_{-\frac{4}{3}}(z) = -\frac{i\,\pi\,z^{4/3}}{6\,2^{2/3}\,\sqrt[3]{3}} \left(\frac{3^{2/3}\,i\left(z^{2/3} + (-1)^{5/6}\left(\sqrt[4]{-1}\;z\right)^{2/3}\right) \operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}}{z^{8/3}\left(\sqrt[4]{-1}\;z\right)^{2/3}} + \right.$$

$$\frac{3^{2/3}\,i\left(z^{2/3} + \sqrt[6]{-1}\left(\sqrt[4]{-1}\;z\right)^{2/3}\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}}{z^{8/3}\left(\sqrt[4]{-1}\;z\right)^{2/3}} -$$

$$\frac{3\,i\,\sqrt[6]{3}\left(z^{2/3} - (-1)^{5/6}\left(\sqrt[4]{-1}\;z\right)^{2/3}\right) \operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}}{z^{8/3}\left(\sqrt[4]{-1}\;z\right)^{2/3}} -$$

$$\frac{3\,i\,\sqrt[6]{3}\left(z^{2/3} - \sqrt[6]{-1}\left(\sqrt[4]{-1}\;z\right)^{2/3}\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}}{z^{8/3}\left(\sqrt[4]{-1}\;z\right)^{2/3}} -$$

$$2\sqrt{3}\left(\frac{1}{\left(\sqrt[4]{-1}\;z\right)^{8/3}} - \frac{\sqrt[3]{-1}}{z^{8/3}}\right) \operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 2\,(-1)^{2/3}\sqrt{3}\left(\frac{1}{z^{8/3}} - \frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\;z\right)^{8/3}}\right) \operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) -$$

$$\left. 2\left(-\frac{1}{\left(\sqrt[4]{-1}\;z\right)^{8/3}} - \frac{\sqrt[3]{-1}}{z^{8/3}}\right) \operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 2\,(-1)^{2/3}\left(\frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\;z\right)^{8/3}} + \frac{1}{z^{8/3}}\right) \operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.19.03.0016.01

$$\operatorname{kei}_{-\frac{2}{3}}(z) = \frac{(1-i)\,(-1)^{3/4}\,\pi}{6\,2^{5/6}\,\sqrt[6]{3}\;z^{4/3}}$$

$$\left(3\left(\sqrt[6]{-1}\;z^{2/3} + \left(\sqrt[4]{-1}\;z\right)^{2/3}\right) \operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 3\left((-1)^{5/6}\,z^{2/3} + \left(\sqrt[4]{-1}\;z\right)^{2/3}\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \right.$$

$$\left. \sqrt{3}\left(\left(\left(\sqrt[4]{-1}\;z\right)^{2/3} - \sqrt[6]{-1}\;z^{2/3}\right) \operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \left(\left(\sqrt[4]{-1}\;z\right)^{2/3} - (-1)^{5/6}\,z^{2/3}\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)$$

03.19.03.0017.01

$$\operatorname{kei}_{-\frac{1}{2}}(z) = \frac{(-1)^{3/8}}{2\sqrt{z}}\, e^{-\sqrt[4]{-1}\;z}\left(-\sqrt[4]{-1} + e^{i\sqrt{2}\;z}\right)\sqrt{\frac{\pi}{2}}$$

03.19.03.0018.01

$$\operatorname{kei}_{-\frac{1}{2}}(z) = \frac{1}{\sqrt{z}}\, e^{-\frac{z}{\sqrt{2}}}\sqrt{\frac{\pi}{2}}\,\cos\!\left(\frac{z}{\sqrt{2}} + \frac{3\,\pi}{8}\right)$$

03.19.03.0019.01

$$\operatorname{kei}_{-\frac{1}{3}}(z) = -\frac{\sqrt[4]{-1}\,\pi}{2\sqrt[3]{6}\,\sqrt[3]{z}\,((1+i)z)^{2/3}} \left(\sqrt{3}\left(i z^{2/3} + \sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\left(z^{2/3} - \sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3} - i z^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - \left(z^{2/3} + \sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.19.03.0020.01

$$\operatorname{kei}_{\frac{1}{3}}(z) = -\frac{\sqrt[4]{-1}\,\pi}{2\sqrt[3]{6}\,\sqrt[3]{z}\,((1+i)z)^{2/3}} \left(\sqrt{3}\left(\sqrt[6]{-1}\,z^{2/3} + \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt[3]{-1}\,\sqrt{3}\left(z^{2/3} - \sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(\left(\sqrt[4]{-1}\,z\right)^{2/3} - \sqrt[6]{-1}\,z^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - \sqrt[3]{-1}\left(z^{2/3} + \sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.19.03.0021.01

$$\operatorname{kei}_{\frac{1}{2}}(z) = \frac{(-1)^{7/8}}{2\sqrt{z}}\sqrt{\frac{\pi}{2}}\,e^{-\sqrt[4]{-1}\,z}\left(\sqrt[4]{-1} + e^{i\sqrt{2}\,z}\right)$$

03.19.03.0022.01

$$\operatorname{kei}_{\frac{1}{2}}(z) = -\frac{1}{\sqrt{z}}\,e^{-\frac{z}{\sqrt{2}}}\sqrt{\frac{\pi}{2}}\sin\left(\frac{z}{\sqrt{2}} + \frac{3\pi}{8}\right)$$

03.19.03.0023.01

$$\operatorname{kei}_{\frac{2}{3}}(z) = -\frac{\pi}{6\sqrt[6]{6}\,z^{2/3}\sqrt[3]{(1+i)z}} \left(-3\left(\sqrt[12]{-1}\,\sqrt[3]{z} + \sqrt[3]{\sqrt[4]{-1}\,z}\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 3\left(\sqrt[3]{\sqrt[4]{-1}\,z} - (-1)^{5/12}\sqrt[3]{z}\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\left(\left(\sqrt[3]{\sqrt[4]{-1}\,z} - \sqrt[12]{-1}\,\sqrt[3]{z}\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left((-1)^{5/12}\sqrt[3]{z} + \sqrt[3]{\sqrt[4]{-1}\,z}\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)\right)$$

03.19.03.0024.01

$$\operatorname{kei}_{\frac{4}{3}}(z) = \frac{i\pi}{3\sqrt[3]{6}\,z^{4/3}((1+i)z)^{8/3}} \left(-3\sqrt[6]{3}\,z^2\left((-1)^{2/3}z^{2/3} + i\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 3\sqrt[6]{3}\left(\sqrt[3]{-1}\,z^{8/3} - i z^2\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 3^{2/3}\,i z^2\left(\sqrt[6]{-1}\,z^{2/3} - \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} - 3^{2/3}z^2\left(\sqrt[3]{-1}\,z^{2/3} + i\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 2\sqrt{3}\left((-1)^{2/3}z^{8/3} + \left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 2\sqrt[3]{-1}\,\sqrt{3}\left(-z^{8/3} - (-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 2\left((-1)^{2/3}z^{8/3} - \left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 2\sqrt[3]{-1}\left((-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3} - z^{8/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.19.03.0025.01

$$\operatorname{kei}_{\frac{3}{2}}(z) = \frac{(-1)^{3/8}}{2\, z^{3/2}}\, e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \left(\sqrt[4]{-1}\, z - e^{i \sqrt{2}\, z} \left(z + \sqrt[4]{-1}\right) + 1\right)$$

03.19.03.0026.01

$$\operatorname{kei}_{\frac{3}{2}}(z) = \frac{\sqrt{\pi}}{4\, z^{3/2}}\, e^{-\frac{z}{\sqrt{2}}} \left(-z \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) + \cos\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) + \left(z + \sqrt{2}\right) \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) - \left(\sqrt{2}\, z + 1\right) \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right)\right)$$

03.19.03.0027.01

$$\operatorname{kei}_{\frac{5}{3}}(z) = \frac{(-1)^{2/3}\, \pi\, z^{2/3}}{36\, 2^{5/6}\, \sqrt[6]{3}\, ((1+i)\, z)^{5/3}} \left(\frac{48\, i \left(2^{2/3}\, (1-i)\, \sqrt[3]{z} + \left(1 + i\sqrt{3}\right) \sqrt[3]{(1+i)\, z}\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) z^{2/3}}{((1+i)\, z)^{5/3}} + \right.$$

$$9 \sqrt[3]{3} \left(4 \sqrt[3]{-1}\, z^{2/3} + 2^{2/3} \left(i + \sqrt{3}\right) ((1+i)\, z)^{2/3}\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$

$$18 \sqrt[3]{3} \left(\left(-i + \sqrt{3}\right) z^{2/3} - 2^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$

$$\frac{6\, ((1+i)\, z)^{8/3} \left((-1)^{7/12} \sqrt{2}\, (2 - 2\, i)\, z^{2/3} + 2^{2/3} \left(i + \sqrt{3}\right) ((1+i)\, z)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)}{z^4} +$$

$$\frac{3\, i \left(4\, (-3)^{5/6}\, z^{8/3} - i \sqrt[3]{\frac{3}{2}} \left(-3\, i + \sqrt{3}\right) ((1+i)\, z)^{8/3}\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)}{z^2} -$$

$$6 \left(2^{2/3}\, 3^{5/6}\, ((1+i)\, z)^{2/3} - 2\, (-3)^{5/6}\, z^{2/3}\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$

$$\frac{8 \sqrt{2} \left(\sqrt[6]{2} \left(-3 - i\sqrt{3}\right) \sqrt[3]{z} + 2 \sqrt[12]{-1}\, \sqrt{3}\, \sqrt[3]{(1+i)\, z}\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)}{\sqrt[3]{z}\, ((1+i)\, z)^{2/3}} +$$

$$\left.\frac{16 \sqrt{6} \left(\sqrt[6]{2}\, \sqrt[3]{z} - (-1)^{7/12} \sqrt[3]{(1+i)\, z}\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)}{\sqrt[3]{z}\, ((1+i)\, z)^{2/3}}\right)$$

03.19.03.0028.01

$$\operatorname{kei}_{\frac{7}{3}}(z) = -\frac{(-1)^{3/4}\, 2^{2/3}\, \pi}{9\sqrt[3]{3}\; z^{7/3}\, ((1+i)\, z)^{14/3}} \left(24\sqrt[6]{3}\; z^4 \left(\sqrt[6]{-1}\; z^{2/3} + \left(\sqrt[4]{-1}\; z\right)^{2/3}\right) \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 24\sqrt[6]{3}\; z^4 \left(\sqrt[3]{-1}\; z^{2/3} - i\left(\sqrt[4]{-1}\; z\right)^{2/3}\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 8\, 3^{2/3}\, z^4 \left(\left(\sqrt[4]{-1}\; z\right)^{2/3} - \sqrt[6]{-1}\; z^{2/3}\right) \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 8\, 3^{2/3}\, i\, z^4 \left((-1)^{5/6}\, z^{2/3} - \left(\sqrt[4]{-1}\; z\right)^{2/3}\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + \sqrt{3}\left(16\left(\left(\sqrt[4]{-1}\; z\right)^{14/3} - \sqrt[6]{-1}\; z^{14/3}\right) - 9\, i\, z^6 \left(\sqrt[6]{-1}\; z^{2/3} + \left(\sqrt[4]{-1}\; z\right)^{2/3}\right)\right) \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \sqrt[3]{-1}\; \sqrt{3}\, \left(16 - 9\, i\, z^2\right) \left(z^{14/3} + \sqrt[6]{-1}\; \left(\sqrt[4]{-1}\; z\right)^{14/3}\right) \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + z^4 \left(16\sqrt[6]{-1}\; z^{2/3} + 9\, (-1)^{2/3}\, z^{8/3} - 9\, i\left(\sqrt[4]{-1}\; z\right)^{2/3} z^2 - 16\left(\sqrt[4]{-1}\; z\right)^{2/3}\right) \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + z^4 \left(-16\sqrt[3]{-1}\; z^{2/3} + 9\, (-1)^{5/6}\, z^{8/3} - 9\left(\sqrt[4]{-1}\; z\right)^{2/3} z^2 - 16\, i\left(\sqrt[4]{-1}\; z\right)^{2/3}\right) \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.19.03.0029.01

$$\operatorname{kei}_{\frac{5}{2}}(z) = \frac{(-1)^{7/8}}{2\, z^{5/2}}\, e^{-\sqrt[4]{-1}\; z} \sqrt{\frac{\pi}{2}} \left(-\sqrt[4]{-1}\; z^2 - 3\, z + 3\, (-1)^{3/4} - e^{i\sqrt{2}\; z}\left(z^2 + 3\sqrt[4]{-1}\; z + 3\, i\right)\right)$$

03.19.03.0030.01

$$\operatorname{kei}_{\frac{8}{3}}(z) = \frac{(1-i)\sqrt[12]{-1}\; \pi}{216\, 2^{5/6}\sqrt[6]{3}\; z^{11/3}} \left(-2\, i\sqrt{3}\left(80\sqrt[6]{-1}\; z^{2/3} + 18\, (-1)^{2/3}\, z^{8/3} - 9\, i\, 2^{2/3}\, ((1+i)\, z)^{2/3}\, z^2 - 40\, 2^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{z} - i\left(-160\sqrt[6]{-1}\; \sqrt{3}\; z^{2/3} + 36\, (-1)^{2/3}\sqrt{3}\; z^{8/3} - 40\, 2^{2/3}\left(3\, i + \sqrt{3}\right) ((1+i)\, z)^{2/3} + \frac{9\left(3\, i + \sqrt{3}\right) ((1+i)\, z)^{8/3}}{\sqrt[3]{2}}\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{z} + \sqrt[3]{3}\; (90 - 90\, i)\left(2^{2/3}\, (1+i)\sqrt[3]{z} + \left(i + \sqrt{3}\right)\sqrt[3]{(1+i)\, z}\right) \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) z^2 + 90\sqrt[3]{3}\left(2^{2/3}\left(1 + i\sqrt{3}\right)\sqrt[3]{z} - (1-i)\left(i + \sqrt{3}\right)\sqrt[3]{(1+i)\, z}\right) \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) z^2 + \sqrt[3]{3}\; (30 - 30\, i)\left(2^{2/3}\sqrt{3}\; (1+i)\sqrt[3]{z} + \left(-3 - i\sqrt{3}\right)\sqrt[3]{(1+i)\, z}\right) \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) z^2 + \sqrt[3]{3}\; (30 - 30\, i)\left(2\, (-1)^{7/12}\sqrt[6]{2}\; \sqrt{3}\; \sqrt[3]{z} + \left(3 + i\sqrt{3}\right)\sqrt[3]{(1+i)\, z}\right) \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) z^2 - 6\left(-40\, i\, 2^{2/3}\, ((1+i)\, z)^{2/3}\sqrt[3]{z} + 9\, 2^{2/3}\, ((1+i)\, z)^{2/3}\, z^{7/3} + 18\sqrt[6]{-1}\; z^3 + 40\left(1 - i\sqrt{3}\right) z\right) \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 3\, i\left(40\, 2^{2/3}\left(1 + i\sqrt{3}\right) ((1+i)\, z)^{2/3}\sqrt[3]{z} + 9\, 2^{2/3}\left(-i + \sqrt{3}\right) ((1+i)\, z)^{2/3}\, z^{7/3} + 36\, (-1)^{2/3}\, z^3 - 80\left(i + \sqrt{3}\right) z\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.19.03.0031.01

$$\operatorname{kei}_{\frac{10}{3}}(z) = \frac{i\,\pi}{108\,\sqrt[3]{6}\,z^{11/3}\,((1+i)\,z)^{2/3}} \Bigg(-16\sqrt{3}\left(-28\,(-1)^{2/3}\,z^{2/3} + 18\,\sqrt[6]{-1}\,z^{8/3} + 9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 - 14\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} +$$
$$16\sqrt{3}\left(-28\,\sqrt[3]{-1}\,z^{2/3} + 18\,(-1)^{5/6}\,z^{8/3} + 9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 + 14\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\right)\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} -$$
$$16\,i\left(28\,\sqrt[6]{-1}\,z^{2/3} + 18\,(-1)^{2/3}\,z^{8/3} - 9\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 - 14\,2^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} -$$
$$16\,i\left(28\,(-1)^{5/6}\,z^{2/3} + 18\,\sqrt[3]{-1}\,z^{8/3} - 14\,2^{2/3}\,((1+i)\,z)^{2/3} + \frac{9\,((1+i)\,z)^{8/3}}{\sqrt[3]{2}}\right)\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} +$$
$$3^{2/3}\,((1+i)\,z)^{2/3}\left(224\,(-1)^{2/3}\,z^{2/3} - 18\,\sqrt[6]{-1}\,z^{8/3} + 9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 - 112\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\right)$$
$$\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} + 3^{2/3}\,((1+i)\,z)^{2/3}$$
$$\left(224\,\sqrt[3]{-1}\,z^{2/3} - 18\,(-1)^{5/6}\,z^{8/3} + 9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 + 112\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} +$$
$$3\,\sqrt[6]{3}\,((1+i)\,z)^{2/3}\left(9\,z^2 - 112\,i\right)\left(2^{2/3}\,((1+i)\,z)^{2/3}\,\sqrt[3]{z} + 2\,\sqrt[6]{-1}\,z\right)\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) +$$
$$3\,\sqrt[6]{3}\,((1+i)\,z)^{4/3}\left(9\,z^2 + 112\,i\right)\left(2^{2/3}\,\sqrt[3]{z} + \sqrt[3]{-1}\,(1+i)\,\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)$$

03.19.03.0032.01

$$\operatorname{kei}_{\frac{7}{2}}(z) = \frac{(-1)^{3/8}}{2\,z^{7/2}}\,e^{-\sqrt[4]{-1}\,z}\sqrt{\frac{\pi}{2}}\left(-\sqrt[4]{-1}\,z^3 - 6\,z^2 + 15\,(-1)^{3/4}\,z + 15\,i + e^{i\sqrt{2}\,z}\left(z^3 + 6\,\sqrt[4]{-1}\,z^2 + 15\,i\,z + 15\,(-1)^{3/4}\right)\right)$$

03.19.03.0033.01

$$\operatorname{kei}_{\frac{11}{3}}(z) = \frac{\sqrt[6]{-\frac{1}{3}}\ \pi\, z^{19/3}}{324\,\sqrt[3]{2}\,((1+i)\,z)^{8/3}\left(\sqrt[4]{-1}\,z\right)^{25/3}}$$

$$\Bigg(120\left(9\,i\,z^2+32\right)\left(\sqrt[3]{2}\left(1+i\sqrt{3}\right)z^{2/3}+\left(i+\sqrt{3}\right)((1+i)\,z)^{2/3}\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z}\,-$$

$$120\,i\left(9\,z^2+32\,i\right)\left(2\,((1+i)\,z)^{2/3}-\sqrt[3]{2}\left(-i+\sqrt{3}\right)z^{2/3}\right)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z}\,+$$

$$40\left(-64\sqrt[3]{-2}\,\sqrt{3}\,z^{2/3}+9\sqrt[3]{2}\left(3-i\sqrt{3}\right)z^{8/3}-9\left(-3\,i+\sqrt{3}\right)((1+i)\,z)^{2/3}\,z^2+32\left(3+i\sqrt{3}\right)((1+i)\,z)^{2/3}\right)$$

$$\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z}+\sqrt[3]{3}\,(9-9\,i)\left(9\,z^2-160\,i\right)$$

$$\left(\left(i+\sqrt{3}\right)(1+i)\sqrt[3]{z}+\sqrt[3]{2}\left(1+i\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2+\sqrt[3]{3}\,(18+18\,i)$$

$$\left((-160-160\,i)\sqrt[3]{z}-(9-9\,i)\,z^{7/3}-9\sqrt[3]{-2}\,\sqrt[3]{(1+i)\,z}\,z^2+80\sqrt[3]{2}\left(-i+\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)$$

$$z^2+\sqrt[3]{3}\,(3+3\,i)\left(9\,z^2-160\,i\right)\left(\left(-3\,i+\sqrt{3}\right)(1+i)\sqrt[3]{z}+\sqrt[3]{2}\left(3\,i+\sqrt{3}\right)i\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2-$$

$$(3-3\,i)\sqrt[3]{3}\left(9\,z^2+160\,i\right)\left(\sqrt{3}\,(2+2\,i)\sqrt[3]{z}+\sqrt[3]{2}\left(3-i\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2+$$

$$40\left(64\sqrt{3}\,((1+i)\,z)^{2/3}\sqrt[3]{z}-18\,i\sqrt{3}\,((1+i)\,z)^{2/3}\,z^{7/3}-9\sqrt[3]{2}\left(3\,i+\sqrt{3}\right)z^3-64\sqrt[3]{-2}\,\sqrt{3}\,i\,z\right)$$

$$\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)$$

03.19.03.0034.01

$$\operatorname{kei}_{\frac{13}{3}}(z) = \frac{2(-1)^{3/4}\, 2^{2/3}\, \pi\, z^{11/3}}{81\sqrt[3]{3}\, ((1+i)z)^{26/3}} \Bigg(28\, 3^{2/3} \left(-160\sqrt[6]{-1}\, z^{2/3} - 18(-1)^{2/3} z^{8/3} + 80\, 2^{2/3} ((1+i)z)^{2/3} + \frac{9((1+i)z)^{8/3}}{\sqrt[3]{2}}\right) i\, \operatorname{Bi}'\!\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) ((1+i)z)^{2/3} +$$

$$28\, 3^{2/3}\, i \left(160\sqrt[3]{-1}\, z^{2/3} - 18(-1)^{5/6} z^{8/3} + 9\, 2^{2/3} ((1+i)z)^{2/3} z^2 + 80\, 2^{2/3}\, i\, ((1+i)z)^{2/3}\right)$$

$$\operatorname{Bi}'\!\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) ((1+i)z)^{2/3} + \sqrt{3}\left(-8960(-1)^{2/3} z^{2/3} + 6048\sqrt[6]{-1}\, z^{8/3} + 162(-1)^{2/3} z^{14/3} +\right.$$

$$\left. 81\, 2^{2/3}\, i\, ((1+i)z)^{2/3} z^4 + 3024\, 2^{2/3} ((1+i)z)^{2/3} z^2 - 4480\, i\, 2^{2/3} ((1+i)z)^{2/3}\right) \operatorname{Ai}\!\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) +$$

$$\sqrt{3}\left(-8960(-1)^{5/6} z^{2/3} - 6048\sqrt[3]{-1}\, z^{8/3} + 162(-1)^{5/6} z^{14/3} + 81\, 2^{2/3} ((1+i)z)^{2/3} z^4 +\right.$$

$$\left. 3024\, 2^{2/3}\, i\, ((1+i)z)^{2/3} z^2 - 4480\, 2^{2/3} ((1+i)z)^{2/3}\right) \operatorname{Ai}\!\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) +$$

$$\frac{84\sqrt[6]{3}\, ((1+i)z)^{4/3} \left(9\, i\, z^2 + 80\right) \left(2^{2/3}\, i\sqrt[3]{z} + \sqrt[6]{-1}\, (1+i)\sqrt[3]{(1+i)z}\right) \operatorname{Ai}'\!\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right)}{\sqrt[3]{z}} -$$

$$\frac{1}{((1+i)z)^{2/3}}\, 168\sqrt[6]{3}\, z^{5/3} \left(80\, 2^{2/3}\, i\sqrt[3]{z} + 9\, 2^{2/3} z^{7/3} + 9\sqrt[3]{-1}\, ((1+i)z)^{4/3} z + (-1)^{5/6} (80 + 80\, i)\sqrt[3]{(1+i)z}\right)$$

$$\operatorname{Ai}'\!\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + \left(8960(-1)^{2/3} z^{2/3} - 6048\sqrt[6]{-1}\, z^{8/3} - 162(-1)^{2/3} z^{14/3} +\right.$$

$$\left. 81\, 2^{2/3}\, i\, ((1+i)z)^{2/3} z^4 + 3024\, 2^{2/3} ((1+i)z)^{2/3} z^2 - 4480\, i\, 2^{2/3} ((1+i)z)^{2/3}\right) \operatorname{Bi}\!\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) +$$

$$\left(8960(-1)^{5/6} z^{2/3} + 6048\sqrt[3]{-1}\, z^{8/3} - 162(-1)^{5/6} z^{14/3} + 81\, 2^{2/3} ((1+i)z)^{2/3} z^4 +\right.$$

$$\left. 3024\, 2^{2/3}\, i\, ((1+i)z)^{2/3} z^2 - 4480\, 2^{2/3} ((1+i)z)^{2/3}\right) \operatorname{Bi}\!\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right)\Bigg)$$

03.19.03.0035.01

$$\operatorname{kei}_{\frac{9}{2}}(z) = \frac{(-1)^{7/8}}{2 z^{9/2}}\, e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \left(\sqrt[4]{-1}\, z^4 + 10 z^3 - 45(-1)^{3/4} z^2 - 105\, i\, z - 105\sqrt[4]{-1} + e^{i\sqrt{2}\, z} \left(z^4 + \sqrt{2}\,(5 + 5\, i) z^3 + 45\, i\, z^2 + 105(-1)^{3/4} z - 105\right)\right)$$

03.19.03.0036.01

$$
\begin{aligned}
&\operatorname{kei}_{\frac{14}{3}}(z) = -\frac{\pi}{486\, 2^{5/6}\, z^{10/3}\, ((1+i)\,z)^{5/3}} \sqrt[6]{-\frac{1}{3}} \\
&\Bigg(\sqrt[6]{2}\,\sqrt[3]{3}\,(144+144\,i)\,z\,\Big(-110\,\sqrt[3]{2}\,\big(-i+\sqrt{3}\big)\,i\,z^{2/3}+9\,\sqrt[3]{2}\,\big(-i+\sqrt{3}\big)\,z^{8/3}-110\,\big(i+\sqrt{3}\big)\,((1+i)\,z)^{2/3}- \\
&\quad 9\,\sqrt[6]{-1}\,((1+i)\,z)^{8/3}\Big)\operatorname{Ai}\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)+ \\
&\sqrt[6]{2}\,\sqrt[3]{3}\,(144+144\,i)\,z\,\big(9\,z^2+110\,i\big)\Big(2\,((1+i)\,z)^{2/3}-\sqrt[3]{2}\,\big(-i+\sqrt{3}\big)\,z^{2/3}\Big)\operatorname{Ai}\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)- \\
&\frac{1}{\sqrt[3]{(1+i)\,z}}\Big(3\,\Big(\sqrt[12]{-1}\,(-28\,160-28\,160\,i)\,z^{2/3}-(8640+8640\,i)\,(-1)^{7/12}\,z^{8/3}+\sqrt[12]{-1}\,(162+162\,i)\,z^{14/3}+ \\
&\quad 81\,\sqrt[6]{2}\,\big(i+\sqrt{3}\big)\,((1+i)\,z)^{2/3}\,z^4+4320\,\sqrt[6]{2}\,\big(1-i\,\sqrt{3}\big)\,((1+i)\,z)^{2/3}\,z^2-14\,080\,\sqrt[6]{2}\,\big(i+\sqrt{3}\big)\,((1+i)\,z)^{2/3}\Big) \\
&\quad \operatorname{Ai}'\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)-\frac{1}{\sqrt[3]{z}}\Big(6\,\sqrt[3]{(1+i)\,z}\,\Big(14\,080\,\sqrt[6]{2}\,i\,\sqrt[3]{z}+4320\,\sqrt[6]{2}\,z^{7/3}-81\,i\,\sqrt[6]{2}\,z^{13/3}+ \\
&\quad 81\,\sqrt[12]{-1}\,\sqrt[3]{(1+i)\,z}\,z^4+4320\,(-1)^{7/12}\,\sqrt[3]{(1+i)\,z}\,z^2-14\,080\,\sqrt[12]{-1}\,\sqrt[3]{(1+i)\,z}\Big)\operatorname{Ai}'\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)+ \\
&\frac{1}{((1+i)\,z)^{4/3}}\Big(96\,i\,\sqrt[6]{2}\,\sqrt[3]{3}\,z^{8/3}\Big(\big(-3\,i+\sqrt{3}\big)\,(110-110\,i)\,\sqrt[3]{z}+\sqrt[6]{-1}\,\sqrt{3}\,(18-18\,i)\,z^{7/3}+ \\
&\quad 110\,\sqrt[3]{2}\,\big(3\,i+\sqrt{3}\big)\,\sqrt[3]{(1+i)\,z}+\frac{9\,\big(3\,i+\sqrt{3}\big)\,((1+i)\,z)^{7/3}}{2^{2/3}}\Big)\operatorname{Bi}\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)+ \\
&\sqrt[6]{2}\,\sqrt[3]{3}\,(48+48\,i)\,z\,\big(9\,z^2+110\,i\big)\Big(\sqrt[3]{2}\,\big(3-i\,\sqrt{3}\big)\,z^{2/3}+2\,\sqrt{3}\,((1+i)\,z)^{2/3}\Big)\operatorname{Bi}\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)+ \\
&\frac{1}{\sqrt[3]{(1+i)\,z}}\Big(\Big(\sqrt[12]{-1}\,\sqrt{3}\,(-28\,160-28\,160\,i)\,z^{2/3}-(8640+8640\,i)\,(-1)^{7/12}\,\sqrt{3}\,z^{8/3}+ \\
&\quad \sqrt[12]{-1}\,\sqrt{3}\,(162+162\,i)\,z^{14/3}-81\,i\,\sqrt[6]{2}\,\big(-3\,i+\sqrt{3}\big)\,((1+i)\,z)^{2/3}\,z^4- \\
&\quad 4320\,\sqrt[6]{2}\,\big(-3\,i+\sqrt{3}\big)\,((1+i)\,z)^{2/3}\,z^2+14\,080\,\sqrt[6]{2}\,\big(3+i\,\sqrt{3}\big)\,((1+i)\,z)^{2/3}\Big)\operatorname{Bi}'\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)+ \\
&\frac{1}{\sqrt[3]{(1+i)\,z}}\Big(2\,\sqrt{3}\,\Big(\sqrt[12]{-1}\,(-14\,080-14\,080\,i)\,z^{2/3}+(-1)^{7/12}\,(4320+4320\,i)\,z^{8/3}+\sqrt[12]{-1}\,(81+81\,i)\,z^{14/3}+ \\
&\quad 81\,\sqrt[6]{2}\,i\,((1+i)\,z)^{2/3}\,z^4-4320\,\sqrt[6]{2}\,((1+i)\,z)^{2/3}\,z^2-14\,080\,i\,\sqrt[6]{2}\,((1+i)\,z)^{2/3}\Big)\operatorname{Bi}'\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)\Bigg)
\end{aligned}
$$

Symbolic rational ν

03.19.03.0037.01

$$\operatorname{kei}_\nu(z) = -\frac{\sqrt[8]{-1}}{2\sqrt{z}}\, e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} \sqrt{\frac{\pi}{2}} \left(\sum_{k=0}^{\left\lfloor \frac{1}{4}(2|\nu|-3)\right\rfloor} \frac{\left(2k+|\nu|+\frac{1}{2}\right)!\, i^{-k} z^{-2k-1}}{2^{2k+1}(2k+1)!\left(-2k+|\nu|-\frac{3}{2}\right)!} \left(1-(-1)^{3/4} e^{i\left(\sqrt{2}\, z+\pi(k+\nu)\right)}\right) + \sum_{k=0}^{\left\lfloor \frac{1}{4}(2|\nu|-1)\right\rfloor} \frac{\left(2k+|\nu|-\frac{1}{2}\right)!\, i^{-k} z^{-2k}}{2^{2k}(2k)!\left(-2k+|\nu|-\frac{1}{2}\right)!} \left(\sqrt[4]{-1} + e^{i\left(\sqrt{2}\, z+\pi\left(k+\nu-\frac{1}{2}\right)\right)}\right)\right) /;\ \nu-\frac{1}{2}\in\mathbb{Z}$$

03.19.03.0038.01

$$\operatorname{kei}_\nu(z) = -\frac{i\, 2^{\frac{1}{2}(\nu+3|\nu|-6)} \sqrt[6]{3}\, e^{\frac{1}{4}(-3)i\pi\nu} \pi z^{-\nu} ((1+i)z)^{-\nu-|\nu|} \csc(\pi\nu)}{\Gamma(1-|\nu|)} \Gamma\!\left(\frac{2}{3}\right) \left(\frac{1}{2}\sqrt[6]{3}\, ((1+i)z)^{2/3} \sum_{k=0}^{|\nu|-\frac{4}{3}} \frac{4^{-k}\left(i z^2\right)^k \left(-k+|\nu|-\frac{4}{3}\right)!}{k!\left(-2k+|\nu|-\frac{4}{3}\right)!\left(\frac{4}{3}\right)_k (1-|\nu|)_k} \left(3 e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} - i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))} e^{\frac{i\pi\nu}{2}} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}'\!\left(-\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right) \operatorname{sgn}(\nu)^2 - \left(3(-1)^k \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} + \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}'\!\left(\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\, e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} + e^{\frac{i\pi\nu}{2}} i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}'\!\left(-\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right)\right) \operatorname{sgn}(\nu) + (-1)^k \sqrt{3} \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} - \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}'\!\left(\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right)\right) + \sum_{k=0}^{|\nu|-\frac{1}{3}} \frac{4^{-k}\left(i z^2\right)^k \left(-k+|\nu|-\frac{1}{3}\right)!}{k!\left(-2k+|\nu|-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k (1-|\nu|)_k} \left(\sqrt{3}\, e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} - i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))} e^{\frac{i\pi\nu}{2}} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}\!\left(-\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right) \operatorname{sgn}(\nu)^2 + \left((-1)^k \sqrt{3} \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} + \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}\!\left(\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right) - e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} + e^{\frac{i\pi\nu}{2}} i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}\!\left(-\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right)\right) \operatorname{sgn}(\nu) - (-1)^k \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} - \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}\!\left(\frac{1}{2} 3^{2/3}((1+i)z)^{2/3}\right)\right)\right) /;\ |\nu|-\frac{1}{3}\in\mathbb{Z}$$

03.19.03.0039.01

$$\operatorname{kei}_\nu(z) = \frac{i\, 2^{|\nu|-3}\, e^{\frac{1}{4}(-3)\, i\, \pi\, \nu}\, \pi\, z^{-\nu} \left(\sqrt[4]{-1}\; z\right)^{-\nu-|\nu|} \csc(\pi\, \nu)\, \Gamma\!\left(\frac{1}{3}\right) \operatorname{sgn}(\nu)}{3^{2/3}\, \Gamma(1-|\nu|)}$$
$$\left(\frac{3^{5/6}\, 3\, ((1+i)\, z)^{4/3}}{8} \sum_{k=0}^{|\nu|-\frac{5}{3}} \frac{4^{-k} \left(i\, z^2\right)^k \left(-k+|\nu|-\frac{5}{3}\right)!}{k! \left(-2\, k+|\nu|-\frac{5}{3}\right)! \left(\frac{5}{3}\right)_k (1-|\nu|)_k} \left(-(-1)^k \sqrt{3} \left(e^{\frac{i\,\pi\,\nu}{2}}\, z^{2\,\nu} + \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \right.\right.$$
$$\sqrt{3}\; e^{i\,\pi\,\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\,\nu} + e^{\frac{i\,\pi\,\nu}{2}}\, i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))} \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) -$$
$$(-1)^k \left(e^{\frac{i\,\pi\,\nu}{2}}\, z^{2\,\nu} - \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{sgn}(\nu) +$$
$$\left. e^{i\,\pi\,\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\,\nu} - i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))}\, e^{\frac{i\,\pi\,\nu}{2}} \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{sgn}(\nu)\right) +$$
$$\sum_{k=0}^{|\nu|-\frac{2}{3}} \frac{4^{-k} \left(i\, z^2\right)^k \left(-k+|\nu|-\frac{2}{3}\right)!}{k! \left(-2\, k+|\nu|-\frac{2}{3}\right)! \left(\frac{2}{3}\right)_k (1-|\nu|)_k} \left(3\, (-1)^k \left(e^{\frac{i\,\pi\,\nu}{2}}\, z^{2\,\nu} + \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - \right.$$
$$3\, e^{i\,\pi\,\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\,\nu} + e^{\frac{i\,\pi\,\nu}{2}}\, i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))} \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$
$$(-1)^k \sqrt{3} \left(e^{\frac{i\,\pi\,\nu}{2}}\, z^{2\,\nu} - \left(\sqrt[4]{-1}\; z\right)^{2\,\nu}\right) \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{sgn}(\nu) +$$
$$\left.\left. \sqrt{3}\; e^{i\,\pi\,\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))}\, e^{\frac{i\,\pi\,\nu}{2}} \left(\sqrt[4]{-1}\; z\right)^{2\,\nu} - i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\,\nu}\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{sgn}(\nu)\right)\right) /;\ |\nu| - \frac{2}{3} \in \mathbb{Z}$$

Values at fixed points

03.19.03.0040.01

$$\operatorname{kei}_0(0) = -\frac{\pi}{4}$$

Values at infinities

03.19.03.0041.01

$$\lim_{x \to \infty} \operatorname{kei}_\nu(x) = 0$$

03.19.03.0042.01

$$\lim_{x \to -\infty} \operatorname{kei}_\nu(x) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$\operatorname{kei}_\nu(z)$ is an analytical function of ν and z, which is defined in $\mathbb{C}^2$.

03.19.04.0001.01

$$(\nu * z) \longrightarrow \operatorname{kei}_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

03.19.04.0002.01

$\text{kei}_{-n}(z) = (-1)^n \, \text{kei}_n(z) \,/;\, n \in \mathbb{Z}$

Mirror symmetry

03.19.04.0003.01

$\text{kei}_{\bar{\nu}}(\bar{z}) == \overline{\text{kei}_\nu(z)} \,/;\, z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, the function $\text{kei}_\nu(z)$ has an essential singularity at $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is a branch point for generic ν.

03.19.04.0004.01

$\mathcal{S}ing_z(\text{kei}_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$

With respect to ν

For fixed z, the function $\text{kei}_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.19.04.0005.01

$\mathcal{S}ing_\nu(\text{kei}_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to z

For fixed ν, the function $\text{kei}_\nu(z)$ has two branch points: $z == 0$, $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is an essential singularity.

03.19.04.0006.01

$\mathcal{BP}_z(\text{kei}_\nu(z)) == \{0, \tilde{\infty}\}$

03.19.04.0007.01

$\mathcal{R}_z(\text{kei}_\nu(z), 0) == \log \,/;\, \nu \notin \mathbb{Q}$

03.19.04.0008.01

$\mathcal{R}_z\left(\text{kei}_{\frac{p}{q}}(z), 0\right) == q \,/;\, p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$

03.19.04.0009.01

$\mathcal{R}_z(\text{kei}_\nu(z), \tilde{\infty}) == \log \,/;\, \nu \notin \mathbb{Q}$

03.19.04.0010.01

$\mathcal{R}_z\left(\text{kei}_{\frac{p}{q}}(z), \tilde{\infty}\right) == q \,/;\, p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$

With respect to ν

For fixed z, the function $\text{kei}_\nu(z)$ does not have branch points.

03.19.04.0011.01

$$\mathcal{BP}_\nu(\text{kei}_\nu(z)) = \{\}$$

Branch cuts

With respect to z

For fixed ν, the function $\text{kei}_\nu(z)$ has one infinitely long branch cut. For fixed ν, the function $\text{kei}_\nu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.19.04.0012.01

$$\mathcal{BC}_z(\text{kei}_\nu(z)) = \{\{(-\infty, 0), -i\}\} /; \nu \notin \mathbb{Z}$$

03.19.04.0013.01

$$\lim_{\epsilon \to +0} \text{kei}_\nu(x + i\,\epsilon) = \text{kei}_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.19.04.0014.01

$$\lim_{\epsilon \to +0} \text{kei}_\nu(x - i\,\epsilon) = \frac{1}{2} e^{-2 i \pi \nu} \pi \left(e^{4 i \pi \nu} \csc(\pi \nu) \text{bei}_{-\nu}(x) - \cot(\pi \nu) \text{bei}_\nu(x) + \text{ber}_\nu(x)\right) /; \nu \notin \mathbb{Z} \bigwedge x \in \mathbb{R} \bigwedge x < 0$$

03.19.04.0015.01

$$\lim_{\epsilon \to +0} \text{kei}_\nu(x - i\,\epsilon) = 2\, i\, \pi \cos(\pi \nu) \text{bei}_{-\nu}(x) + e^{-2 i \pi \nu} \text{kei}_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.19.04.0016.01

$$\lim_{\epsilon \to +0} \text{kei}_\nu(x - i\,\epsilon) = 2\, i\, \pi\, \text{bei}_\nu(x) + \text{kei}_\nu(x) /; \nu \in \mathbb{Z} \land x \in \mathbb{R} \land x < 0$$

With respect to ν

For fixed z, the function $\text{kei}_\nu(z)$ is an entire function of ν and does not have branch cuts.

03.19.04.0017.01

$$\mathcal{BC}_\nu(\text{kei}_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at $\nu = \pm n$

03.19.06.0001.01

$$\text{kei}_\nu(z) \propto \text{kei}_n(z) + \left(-\frac{\pi}{2} \text{ker}_n(z) + \frac{\pi\, n!}{4} \sum_{k=0}^{n-1} \frac{\left(\frac{z}{2}\right)^{k-n}}{k!\,(n-k)} \left(\cos\left(\frac{3}{4}(k-n)\pi\right) \text{ber}_k(z) - \sin\left(\frac{3}{4}(k-n)\pi\right) \text{bei}_k(z)\right) + \frac{(-1)^n}{4} \text{bei}_{-n}^{(2,0)}(z) - \frac{1}{4} \text{bei}_n^{(2,0)}(z)\right) (\nu - n) + \ldots /; (\nu \to n) \land n \in \mathbb{N}$$

03.19.06.0002.01

$$\operatorname{kei}_\nu(z) \propto (-1)^n \operatorname{kei}_n(z) + \left(\frac{(-1)^{n-1}\pi}{2}\operatorname{ker}_n(z) - \frac{\pi(-1)^n n!}{4}\sum_{k=0}^{n-1}\frac{\left(\frac{z}{2}\right)^{k-n}}{k!\,(n-k)}\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\operatorname{ber}_k(z) - \sin\left(\frac{3}{4}(k-n)\pi\right)\operatorname{bei}_k(z)\right) - \frac{1}{4}\operatorname{bei}_{-n}^{(2,0)}(z) + \frac{(-1)^n}{4}\operatorname{bei}_n^{(2,0)}(z)\right)(n+\nu) + \ldots\ /;\ (\nu \to -n) \land n \in \mathbb{N}$$

Expansions at generic point $z == z_0$

03.19.06.0003.01

$$\operatorname{kei}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\operatorname{kei}_\nu(z_0) - 2\,i\,\pi\cos(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\operatorname{bei}_{-\nu}(z_0) - \frac{1}{2\sqrt{2}}\left(\left(\frac{1}{z_0}\right)^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(\operatorname{kei}_{\nu-1}(z_0) - \operatorname{kei}_{\nu+1}(z_0) - \operatorname{ker}_{\nu-1}(z_0) + \operatorname{ker}_{\nu+1}(z_0)) - 2\,i\,\pi\cos(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor(\operatorname{bei}_{-\nu-1}(z_0) - \operatorname{bei}_{1-\nu}(z_0) - \operatorname{ber}_{-\nu-1}(z_0) + \operatorname{ber}_{1-\nu}(z_0))\right)(z-z_0) - \frac{1}{8}\left(\left(\frac{1}{z_0}\right)^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(\operatorname{ker}_{\nu-2}(z_0) - 2\operatorname{ker}_\nu(z_0) + \operatorname{ker}_{\nu+2}(z_0)) - 2\,i\,\pi\cos(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor(\operatorname{ber}_{-\nu-2}(z_0) + \operatorname{ber}_{2-\nu}(z_0) - 2\operatorname{ber}_{-\nu}(z_0))\right)(z-z_0)^2 + \ldots\ /;\ (z \to z_0)$$

03.19.06.0004.01

$$\operatorname{kei}_\nu(z) = \sum_{k=0}^{\infty}\frac{\operatorname{kei}_\nu^{(0,k)}(z_0)\,(z-z_0)^k}{k!}\ /;\ |\arg(z_0)| < \pi$$

03.19.06.0005.01

$$\operatorname{kei}_\nu(z) = -\frac{1}{4}\sum_{k=0}^{\infty}\frac{1}{k!}\,G_{5,9}^{4,4}\!\left(\frac{z_0}{4}, \frac{1}{4}\,\middle|\,\begin{matrix} -\frac{k}{4}, \frac{1-k}{4}, \frac{2-k}{4}, \frac{3-k}{4}, \frac{2\nu-k}{4} \\ \frac{1}{4}(k-n+\nu), \frac{\nu-k}{4}, \frac{2-k-\nu}{4}, -\frac{k+\nu}{4}, \frac{2\nu-k}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\end{matrix}\right)(z-z_0)^k\ /;\ |\arg(z_0)| < \pi$$

03.19.06.0006.01

$$\operatorname{kei}_\nu(z) = \frac{i\,\pi^{3/2}}{4} \sum_{k=0}^{\infty} \frac{2^k z_0^{-k}}{k!} \left(2^{2\nu} z_0^{-\nu} \csc(\pi\nu)\,\Gamma(1-\nu) \left(\frac{1}{z_0}\right)^{-\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{-\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(-e^{-\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1-k-\nu}{2}, \frac{2-k-\nu}{2}, 1-\nu; \frac{i z_0^2}{4}\right) + e^{\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1-k-\nu}{2}, \frac{2-k-\nu}{2}, 1-\nu; -\frac{i z_0^2}{4}\right)\right) + 2^{-2\nu} z_0^{\nu} (i + \cot(\pi\nu))\,\Gamma(\nu+1) \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(e^{-\frac{5 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; \frac{i z_0^2}{4}\right) - e^{-\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; -\frac{i z_0^2}{4}\right)\right)\right) (z-z_0)^k \;/;\; \nu \notin \mathbb{Z}$$

03.19.06.0007.01

$$\operatorname{kei}_\nu(z) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}} (i-1)^k}{k!} \left(\sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \left(\left(1+i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu}(z_0) - 2 i \pi (-1)^k \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu}(z_0) \right) - i\left(1-i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu}(z_0) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu}(z_0) \right) \right) - \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \left(\left(1+i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu+2}(z_0) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu-2}(z_0) \right) - i\left(1-i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu+2}(z_0) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu-2}(z_0) \right) \right) \right) (z-z_0)^k$$

03.19.06.0008.01

$$\operatorname{kei}_\nu(z) = \sum_{k=0}^{\infty} \frac{z_0^{-k}}{k!} \sum_{m=0}^{k} (-1)^{k+m} \binom{k}{m} (-\nu)_{k-m}$$

$$\sum_{i=0}^{m} \frac{(-1)^i \, 2^{2i-m} \, (-m)_{2(m-i)} \, (\nu)_i}{(m-i)!} \left(-\frac{z^2}{4} \operatorname{ker}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{i-1}{2} \right\rfloor} \frac{(-1)^j \, (i-2j-1)!}{(2j+1)! \, (i-4j-2)! \, (-i-\nu+1)_{2j+1} \, (\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} + \right.$$

$$\frac{z}{2\sqrt{2}} (\operatorname{kei}_{\nu-1}(z) - \operatorname{ker}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{i-1}{2} \right\rfloor} \frac{(-1)^j \, (i-2j-1)!}{(2j)! \, (i-4j-1)! \, (-i-\nu+1)_{2j} \, (\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} -$$

$$\frac{z^3}{8\sqrt{2}} (\operatorname{kei}_{\nu-1}(z) + \operatorname{ker}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{i-2}{2} \right\rfloor} \frac{(-1)^j \, (i-2j-2)!}{(2j+1)! \, (i-4j-3)! \, (-i-\nu+1)_{2j+1} \, (\nu)_{2j+2}} \left(\frac{z}{2}\right)^{4j} +$$

$$\left. \operatorname{kei}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{i}{2} \right\rfloor} \frac{(-1)^j \, (i-2j)!}{(2j)! \, (i-4j)! \, (-i-\nu+1)_{2j} \, (\nu)_{2j}} \left(\frac{z}{2}\right)^{4j} \right) (z-z_0)^k \; /; \; |\arg(z_0)| < \pi$$

03.19.06.0009.01

$$\operatorname{kei}_\nu(z) \propto \left(\operatorname{kei}_\nu(z_0) \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} - 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{bei}_{-\nu}(z_0) \right) (1 + O(z-z_0))$$

Expansions on branch cuts

03.19.06.0010.01

$$\operatorname{kei}_\nu(z) \propto -2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{bei}_{-\nu}(x) +$$

$$e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{kei}_\nu(x) - \frac{1}{2\sqrt{2}} \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (\operatorname{kei}_{\nu-1}(x) - \operatorname{kei}_{\nu+1}(x) - \operatorname{ker}_{\nu-1}(x) + \operatorname{ker}_{\nu+1}(x)) - \right.$$

$$\left. 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor (\operatorname{bei}_{-\nu-1}(x) - \operatorname{bei}_{1-\nu}(x) - \operatorname{ber}_{-\nu-1}(x) + \operatorname{ber}_{1-\nu}(x)) \right) (z-x) -$$

$$\frac{1}{8} \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (\operatorname{ker}_{\nu-2}(x) - 2 \operatorname{ker}_\nu(x) + \operatorname{ker}_{\nu+2}(x)) - 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor (\operatorname{ber}_{-\nu-2}(x) + \operatorname{ber}_{2-\nu}(x) - 2 \operatorname{ber}_{-\nu}(x)) \right)$$

$$(z-x)^2 + \ldots \; /; \; (z \to x) \land x \in \mathbb{R} \land x < 0$$

03.19.06.0011.01

$$\operatorname{kei}_\nu(z) = \frac{i\,\pi^{3/2}}{4} \sum_{k=0}^{\infty} \frac{2^k x^{-k}}{k!} \left(2^{2\nu} e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \csc(\pi \nu)\, \Gamma(1-\nu) \left(e^{\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-k-\nu+1), \frac{1}{2}(-k-\nu+2), 1-\nu; -\frac{i x^2}{4}\right) - e^{-\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-k-\nu+1), \frac{1}{2}(-k-\nu+2), 1-\nu; \frac{i x^2}{4}\right)\right) x^{-\nu} + 2^{-2\nu} e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (i + \cot(\pi \nu))\, \Gamma(\nu+1) \left(e^{-\frac{5 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+1; \frac{i x^2}{4}\right) - e^{-\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+1; -\frac{i x^2}{4}\right)\right) x^{\nu}\right) (z-x)^k \text{ /; } \nu \notin \mathbb{Z} \bigwedge x \in \mathbb{R} \bigwedge x < 0$$

03.19.06.0012.01

$$\operatorname{kei}_\nu(z) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}} (i-1)^k}{k!} \left(\sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \left(\left(1+i^k\right)\left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu}(x) - 2 i \pi (-1)^k \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu}(x)\right) - i\left(1-i^k\right)\left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu}(x) - (-1)^k\, 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu}(x)\right)\right) - \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \left(\left(1+i^k\right)\left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu+2}(x) - (-1)^k\, 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu-2}(x)\right) - i\left(1-i^k\right)\left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu+2}(x) - (-1)^k\, 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu-2}(x)\right)\right)\right) (z-x)^k \text{ /; } x \in \mathbb{R} \land x < 0$$

03.19.06.0013.01

$$\operatorname{kei}_\nu(z) \propto \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{kei}_\nu(x) - 2 i \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{bei}_{-\nu}(x)\right) (1 + O(z-x)) \text{ /; } x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

General case

03.19.06.0014.01

$$\operatorname{kei}_\nu(z) \propto -2^{\nu-3}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu-1)\,z^{2-\nu}\left(1-\frac{z^4}{96\,(\nu-3)\,(\nu-2)}+\frac{z^8}{30\,720\,(\nu-5)\,(\nu-4)\,(\nu-3)\,(\nu-2)}+\ldots\right)-$$
$$2^{\nu-1}\,\Gamma(\nu)\sin\left(\frac{3\pi\nu}{4}\right)z^{-\nu}\left(1-\frac{z^4}{32\,(\nu-2)\,(\nu-1)}+\frac{z^8}{6144\,(\nu-4)\,(\nu-3)\,(\nu-2)\,(\nu-1)}+\ldots\right)-$$
$$2^{-\nu-1}\sin\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\left(1-\frac{z^4}{32\,(\nu+1)\,(\nu+2)}+\frac{z^8}{6144\,(\nu+1)\,(\nu+2)\,(\nu+3)\,(\nu+4)}+\ldots\right)-$$
$$2^{-\nu-3}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu-1)\,z^{\nu+2}\left(1-\frac{z^4}{96\,(\nu+2)\,(\nu+3)}+\frac{z^8}{30\,720\,(\nu+2)\,(\nu+3)\,(\nu+4)\,(\nu+5)}+\ldots\right)\;/;\;(z\to 0)\land \nu\notin\mathbb{Z}$$

03.19.06.0015.01

$$\operatorname{kei}_\nu(z) = -z^{\nu}\,\frac{\Gamma(-\nu)}{2^{\nu+1}}\sum_{k=0}^{\infty}\frac{1}{(\nu+1)_k\,k!}\sin\left(\frac{\pi}{4}(\nu-2k)\right)\left(\frac{z}{2}\right)^{2k}-z^{-\nu}\,\frac{\Gamma(\nu)}{2^{-\nu+1}}\sum_{k=0}^{\infty}\frac{1}{(1-\nu)_k\,k!}\sin\left(\frac{\pi}{4}(3\nu-2k)\right)\left(\frac{z}{2}\right)^{2k}\;/;\;\nu\notin\mathbb{Z}$$

03.19.06.0016.01

$$\operatorname{kei}_\nu(z) = -2^{\nu-1}\,\Gamma(\nu)\sin\left(\frac{3\pi\nu}{4}\right)z^{-\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{1-\nu}{2}\right)_k\left(1-\frac{\nu}{2}\right)_k\left(\frac{1}{2}\right)_k k!}-2^{-\nu-1}\sin\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{1}{2}\right)_k k!}-$$
$$2^{\nu-3}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu-1)\,z^{2-\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(1-\frac{\nu}{2}\right)_k\left(\frac{3-\nu}{2}\right)_k\left(\frac{3}{2}\right)_k k!}-2^{-\nu-3}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu-1)\,z^{\nu+2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k\left(\frac{3}{2}\right)_k k!}\;/;\;\nu\notin\mathbb{Z}$$

03.19.06.0017.01

$$\operatorname{kei}_\nu(z) = -2^{\nu-3}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu-1)\,z^{2-\nu}\,{}_0F_3\left(;\frac{3}{2},1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2};-\frac{z^4}{256}\right)-$$
$$2^{\nu-1}\,\Gamma(\nu)\sin\left(\frac{3\pi\nu}{4}\right)z^{-\nu}\,{}_0F_3\left(;\frac{1}{2},\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)-2^{-\nu-1}\sin\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\,{}_0F_3\left(;\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-$$
$$2^{-\nu-3}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu-1)\,z^{\nu+2}\,{}_0F_3\left(;\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{z^4}{256}\right)\;/;\;\nu\notin\mathbb{Z}$$

03.19.06.0018.01

$$\operatorname{kei}_\nu(z) = 2^{2\nu-5}\,\pi^2\cos\left(\frac{3\pi\nu}{4}\right)\csc(\pi\nu)\,z^{2-\nu}\,{}_0\tilde{F}_3\left(;\frac{3}{2},1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2};-\frac{z^4}{256}\right)-$$
$$2^{2\nu-1}\,\pi^2\csc(\pi\nu)\sin\left(\frac{3\pi\nu}{4}\right)z^{-\nu}\,{}_0\tilde{F}_3\left(;\frac{1}{2},\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)+2^{-2\nu-1}\,\pi^2\csc(\pi\nu)\sin\left(\frac{\pi\nu}{4}\right)z^{\nu}$$
$${}_0\tilde{F}_3\left(;\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-2^{-2\nu-5}\,\pi^2\cos\left(\frac{\pi\nu}{4}\right)\csc(\pi\nu)\,z^{\nu+2}\,{}_0\tilde{F}_3\left(;\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{z^4}{256}\right)\;/;\;\nu\notin\mathbb{Z}$$

03.19.06.0019.01

$$\operatorname{kei}_\nu(z) \propto -2^{\nu-1}\,\Gamma(\nu)\sin\left(\frac{3\pi\nu}{4}\right)z^{-\nu}\left(1+O\left(z^2\right)\right)-2^{-\nu-1}\sin\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\left(1+O\left(z^2\right)\right)\;/;\;\nu\notin\mathbb{Z}$$

03.19.06.0020.01

$$\operatorname{kei}_\nu(z) \propto \begin{cases} -\frac{\pi}{4} & \nu = 0 \\ -(-1)^{\frac{|\nu|}{4}} 2^{|\nu|-3} z^{2-|\nu|} (|\nu|-2)! & \frac{\nu}{4} \in \mathbb{Z} \\ (-1+i)(-1)^{\nu-1} (-1)^{\nu/4} 2^{|\nu|-2} z^{-|\nu|} (|\nu|-1)! & \frac{\nu-1}{4} \in \mathbb{Z} \\ (-1)^{\theta\left(\frac{\nu-2}{4}\right)} i (-1)^{\nu/4} 2^{|\nu|-1} z^{-|\nu|} (|\nu|-1)! & \frac{\nu-2}{4} \in \mathbb{Z} \\ (1+i)(-1)^{\nu-1} (-1)^{\nu/4} 2^{|\nu|-2} z^{-|\nu|} (|\nu|-1)! & \frac{\nu-3}{4} \in \mathbb{Z} \\ -2^{\nu-1} \Gamma(\nu) \sin\left(\frac{3\pi\nu}{4}\right) z^{-\nu} - 2^{-\nu-1} \Gamma(-\nu) \sin\left(\frac{\pi\nu}{4}\right) z^{\nu} & \text{True} \end{cases} /; (z \to 0)$$

03.19.06.0021.01

$$\operatorname{kei}_\nu(z) = F_\infty(z, \nu) /; \left(\left(F_n(z, \nu) = -\frac{z^\nu \Gamma(-\nu)}{2^{\nu+1}} \sum_{k=0}^{n} \frac{\sin\left(\frac{1}{4}\pi(\nu-2k)\right)}{(\nu+1)_k\, k!} \left(\frac{z}{2}\right)^{2k} - \frac{z^{-\nu}\Gamma(\nu)}{2^{1-\nu}} \sum_{k=0}^{n} \frac{\sin\left(\frac{1}{4}\pi(3\nu-2k)\right)}{(1-\nu)_k\, k!} \left(\frac{z}{2}\right)^{2k} = \right.\right.$$

$$\operatorname{kei}_\nu(z) - \left((-i)^n 2^{-2n+\nu-4} e^{\frac{3i\pi\nu}{4}} \pi z^{2n-\nu+2} \csc(\pi\nu)\, {}_1\tilde{F}_2\left(1; n+2, n-\nu+2; -\frac{i z^2}{4}\right) - \right.$$

$$(-i)^n 2^{-2n-\nu-4} e^{\frac{i\pi\nu}{4}} \pi z^{2n+\nu+2} \csc(\pi\nu)\, {}_1\tilde{F}_2\left(1; n+2, n+\nu+2; -\frac{i z^2}{4}\right) + 2^{-2n+\nu-4} e^{-\frac{3i\pi\nu}{4}} i^n \pi z^{2n-\nu+2} \csc(\pi\nu)$$

$$\left.\left.\left. {}_1\tilde{F}_2\left(1; n+2, n-\nu+2; \frac{i z^2}{4}\right) - i^n 2^{-2n-\nu-4} e^{-\frac{i\pi\nu}{4}} \pi z^{2n+\nu+2} \csc(\pi\nu)\, {}_1\tilde{F}_2\left(1; n+2, n+\nu+2; \frac{i z^2}{4}\right)\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Logarithmic cases

03.19.06.0022.01

$$\operatorname{kei}_0(z) = -\frac{\pi}{4}\left(1 - \frac{z^4}{64} + \frac{z^8}{147456} + \ldots\right) + \frac{z^2}{4}\left(-\log\left(\frac{z}{2}\right) - \gamma + 1 + \frac{6\log\left(\frac{z}{2}\right) + 6\gamma - 11}{3456} z^4 - \frac{2\log\left(\frac{z}{2}\right) - \frac{137}{30} + 2\gamma}{7372800} z^8 + \ldots\right) /;$$

$$(z \to 0)$$

03.19.06.0023.01

$$\operatorname{kei}_1(z) \propto -\frac{1}{\sqrt{2}\, z} - \frac{z}{8}\left(\sqrt{2}\left(2\log\left(\frac{z}{2}\right) + 2\gamma - 1\right) - \frac{2\log\left(\frac{z}{2}\right) - \frac{5}{2} + 2\gamma}{4\sqrt{2}} z^2 - \frac{2\log\left(\frac{z}{2}\right) - \frac{10}{3} + 2\gamma}{96\sqrt{2}} z^4 + \ldots\right) +$$

$$\frac{\pi z}{8\sqrt{2}}\left(1 - \frac{z^4}{192} + \frac{z^8}{737280} + \ldots\right) + \frac{\pi z^3}{64\sqrt{2}}\left(1 - \frac{z^4}{1152} + \frac{z^8}{11059200} + \ldots\right) /; (z \to 0)$$

03.19.06.0024.01

$$\operatorname{kei}_2(z) \propto \frac{2}{z^2} + \frac{z^2}{16}\left(2\log\left(\frac{z}{2}\right) - \frac{3}{2} + 2\gamma - \frac{1}{384}\left(2\log\left(\frac{z}{2}\right) - \frac{43}{12} + 2\gamma\right) z^4 + \ldots\right) - \frac{\pi z^4}{384}\left(1 - \frac{z^4}{1920} + \frac{z^8}{25804800} + \ldots\right) /; (z \to 0)$$

03.19.06.0025.01

$$\operatorname{kei}_n(z) \propto \frac{i}{4}\left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{\left(e^{\frac{3 i \pi n}{4}} - (-1)^k e^{-\frac{1}{4}(3 i \pi n)}\right)(n-k-1)!}{k!} \left(\frac{i z^2}{4}\right)^k + (-1)^n 2^{-n-2} z^n \left(\frac{2}{n!} \sin\left(\frac{n\pi}{4}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(n+1) + \gamma\right) - \frac{1}{2(n+1)!} \cos\left(\frac{n\pi}{4}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(n+2) + \gamma - 1\right) z^2 - \frac{1}{16(n+2)!} \sin\left(\frac{n\pi}{4}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(n+3) - \frac{3}{2} + \gamma\right) z^4 + \ldots\right) - \frac{\pi 2^{-n-2} z^n \cos\left(\frac{3n\pi}{4}\right)}{n!}\left(1 - \frac{z^4}{32(n+1)(n+2)} + \frac{z^8}{6144(n+1)(n+2)(n+3)(n+4)} + \ldots\right) + \frac{\pi 2^{-n-4} z^{n+2} \sin\left(\frac{3n\pi}{4}\right)}{(n+1)!}\left(1 - \frac{z^4}{96(n+2)(n+3)} + \frac{z^8}{30\,720(n+2)(n+3)(n+4)(n+5)} + \ldots\right) /; (z \to 0) \land n \in \mathbb{N}$$

03.19.06.0026.01

$$\operatorname{kei}_n(z) = \frac{i}{4}\left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{\left(e^{\frac{3 i \pi n}{4}} - (-1)^k e^{-\frac{3 i \pi n}{4}}\right)(n-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k - \frac{\pi}{4} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4}\pi(2k+3n)\right)}{k!(k+n)!}\left(\frac{z}{2}\right)^{2k+n} + (-1)^n 2^{-n-2} i z^n \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{i\pi n}{4}} - (-1)^k e^{\frac{i \pi n}{4}}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(k+1) - \psi(k+n+1)\right)}{k!(k+n)!}\left(\frac{i z^2}{4}\right)^k /; n \in \mathbb{N}$$

03.19.06.0027.01

$$\operatorname{kei}_\nu(z) = \frac{i}{4}\left(\frac{z}{2}\right)^{-|\nu|} \sum_{k=0}^{|\nu|-1} \frac{\left(e^{\frac{1}{4} i\pi(2\nu+|\nu|)} - (-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k - \frac{\pi}{4} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4}\pi(2(k+\nu)+|\nu|)\right)}{k!(k+|\nu|)!}\left(\frac{z}{2}\right)^{2k+|\nu|} + \frac{i}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)} - (-1)^k e^{\frac{1}{4} i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(k+1) - \psi(k+|\nu|+1)\right)}{k!(k+|\nu|)!}\left(\frac{i z^2}{4}\right)^k /; \nu \in \mathbb{Z}$$

03.19.06.0028.01

$$\operatorname{kei}_n(z) = \frac{1}{8}\left(4 K_n\left(\sqrt[4]{-1}\, z\right)(-i)^{n+1} - 2(-1)^n i \pi Y_n\left(\sqrt[4]{-1}\, z\right) - i^{n+1} I_n\left(\sqrt[4]{-1}\, z\right)\left(-i\pi - 4\log(z) + 4\log\left(\sqrt[4]{-1}\, z\right)\right) - (-1)^n i J_n\left(\sqrt[4]{-1}\, z\right)\left(-i\pi + 4\log(z) - 4\log\left(\sqrt[4]{-1}\, z\right)\right) - (-1)^n i\, n! \sum_{k=0}^{n-1} \frac{2^{-k+n+1} i^{\frac{k-n}{2}} z^{k-n}}{(k-n)\, k!}\left((-1)^k i^n I_k\left(\sqrt[4]{-1}\, z\right) - J_k\left(\sqrt[4]{-1}\, z\right)\right) + i^{n+1} \sum_{k=0}^{n-1} \frac{2^{-2k+n+1} i^{k-\frac{n}{2}}\left(-(-1)^{k+n} + i^n\right)(n-k-1)!\, z^{2k-n}}{k!} - \frac{i\, 2^{1-n} e^{\frac{3 i n \pi}{4}} z^n}{n!} \sum_{j=1}^{n} \frac{1}{j}\left(i^n\, {}_1F_2\left(j; j+1, n+1; -\frac{1}{4}\left(i z^2\right)\right) - {}_1F_2\left(j; j+1, n+1; \frac{i z^2}{4}\right)\right)\right) /; n \in \mathbb{N}$$

03.19.06.0029.01

$$\operatorname{kei}_\nu(z) = \frac{i}{4}\left(\frac{z}{2}\right)^{-|\nu|} \sum_{k=0}^{|\nu|-1} \frac{\left(e^{\frac{1}{4} i \pi (2\nu+|\nu|)} - (-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!}{k!} \left(\frac{i z^2}{4}\right)^k - \frac{2^{-|\nu|-2} \pi z^{|\nu|} \cos\left(\frac{1}{4}\pi(2\nu+|\nu|)\right)}{\Gamma(|\nu|+1)} {}_0F_3\left(;\frac{1}{2}, \frac{1}{2}(|\nu|+1), \frac{1}{2}(|\nu|+2); -\frac{z^4}{256}\right) + \frac{2^{-|\nu|-4} \pi z^{|\nu|+2} \sin\left(\frac{1}{4}\pi(2\nu+|\nu|)\right)}{\Gamma(|\nu|+2)} {}_0F_3\left(;\frac{3}{2}, \frac{1}{2}(|\nu|+2), \frac{1}{2}(|\nu|+3); -\frac{z^4}{256}\right) + \frac{i}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)} - (-1)^k e^{\frac{1}{4} i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(k+1) - \psi(k+|\nu|+1)\right)}{k!\,(k+|\nu|)!} \left(\frac{i z^2}{4}\right)^k /; \nu \in \mathbb{Z}$$

03.19.06.0030.01

$$\operatorname{kei}_\nu(z) = \frac{i}{4}\left(\frac{z}{2}\right)^{-|\nu|} \sum_{k=0}^{|\nu|-1} \frac{\left(e^{\frac{1}{4} i \pi (2\nu+|\nu|)} - (-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!}{k!} \left(\frac{i z^2}{4}\right)^k - 4^{-|\nu|-1} \pi^2 z^{|\nu|} \cos\left(\frac{1}{4}\pi(2\nu+|\nu|)\right) {}_0\tilde{F}_3\left(;\frac{1}{2}, \frac{1}{2}(|\nu|+1), \frac{1}{2}(|\nu|+2); -\frac{z^4}{256}\right) + 4^{-|\nu|-3} \pi^2 z^{|\nu|+2} \sin\left(\frac{1}{4}\pi(2\nu+|\nu|)\right) {}_0\tilde{F}_3\left(;\frac{3}{2}, \frac{1}{2}(|\nu|+2), \frac{1}{2}(|\nu|+3); -\frac{z^4}{256}\right) + \frac{i}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)} - (-1)^k e^{\frac{1}{4} i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(k+1) - \psi(k+|\nu|+1)\right)}{k!\,(k+|\nu|)!} \left(\frac{i z^2}{4}\right)^k /; \nu \in \mathbb{Z}$$

03.19.06.0031.01

$$\operatorname{kei}_n(z) = \frac{1}{4} i \left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{\left(e^{\frac{3 i\pi n}{4}} - (-1)^k e^{-\frac{1}{4}(3 i\pi n)}\right)(n-k-1)!}{k!} \left(\frac{i z^2}{4}\right)^k - \frac{2^{-n-2} \pi z^n}{n!} \cos\left(\frac{3 n\pi}{4}\right) {}_0F_3\left(;\frac{1}{2}, \frac{n}{2}+\frac{1}{2}, \frac{n}{2}+1; -\frac{z^4}{256}\right) + \frac{2^{-n-4} \pi z^{n+2}}{(n+1)!} \sin\left(\frac{3 n\pi}{4}\right) {}_0F_3\left(;\frac{3}{2}, \frac{n}{2}+1, \frac{n}{2}+\frac{3}{2}; -\frac{z^4}{256}\right) + (-1)^n 2^{-n-2} i z^n \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{i\pi n}{4}} - (-1)^k e^{\frac{i\pi n}{4}}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(k+1) - \psi(k+n+1)\right)}{k!\,(k+n)!} \left(\frac{i z^2}{4}\right)^k /; n \in \mathbb{N}$$

03.19.06.0032.01

$$\operatorname{kei}_\nu(z) = \frac{i}{4}\left(\frac{z}{2}\right)^{-|\nu|}\sum_{k=0}^{|\nu|-1}\frac{\left(e^{\frac{1}{4} i\pi(2\nu+|\nu|)} - (-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k -$$

$$\frac{1}{8} e^{-\frac{1}{2} i\pi(\nu+|\nu|)}\pi\left(e^{\frac{1}{2} i\pi(2\nu+|\nu|)} I_{|\nu|}\left(\sqrt[4]{-1}\, z\right) + J_{|\nu|}\left(\sqrt[4]{-1}\, z\right)\right) +$$

$$\frac{i}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}}\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)} - (-1)^k e^{\frac{1}{4} i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right) - \psi(k+1) - \psi(k+|\nu|+1)\right)}{k!\,(k+|\nu|)!}\left(\frac{i z^2}{4}\right)^k \;/;\; \nu \in \mathbb{Z}$$

03.19.06.0033.01

$$\operatorname{kei}_0(z) \propto -\frac{\pi}{4}\left(1 + O\left(z^2\right)\right) - \frac{z^2}{4}\log(z)\left(1 + O\left(z^4\right)\right)$$

03.19.06.0034.01

$$\operatorname{kei}_1(z) \propto -\frac{1}{\sqrt{2}\, z}\left(1 + O\left(z^2\right)\right) - \frac{z\log(z)}{2\sqrt{2}}\left(1 + O\left(z^2\right)\right)$$

03.19.06.0035.01

$$\operatorname{kei}_2(z) \propto \frac{2}{z^2}\left(1 + O\left(z^4\right)\right) + \frac{z^2\log(z)}{8}\left(1 + O\left(z^4\right)\right)$$

Asymptotic series expansions

Expansions for any *z* in exponential form

Using exponential function with branch cut-free arguments

General case

03.19.06.0036.01

$$\operatorname{kei}_\nu(z) \propto \frac{\sqrt{\pi}\,\csc(\pi\nu)}{4\sqrt{2}}$$

$$\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(z^\nu\Bigg(e^{\frac{3 i\pi\nu}{4} - \frac{i z}{\sqrt{2}}}\left((-1)^{3/4}\sin(\pi\nu) - \frac{i\sqrt{-i z^2}\,\cos(\pi\nu)}{z}\right)\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} + (-1)^{3/4} e^{\frac{i z}{\sqrt{2}} + \frac{i\pi\nu}{4}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\Bigg) -$$

$$z^{-\nu}\left((-1)^{3/4} e^{\frac{i z}{\sqrt{2}} - \frac{5 i\pi\nu}{4}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}} - e^{\frac{i\pi\nu}{4} - \frac{i z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}\left(\frac{i\sqrt{-i z^2}\,\cos(\pi\nu)}{z} + (-1)^{3/4}\sin(\pi\nu)\right)\right)\Bigg) +$$

$$e^{-\frac{z}{\sqrt{2}}}\Bigg(z^{-\nu}\left((-1)^{3/4} e^{\frac{i z}{\sqrt{2}} + \frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}} + e^{-\frac{i z}{\sqrt{2}} - \frac{5 i\pi\nu}{4}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt{i z^2}\,\cos(\pi\nu)}{z} - (-1)^{3/4}\sin(\pi\nu)\right)\right) -$$

$$z^\nu\left((-1)^{3/4} e^{\frac{i z}{\sqrt{2}} + \frac{3 i\pi\nu}{4}}\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} + e^{\frac{i\pi\nu}{4} - \frac{i z}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{i z^2}\,\cos(\pi\nu)}{z} + (-1)^{3/4}\sin(\pi\nu)\right)\right)\Bigg) +$$

$$\frac{4\nu^2-1}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(\left(\left(i\,e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}-e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{-iz^2}\cos(\pi\nu)}{z}-\sin(\pi\nu)\right)\right)\right)z^{-\nu}+\right.\right.$$

$$\left.\left(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{-iz^2}\cos(\pi\nu)}{z}+\sin(\pi\nu)\right)-i\,e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\right)z^{\nu}\right)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(z^{\nu}\left(e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}+e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{iz^2}\cos(\pi\nu)}{z}-i\sin(\pi\nu)\right)\right)-\right.$$

$$\left.\left.z^{-\nu}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}+e^{-\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{iz^2}\cos(\pi\nu)}{z}+i\sin(\pi\nu)\right)\right)\right)\right)+$$

$$\frac{16\nu^4-40\nu^2+9}{128z^2}\left(e^{\frac{z}{\sqrt{2}}}\left(z^{\nu}\left(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(\frac{\sqrt{-iz^2}\cos(\pi\nu)}{z}-\sqrt[4]{-1}\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}+\right.\right.\right.$$

$$\left.\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\right)-$$

$$\left.z^{-\nu}\left(e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(\frac{\sqrt{-iz^2}\cos(\pi\nu)}{z}+\sqrt[4]{-1}\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}+\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\right)\right)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(z^{\nu}\left(\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}+e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\left(\frac{i\sqrt{iz^2}\cos(\pi\nu)}{z}-\sqrt[4]{-1}\sin(\pi\nu)\right)\right)-\right.$$

$$\left.\left.z^{-\nu}\left(\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}+e^{-\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\left(\frac{i\sqrt{iz^2}\cos(\pi\nu)}{z}+\sqrt[4]{-1}\sin(\pi\nu)\right)\right)\right)\right)+$$

$$\frac{64\nu^6-560\nu^4+1036\nu^2-225}{3072z^3}\left(e^{\frac{z}{\sqrt{2}}}\left(z^{-\nu}\left(e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(\frac{\sqrt[4]{-1}\sqrt{-iz^2}\cos(\pi\nu)}{z}+i\sin(\pi\nu)\right)\right.\right.\right.$$

$$\left.\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}+e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\right)-$$

$$\left.z^{\nu}\left(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(\frac{\sqrt[4]{-1}\left(\sqrt{-iz^2}\cos(\pi\nu)\right)}{z}-i\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}+e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\right)\right)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(z^{\nu}\left(e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{4}}\, i\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}+e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z}-\sin(\pi \nu)\right)\right)-\right.$$

$$z^{-\nu}\left(e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{4}}\, i\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}+e^{-\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\right.$$

$$\left.\left.\left.\left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z}+\sin(\pi \nu)\right)\right)\right)+\ldots\right) /; (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.19.06.0037.01

$$\operatorname{kei}_{\nu}(z) \propto \frac{\sqrt{\pi}\, \csc(\pi \nu)}{4 \sqrt{2}}$$

$$\left(z^{-\nu}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}\left(\frac{i \sqrt{-i z^2}\, \cos(\pi \nu)}{z}-\frac{1}{\sqrt[4]{-1}} \sin(\pi \nu)\right)\left(\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k}\left(\nu+\frac{1}{2}\right)_{2 k}}{(2 k)!}\left(\frac{i}{z^2}\right)^k+\right.\right.\right.\right.$$

$$\left.\left.O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\right)\right)+\frac{1}{\sqrt[4]{-1}}\, e^{\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\left(\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k}\left(\nu+\frac{1}{2}\right)_{2 k}}{(2 k)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\right)\right)\right)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(e^{-\frac{i z}{\sqrt{2}}}\, e^{-\frac{1}{4}(5 i \pi \nu)}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt{i z^2}\, \cos(\pi \nu)}{z}+\frac{1}{\sqrt[4]{-1}} \sin(\pi \nu)\right)\left(\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k}\left(\nu+\frac{1}{2}\right)_{2 k}}{(2 k)!}\left(-\frac{i}{z^2}\right)^k+\right.\right.$$

$$\left.\left.O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\right)\right)-\frac{1}{\sqrt[4]{-1}}\, e^{\frac{i z}{\sqrt{2}}}\, e^{\frac{i \pi \nu}{4}}\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}\left(\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k}\left(\nu+\frac{1}{2}\right)_{2 k}}{(2 k)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\right)\right)\right)+$$

$$\frac{i}{2 z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\, \sqrt{-i z^2}\, \cos(\pi \nu)}{z}+i \sin(\pi \nu)\right)\right.\right.$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k+1}\left(\nu+\frac{1}{2}\right)_{2 k+1}}{(2 k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2}\right\rfloor+2}}\right)\right)-e^{\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}$$

$$\left.\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k+1}\left(\nu+\frac{1}{2}\right)_{2 k+1}}{(2 k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2}\right\rfloor+2}}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{-\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\right.$$

$$\left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z}+\sin(\pi \nu)\right)\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{2^{-2 k}\left(\frac{1}{2}-\nu\right)_{2 k+1}\left(\nu+\frac{1}{2}\right)_{2 k+1}}{(2 k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2}\right\rfloor+2}}\right)\right)-$$

$$i\, e^{\frac{i z}{\sqrt{2}}+\frac{i\pi\nu}{4}} \left(-\sqrt[4]{-1}\; z\right)^{\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)\Bigg)\Bigg)\Bigg) -$$

$$z^\nu \Bigg(e^{\frac{z}{\sqrt{2}}} \Bigg(e^{\frac{3 i\pi\nu}{4}-\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1}\; z\right)^{-\nu-\frac{1}{2}} \left(\frac{i\sqrt{-i z^2}\,\cos(\pi\nu)}{z} + \frac{1}{\sqrt[4]{-1}}\sin(\pi\nu)\right) \Bigg(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k +$$

$$O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\Bigg) + \frac{1}{\sqrt[4]{-1}}\, e^{\frac{i z}{\sqrt{2}}+\frac{i\pi\nu}{4}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)\Bigg) +$$

$$e^{-\frac{z}{\sqrt{2}}} \Bigg(e^{\frac{i\pi\nu}{4}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\cos(\pi\nu)\sqrt{i z^2}}{z} - \frac{1}{\sqrt[4]{-1}}\sin(\pi\nu)\right) \Bigg(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k +$$

$$O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\Bigg) - \frac{1}{\sqrt[4]{-1}}\, e^{\frac{i z}{\sqrt{2}}+\frac{3 i\pi\nu}{4}} \left(-\sqrt[4]{-1}\; z\right)^{-\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)\Bigg) +$$

$$\frac{1}{2z}\Bigg(e^{\frac{z}{\sqrt{2}}} \Bigg(\left(\frac{(-1)^{3/4}\sqrt{-i z^2}\,\cos(\pi\nu)}{z} + \sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\; z\right)^{-\nu-\frac{1}{2}} e^{\frac{3 i\pi\nu}{4}-\frac{i z}{\sqrt{2}}}$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right) -$$

$$i\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} e^{\frac{i z}{\sqrt{2}}+\frac{i\pi\nu}{4}} \left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)\Bigg) +$$

$$e^{-\frac{z}{\sqrt{2}}} \Bigg(e^{\frac{i z}{\sqrt{2}}+\frac{3 i\pi\nu}{4}} \left(-\sqrt[4]{-1}\; z\right)^{-\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right) +$$

$$e^{\frac{i\pi\nu}{4}-\frac{i z}{\sqrt{2}}}\, i\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}\,\cos(\pi\nu)}{z} - \sin(\pi\nu)\right)$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)\Bigg)\Bigg)\Bigg)\Bigg) /;\ (|z| \to \infty) \land \nu \notin \mathbb{Z} \land n \in \mathbb{N}$$

03.19.06.0038.01

$$\operatorname{kei}_{\nu}(z) \propto \frac{\sqrt{\pi}\,\csc(\pi\,\nu)}{4\sqrt{2}} \Bigg(z^{-\nu} \Bigg(-e^{\frac{i\pi\nu}{4}} \left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}} \Bigg(\frac{1}{\sqrt[4]{-1}}\, e^{(-1)^{3/4} z} - e^{-(-1)^{3/4} z} \Bigg(\frac{i\sqrt{-i\,z^2}\,\cos(\pi\,\nu)}{z} - \frac{1}{\sqrt[4]{-1}}\sin(\pi\,\nu) \Bigg)\Bigg)$$

$$\Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{z^2}\right)^k + O\Bigg(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\Bigg)\Bigg) +$$

$$e^{-\frac{5i\pi\nu}{4}} \left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}} \Bigg(e^{-\sqrt[4]{-1}\,z} \Bigg(\frac{\sqrt{i\,z^2}\,\cos(\pi\,\nu)}{z} + \frac{1}{\sqrt[4]{-1}}\sin(\pi\,\nu)\Bigg) + \frac{1}{\sqrt[4]{-1}}\, e^{\sqrt[4]{-1}\,z}\Bigg)$$

$$\Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{z^2}\right)^k + O\Bigg(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\Bigg)\Bigg) +$$

$$\frac{i}{2z} \Bigg(e^{-\frac{5i\pi\nu}{4}} \left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}} \Bigg(e^{-\sqrt[4]{-1}\,z} \Bigg(\frac{\sqrt[4]{-1}\,\sqrt{i\,z^2}\,\cos(\pi\,\nu)}{z} + \sin(\pi\,\nu)\Bigg) - e^{\sqrt[4]{-1}\,z}\Bigg)$$

$$\Bigg(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(-\frac{i}{z^2}\right)^k + O\Bigg(\frac{1}{z^{2\left\lfloor \frac{n-1}{2}\right\rfloor+2}}\Bigg)\Bigg) -$$

$$e^{\frac{i\pi\nu}{4}} \left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}} \Bigg(i\, e^{(-1)^{3/4} z} - e^{-(-1)^{3/4} z} \Bigg(\frac{\sqrt[4]{-1}\,\sqrt{-i\,z^2}\,\cos(\pi\,\nu)}{z} + i\sin(\pi\,\nu)\Bigg)\Bigg)$$

$$\Bigg(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{z^2}\right)^k + O\Bigg(\frac{1}{z^{2\left\lfloor \frac{n-1}{2}\right\rfloor+2}}\Bigg)\Bigg)\Bigg)\Bigg) -$$

$$z^{\nu} \Bigg(e^{\frac{3i\pi\nu}{4}-(-1)^{3/4} z} \left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}} \Bigg(\frac{i\sqrt{-i\,z^2}\,\cos(\pi\,\nu)}{z} - \frac{1}{\sqrt[4]{-1}}\, e^{2(-1)^{3/4} z} + \frac{1}{\sqrt[4]{-1}}\sin(\pi\,\nu)\Bigg)$$

$$\Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{z^2}\right)^k + O\Bigg(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\Bigg)\Bigg) + e^{\frac{i\pi\nu}{4}-\sqrt[4]{-1}\,z} \left((-1)^{3/4}\,z\right)^{-\nu-\frac{1}{2}}$$

$$\Bigg(\frac{\sqrt{i\,z^2}\,\cos(\pi\,\nu)}{z} + \frac{1}{\sqrt[4]{-1}}\, e^{2\sqrt[4]{-1}\,z} - \frac{1}{\sqrt[4]{-1}}\sin(\pi\,\nu)\Bigg)\Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{z^2}\right)^k + O\Bigg(\frac{1}{z^{2\left\lfloor \frac{n}{2}\right\rfloor+2}}\Bigg)\Bigg) +$$

$$\frac{1}{2z} \Bigg(e^{\frac{3i\pi\nu}{4}-(-1)^{3/4} z} \left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}} \Bigg(\frac{(-1)^{3/4}\sqrt{-i\,z^2}\,\cos(\pi\,\nu)}{z} + e^{2(-1)^{3/4} z} + \sin(\pi\,\nu)\Bigg)$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right) +$$

$$e^{\frac{i\pi\nu}{4}-\sqrt[4]{-1}\,z}\, i\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\sqrt{i z^2}\cos(\pi\nu)}{z} - e^{2\sqrt[4]{-1}\,z} - \sin(\pi\nu)\right)$$

$$\left.\left.\left.\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)\right)\right)\right) /; (|z|\to\infty)\bigwedge \nu\notin\mathbb{Z}\bigwedge n\in\mathbb{N}$$

03.19.06.0039.01

$$\mathrm{kei}_\nu(z) \propto \frac{1}{64}\sqrt{\frac{\pi}{2}}\,(i\cot(\pi\nu)+1)\sqrt{\pi}\, e^{\frac{i\pi\nu}{4}-\frac{(1+i)z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(4\nu^2-1\right)z^{-\nu-2}$$

$$\left(e^{\sqrt{2}\,z+\frac{5i\pi\nu}{2}}\left(\sqrt[4]{-1}\sqrt{-i z^2}-z\right)z^{2\nu} + e^{\sqrt{2}\,z+\frac{i\pi\nu}{2}}\left(z+\sqrt[4]{-1}\sqrt{-i z^2}\right)z^{2\nu} - 2e^{\sqrt{2}\,i z+\frac{3i\pi\nu}{2}}\, i z^{2\nu+1} + 2e^{i\left(\sqrt{2}\,z+\pi\nu\right)}\, i\right.$$

$$\left.\left(-\sqrt[4]{-1}\,z\right)^{2\nu} z - e^{\sqrt{2}\,z+2i\pi\nu}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}\left(z+\sqrt[4]{-1}\sqrt{-i z^2}\right) + e^{\sqrt{2}\,z}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}\left(z-\sqrt[4]{-1}\sqrt{-i z^2}\right)\right)$$

$${}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right) + \frac{1}{64}\sqrt{\frac{\pi}{2}}\,(i\cot(\pi\nu)+1)\sqrt{\pi}\, e^{-\frac{(1+i)z}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}$$

$$\left(4\nu^2-1\right)z^{-\nu-2}\left(e^{\frac{3i\pi\nu}{2}}\left(\sqrt[4]{-1}\sqrt{i z^2}-i z\right)z^{2\nu} + e^{\frac{7i\pi\nu}{2}}\left(i z+\sqrt[4]{-1}\sqrt{i z^2}\right)z^{2\nu} - 2e^{\sqrt{2}\,(1+i)z+\frac{5i\pi\nu}{2}} z^{2\nu+1} +\right.$$

$$\left.2e^{\sqrt{2}\,(1+i)z+i\pi\nu}\left((-1)^{3/4}z\right)^{2\nu} z - \left((-1)^{3/4}z\right)^{2\nu}\left(i z+\sqrt[4]{-1}\sqrt{i z^2}\right) + e^{2i\pi\nu}\left((-1)^{3/4}z\right)^{2\nu}\left(i z-\sqrt[4]{-1}\sqrt{i z^2}\right)\right)$$

$${}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right) + \frac{1}{4\sqrt{2}}\sqrt{\pi}\, e^{-\frac{(1+i)z}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}$$

$$z^{-\nu-1}\left(-e^{\frac{5i\pi\nu}{2}}\cos(\pi\nu)\left((-1)^{3/4}z - i\sqrt{i z^2} + \sqrt{i z^2}\cot(\pi\nu)\right)z^{2\nu} +\right.$$

$$\sqrt[4]{-1}\left(e^{\sqrt{2}\,(1+i)z+\frac{5i\pi\nu}{2}} z^{2\nu} - e^{\frac{5i\pi\nu}{2}}\sin(\pi\nu)\, z^{2\nu} - i\left((-1)^{3/4}z\right)^{2\nu} - i\, e^{(1+i)\sqrt{2}\,z}\left((-1)^{3/4}z\right)^{2\nu}\csc(\pi\nu)\right)z +$$

$$\left.\left(\sqrt{i z^2}\left((-1)^{3/4}z\right)^{2\nu} + (-1)^{3/4}e^{\sqrt{2}\,(1+i)z+\frac{5i\pi\nu}{2}} z^{2\nu+1}\right)\cot(\pi\nu)\right){}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right) +$$

$$\frac{1}{4\sqrt{2}}\sqrt{\pi}\, e^{-\frac{(1+i)z}{\sqrt{2}}-\frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{3}{2}} z^{-\nu}\left(e^{\sqrt{2}\,z}\cos(\pi\nu)\left(z+(-1)^{3/4}\sqrt{-i z^2}\cot(\pi\nu) - \sqrt[4]{-1}\sqrt{-i z^2}\right)z^{2\nu} +\right.$$

$$\left(e^{i\sqrt{2}\,z}(-i)z^{2\nu} + e^{\sqrt{2}\,z}\, i\sin(\pi\nu)\, z^{2\nu} + e^{\sqrt{2}\,z+\frac{i\pi\nu}{2}}\left(-\sqrt[4]{-1}\,z\right)^{2\nu} + e^{\frac{1}{2}i\left(2\sqrt{2}\,z+\pi\nu\right)}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}\csc(\pi\nu)\right)z -$$

$$\left.\left(e^{\sqrt{2}\,z+\frac{i\pi\nu}{2}} z\sqrt{-i z^2}\left(-\sqrt[4]{-1}\,z\right)^{2\nu-1} + e^{i\sqrt{2}\,z} z^{2\nu+1}\right)\cot(\pi\nu)\right)$$

$${}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right) /; (|z|\to\infty)\bigwedge \nu\notin\mathbb{Z}$$

03.19.06.0040.01

$$\operatorname{kei}_\nu(z) \propto \frac{1}{64}\sqrt{\frac{\pi}{2}}\,(i\cot(\pi\nu)+1)\sqrt{\pi}\;e^{\frac{i\pi\nu}{4}-\frac{(1+i)z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(4\nu^2-1\right)z^{-\nu-2}$$

$$\Big(e^{\sqrt{2}\,z+\frac{5i\pi\nu}{2}}\left(\sqrt[4]{-1}\,\sqrt{-iz^2}-z\right)z^{2\nu}+e^{\sqrt{2}\,z+\frac{i\pi\nu}{2}}\left(z+\sqrt[4]{-1}\,\sqrt{-iz^2}\right)z^{2\nu}-2\,e^{\sqrt{2}\,iz+\frac{3i\pi\nu}{2}}\,i\,z^{2\nu+1}+2\,e^{i\left(\sqrt{2}\,z+\pi\nu\right)}\,i$$

$$\left(-\sqrt[4]{-1}\,z\right)^{2\nu}z-e^{\sqrt{2}\,z+2i\pi\nu}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}\left(z+\sqrt[4]{-1}\,\sqrt{-iz^2}\right)+e^{\sqrt{2}\,z}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}\left(z-\sqrt[4]{-1}\,\sqrt{-iz^2}\right)\Big)$$

$${}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+\frac{1}{64}\sqrt{\frac{\pi}{2}}\,(i\cot(\pi\nu)+1)\sqrt{\pi}\;e^{-\frac{(1+i)z}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}$$

$$\left(4\nu^2-1\right)z^{-\nu-2}\Big(e^{\frac{3i\pi\nu}{2}}\left(\sqrt[4]{-1}\,\sqrt{iz^2}-iz\right)z^{2\nu}+e^{\frac{7i\pi\nu}{2}}\left(iz+\sqrt[4]{-1}\,\sqrt{iz^2}\right)z^{2\nu}-2\,e^{\sqrt{2}\,(1+i)z+\frac{5i\pi\nu}{2}}\,z^{2\nu+1}+$$

$$2\,e^{\sqrt{2}\,(1+i)z+i\pi\nu}\left((-1)^{3/4}z\right)^{2\nu}z-\left((-1)^{3/4}z\right)^{2\nu}\left(iz+\sqrt[4]{-1}\,\sqrt{iz^2}\right)+e^{2i\pi\nu}\left((-1)^{3/4}z\right)^{2\nu}\left(iz-\sqrt[4]{-1}\,\sqrt{iz^2}\right)\Big)$$

$${}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)+\frac{1}{4\sqrt{2}}\sqrt{\pi}\;e^{-\frac{(1+i)z}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}$$

$$z^{-\nu-1}\Big(-e^{\frac{5i\pi\nu}{2}}\cos(\pi\nu)\left((-1)^{3/4}z-i\sqrt{iz^2}+\sqrt{iz^2}\,\cot(\pi\nu)\right)z^{2\nu}+$$

$$\sqrt[4]{-1}\,\left(e^{\sqrt{2}\,(1+i)z+\frac{5i\pi\nu}{2}}z^{2\nu}-e^{\frac{5i\pi\nu}{2}}\sin(\pi\nu)\,z^{2\nu}-i\left((-1)^{3/4}z\right)^{2\nu}-i\,e^{(1+i)\sqrt{2}\,z}\left((-1)^{3/4}z\right)^{2\nu}\csc(\pi\nu)\right)z+$$

$$\left(\sqrt{iz^2}\,\left((-1)^{3/4}z\right)^{2\nu}+(-1)^{3/4}e^{\sqrt{2}\,(1+i)z+\frac{5i\pi\nu}{2}}z^{2\nu+1}\right)\cot(\pi\nu)\Big)\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)+$$

$$\frac{1}{4\sqrt{2}}\sqrt{\pi}\;e^{-\frac{(1+i)z}{\sqrt{2}}-\frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{3}{2}}z^{-\nu}\Big(e^{\sqrt{2}\,z}\cos(\pi\nu)\left(z+(-1)^{3/4}\sqrt{-iz^2}\,\cot(\pi\nu)-\sqrt[4]{-1}\,\sqrt{-iz^2}\right)z^{2\nu}+$$

$$\left(e^{i\sqrt{2}\,z}(-i)\,z^{2\nu}+e^{\sqrt{2}\,z}\,i\sin(\pi\nu)\,z^{2\nu}+e^{\sqrt{2}\,z+\frac{i\pi\nu}{2}}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}+e^{\frac{1}{2}i\left(2\sqrt{2}\,z+\pi\nu\right)}\left(-\sqrt[4]{-1}\,z\right)^{2\nu}\csc(\pi\nu)\right)z-$$

$$\left(e^{\sqrt{2}\,z+\frac{i\pi\nu}{2}}z\sqrt{-iz^2}\,\left(-\sqrt[4]{-1}\,z\right)^{2\nu-1}+e^{i\sqrt{2}\,z}z^{2\nu+1}\right)\cot(\pi\nu)\Big)$$

$${}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)\ /;\ (|z|\to\infty)\bigwedge\nu\notin\mathbb{Z}$$

03.19.06.0041.01

$$\operatorname{kei}_\nu(z) \propto$$

$$\frac{1}{4}\sqrt{\frac{\pi}{2}}\left(e^{\frac{z}{\sqrt{2}}}\left(\sqrt[4]{-1}\, e^{-\frac{i\pi\nu}{4}} z^\nu \left(e^{-\frac{iz}{\sqrt{2}}}(i\cot(\pi\nu)-1)\left(\frac{(-1)^{3/4}\sqrt{-i z^2}\,\cos(\pi\nu)}{z}+\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}\left(1+O\left(\frac{1}{z^2}\right)\right)+\right.\right.\right.$$

$$\left. e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}(i\cot(\pi\nu)+1)\left(1+O\left(\frac{1}{z^2}\right)\right)\right)-$$

$$(-1)^{3/4} e^{\frac{i\pi\nu}{4}} z^{-\nu}\csc(\pi\nu)\left(e^{-\frac{iz}{\sqrt{2}}}\left(\frac{(-1)^{3/4}\sqrt{-i z^2}\,\cos(\pi\nu)}{z}-\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}\left(1+O\left(\frac{1}{z^2}\right)\right)+\right.$$

$$\left.\left. e^{\frac{iz}{\sqrt{2}}-\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(\sqrt[4]{-1}\, e^{-\frac{i\pi\nu}{4}} z^\nu\left(e^{\frac{iz}{\sqrt{2}}}(1-i\cot(\pi\nu))\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\right.\right.$$

$$\left.(i\cot(\pi\nu)+1)\left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}\,\cos(\pi\nu)}{z}-\sin(\pi\nu)\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\right)-$$

$$(-1)^{3/4} e^{\frac{i\pi\nu}{4}} z^{-\nu}\csc(\pi\nu)\left(e^{-\frac{iz}{\sqrt{2}}-\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}\,\cos(\pi\nu)}{z}+\sin(\pi\nu)\right)\left(1+O\left(\frac{1}{z^2}\right)\right)-\right.$$

$$\left.\left.\left. e^{\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right)\right)\;/;\;(|z|\to\infty)\land \nu\notin\mathbb{Z}$$

03.19.06.0042.01

$$\operatorname{kei}_\nu(z) \propto$$

$$\begin{cases}
\frac{\sqrt{\pi}\,(-1)^{3/8}}{2\sqrt{2z}}\left(-e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}}+\sqrt[4]{-1}\, e^{(-1)^{3/4} z+\frac{i\pi\nu}{2}}\right) & \arg(z)\le\frac{\pi}{4} \\
-\frac{(-1)^{3/8}\sqrt{\pi}}{2\sqrt{2z}}\left(e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}}+i\, e^{\sqrt[4]{-1}\, z+\frac{i\pi\nu}{2}}-\sqrt[4]{-1}\, e^{(-1)^{3/4} z+\frac{i\pi\nu}{2}}+i\, e^{\sqrt[4]{-1}\, z-\frac{3i\pi\nu}{2}}\right) & \frac{\pi}{4}<\arg(z)\le \\
-\frac{\sqrt{\pi}\,\sqrt[8]{-1}}{2\sqrt{2z}}\left(\sqrt[4]{-1}\, e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}}+e^{-(-1)^{3/4} z-\frac{i\pi\nu}{2}}+\right. & \text{True} \\
\quad\left.(-1)^{3/4} e^{\sqrt[4]{-1}\, z+\frac{i\pi\nu}{2}}-i\, e^{(-1)^{3/4} z+\frac{i\pi\nu}{2}}+(-1)^{3/4} e^{\sqrt[4]{-1}\, z-\frac{3i\pi\nu}{2}}+e^{\frac{3i\pi\nu}{2}-(-1)^{3/4} z}\right) &
\end{cases}$$

Logarithmic cases

03.19.06.0043.01

$$\operatorname{kei}_\nu(z) \propto \frac{1}{8\sqrt{2\pi}} \Bigg(e^{\frac{z}{\sqrt{2}}} \Bigg(\frac{e^{\frac{3 i \pi \nu}{2} - \frac{i z}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \Bigg(\frac{4 i \sqrt{-i z^2}\, \big(\log\big(-\sqrt[4]{-1}\, z\big) - \log(z)\big)}{z} + \frac{\pi \sqrt{-i z^2}}{z} + 4 (-1)^{3/4} \pi \Bigg) +$$

$$\frac{e^{\frac{i z}{\sqrt{2}} + \frac{i \pi \nu}{2}}}{\sqrt{(-1)^{3/4} z}} \big(3 (-1)^{-3/4} \pi - 4 (-1)^{3/4} \big(\log\big((-1)^{3/4} z\big) - \log(z)\big)\big) \Bigg) +$$

$$e^{-\frac{z}{\sqrt{2}}} \Bigg(\frac{e^{\frac{3 i \pi \nu}{2} - \frac{i z}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \Bigg(-4 (-1)^{3/4} \pi - \frac{3 \pi i \sqrt{i z^2}}{z} + \frac{4 \big(\log\big((-1)^{3/4} z\big) - \log(z)\big) \sqrt{i z^2}}{z} \Bigg) + \frac{e^{\frac{i z}{\sqrt{2}} + \frac{i \pi \nu}{2}}}{\sqrt{-\sqrt[4]{-1}\, z}}$$

$$\big(4 (-1)^{3/4} \big(\log\big(-\sqrt[4]{-1}\, z\big) - \log(z)\big) + \sqrt[4]{-1}\, \pi\big) \Bigg) \Bigg) \left(1 + O\left(\frac{1}{z^4}\right)\right) + \frac{e^{\frac{\pi i (\nu + 1)}{2} - \frac{(1 + i) z}{\sqrt{2}}}}{8 \sqrt{2\pi}\, \sqrt{-\sqrt[4]{-1}\, z}\, \big((-1)^{3/4} z\big)^{3/2}}$$

$$\Bigg(-\frac{(-1)^{3/4} (1 - 4 \nu^2)}{8 z} \Bigg(b - \sqrt{(-1)^{3/4} z}\, \Bigg(\frac{1}{2} \Big((1 + i) (-1)^\nu e^{\sqrt{2}\, z} \pi \big((4 + 4 i) z - i \sqrt{2}\, \sqrt{-i z^2} \big) - 2 e^{i \sqrt{2}\, z} \pi z \Big) +$$

$$4 \Big((-1)^{\nu + \frac{1}{4}} e^{\sqrt{2}\, z} \sqrt{-i z^2} - i e^{i \sqrt{2}\, z} z \Big) \big(\log\big(-\sqrt[4]{-1}\, z\big) - \log(z)\big) \Bigg) +$$

$$\sqrt{-\sqrt[4]{-1}\, z}\, \Big(\pi \Big(-3 i e^{(1 + i) \sqrt{2}\, z} z - 4 (-1)^\nu z + 3 (-1)^{\nu + \frac{3}{4}} \sqrt{i z^2} \Big) +$$

$$4 \Big(e^{(1 + i) \sqrt{2}\, z} z - (-1)^{\nu + \frac{1}{4}} \sqrt{i z^2} \Big) \big(\log\big((-1)^{3/4} z\big) - \log(z)\big) \Big) \Bigg) \left(1 + O\left(\frac{1}{z^4}\right)\right) -$$

$$\frac{i (16 \nu^4 - 40 \nu^2 + 9)}{128 z^2} \Bigg(\sqrt{(-1)^{3/4} z}\, \Bigg(4 \Big(e^{i \sqrt{2}\, z} z - (-1)^{\nu + \frac{3}{4}} e^{\sqrt{2}\, z} \sqrt{-i z^2} \Big) \big(\log\big(-\sqrt[4]{-1}\, z\big) - \log(z)\big) -$$

$$\frac{\pi}{\sqrt{2}} \Big(\sqrt{2}\, e^{i \sqrt{2}\, z} i z + (-1)^\nu e^{\sqrt{2}\, z} (1 + i) \Big(\sqrt{2}\, (-2 + 2 i) z + \sqrt{-i z^2} \Big) \Big) \Bigg) +$$

$$\sqrt{-\sqrt[4]{-1}\, z}\, \Bigg(\pi \Bigg(\frac{(-1)^\nu \sqrt{i z^2}\, (3 - 3 i)}{\sqrt{2}} - 3 i e^{(1 + i) \sqrt{2}\, z} z + 4 (-1)^\nu z \Bigg) + 4 \Big(e^{(1 + i) \sqrt{2}\, z} z + (-1)^{\nu + \frac{1}{4}} \sqrt{i z^2} \Big)$$

$$\big(\log\big((-1)^{3/4} z\big) - \log(z)\big) \Bigg) \Bigg) \left(1 + O\left(\frac{1}{z^4}\right)\right) + \frac{\sqrt[4]{-1}\, (64 \nu^6 - 560 \nu^4 + 1036 \nu^2 - 225)}{3072 z^3}$$

$$\Bigg(\sqrt{(-1)^{3/4} z}\, \Bigg(\frac{1}{2} \Big((1 + i) (-1)^\nu e^{\sqrt{2}\, z} \pi \big((4 + 4 i) z - i \sqrt{2}\, \sqrt{-i z^2} \big) - 2 e^{i \sqrt{2}\, z} \pi z \Big) +$$

$$4 \Big((-1)^{\nu + \frac{1}{4}} e^{\sqrt{2}\, z} \sqrt{-i z^2} - i e^{i \sqrt{2}\, z} z \Big) \big(\log\big(-\sqrt[4]{-1}\, z\big) - \log(z)\big) \Bigg) +$$

$$\sqrt{-\sqrt[4]{-1}\, z}\, \Big(\pi \Big(-3 i e^{(1 + i) \sqrt{2}\, z} z - 4 (-1)^\nu z + 3 (-1)^{\nu + \frac{3}{4}} \sqrt{i z^2} \Big) +$$

03.19.06.0044.01

$$\operatorname{kei}_\nu(z) \propto \frac{e^{-\frac{(1+i)z}{\sqrt{2}}+\frac{1}{2} i \pi (\nu+1)}}{8\sqrt{2\pi}\sqrt{-\sqrt[4]{-1}\,z}\left((-1)^{3/4} z\right)^{3/2}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{4z^2}\right)^k \left(\frac{\pi}{\sqrt{2}}\left((-1)^{k+\frac{3}{4}}\sqrt{2}\left(4(-1)^\nu - 3 i e^{(1+i)\sqrt{2}\,z}\right)\left(-\sqrt[4]{-1}\,z\right)^{3/2} + 3(-1)^{k+\nu}(1-i)\sqrt{i z^2}\sqrt{-\sqrt[4]{-1}\,z} - \sqrt{(-1)^{3/4} z}\left(\sqrt{2}\, e^{i\sqrt{2}\,z}(-i) z - (1+i)(-1)^\nu e^{\sqrt{2}\,z}\left(2\sqrt{2}(-1+i) z + \sqrt{-i z^2}\right)\right)\right) - 4\sqrt{(-1)^{3/4} z}\left(e^{i\sqrt{2}\,z} z - (-1)^{\nu+\frac{3}{4}} e^{\sqrt{2}\,z}\sqrt{-i z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right) + 4(-1)^k\sqrt{-\sqrt[4]{-1}\,z}\left(e^{(1+i)\sqrt{2}\,z} z + (-1)^{\nu+\frac{1}{4}}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\right) - \frac{(-1)^{3/4}}{2z}\sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{4z^2}\right)^k \left(\frac{(1+i)\pi}{2}\left((-1)^{k+\frac{3}{4}}\left(4(-1)^\nu + 3 i e^{(1+i)\sqrt{2}\,z}\right)(-1+i)\left(-\sqrt[4]{-1}\,z\right)^{3/2} + 3(-1)^{k+\nu+\frac{3}{4}}(1-i)\sqrt{i z^2}\sqrt{-\sqrt[4]{-1}\,z} - \sqrt{(-1)^{3/4} z}\left(e^{i\sqrt{2}\,z}(-1+i) z + (-1)^\nu e^{\sqrt{2}\,z}\left(4(1+i) z - i\sqrt{2}\sqrt{-i z^2}\right)\right)\right) - 4\sqrt{(-1)^{3/4} z}\left((-1)^{\nu+\frac{1}{4}} e^{\sqrt{2}\,z}\sqrt{-i z^2} - i e^{i\sqrt{2}\,z} z\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right) + 4(-1)^k\sqrt{-\sqrt[4]{-1}\,z}\left(e^{(1+i)\sqrt{2}\,z} z - (-1)^{\nu+\frac{1}{4}}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\right) + \ldots\right) /;\ (|z|\to\infty)\land \nu\in\mathbb{Z}\land n\in\mathbb{N}$$

03.19.06.0045.01

$$\operatorname{kei}_\nu(z) \propto \frac{e^{\frac{\pi i(\nu+1)}{2}-\frac{(1+i)z}{\sqrt{2}}}}{8\sqrt{2\pi}\sqrt{-\sqrt[4]{-1}\,z}\left((-1)^{3/4} z\right)^{3/2}}\left(\left(\left(-\sqrt{(-1)^{3/4} z}\left(4\left(e^{i\sqrt{2}\,z} z - (-1)^{\nu+\frac{3}{4}} e^{\sqrt{2}\,z}\sqrt{-i z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right) - \frac{\pi}{\sqrt{2}}\left(\sqrt{2}\, e^{i\sqrt{2}\,z} i z + (-1)^\nu e^{\sqrt{2}\,z}(1+i)\left(\sqrt{2}(-2+2i) z + \sqrt{-i z^2}\right)\right)\right) + \sqrt{-\sqrt[4]{-1}\,z}\left(\pi\left(\frac{(-1)^\nu\sqrt{i z^2}(3-3i)}{\sqrt{2}} - 3 i e^{(1+i)\sqrt{2}\,z} z + 4(-1)^\nu z\right) + 4\left(e^{(1+i)\sqrt{2}\,z} z + (-1)^{\nu+\frac{1}{4}}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(1-2\nu), \frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(2\nu+1), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7); \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{16}{z^4}\right) - \frac{(-1)^{3/4}\left(1-4\nu^2\right)}{8z}\left(\sqrt{-\sqrt[4]{-1}\,z}\left(\pi\left(-3 i e^{(1+i)\sqrt{2}\,z} z - 4(-1)^\nu z + 3(-1)^{\nu+\frac{3}{4}}\sqrt{i z^2}\right) + \right.\right.\right.$$

$$4\left(e^{(1+i)\sqrt{2}\,z}\,z-(-1)^{\nu+\frac{1}{4}}\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)-$$

$$\sqrt{(-1)^{3/4}\,z}\,\left(\frac{1}{2}\left((1+i)\,(-1)^{\nu}\,e^{\sqrt{2}\,z}\,\pi\left((4+4\,i)\,z-i\,\sqrt{2}\,\sqrt{-i\,z^2}\right)-2\,e^{i\sqrt{2}\,z}\,\pi\,z\right)+\right.$$

$$\left.4\left((-1)^{\nu+\frac{1}{4}}\,e^{\sqrt{2}\,z}\,\sqrt{-i\,z^2}-i\,e^{i\sqrt{2}\,z}\,z\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)\right)\right)\,{}_8F_3\left(\frac{1}{8}\,(3-2\,\nu),\,\frac{1}{8}\,(5-2\,\nu),\right.$$

$$\left.\frac{1}{8}\,(7-2\,\nu),\,\frac{1}{8}\,(9-2\,\nu),\,\frac{1}{8}\,(2\,\nu+3),\,\frac{1}{8}\,(2\,\nu+5),\,\frac{1}{8}\,(2\,\nu+7),\,\frac{1}{8}\,(2\,\nu+9);\,\frac{1}{2},\,\frac{3}{4},\,\frac{5}{4};\,-\frac{16}{z^4}\right)-$$

$$\frac{i\left(16\,\nu^4-40\,\nu^2+9\right)}{128\,z^2}\left(\sqrt{(-1)^{3/4}\,z}\,\left(4\left(e^{i\sqrt{2}\,z}\,z-(-1)^{\nu+\frac{3}{4}}\,e^{\sqrt{2}\,z}\,\sqrt{-i\,z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)-\right.\right.$$

$$\left.\frac{\pi}{\sqrt{2}}\left(\sqrt{2}\,e^{i\sqrt{2}\,z}\,i\,z+(-1)^{\nu}\,e^{\sqrt{2}\,z}\,(1+i)\left(\sqrt{2}\,(-2+2\,i)\,z+\sqrt{-i\,z^2}\right)\right)\right)+$$

$$\sqrt{-\sqrt[4]{-1}\,z}\,\left(\pi\left(\frac{(-1)^{\nu}\,\sqrt{i\,z^2}\,(3-3\,i)}{\sqrt{2}}-3\,i\,e^{(1+i)\sqrt{2}\,z}\,z+4\,(-1)^{\nu}\,z\right)+\right.$$

$$\left.\left.4\left(e^{(1+i)\sqrt{2}\,z}\,z+(-1)^{\nu+\frac{1}{4}}\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)\right)$$

$${}_8F_3\left(\frac{1}{8}\,(5-2\,\nu),\,\frac{1}{8}\,(7-2\,\nu),\,\frac{1}{8}\,(9-2\,\nu),\,\frac{1}{8}\,(11-2\,\nu),\,\frac{1}{8}\,(2\,\nu+5),\,\frac{1}{8}\,(2\,\nu+7),\,\frac{1}{8}\,(2\,\nu+9),\right.$$

$$\left.\frac{1}{8}\,(2\,\nu+11);\,\frac{3}{4},\,\frac{5}{4},\,\frac{3}{2};\,-\frac{16}{z^4}\right)+\frac{\sqrt[4]{-1}\,\left(64\,\nu^6-560\,\nu^4+1036\,\nu^2-225\right)}{3072\,z^3}$$

$$\left(\sqrt{(-1)^{3/4}\,z}\,\left(\frac{1}{2}\left((1+i)\,(-1)^{\nu}\,e^{\sqrt{2}\,z}\,\pi\left((4+4\,i)\,z-i\,\sqrt{2}\,\sqrt{-i\,z^2}\right)-2\,e^{i\sqrt{2}\,z}\,\pi\,z\right)+\right.\right.$$

$$\left.4\left((-1)^{\nu+\frac{1}{4}}\,e^{\sqrt{2}\,z}\,\sqrt{-i\,z^2}-i\,e^{i\sqrt{2}\,z}\,z\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)\right)+$$

$$\sqrt{-\sqrt[4]{-1}\,z}\,\left(\pi\left(-3\,i\,e^{(1+i)\sqrt{2}\,z}\,z-4\,(-1)^{\nu}\,z+3\,(-1)^{\nu+\frac{3}{4}}\sqrt{i\,z^2}\right)+\right.$$

$$\left.\left.4\left(e^{(1+i)\sqrt{2}\,z}\,z-(-1)^{\nu+\frac{1}{4}}\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)\right)$$

$${}_8F_3\left(\frac{1}{8}\,(7-2\,\nu),\,\frac{1}{8}\,(9-2\,\nu),\,\frac{1}{8}\,(11-2\,\nu),\,\frac{1}{8}\,(13-2\,\nu),\,\frac{1}{8}\,(2\,\nu+7),\,\frac{1}{8}\,(2\,\nu+9),\,\frac{1}{8}\,(2\,\nu+11),\right.$$

$$\left.\left.\frac{1}{8}\,(2\,\nu+13);\,\frac{5}{4},\,\frac{3}{2},\,\frac{7}{4};\,-\frac{16}{z^4}\right)\right)\,/;\,(|z|\to\infty)\land\nu\in\mathbb{Z}$$

03.19.06.0046.01

$$\operatorname{kei}_\nu(z) \propto \frac{1}{8\sqrt{2\pi}} \Bigg(e^{\frac{z}{\sqrt{2}}} \Bigg(\frac{e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(\frac{4\,i\sqrt{-i z^2}\left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z)\right)}{z} + \frac{\pi\sqrt{-i z^2}}{z} + 4(-1)^{3/4}\pi \right) + \frac{e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}}{\sqrt{(-1)^{3/4} z}} \left(3(-1)^{-3/4}\pi - 4(-1)^{3/4}\left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\right) \Bigg) + e^{-\frac{z}{\sqrt{2}}} \Bigg(\frac{e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \left(-4(-1)^{3/4}\pi - \frac{3\pi\, i\sqrt{i z^2}}{z} + \frac{4\left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\sqrt{i z^2}}{z} \right) + \frac{e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(4(-1)^{3/4}\left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z)\right) + \sqrt[4]{-1}\,\pi\right) \Bigg) \Bigg) \left(1 + O\left(\frac{1}{z}\right)\right) /;\ (|z| \to \infty) \land \nu \in \mathbb{Z}$$

03.19.06.0047.01

$$\operatorname{kei}_\nu(z) \propto \begin{cases} \frac{\sqrt{\pi}\,(-1)^{5/8}}{2\sqrt{2z}}\left(e^{(-1)^{3/4} z + \frac{i\pi\nu}{2}} + (-1)^{3/4} e^{\frac{3i\pi\nu}{2} - \sqrt[4]{-1}\, z}\right) & \arg(z) \le \frac{\pi}{4} \\ -\frac{\sqrt{\pi}\,\sqrt[8]{-1}}{2\sqrt{2z}}\left(2(-1)^{3/4} e^{\sqrt[4]{-1}\, z + \frac{i\pi\nu}{2}} - i\, e^{(-1)^{3/4} z + \frac{i\pi\nu}{2}} + \sqrt[4]{-1}\, e^{\frac{3i\pi\nu}{2} - \sqrt[4]{-1}\, z}\right) & \frac{\pi}{4} < \arg(z) \le \frac{3\pi}{4} \\ -\frac{\sqrt{\pi}\,\sqrt[8]{-1}}{2\sqrt{2z}}\left(2(-1)^{3/4} e^{\sqrt[4]{-1}\, z + \frac{i\pi\nu}{2}} - i\, e^{(-1)^{3/4} z + \frac{i\pi\nu}{2}} + \sqrt[4]{-1}\, e^{\frac{3i\pi\nu}{2} - \sqrt[4]{-1}\, z} + 2\, e^{\frac{3i\pi\nu}{2} - (-1)^{3/4} z}\right) & \text{True} \end{cases} /;$$

$$(|z| \to \infty) \land \nu \in \mathbb{Z}$$

Residue representations

03.19.06.0048.01

$$\operatorname{kei}_\nu(z) = -\frac{1}{4}\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s}\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(s-\frac{\nu}{4}\right)\Gamma\left(s+\frac{2-\nu}{4}\right)}{\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(1-\frac{\nu}{2}-s\right)}\Gamma\left(\frac{\nu+2}{4}+s\right)\right)\left(-j-\frac{\nu+2}{4}\right) - \frac{1}{4}\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s}\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(s-\frac{\nu}{4}\right)\Gamma\left(s+\frac{\nu+2}{4}\right)}{\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(1-\frac{\nu}{2}-s\right)}\Gamma\left(s+\frac{2-\nu}{4}\right)\right)\left(-j-\frac{2-\nu}{4}\right) - \frac{1}{4}\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s}\Gamma\left(s-\frac{\nu}{4}\right)\Gamma\left(\frac{\nu+2}{4}+s\right)\Gamma\left(s+\frac{2-\nu}{4}\right)}{\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(1-\frac{\nu}{2}-s\right)}\Gamma\left(s+\frac{\nu}{4}\right)\right)\left(-j-\frac{\nu}{4}\right) - \frac{1}{4}\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s}\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(\frac{\nu+2}{4}+s\right)\Gamma\left(s+\frac{2-\nu}{4}\right)}{\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(1-\frac{\nu}{2}-s\right)}\Gamma\left(s-\frac{\nu}{4}\right)\right)\left(-j+\frac{\nu}{4}\right) /;\ \nu \notin \mathbb{Z}$$

Integral representations

On the real axis

Contour integral representations

03.19.07.0001.01

$$\operatorname{kei}_\nu(z) == -\frac{1}{8\pi i}\int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(s-\frac{\nu}{4}\right)\Gamma\left(\frac{\nu+2}{4}+s\right)\Gamma\left(s+\frac{2-\nu}{4}\right)}{\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(1-\frac{\nu}{2}-s\right)}\left(\frac{z}{4}\right)^{-4s} ds$$

Limit representations

Generating functions

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.19.13.0001.01

$$w^{(4)}(z)\,z^4 + 2\,w^{(3)}(z)\,z^3 - \left(2\nu^2+1\right)w''(z)\,z^2 + \left(2\nu^2+1\right)w'(z)\,z + \left(z^4+\nu^4-4\nu^2\right)w(z) = 0\;/;\; w(z) = \operatorname{ber}_\nu(z)\,c_1 + \operatorname{bei}_\nu(z)\,c_2 + \operatorname{ker}_\nu(z)\,c_3 + \operatorname{kei}_\nu(z)\,c_4$$

03.19.13.0002.01

$$W_z(\operatorname{ber}_\nu(z), \operatorname{bei}_\nu(z), \operatorname{ker}_\nu(z), \operatorname{kei}_\nu(z)) == -\frac{1}{z^2}$$

03.19.13.0003.01

$$g(z)^4\, g'(z)^3\, w^{(4)}(z) + 2\,g(z)^3\left(g'(z)^2 - 3\,g(z)\,g''(z)\right)g'(z)^2\, w^{(3)}(z) - g(z)^2\left(\left(2\nu^2+1\right)g'(z)^4 + 6\,g(z)\,g''(z)\,g'(z)^2 + 4\,g(z)^2\,g^{(3)}(z)\,g'(z) - 15\,g(z)^2\,g''(z)^2\right)g'(z)\,w''(z) + g(z)\left(\left(2\nu^2+1\right)g'(z)^6 + \left(2\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4 - 2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3 + g(z)^2\left(6\,g''(z)^2 - g(z)\,g^{(4)}(z)\right)g'(z)^2 + 10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z) - 15\,g(z)^3\,g''(z)^3\right)w'(z) + \left(\nu^4 - 4\nu^2 + g(z)^4\right)g'(z)^7\,w(z) = 0\;/;\; w(z) = c_1\operatorname{ber}_\nu(g(z)) + c_2\operatorname{bei}_\nu(g(z)) + c_3\operatorname{ker}_\nu(g(z)) + c_4\operatorname{kei}_\nu(g(z))$$

03.19.13.0004.01

$$W_z(\operatorname{ber}_\nu(g(z)), \operatorname{bei}_\nu(g(z)), \operatorname{ker}_\nu(g(z)), \operatorname{kei}_\nu(g(z))) == -\frac{g'(z)^6}{g(z)^2}$$

03.19.13.0005.01

$$g(z)^4\, g'(z)^3\, h(z)^4\, w^{(4)}(z) + 2\, g(z)^3\, g'(z)^2 \left(h(z) \left(g'(z)^2 - 3\, g(z)\, g''(z)\right) - 2\, g(z)\, g'(z)\, h'(z)\right) h(z)^3\, w^{(3)}(z) + g(z)^2\, g'(z) \left(-\left(\left(2\, \nu^2 + 1\right) g'(z)^4 + 6\, g(z)\, g''(z)\, g'(z)^2 + 4\, g(z)^2\, g^{(3)}(z)\, g'(z) - 15\, g(z)^2\, g''(z)^2\right) h(z)^2 - 6\, g(z)\, g'(z) \left(h'(z)\, g'(z)^2 + g(z)\, h''(z)\, g'(z) - 3\, g(z)\, h'(z)\, g''(z)\right) h(z) + 12\, g(z)^2\, g'(z)^2\, h'(z)^2\right) h(z)^2\, w''(z) + g(z) \left(\left(\left(\left(2\, \nu^2 + 1\right) g'(z)^6 + \left(2\, \nu^2 + 1\right) g(z)\, g''(z)\, g'(z)^4 - 2\, g(z)^2\, g^{(3)}(z)\, g'(z)^3 + g(z)^2 \left(6\, g''(z)^2 - g(z)\, g^{(4)}(z)\right) g'(z)^2 + 10\, g(z)^3\, g''(z)\, g^{(3)}(z)\, g'(z) - 15\, g(z)^3\, g''(z)^3\right) h(z)^3 + 2\, g(z)\, g'(z) \left(\left(2\, \nu^2 + 1\right) h'(z)\, g'(z)^4 - 3\, g(z)\, h''(z)\, g'(z)^3 - 2\, g(z) \left(g(z)\, h^{(3)}(z) - 3\, h'(z)\, g''(z)\right) g'(z)^2 + g(z)^2 \left(9\, g''(z)\, h''(z) + 4\, h'(z)\, g^{(3)}(z)\right) g'(z) - 15\, g(z)^2\, h'(z)\, g''(z)^2\right) h(z)^2 + 12\, g(z)^2\, g'(z)^2\, h'(z) \left(h'(z)\, g'(z)^2 + 2\, g(z)\, h''(z)\, g'(z) - 3\, g(z)\, h'(z)\, g''(z)\right) h(z) - 24\, g(z)^3\, g'(z)^3\, h'(z)^3\right) h(z)\, w'(z) + \left(\left(\nu^4 - 4\, \nu^2 + g(z)^4\right) h(z)^4\, g'(z)^7 + g(z)^4 \left(24\, h'(z)^4 - 36\, h(z)\, h''(z)\, h'(z)^2 + 8\, h(z)^2\, h^{(3)}(z)\, h'(z) + h(z)^2 \left(6\, h''(z)^2 - h(z)\, h^{(4)}(z)\right)\right) g'(z)^3 - 2\, g(z)^3\, h(z) \left(g'(z)^2 - 3\, g(z)\, g''(z)\right) \left(6\, h'(z)^3 - 6\, h(z)\, h''(z)\, h'(z) + h(z)^2\, h^{(3)}(z)\right) g'(z)^2 + g(z)^2\, h(z)^2 \left(h(z)\, h''(z) - 2\, h'(z)^2\right) \left(\left(2\, \nu^2 + 1\right) g'(z)^4 + 6\, g(z)\, g''(z)\, g'(z)^2 + 4\, g(z)^2\, g^{(3)}(z)\, g'(z) - 15\, g(z)^2\, g''(z)^2\right) g'(z) - g(z)\, h(z)^3\, h'(z) \left(\left(2\, \nu^2 + 1\right) g'(z)^6 + \left(2\, \nu^2 + 1\right) g(z)\, g''(z)\, g'(z)^4 - 2\, g(z)^2\, g^{(3)}(z)\, g'(z)^3 + g(z)^2 \left(6\, g''(z)^2 - g(z)\, g^{(4)}(z)\right) g'(z)^2 + 10\, g(z)^3\, g''(z)\, g^{(3)}(z)\, g'(z) - 15\, g(z)^3\, g''(z)^3\right)\right) w(z) = 0 \;/;\; w(z) = c_1\, h(z)\, \mathrm{ber}_\nu(g(z)) + c_2\, h(z)\, \mathrm{bei}_\nu(g(z)) + c_3\, h(z)\, \mathrm{ker}_\nu(g(z)) + c_4\, h(z)\, \mathrm{kei}_\nu(g(z))$$

03.19.13.0006.01

$$W_z(h(z)\, \mathrm{ber}_\nu(g(z)),\, h(z)\, \mathrm{bei}_\nu(g(z)),\, h(z)\, \mathrm{ker}_\nu(g(z)),\, h(z)\, \mathrm{kei}_\nu(g(z))) = -\frac{h(z)^4\, g'(z)^6}{g(z)^2}$$

03.19.13.0007.01

$$z^4\, w^{(4)}(z) + (6 - 4\, r - 4\, s)\, z^3\, w^{(3)}(z) + \left(7 - 2\left(\nu^2 - 2\right) r^2 + 12\, (s-1)\, r + 6\, (s-2)\, s\right) z^2\, w''(z) + (2\, r + 2\, s - 1) \left(2\, r^2\, \nu^2 - 2\, (s-1)\, s + r\, (2 - 4\, s) - 1\right) z\, w'(z) + \left(\left(a^4\, z^{4\, r} + \nu^4 - 4\, \nu^2\right) r^4 - 4\, s\, \nu^2\, r^3 - 2\, s^2 \left(\nu^2 - 2\right) r^2 + 4\, s^3\, r + s^4\right) w(z) = 0 \;/;\; w(z) = c_1\, z^s\, \mathrm{ber}_\nu(a\, z^r) + c_2\, z^s\, \mathrm{bei}_\nu(a\, z^r) + c_3\, z^s\, \mathrm{ker}_\nu(a\, z^r) + c_4\, z^s\, \mathrm{kei}_\nu(a\, z^r)$$

03.19.13.0008.01

$$W_z(z^s\, \mathrm{ber}_\nu(a\, z^r),\, z^s\, \mathrm{bei}_\nu(a\, z^r),\, z^s\, \mathrm{ker}_\nu(a\, z^r),\, z^s\, \mathrm{kei}_\nu(a\, z^r)) = -a^4\, r^6\, z^{4\, r + 4\, s - 6}$$

03.19.13.0009.01

$$w^{(4)}(z) - 4\, (\log(r) + \log(s))\, w^{(3)}(z) + 2\left(-\left(\nu^2 - 2\right) \log^2(r) + 6 \log(s) \log(r) + 3 \log^2(s)\right) w''(z) + 4\, (\log(r) + \log(s)) \left(\nu^2 \log^2(r) - 2 \log(s) \log(r) - \log^2(s)\right) w'(z) + \left(\left(a^4\, r^{4\, z} + \nu^4 - 4\, \nu^2\right) \log^4(r) - 4\, \nu^2 \log(s) \log^3(r) - 2\left(\nu^2 - 2\right) \log^2(s) \log^2(r) + 4 \log^3(s) \log(r) + \log^4(s)\right) w(z) = 0 \;/;\; w(z) = c_1\, s^z\, \mathrm{ber}_\nu(a\, r^z) + c_2\, s^z\, \mathrm{bei}_\nu(a\, r^z) + c_3\, s^z\, \mathrm{ker}_\nu(a\, r^z) + c_4\, s^z\, \mathrm{kei}_\nu(a\, r^z)$$

03.19.13.0010.01

$$W_z(s^z\, \mathrm{ber}_\nu(a\, r^z),\, s^z\, \mathrm{bei}_\nu(a\, r^z),\, s^z\, \mathrm{ker}_\nu(a\, r^z),\, s^z\, \mathrm{kei}_\nu(a\, r^z)) = -a^4\, r^{4\, z}\, s^{4\, z} \log^6(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.19.16.0001.01

$$\mathrm{kei}_\nu(-z) = (-z)^\nu\, \mathrm{kei}_\nu(z)\, z^{-\nu} + \frac{1}{2}\, \pi\, ((-z)^{-\nu}\, z^\nu - (-z)^\nu\, z^{-\nu})\, \csc(\pi\, \nu)\, \mathrm{bei}_{-\nu}(z) \;/;\; \nu \notin \mathbb{Z}$$

03.19.16.0002.01

$$\mathrm{kei}_\nu(-z) = (-1)^\nu\, \mathrm{kei}_\nu(z) + (-1)^\nu\, \mathrm{bei}_\nu(z)\, (\log(z) - \log(-z)) \;/;\; \nu \in \mathbb{Z}$$

03.19.16.0003.01

$$\operatorname{kei}_\nu(i z) = \frac{1}{2}\pi\csc(\pi\nu)\left((i z)^\nu z^{-\nu}\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{bei}_\nu(z) - \operatorname{ber}_\nu(z)\sin\left(\frac{\pi\nu}{2}\right)\right) - (i z)^{-\nu} z^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_{-\nu}(z) + \operatorname{ber}_{-\nu}(z)\sin\left(\frac{3\pi\nu}{2}\right)\right)\right) /; \nu \notin \mathbb{Z}$$

03.19.16.0004.01

$$\operatorname{kei}_\nu(i z) = -i^\nu\left(\cos\left(\frac{\nu\pi}{2}\right)\operatorname{kei}_\nu(z) + \sin\left(\frac{\nu\pi}{2}\right)\operatorname{ker}_\nu(z)\right) - \frac{1}{2} i^\nu\left(\pi\cos\left(\frac{\nu\pi}{2}\right) + 2(\log(z) - \log(i z))\sin\left(\frac{\nu\pi}{2}\right)\right)\operatorname{ber}_\nu(z) + \frac{1}{2} i^\nu\left(2\cos\left(\frac{\nu\pi}{2}\right)(\log(i z) - \log(z)) + \pi\sin\left(\frac{\nu\pi}{2}\right)\right)\operatorname{bei}_\nu(z) /; \nu \in \mathbb{Z}$$

03.19.16.0005.01

$$\operatorname{ker}_\nu(-i z) = \frac{1}{2}\pi\csc(\pi\nu)\left((-i z)^\nu z^{-\nu}\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{bei}_\nu(z) - \operatorname{ber}_\nu(z)\sin\left(\frac{\pi\nu}{2}\right)\right) - (-i z)^{-\nu} z^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_{-\nu}(z) + \operatorname{ber}_{-\nu}(z)\sin\left(\frac{3\pi\nu}{2}\right)\right)\right) /; \nu \notin \mathbb{Z}$$

03.19.16.0006.01

$$\operatorname{kei}_\nu(-i z) = -\frac{1}{2} i^\nu\left((-1)^\nu\pi\cos\left(\frac{\pi\nu}{2}\right) + 2\left((-1)^\nu\log(z) + \log(-i z) - (1 + (-1)^\nu)\log(i z)\right)\sin\left(\frac{\pi\nu}{2}\right)\right)\operatorname{ber}_\nu(z) + \frac{1}{2} i^\nu\left(2\cos\left(\frac{\pi\nu}{2}\right)\left(-(-1)^\nu\log(z) + \log(-i z) + (-1 + (-1)^\nu)\log(i z)\right) + (-1)^\nu\pi\sin\left(\frac{\pi\nu}{2}\right)\right)\operatorname{bei}_\nu(z) - (-i)^\nu\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{kei}_\nu(z) + \operatorname{ker}_\nu(z)\sin\left(\frac{\pi\nu}{2}\right)\right) /; \nu \in \mathbb{Z}$$

03.19.16.0007.01

$$\operatorname{kei}_\nu\left(\frac{1}{\sqrt[4]{-1}} z\right) = \frac{1}{2}\pi\csc(\pi\nu)\left(\left(\frac{1}{\sqrt[4]{-1}} z\right)^\nu\left(\sqrt[4]{-1}\, z\right)^{-\nu}\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) - \sin\left(\frac{\pi\nu}{2}\right)\operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right)\right) - \left(\frac{1}{\sqrt[4]{-1}} z\right)^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_{-\nu}\left(\sqrt[4]{-1}\, z\right) + \operatorname{ber}_{-\nu}\left(\sqrt[4]{-1}\, z\right)\sin\left(\frac{3\pi\nu}{2}\right)\right)\right) /; \nu \notin \mathbb{Z}$$

03.19.16.0008.01

$$\operatorname{kei}_\nu\left(\frac{1}{\sqrt[4]{-1}} z\right) = -(-i)^\nu\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{kei}_\nu\left(\sqrt[4]{-1}\, z\right) + \operatorname{ker}_\nu\left(\sqrt[4]{-1}\, z\right)\sin\left(\frac{\pi\nu}{2}\right)\right) + \frac{1}{2} i^\nu\left(2\cos\left(\frac{\pi\nu}{2}\right)\left(-(-1)^\nu\log\left(\sqrt[4]{-1}\, z\right) + \log\left(-(-1)^{3/4} z\right) + (-1 + (-1)^\nu)\log\left((-1)^{3/4} z\right)\right) + (-1)^\nu\pi\sin\left(\frac{\pi\nu}{2}\right)\right)\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{2} i^\nu\left((-1)^\nu\pi\cos\left(\frac{\pi\nu}{2}\right) + 2\left((-1)^\nu\log\left(\sqrt[4]{-1}\, z\right) + \log\left(-(-1)^{3/4} z\right) - (1 + (-1)^\nu)\log\left((-1)^{3/4} z\right)\right)\sin\left(\frac{\pi\nu}{2}\right)\right)\operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) /; \nu \in \mathbb{Z}$$

03.19.16.0009.01

$$\operatorname{kei}_\nu\left((-1)^{-3/4} z\right) = \left((-1)^{-3/4} z\right)^\nu\operatorname{kei}_\nu\left(\sqrt[4]{-1}\, z\right)\left(\sqrt[4]{-1}\, z\right)^{-\nu} + \frac{1}{2}\pi\left(\left((-1)^{-3/4} z\right)^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu - \left((-1)^{-3/4} z\right)^\nu\left(\sqrt[4]{-1}\, z\right)^{-\nu}\right)\csc(\pi\nu)\operatorname{bei}_{-\nu}\left(\sqrt[4]{-1}\, z\right) /; \nu \notin \mathbb{Z}$$

03.19.16.0010.01

$$\operatorname{kei}_\nu\left((-1)^{-3/4} z\right) = (-1)^\nu\operatorname{kei}_\nu\left(\sqrt[4]{-1}\, z\right) + (-1)^\nu\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right)\left(\log\left(\sqrt[4]{-1}\, z\right) - \log\left(-\sqrt[4]{-1}\, z\right)\right) /; \nu \in \mathbb{Z}$$

03.19.16.0011.01

$$\operatorname{kei}_\nu\left((-1)^{3/4} z\right) = \frac{1}{2}\pi \csc(\pi\nu)\left(\left((-1)^{3/4} z\right)^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu}\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) - \sin\left(\frac{\pi\nu}{2}\right)\operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right)\right) - \left((-1)^{3/4} z\right)^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_{-\nu}\left(\sqrt[4]{-1}\, z\right) + \operatorname{ber}_{-\nu}\left(\sqrt[4]{-1}\, z\right)\sin\left(\frac{3\pi\nu}{2}\right)\right)\right) /;\ \nu \notin \mathbb{Z}$$

03.19.16.0012.01

$$\operatorname{kei}_\nu\left((-1)^{3/4} z\right) = \frac{1}{2} i^\nu \left(2\cos\left(\frac{\pi\nu}{2}\right)\left(\log\left((-1)^{3/4} z\right) - \log\left(\sqrt[4]{-1}\, z\right)\right) + \pi\sin\left(\frac{\pi\nu}{2}\right)\right)\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) - i^\nu\left(\cos\left(\frac{\pi\nu}{2}\right)\operatorname{kei}_\nu\left(\sqrt[4]{-1}\, z\right) + \sin\left(\frac{\pi\nu}{2}\right)\operatorname{ker}_\nu\left(\sqrt[4]{-1}\, z\right)\right) - \frac{1}{2} i^\nu\left(\pi\cos\left(\frac{\pi\nu}{2}\right) + 2\left(\log\left(\sqrt[4]{-1}\, z\right) - \log\left((-1)^{3/4} z\right)\right)\sin\left(\frac{\pi\nu}{2}\right)\right)\operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) /;\ \nu \in \mathbb{Z}$$

03.19.16.0013.01

$$\operatorname{kei}_\nu\left(\sqrt[4]{z^4}\right) = \frac{1}{8}\pi z^{-\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right)\left(2\left(z^2 + \sqrt{z^4}\right)\cot(\pi\nu) + \left(\sqrt{z^4} - z^2\right)\csc\left(\frac{\pi\nu}{2}\right)\right)\operatorname{bei}_\nu(z) + \frac{1}{8}\pi z^{-\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(\left(\sqrt{z^4} - z^2\right)\left(\left(z^4\right)^{\nu/2} + z^{2\nu}\right)\sec\left(\frac{\pi\nu}{2}\right) - 2\left(z^2 + \sqrt{z^4}\right)\left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right)\right)\operatorname{ber}_\nu(z) + \frac{1}{2}\sin\left(\frac{3\pi\nu}{2}\right) z^{\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(\sqrt{z^4} - z^2\right)\operatorname{ker}_\nu(z) + z^{\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(\sqrt{z^4}\cos^2\left(\frac{3\pi\nu}{4}\right) + z^2\sin^2\left(\frac{3\pi\nu}{4}\right)\right)\operatorname{kei}_\nu(z) /;\ \nu \notin \mathbb{Z}$$

03.19.16.0014.01

$$\operatorname{kei}_\nu\left(\sqrt[4]{z^4}\right) = \frac{1}{32} z^{-\nu-2}\left(z^4\right)^{-\frac{\nu}{4}} \left(4\, i\, i^\nu\left(-1 + (-1)^\nu\right)\pi z^{2\nu}\left(\sqrt{z^4} - z^2\right) + \left(\left(2 + i^\nu + e^{\frac{3i\nu\pi}{2}}\right)\sqrt{z^4} - \left(-2 + i^\nu + e^{\frac{3i\nu\pi}{2}}\right)z^2\right)\left(\left(z^4\right)^{\nu/2} + z^{2\nu}\right)\left(4\log(z) - \log\left(z^4\right)\right)\right)$$
$$\operatorname{bei}_\nu(z) + \frac{1}{32} z^{-\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(4\pi\left(z^2\left(z^4\right)^{\nu/2} + \left(z^4\right)^{\frac{\nu+1}{2}} + \left(-1 + i^\nu + e^{\frac{3i\nu\pi}{2}}\right)z^{2\nu}\sqrt{z^4} - \left(1 + i^\nu + e^{\frac{3i\nu\pi}{2}}\right)z^{2\nu+2}\right) - i\, i^\nu\left(-1 + (-1)^\nu\right)\left(\sqrt{z^4} - z^2\right)\left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right)\left(4\log(z) - \log\left(z^4\right)\right)\right)\operatorname{ber}_\nu(z) +$$
$$\frac{1}{8} z^{-\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(\sqrt{z^4} - z^2\right)\left(i\, i^\nu\left(-1 + (-1)^\nu\right)\left(\left(z^4\right)^{\nu/2} + z^{2\nu}\right) + 4 z^{2\nu}\sin\left(\frac{3\nu\pi}{2}\right)\right)\operatorname{ker}_\nu(z) +$$
$$\frac{1}{8} z^{-\nu-2}\left(z^4\right)^{-\frac{\nu}{4}}\left(8\left(\sqrt{z^4}\cos^2\left(\frac{3\nu\pi}{4}\right) + z^2\sin^2\left(\frac{3\nu\pi}{4}\right)\right)z^{2\nu} + \left(\left(2 + i^\nu + e^{\frac{3i\nu\pi}{2}}\right)\sqrt{z^4} - \left(-2 + i^\nu + e^{\frac{3i\nu\pi}{2}}\right)z^2\right)\left(\left(z^4\right)^{\nu/2} - z^{2\nu}\right)\right)\operatorname{kei}_\nu(z) /;\ \nu \in \mathbb{Z}$$

03.19.16.0015.01

$$\operatorname{kei}_{-\nu}(z) = \cos(\pi\nu)\operatorname{kei}_\nu(z) + \sin(\pi\nu)\operatorname{ker}_\nu(z)$$

Addition formulas

03.19.16.0016.01

$$\operatorname{kei}_\nu(z) + i\operatorname{ker}_\nu(z) = \frac{\pi\, i\, \csc(\pi\nu)}{2}\left(\frac{e^{\frac{3i\pi\nu}{4}}\left((-1)^{3/4} z\right)^\nu}{z^\nu} I_{-\nu}\left((-1)^{3/4} z\right) - \frac{e^{\frac{i\pi\nu}{4}} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right)\right)$$

03.19.16.0017.01

$$\operatorname{kei}_\nu(z) + i\operatorname{ker}_\nu(z) = \frac{\pi\, i\, \csc(\pi\nu)}{2}\left(\frac{e^{\frac{3i\pi\nu}{4}}\left((-1)^{3/4} z\right)^\nu}{z^\nu} I_{-\nu}\left((-1)^{3/4} z\right) - \frac{e^{\frac{i\pi\nu}{4}} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right)\right)$$

Multiple arguments

03.19.16.0018.01

$$\operatorname{kei}_\nu(z) + i \operatorname{ker}_\nu(z) = \frac{\pi i \csc(\pi \nu)}{2} \left(\frac{e^{\frac{3 i \pi \nu}{4}} \left((-1)^{3/4} z\right)^\nu}{z^\nu} I_{-\nu}\left((-1)^{3/4} z\right) - \frac{e^{\frac{i \pi \nu}{4}} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right) \right)$$

Related transformations

Involving $\operatorname{ker}_\nu(z)$

03.19.16.0019.01

$$\operatorname{kei}_\nu(z) + i \operatorname{ker}_\nu(z) = \frac{\pi i \csc(\pi \nu)}{2} \left(\frac{e^{\frac{3 i \pi \nu}{4}} \left((-1)^{3/4} z\right)^\nu}{z^\nu} I_{-\nu}\left((-1)^{3/4} z\right) - \frac{e^{\frac{i \pi \nu}{4}} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right) \right) /; \nu \notin \mathbb{Z}$$

03.19.16.0020.01

$$\operatorname{kei}_\nu(z) + i \operatorname{ker}_\nu(z) = \frac{1}{4} (-(i (-1)^\nu)) \left(2 \pi Y_\nu\left(\sqrt[4]{-1}\, z\right) + J_\nu\left(\sqrt[4]{-1}\, z\right) \left(-i \pi + 4 \log(z) - 4 \log\left(\sqrt[4]{-1}\, z\right)\right)\right) /; \nu \in \mathbb{Z}$$

03.19.16.0021.01

$$\operatorname{kei}_\nu(z) - i \operatorname{ker}_\nu(z) = \frac{\pi i \csc(\pi \nu)}{2} \left(\frac{z^\nu}{e^{\frac{i \pi \nu}{4}} \left(\sqrt[4]{-1}\, z\right)^\nu} I_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{e^{\frac{1}{4}(-3) i \pi \nu} \left(\sqrt[4]{-1}\, z\right)^\nu}{z^\nu} I_{-\nu}\left(\sqrt[4]{-1}\, z\right) \right) /; \nu \notin \mathbb{Z}$$

03.19.16.0022.01

$$\operatorname{kei}_\nu(z) - i \operatorname{ker}_\nu(z) = (-i)^{\nu+1} K_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{4} i^{\nu+1} (-i \pi - \log(4) - 4 \log(z) + 4 \log((1+i) z)) I_\nu\left(\sqrt[4]{-1}\, z\right) /; \nu \in \mathbb{Z}$$

Identities

Recurrence identities

Consecutive neighbors

03.19.17.0001.01

$$\operatorname{kei}_\nu(z) = -\operatorname{kei}_{\nu+2}(z) - \frac{\sqrt{2}\, (\nu+1)}{z} (\operatorname{kei}_{\nu+1}(z) + \operatorname{ker}_{\nu+1}(z))$$

03.19.17.0002.01

$$\operatorname{kei}_\nu(z) = -\operatorname{kei}_{\nu-2}(z) - \frac{\sqrt{2}\, (\nu-1)}{z} (\operatorname{kei}_{\nu-1}(z) + \operatorname{ker}_{\nu-1}(z))$$

Distant neighbors

Increasing

03.19.17.0003.01

$$\operatorname{kei}_\nu(z) = (\nu+1)_{n-1}\left((n+\nu)\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{(-1)^k (n-k)!\, 2^{n-2k} z^{2k-n}}{k!\,(n-2k)!\,(-n-\nu)_k (\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k-3n)\pi\right)\operatorname{kei}_{n+\nu}(z) + \sin\left(\frac{1}{4}(2k-3n)\pi\right)\operatorname{ker}_{n+\nu}(z)\right) + \sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{(-1)^k (-k+n-1)!\, 2^{-2k+n-1} z^{2k-n+1}}{k!\,(-2k+n-1)!\,(-n-\nu+1)_k (\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k-3n-1)\pi\right)\operatorname{kei}_{n+\nu+1}(z) + \sin\left(\frac{1}{4}(2k-3n-1)\pi\right)\operatorname{ker}_{n+\nu+1}(z)\right)\right) /;\; n \in \mathbb{N}$$

03.19.17.0004.01

$$\operatorname{kei}_\nu(z) = -\operatorname{kei}_{\nu+2}(z) + \frac{\sqrt{2}\,(\nu+1)}{z}\operatorname{kei}_{\nu+3}(z) + \frac{4(\nu+1)(\nu+2)}{z^2}\operatorname{ker}_{\nu+2}(z) + \frac{\sqrt{2}\,(\nu+1)}{z}\operatorname{ker}_{\nu+3}(z)$$

03.19.17.0005.01

$$\operatorname{kei}_\nu(z) = \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2\nu^2+8\nu+6\right)}{z^3}\operatorname{kei}_{\nu+3}(z) + \operatorname{kei}_{\nu+4}(z) + \frac{2\sqrt{2}\,(\nu+2)\left(z^2-2\left(\nu^2+4\nu+3\right)\right)}{z^3}\operatorname{ker}_{\nu+3}(z) - \frac{4(\nu+1)(\nu+2)}{z^2}\operatorname{ker}_{\nu+4}(z)$$

03.19.17.0006.01

$$\operatorname{kei}_\nu(z) = \frac{\left(z^4-16\left(\nu^4+10\nu^3+35\nu^2+50\nu+24\right)\right)}{z^4}\operatorname{kei}_{\nu+4}(z) + \frac{2\sqrt{2}\,(\nu+2)\left(-z^2+2\nu^2+8\nu+6\right)}{z^3}\operatorname{ker}_{\nu+5}(z) - \frac{12(\nu+2)(\nu+3)}{z^2}\operatorname{ker}_{\nu+4}(z) - \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2\nu^2+8\nu+6\right)}{z^3}\operatorname{kei}_{\nu+5}(z)$$

03.19.17.0007.01

$$\operatorname{kei}_\nu(z) = \frac{\sqrt{2}\,(\nu+3)\left(-3z^4-16\left(\nu^2+6\nu+8\right)z^2+16\left(\nu^4+12\nu^3+49\nu^2+78\nu+40\right)\right)}{z^5}\operatorname{kei}_{\nu+5}(z) + \frac{\sqrt{2}\,(\nu+3)\left(-3z^4+16\left(\nu^2+6\nu+8\right)z^2+16\left(\nu^4+12\nu^3+49\nu^2+78\nu+40\right)\right)}{z^5}\operatorname{ker}_{\nu+5}(z) + \frac{12(\nu+2)(\nu+3)}{z^2}\operatorname{ker}_{\nu+6}(z) - \frac{\left(z^4-16\left(\nu^4+10\nu^3+35\nu^2+50\nu+24\right)\right)}{z^4}\operatorname{kei}_{\nu+6}(z)$$

Decreasing

03.19.17.0008.01

$$\operatorname{kei}_\nu(z) = (1-\nu)_{n-1}\left((n-\nu)\sum_{k=0}^{\lfloor \frac{n}{2}\rfloor} \frac{(n-k)!\,(-1)^k\, 2^{n-2k}\, z^{2k-n}}{k!\,(n-2k)!\,(1-\nu)_k\,(\nu-n)_k}\left(\cos\left(\frac{1}{4}(2k+n)\pi\right)\operatorname{kei}_{\nu-n}(z)+\sin\left(\frac{1}{4}(2k+n)\pi\right)\operatorname{ker}_{\nu-n}(z)\right) - \sum_{k=0}^{\lfloor \frac{n-1}{2}\rfloor} \frac{(-k+n-1)!\,(-1)^k\, 2^{-2k+n-1}\, z^{2k-n+1}}{k!\,(-2k+n-1)!\,(1-\nu)_k\,(-n+\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k+n-1)\pi\right)\operatorname{kei}_{-n+\nu-1}(z)+\sin\left(\frac{1}{4}(2k+n-1)\pi\right)\operatorname{ker}_{-n+\nu-1}(z)\right)\right)\,/;\, n\in\mathbb{N}$$

03.19.17.0009.01

$$\operatorname{kei}_\nu(z) = \frac{\sqrt{2}\,(\nu-1)}{z}\operatorname{kei}_{\nu-3}(z) - \operatorname{kei}_{\nu-2}(z) + \frac{\sqrt{2}\,(\nu-1)}{z}\operatorname{ker}_{\nu-3}(z) + \frac{4(\nu-2)(\nu-1)}{z^2}\operatorname{ker}_{\nu-2}(z)$$

03.19.17.0010.01

$$\operatorname{kei}_\nu(z) = \operatorname{kei}_{\nu-4}(z) + \frac{2\sqrt{2}\,(\nu-2)\left(z^2+2\nu^2-8\nu+6\right)}{z^3}\operatorname{kei}_{\nu-3}(z) + \frac{2\sqrt{2}\,(\nu-2)\left(z^2-2\nu^2+8\nu-6\right)}{z^3}\operatorname{ker}_{\nu-3}(z) - \frac{4(\nu-2)(\nu-1)}{z^2}\operatorname{ker}_{\nu-4}(z)$$

03.19.17.0011.01

$$\operatorname{kei}_\nu(z) = -\frac{2\sqrt{2}\,(\nu-2)\left(z^2+2\nu^2-8\nu+6\right)}{z^3}\operatorname{kei}_{\nu-5}(z) + \frac{\left(z^4-16\left(\nu^4-10\nu^3+35\nu^2-50\nu+24\right)\right)}{z^4}\operatorname{kei}_{\nu-4}(z) - \frac{12(\nu-3)(\nu-2)}{z^2}\operatorname{ker}_{\nu-4}(z) - \frac{2\sqrt{2}\,(\nu-2)\left(z^2-2\nu^2+8\nu-6\right)}{z^3}\operatorname{ker}_{\nu-5}(z)$$

03.19.17.0012.01

$$\operatorname{kei}_\nu(z) = -\frac{\left(z^4-16\left(\nu^4-10\nu^3+35\nu^2-50\nu+24\right)\right)}{z^4}\operatorname{kei}_{\nu-6}(z) + \frac{\sqrt{2}\,(\nu-3)\left(-3z^4-16\left(\nu^2-6\nu+8\right)z^2+16\left(\nu^4-12\nu^3+49\nu^2-78\nu+40\right)\right)}{z^5}\operatorname{kei}_{\nu-5}(z) + \frac{12(\nu-3)(\nu-2)}{z^2}\operatorname{ker}_{\nu-6}(z) + \frac{\sqrt{2}\,(\nu-3)\left(-3z^4+16\left(\nu^2-6\nu+8\right)z^2+16\left(\nu^4-12\nu^3+49\nu^2-78\nu+40\right)\right)}{z^5}\operatorname{ker}_{\nu-5}(z)$$

Functional identities

Relations between contiguous functions

03.19.17.0013.01

$$\operatorname{kei}_\nu(z) = \frac{z}{2\sqrt{2}\,\nu}\left(\operatorname{ker}_{\nu-1}(z)+\operatorname{ker}_{\nu+1}(z)-\operatorname{kei}_{\nu-1}(z)-\operatorname{kei}_{\nu+1}(z)\right)$$

Differentiation

Low-order differentiation

With respect to ν

03.19.20.0001.01

$$\operatorname{kei}_\nu^{(1,0)}(z)=\frac{1}{2}\pi\left(2^\nu\csc(\pi\nu)z^{-\nu}\sum_{k=0}^{\infty}\frac{2^{-2k}z^{2k}\psi(k-\nu+1)\sin\left(\frac{1}{4}\pi(2k-3\nu)\right)}{k!\,\Gamma(k-\nu+1)}+2^{-\nu}\csc(\pi\nu)z^\nu\sum_{k=0}^{\infty}\frac{\left(2^{-2k}z^{2k}\psi(k+\nu+1)\right)\sin\left(\frac{1}{4}\pi(2k-\nu)\right)}{k!\,\Gamma(k+\nu+1)}-\frac{3\pi}{4}\csc(\pi\nu)\operatorname{ber}_{-\nu}(z)-\frac{2}{\pi}\left(\pi\cot(\pi\nu)+\log\left(\frac{z}{2}\right)\right)\operatorname{kei}_\nu(z)+\left(\frac{1}{4}\pi\cot(\pi\nu)+2\log\left(\frac{z}{2}\right)\right)\operatorname{ber}_\nu(z)+\frac{1}{4}\left(\pi-8\cot(\pi\nu)\log\left(\frac{z}{2}\right)\right)\operatorname{bei}_\nu(z)\right)\;/;\;\nu\notin\mathbb{Z}$$

03.19.20.0002.01

$$\operatorname{kei}_n^{(1,0)}(z)=-\frac{\pi}{2}\operatorname{ker}_n(z)+\frac{\pi\,n!}{4}\sum_{k=0}^{n-1}\frac{1}{k!\,(n-k)}\left(\frac{z}{2}\right)^{k-n}\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\operatorname{ber}_k(z)-\sin\left(\frac{3}{4}(k-n)\pi\right)\operatorname{bei}_k(z)\right)+\frac{1}{4}(-1)^n\operatorname{bei}_{-n}^{(1,0)}(z)-\frac{1}{4}\operatorname{bei}_n^{(1,0)}(z)\;/;\;n\in\mathbb{N}$$

03.19.20.0003.01

$$\operatorname{kei}_{-n}^{(1,0)}(z)=\frac{(-1)^{n-1}\pi}{2}\operatorname{ker}_n(z)-\frac{1}{4}\left(\pi(-1)^n n!\right)\sum_{k=0}^{n-1}\frac{1}{k!\,(n-k)}\left(\frac{z}{2}\right)^{k-n}\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\operatorname{ber}_k(z)-\sin\left(\frac{3}{4}(k-n)\pi\right)\operatorname{bei}_k(z)\right)-\frac{1}{4}\operatorname{bei}_{-n}^{(1,0)}(z)+\frac{1}{4}(-1)^n\operatorname{bei}_n^{(1,0)}(z)\;/;\;n\in\mathbb{N}$$

03.19.20.0004.01

$$\operatorname{kei}_{n+\frac{1}{2}}^{(1,0)}(z)=\frac{1}{8}\pi\left(\pi\operatorname{bei}_{n+\frac{1}{2}}(z)-3(-1)^n\pi\operatorname{ber}_{-n-\frac{1}{2}}(z)-4\left(\log(z)-\log\left(\sqrt[4]{-1}\,z\right)\right)\left((-1)^n\operatorname{bei}_{-n-\frac{1}{2}}(z)-\operatorname{ber}_{n+\frac{1}{2}}(z)\right)\right)-$$
$$\frac{(-1)^{3/8}2^{-n-\frac{5}{2}}e^{\frac{i n\pi}{4}}\sqrt{\pi}\,z^{-n-\frac{1}{2}}}{n!}e^{-\sqrt[4]{-1}\,z}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2k}\binom{n}{2k}(2n-2k)!\left((-1)^n\sqrt{2}\,(-1+i)\left(\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)+e^{2\sqrt[4]{-1}\,z}\left(\operatorname{Chi}\left((1+i)\sqrt{2}\,z\right)-\operatorname{Shi}\left((1+i)\sqrt{2}\,z\right)\right)\right)+2(-1)^k e^{\frac{i\pi n}{2}+\sqrt{2}\,z}\,i\left(\operatorname{Ci}\left((1+i)\sqrt{2}\,z\right)+e^{2(-1)^{3/4}z}\left(\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)+i\operatorname{Si}\left((1+i)\sqrt{2}\,z\right)\right)\right)i^k z^{2k}-$$
$$\frac{(-1)^{5/8}2^{-n-\frac{1}{2}}e^{\frac{i n\pi}{4}}\sqrt{\pi}\,z^{\frac{1}{2}-n}}{n!}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\left((-1)^{3/4}(-1)^n e^{i(-1)^{3/4}z}\left(-e^{(1+i)\sqrt{2}\,z}\operatorname{Chi}\left((1+i)\sqrt{2}\,z\right)+\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)+e^{2\sqrt[4]{-1}\,z}\operatorname{Shi}\left(2\sqrt[4]{-1}\,z\right)\right)+(-1)^k e^{\frac{i n\pi}{2}}\,i\sin\left(\sqrt[4]{-1}\,z\right)\left(\operatorname{Ci}\left((1+i)\sqrt{2}\,z\right)+\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)+i\operatorname{Si}\left((1+i)\sqrt{2}\,z\right)\right)-(-1)^k e^{\frac{i n\pi}{2}}\cos\left(\sqrt[4]{-1}\,z\right)\left(\operatorname{Ci}\left((1+i)\sqrt{2}\,z\right)-\psi\left(k+\frac{3}{2}\right)+\psi\left(k-n+\frac{1}{2}\right)+i\operatorname{Si}\left((1+i)\sqrt{2}\,z\right)\right)\right)i^k z^{2k}\;/;\;n\in\mathbb{N}$$

03.19.20.0005.01

$$\mathrm{kei}^{(1,0)}_{-n-\frac{1}{2}}(z) = \frac{1}{8}\pi\left(\pi\,\mathrm{bei}_{-n-\frac{1}{2}}(z) + (-1)^n\left(3\pi\,\mathrm{ber}_{n+\frac{1}{2}}(z) + 4\,\mathrm{bei}_{n+\frac{1}{2}}(z)\left(\log(z) - \log\left(\sqrt[4]{-1}\,z\right)\right)\right) + 4\,\mathrm{ber}_{-n-\frac{1}{2}}(z)\left(\log(z) - \log\left(\sqrt[4]{-1}\,z\right)\right)\right) -$$
$$\frac{(-1)^{7/8}\,2^{-n-\frac{3}{2}}\,e^{\frac{i n\pi}{4}}\sqrt{\pi}\,z^{-n-\frac{1}{2}}}{n!}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} 2^{2k}\binom{n}{2k}(2n-2k)!\,i^k$$
$$\left(\frac{1}{\sqrt[4]{-1}}\left(\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{1}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right) - \cosh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) +\right.$$
$$\frac{1}{\sqrt[4]{-1}}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right) + \psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) - \sinh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) +$$
$$(-1)^{k+n}\,e^{\frac{i n\pi}{2}}\,i\left(\cos\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right) + \psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) + \sin\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right) +$$
$$\left.(-1)^{k+n}\,e^{\frac{i n\pi}{2}}\left(\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{1}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right) - \cos\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right) z^{2k} -$$
$$\frac{\sqrt[8]{-1}\,2^{-n-\frac{1}{2}}\,e^{\frac{i n\pi}{4}}\sqrt{\pi}\,z^{\frac{1}{2}-n}}{n!}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} 2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\,i^k$$
$$\left(\frac{1}{\sqrt[4]{-1}}\left(\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right) + \psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right) - \cosh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) +\right.$$
$$\frac{1}{\sqrt[4]{-1}}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right) - \sinh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) +$$
$$(-1)^{k+n}\,e^{\frac{i n\pi}{2}}\left(\cos\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right) + \sin\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right) -$$
$$\left.i\,(-1)^{k+n}\,e^{\frac{i n\pi}{2}}\left(\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right) + \psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right) - \cos\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right) z^{2k} \;/;\; n \in \mathbb{N}$$

With respect to *z*

03.19.20.0006.01

$$\frac{\partial\,\mathrm{kei}_\nu(z)}{\partial z} = \frac{1}{\sqrt{2}\,z}\left(-z\,\mathrm{kei}_{\nu-1}(z) - \sqrt{2}\,\nu\,\mathrm{kei}_\nu(z) + z\,\mathrm{ker}_{\nu-1}(z)\right)$$

03.19.20.0007.01

$$\frac{\partial\,\mathrm{kei}_\nu(z)}{\partial z} = \frac{1}{2\sqrt{2}}\left(-\mathrm{kei}_{\nu-1}(z) + \mathrm{kei}_{\nu+1}(z) + \mathrm{ker}_{\nu-1}(z) - \mathrm{ker}_{\nu+1}(z)\right)$$

03.19.20.0008.01

$$\frac{\partial\left(z^\nu\,\mathrm{kei}_\nu(z)\right)}{\partial z} = \frac{z^\nu}{\sqrt{2}}\left(\mathrm{ker}_{\nu-1}(z) - \mathrm{kei}_{\nu-1}(z)\right)$$

03.19.20.0009.01

$$\frac{\partial\left(z^{-\nu}\,\mathrm{kei}_\nu(z)\right)}{\partial z} = \frac{z^{-\nu}}{\sqrt{2}}\left(\mathrm{kei}_{\nu+1}(z) - \mathrm{ker}_{\nu+1}(z)\right)$$

03.19.20.0010.01

$$\frac{\partial^2 \operatorname{kei}_\nu(z)}{\partial z^2} = \frac{1}{4}\left(-\operatorname{ker}_{\nu-2}(z) + 2\operatorname{ker}_\nu(z) - \operatorname{ker}_{\nu+2}(z)\right)$$

03.19.20.0011.01

$$\frac{\partial^2 \operatorname{kei}_\nu(z)}{\partial z^2} = \frac{\operatorname{kei}_{\nu-1}(z)}{\sqrt{2}\, z} + \frac{(\nu(\nu+1))\operatorname{kei}_\nu(z)}{z^2} + \operatorname{ker}_\nu(z) - \frac{\operatorname{ker}_{\nu-1}(z)}{\sqrt{2}\, z}$$

Symbolic differentiation

With respect to ν

03.19.20.0012.01

$$\operatorname{kei}_\nu^{(m,0)}(z) = \frac{\pi}{2}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z}{2}\right)^{2k}\frac{\partial^m \frac{\left(\frac{z}{2}\right)^{\nu}\cos\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)}}{\partial\nu^m} + \right.$$

$$\sum_{j=0}^{m}\binom{m}{j}\left(\pi^{m-j}(-i)^{-j+m+1}\sum_{i=0}^{m-j}\frac{(-1)^i\, i!\, \mathcal{S}_{m-j}^{(i)}}{2^i}\left(\left(i\cot\left(\frac{\pi\nu}{2}\right)+1\right)^i\left(i\cot\left(\frac{\pi\nu}{2}\right)-1\right) - 2^{m-j}(i\cot(\pi\nu)+1)^i(i\cot(\pi\nu)-1)\right)\right)$$

$$\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z}{2}\right)^{2k}\frac{\partial^j \frac{\left(\frac{z}{2}\right)^{-\nu}\sin\left(\frac{1}{4}\pi(2k-3\nu)\right)}{\Gamma(k-\nu+1)}}{\partial\nu^j} -$$

$$\sum_{j=0}^{m}\binom{m}{j}\pi^{m-j}\left((-i)^{-j+m+1}2^{m-j}(i\cot(\pi\nu)-1)\sum_{i=0}^{m-j}\frac{(-1)^i\, i!\, \mathcal{S}_{m-j}^{(i)}(i\cot(\pi\nu)+1)^i}{2^i} - \delta_{m-j}\, i\right)$$

$$\left.\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z}{2}\right)^{2k}\frac{\partial^j \frac{\left(\frac{z}{2}\right)^{\nu}\sin\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)}}{\partial\nu^j}\right) /;\ \nu \notin \mathbb{Z}$$

With respect to z

03.19.20.0013.01

$$\frac{\partial^n \mathrm{kei}_\nu(z)}{\partial z^n} = z^{-n} \sum_{m=0}^{n} (-1)^{m+n} \binom{n}{m} (-\nu)_{n-m} \sum_{k=0}^{m} \frac{(-1)^k\, 2^{2k-m}\, (-m)_{2(m-k)}\, (\nu)_k}{(m-k)!} \left(\mathrm{kei}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \frac{(-1)^j\, (k-2j)!}{(2j)!\, (k-4j)!\, (-k-\nu+1)_{2j}\, (\nu)_{2j}} \left(\frac{z}{2}\right)^{4j} + \frac{z}{2\sqrt{2}} (\mathrm{kei}_{\nu-1}(z) - \mathrm{ker}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{\left((-1)^j\, (-2j+k-1)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j)!\, (-4j+k-1)!\, (-k-\nu+1)_{2j}\, (\nu)_{2j+1}} - \frac{1}{4} z^2\, \mathrm{ker}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^j\, (-2j+k-1)!}{(2j+1)!\, (-4j+k-2)!\, (-k-\nu+1)_{2j+1}\, (\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} - \frac{z^3}{8\sqrt{2}} (\mathrm{kei}_{\nu-1}(z) + \mathrm{ker}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{k-2}{2} \right\rfloor} \frac{\left((-1)^j\, (-2j+k-2)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j+1)!\, (-4j+k-3)!\, (-k-\nu+1)_{2j+1}\, (\nu)_{2j+2}} \right) /;\ n \in \mathbb{N}$$

03.19.20.0014.01

$$\frac{\partial^n \mathrm{kei}_\nu(z)}{\partial z^n} = -2^{n+2\nu-2}\, i\, e^{\frac{1}{4}(-3) i \pi \nu} \pi^{3/2} \csc(\pi \nu)\, \Gamma(1-\nu)\, z^{-n-\nu}\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-n-\nu+1), \frac{1}{2}(-n-\nu+2), 1-\nu; \frac{i z^2}{4}\right) + 2^{n+2\nu-2}\, e^{\frac{3 i \pi \nu}{4}}\, i\, \pi^{3/2} \csc(\pi \nu)\, \Gamma(1-\nu)\, z^{-n-\nu}\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-n-\nu+1), \frac{1}{2}(-n-\nu+2), 1-\nu; -\frac{1}{4}\left(i z^2\right)\right) + 2^{n-2\nu-2}\, e^{\frac{3 i \pi \nu}{4}}\, i\, \pi^{3/2} (-i + \cot(\pi \nu))\, \Gamma(\nu+1)\, z^{\nu-n}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1; \frac{i z^2}{4}\right) - 2^{n-2\nu-2}\, i\, e^{\frac{1}{4}(-3) i \pi \nu} \pi^{3/2} (i + \cot(\pi \nu))\, \Gamma(\nu+1)\, z^{\nu-n}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1; -\frac{1}{4}\left(i z^2\right)\right) /;\ \nu \notin \mathbb{Z} \land n \in \mathbb{N}$$

03.19.20.0015.01

$$\frac{\partial^n \mathrm{kei}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{2k} \left((1+i^n)\, \mathrm{kei}_{4k-n+\nu}(z) - i\,(1-i^n)\, \mathrm{ker}_{4k-n+\nu}(z)\right) + \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2k+1} \left(i\,(1-i^n)\, \mathrm{ker}_{4k-n+\nu+2}(z) - (1+i^n)\, \mathrm{kei}_{4k-n+\nu+2}(z)\right) \right) /;\ n \in \mathbb{N}$$

03.19.20.0016.01

$$\frac{\partial^n \mathrm{kei}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \left(\frac{n+1}{2k+1} \binom{n}{2k} \left((1+i^n)\, \mathrm{kei}_{4k-n+\nu}(z) + \left(-i + i^{n+1}\right) \mathrm{ker}_{4k-n+\nu}(z)\right) + \frac{\sqrt{2}\,(1+i)(4k-n+\nu+1)}{z} \binom{n}{2k+1} \left(\left(1-i^{n+1}\right) \mathrm{kei}_{4k-n+\nu+1}(z) + (-i+i^n)\, \mathrm{ker}_{4k-n+\nu+1}(z)\right) \right) /;\ n \in \mathbb{N}$$

03.19.20.0017.01

$$\frac{\partial^n \operatorname{kei}_\nu(z)}{\partial z^n} = -\frac{1}{4} G_{5,9}^{4,4}\left(\frac{z}{4}, \frac{1}{4} \middle| \begin{array}{c} -\frac{n}{4}, \frac{1-n}{4}, \frac{2-n}{4}, \frac{3-n}{4}, \frac{1}{4}(-n+2\nu) \\ \frac{1}{4}(-n+\nu+2), \frac{\nu-n}{4}, \frac{1}{4}(-n-\nu+2), \frac{1}{4}(-n-\nu), \frac{1}{4}(-n+2\nu), 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right) /; n \in \mathbb{Z} \land n \geq 3$$

Fractional integro-differentiation

With respect to z

03.19.20.0018.01

$$\frac{\partial^\alpha \operatorname{kei}_\nu(z)}{\partial z^\alpha} = \frac{i\, 2^{\nu-2} e^{-\frac{3 i \pi \nu}{4}} \pi z^{-\alpha-\nu} \csc(\pi \nu)}{\Gamma(1-\alpha-\nu)} \left(e^{\frac{3 i \pi \nu}{2}} {}_2F_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; 1-\nu, \frac{1-\alpha-\nu}{2}, 1-\frac{\alpha+\nu}{2}; -\frac{i z^2}{4}\right) - {}_2F_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; 1-\nu, \frac{1-\alpha-\nu}{2}, 1-\frac{\alpha+\nu}{2}; \frac{i z^2}{4}\right)\right) - \frac{i\, 2^{-\nu-2} e^{-\frac{i \pi \nu}{4}} \pi z^{\nu-\alpha} \csc(\pi \nu)}{\Gamma(1-\alpha+\nu)} \left(e^{\frac{i \pi \nu}{2}} {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \nu+1, \frac{1-\alpha+\nu}{2}, 1-\frac{\alpha-\nu}{2}; -\frac{i z^2}{4}\right) - {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \nu+1, \frac{1-\alpha+\nu}{2}, 1-\frac{\alpha-\nu}{2}; \frac{i z^2}{4}\right)\right) /; \nu \notin \mathbb{Z}$$

03.19.20.0019.01

$$\frac{\partial^\alpha \operatorname{kei}_\nu(z)}{\partial z^\alpha} = 2^{|\nu|-2} z^{-\alpha-|\nu|} \sum_{k=\left\lfloor \frac{|\nu|-1}{2} \right\rfloor+1}^{|\nu|-1} \frac{\left(e^{\frac{1}{4} i \pi (2\nu+|\nu|)} - (-1)^k e^{-\frac{1}{4}(i \pi (2\nu+|\nu|))}\right)(|\nu|-k-1)!\, \Gamma(2k-|\nu|+1)}{k!\, \Gamma(2k-\alpha-|\nu|+1)} \left(\frac{i z^2}{4}\right)^k + (-1)^{|\nu|-1} 2^{|\nu|-2} z^{-\alpha-|\nu|} \sum_{k=0}^{\left\lfloor \frac{|\nu|-1}{2} \right\rfloor} \frac{\left(e^{\frac{1}{4} i \pi (2\nu+|\nu|)} - (-1)^k e^{-\frac{1}{4}(i \pi (2\nu+|\nu|))}\right)(|\nu|-k-1)!\, (\log(z) - \psi(2k-\alpha-|\nu|+1) + \psi(|\nu|-2k))}{k!\, (|\nu|-2k-1)!\, \Gamma(2k-\alpha-|\nu|+1)} \left(\frac{i z^2}{4}\right)^k - 2^{-|\nu|-2} \pi z^{|\nu|-\alpha} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4}\pi (2(k+\nu)+|\nu|)\right) \Gamma(2k+|\nu|+1)}{k!\, (k+|\nu|)!\, \Gamma(2k-\alpha+|\nu|+1)} \left(\frac{z}{2}\right)^{2k} + i^{\nu+|\nu|} 2^{-|\nu|-1} z^{|\nu|-\alpha} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i \pi |\nu|)} - (-1)^k e^{\frac{1}{4} i \pi |\nu|}\right) \mathcal{FC}_{\log}^{(\alpha)}(z, 2k+|\nu|)}{k!\, (k+|\nu|)!} \left(\frac{i z^2}{4}\right)^k + 2^{-|\nu|-2} i^{\nu+|\nu|} z^{|\nu|-\alpha} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i \pi |\nu|)} - (-1)^k e^{\frac{1}{4} i \pi |\nu|}\right) \Gamma(2k+|\nu|+1)\, (2\log(2) + \psi(k+1) + \psi(k+|\nu|+1))}{k!\, (k+|\nu|)!\, \Gamma(2k-\alpha+|\nu|+1)} \left(\frac{i z^2}{4}\right)^k /; \nu \in \mathbb{Z}$$

Integration

Indefinite integration

03.19.21.0001.01

$$\int \operatorname{kei}_\nu(a z)\, dz = -\frac{1}{16} z\, G_{2,6}^{4,1}\left(\frac{a z}{4}, \frac{1}{4} \middle| \begin{array}{c} \frac{3}{4}, \frac{\nu}{2} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}, \frac{\nu}{2} \end{array}\right)$$

Definite integration

03.19.21.0002.01

$$\int_0^\infty t^{\alpha-1}\, e^{-p\,t}\,\mathrm{kei}_\nu(t)\,dt = 2^{-\nu-3}\,p^{-\alpha-\nu}\Bigg(4^\nu\,\Gamma(\alpha-\nu)\,\Gamma(\nu-1)$$
$$\Bigg(-\frac{(\alpha-\nu)(\alpha-\nu+1)\cos\left(\frac{3\pi\nu}{4}\right)}{p^2}\,{}_4F_3\left(\frac{\alpha}{4}+\frac{1}{2}-\frac{\nu}{4},\frac{\alpha}{4}-\frac{\nu}{4}+\frac{3}{4},\frac{\alpha}{4}-\frac{\nu}{4}+1,\frac{\alpha}{4}-\frac{\nu}{4}+\frac{5}{4};\frac{3}{2},1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2};-\frac{1}{p^4}\right)-$$
$$4(\nu-1)\,{}_4F_3\left(\frac{\alpha}{4}-\frac{\nu}{4},\frac{\alpha}{4}-\frac{\nu}{4}+\frac{1}{4},\frac{\alpha}{4}+\frac{1}{2}-\frac{\nu}{4},\frac{\alpha}{4}-\frac{\nu}{4}+\frac{3}{4};\frac{1}{2},\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{1}{p^4}\right)\sin\left(\frac{3\pi\nu}{4}\right)\Bigg)p^{2\nu}+$$
$$\Gamma(-\nu-1)\,\Gamma(\alpha+\nu)\Bigg(4(\nu+1)\,{}_4F_3\left(\frac{\alpha}{4}+\frac{\nu}{4},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{4},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{2},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{3}{4};\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{1}{p^4}\right)\sin\left(\frac{\pi\nu}{4}\right)-$$
$$\frac{(\alpha+\nu)(\alpha+\nu+1)\cos\left(\frac{\pi\nu}{4}\right)}{p^2}\,{}_4F_3\left(\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{2},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{3}{4},\frac{\alpha}{4}+\frac{\nu}{4}+1,\frac{\alpha}{4}+\frac{\nu}{4}+\frac{5}{4};\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{1}{p^4}\right)\Bigg)\Bigg)\;/;$$
$$\mathrm{Re}(\alpha+\nu)>0\bigwedge\mathrm{Re}(\alpha-\nu)>0\bigwedge\mathrm{Re}(p)>-\frac{1}{\sqrt{2}}\bigwedge\nu\notin\mathbb{Z}$$

Integral transforms

Laplace transforms

03.19.22.0001.01

$$\mathcal{L}_t[\mathrm{kei}_\nu(t)](z)=$$
$$2^{-\nu-3}\,\pi\,z^{-\nu-3}\csc(\pi\nu)\Bigg(\left(2^{2\nu+1}-4^\nu\,\nu\right)\cos\left(\frac{3\pi\nu}{4}\right)z^{2\nu}\,{}_4F_3\left(\frac{3}{4}-\frac{\nu}{4},1-\frac{\nu}{4},\frac{5}{4}-\frac{\nu}{4},\frac{3}{2}-\frac{\nu}{4};\frac{3}{2},1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2};-\frac{1}{z^4}\right)+$$
$$4\Bigg(\sin\left(\frac{\pi\nu}{4}\right){}_4F_3\left(\frac{\nu}{4}+\frac{1}{4},\frac{\nu}{4}+\frac{1}{2},\frac{\nu}{4}+\frac{3}{4},\frac{\nu}{4}+1;\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{1}{z^4}\right)-$$
$$4^\nu\,z^{2\nu}\sin\left(\frac{3\pi\nu}{4}\right){}_4F_3\left(\frac{1}{4}-\frac{\nu}{4},\frac{1}{2}-\frac{\nu}{4},\frac{3}{4}-\frac{\nu}{4},1-\frac{\nu}{4};\frac{1}{2},\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{1}{z^4}\right)\Bigg)z^2-$$
$$(\nu+2)\cos\left(\frac{\pi\nu}{4}\right){}_4F_3\left(\frac{\nu}{4}+\frac{3}{4},\frac{\nu}{4}+1,\frac{\nu}{4}+\frac{5}{4},\frac{\nu}{4}+\frac{3}{2};\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{1}{z^4}\right)\Bigg)\;/;\;|\mathrm{Re}(\nu)|<1\bigwedge\mathrm{Re}(z)>-\frac{1}{\sqrt{2}}$$

Mellin transforms

03.19.22.0002.01

$$\mathcal{M}_t[\mathrm{kei}_\nu(t)](z)=-2^{z-2}\,\Gamma\left(\frac{z-\nu}{2}\right)\Gamma\left(\frac{z+\nu}{2}\right)\sin\left(\frac{1}{4}\pi(z+2\nu)\right)\;/;\;\mathrm{Re}(z+\nu)>0\land\mathrm{Re}(z-\nu)>0$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

03.19.26.0001.01

$$\operatorname{kei}_\nu(z) = 2^{-2\nu-5}\pi^2 \csc(\pi\nu) \left(2^{4\nu}\cos\left(\frac{3\pi\nu}{4}\right) z^{2-\nu}\, {}_0\tilde{F}_3\left(;\frac{3}{2}, \frac{3-\nu}{2}, 1-\frac{\nu}{2}; -\frac{z^4}{256}\right) - 2^{4\nu+4}\sin\left(\frac{3\pi\nu}{4}\right) z^{-\nu}\, {}_0\tilde{F}_3\left(;\frac{1}{2}, \frac{1-\nu}{2}, 1-\frac{\nu}{2}; -\frac{z^4}{256}\right) + 16\sin\left(\frac{\pi\nu}{4}\right) z^{\nu}\, {}_0\tilde{F}_3\left(;\frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu+2}{2}; -\frac{z^4}{256}\right) - \cos\left(\frac{\pi\nu}{4}\right) z^{\nu+2}\, {}_0\tilde{F}_3\left(;\frac{3}{2}, \frac{\nu+3}{2}, \frac{\nu+2}{2}; -\frac{z^4}{256}\right)\right) /; \neg\, \nu \in \mathbb{Z}$$

Involving ${}_pF_q$

03.19.26.0002.01

$$\operatorname{kei}_\nu(z) == -2^{\nu-3}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu-1)\, z^{2-\nu}\, {}_0F_3\left(;\frac{3}{2}, \frac{3-\nu}{2}, 1-\frac{\nu}{2}; -\frac{z^4}{256}\right) - 2^{\nu-1}\Gamma(\nu)\sin\left(\frac{3\pi\nu}{4}\right) z^{-\nu}\, {}_0F_3\left(;\frac{1}{2}, \frac{1-\nu}{2}, 1-\frac{\nu}{2}; -\frac{z^4}{256}\right) - 2^{-\nu-1}\Gamma(-\nu)\sin\left(\frac{\pi\nu}{4}\right) z^{\nu}\, {}_0F_3\left(;\frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu+2}{2}; -\frac{z^4}{256}\right) - 2^{-\nu-3}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu-1)\, z^{\nu+2}\, {}_0F_3\left(;\frac{3}{2}, \frac{\nu+3}{2}, \frac{\nu+2}{2}; -\frac{z^4}{256}\right) /; \nu \notin \mathbb{Z}$$

Involving hypergeometric U

03.19.26.0003.01

$$\operatorname{kei}_\nu(z) = -2^{-\nu-2}\pi\, i\, e^{-\frac{3i\pi\nu}{4}} \csc(\pi\nu)\, z^{-\nu}\left(\left(\sqrt[4]{-1}\, z\right)^{2\nu} - e^{\frac{i\pi\nu}{2}} z^{2\nu}\right) {}_0\tilde{F}_1\left(;\nu+1; \frac{i z^2}{4}\right) - 2^{-\nu-2}\pi\, i\, e^{\frac{i\pi\nu}{4}}\csc(\pi\nu)\, z^{-\nu}\left(z^{2\nu} - e^{\frac{i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{2\nu}\right) {}_0\tilde{F}_1\left(;\nu+1; -\frac{i z^2}{4}\right) - i\, 2^{\nu-1} e^{-\sqrt[4]{-1}\, z - \frac{3i\pi\nu}{4}}\sqrt{\pi}\, z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^{2\nu} U\left(\nu+\frac{1}{2}, 2\nu+1, 2\sqrt[4]{-1}\, z\right) + 2^{\nu-1} e^{\frac{3i\pi\nu}{4} - (-1)^{3/4} z}\, i\sqrt{\pi}\, z^{-\nu}\left((-1)^{3/4} z\right)^{2\nu} U\left(\nu+\frac{1}{2}, 2\nu+1, 2(-1)^{3/4} z\right) /; \nu \notin \mathbb{Z}$$

03.19.26.0004.01

$$\operatorname{kei}_\nu(z) = -2^{-\nu-3} e^{\frac{3i\pi\nu}{4}} z^{\nu}\left(-4i\log(z) + 4i\log\left(\sqrt[4]{-1}\, z\right) + \pi\right) {}_0\tilde{F}_1\left(;\nu+1; \frac{i z^2}{4}\right) - (-1)^{\frac{5\nu}{4}} 2^{-\nu-3} z^{\nu}\left(4i\log(z) - 4i\log\left((-1)^{3/4} z\right) + \pi\right) {}_0\tilde{F}_1\left(;\nu+1; -\frac{i z^2}{4}\right) - 2^{\nu-1} e^{-\sqrt[4]{-1}\, z - \frac{i\pi\nu}{4}}\, i\sqrt{\pi}\, z^{\nu}\, U\left(\nu+\frac{1}{2}, 2\nu+1, 2\sqrt[4]{-1}\, z\right) + (-1)^{\frac{\nu}{4}} 2^{\nu-1} e^{-(-1)^{3/4} z}\, i\sqrt{\pi}\, z^{\nu}\, U\left(\nu+\frac{1}{2}, 2\nu+1, 2(-1)^{3/4} z\right) /; \nu \in \mathbb{Z}$$

Through Meijer G

Classical cases for the direct function itself

03.19.26.0005.01

$$\operatorname{kei}_\nu(z) = -\frac{1}{4} G_{1,5}^{4,0}\left(\frac{z^4}{256} \,\middle|\, \begin{matrix} \frac{\nu}{2} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{2-\nu}{4}, \frac{\nu+2}{4}, \frac{\nu}{2} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

03.19.26.0006.01

$$\operatorname{kei}_{-\nu}(z)+\operatorname{kei}_{\nu}(z)=-\frac{1}{2}\cos\left(\frac{\pi\nu}{2}\right)G_{1,5}^{4,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}0\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{\nu+2}{4},0\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

03.19.26.0007.01

$$\operatorname{kei}_{\nu}(z)-\operatorname{kei}_{-\nu}(z)=-\frac{1}{2}\sin\left(\frac{\pi\nu}{2}\right)G_{1,5}^{4,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}\frac{1}{2}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Classical cases for powers of kei

03.19.26.0008.01

$$\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)^2=\frac{1}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)-\frac{\sqrt{\pi}}{2^{7/2}}G_{3,7}^{6,0}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0009.01

$$\operatorname{kei}_{\nu}(z)^2=\frac{1}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)-\frac{\sqrt{\pi}}{2^{7/2}}G_{3,7}^{6,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases for products of kei

03.19.26.0010.01

$$\operatorname{kei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)=\frac{\cos(\pi\nu)}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{5,0}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right)$$

03.19.26.0011.01

$$\operatorname{kei}_{-\nu}(z)\operatorname{kei}_{\nu}(z)=\frac{\cos(\pi\nu)}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Classical cases involving bei

03.19.26.0012.01

$$\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0013.01

$$\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{2,6}^{3,1}\left(\frac{z}{64}\,\middle|\,\begin{matrix}-\frac{\nu}{2},\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{3,7}^{4,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu+\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

03.19.26.0014.01

$$\operatorname{bei}_{\nu}(z)\operatorname{kei}_{\nu}(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0015.01

$$\operatorname{bei}_{-\nu}(z)\operatorname{kei}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{2,6}^{3,1}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{\nu}{2},\frac{1-\nu}{2}\\0,\frac{1}{2},-\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{3,7}^{4,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu+\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving ber

03.19.26.0016.01

$$\operatorname{ber}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)=-\frac{\sqrt{\pi}}{8}\,G_{1,5}^{3,0}\!\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0017.01

$$\operatorname{ber}_{-\nu}\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)=-\frac{\sqrt{\pi}}{8}\,G_{0,4}^{2,0}\!\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{3,7}^{4,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu,\frac{\nu+1}{2}\end{matrix}\right)$$

03.19.26.0018.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=-\frac{\sqrt{\pi}}{8}\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0019.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{kei}_\nu(z)=-\frac{\sqrt{\pi}}{8}\,G_{0,4}^{2,0}\!\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{3,7}^{4,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu,\frac{\nu+1}{2}\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving powers of ker

03.19.26.0020.01

$$\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)^2+\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right)^2=\frac{1}{8\sqrt{\pi}}\,G_{0,4}^{4,0}\!\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0021.01

$$\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)^2-\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right)^2=-\frac{\sqrt{\pi}}{2^{5/2}}\,G_{3,7}^{6,0}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0022.01

$$\operatorname{kei}_\nu(z)^2+\operatorname{ker}_\nu(z)^2=\frac{1}{8\sqrt{\pi}}\,G_{0,4}^{4,0}\!\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0023.01

$$\operatorname{kei}_\nu(z)^2-\operatorname{ker}_\nu(z)^2=-\frac{\sqrt{\pi}}{2^{5/2}}\,G_{3,7}^{6,0}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ker

03.19.26.0024.01

$$\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)=-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{3,7}^{6,0}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\nu\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0025.01

$$\operatorname{ker}_{-\nu}\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)=-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{5,0}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{matrix}\right)-\frac{\sin(\pi\nu)}{16\sqrt{\pi}}\,G_{0,4}^{4,0}\!\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)$$

03.19.26.0026.01

$$\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)+\operatorname{ker}_{-\nu}\!\left(\sqrt[4]{z}\right)\operatorname{kei}_{-\nu}\!\left(\sqrt[4]{z}\right)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\cos(\pi\nu)\,G_{2,6}^{5,0}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0027.01

$$\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right)-\operatorname{ker}_{-\nu}\!\left(\sqrt[4]{z}\right)\operatorname{kei}_{-\nu}\!\left(\sqrt[4]{z}\right)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\sin(\pi\nu)\,G_{2,6}^{5,0}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0028.01

$$\operatorname{ker}_\nu(z)\operatorname{kei}_\nu(z)=-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{3,7}^{6,0}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\nu\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0029.01

$$\operatorname{ker}_{-\nu}(z)\operatorname{kei}_\nu(z)=-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{5,0}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{matrix}\right)-\frac{\sin(\pi\nu)}{16\sqrt{\pi}}\,G_{0,4}^{4,0}\!\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

03.19.26.0030.01

$$\operatorname{ker}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{ker}_{-\nu}(z)\operatorname{kei}_{-\nu}(z)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\cos(\pi\nu)\,G_{2,6}^{5,0}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0031.01

$$\operatorname{ker}_\nu(z)\operatorname{kei}_\nu(z) - \operatorname{ker}_{-\nu}(z)\operatorname{kei}_{-\nu}(z) = -2^{-\frac{5}{2}}\sqrt{\pi}\,\sin(\pi\nu)\,G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ber, bei and ker

03.19.26.0032.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)+\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0033.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)-\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0034.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)+\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0035.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)-\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0036.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0037.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0038.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.19.26.0039.01

$$\mathrm{bei}_\nu(z)\,\mathrm{ker}_\nu(z)-\mathrm{ber}_\nu(z)\,\mathrm{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving Bessel *J*

03.19.26.0040.01

$$J_\nu\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(-i\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{\pi\sqrt{2}}\left(G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{matrix}\right)-i\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.19.26.0041.01

$$J_{-\nu}\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=\frac{1}{8}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{e^{i\pi\nu}}{\sqrt{2}\,\pi}\left(i\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)-G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)-e^{-i\pi\nu}\left(G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)+i\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving Bessel *I*

03.19.26.0042.01

$$I_\nu\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=\frac{1}{8}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(i\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{\pi\sqrt{2}}\left(i\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.19.26.0043.01

$$I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(e^{i\pi\nu}\left(i\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)-\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(i\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving Bessel *K*

03.19.26.0044.01

$$K_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{kei}_\nu(z)=\frac{1}{16}\left(i\sqrt{\pi}\,e^{\frac{3i\pi\nu}{4}}z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\csc(\pi\nu)\right)G_{0,4}^{3,0}\left(-\frac{z^4}{64}\,\middle|\,0,\tfrac{1}{2},-\tfrac{\nu}{2},\tfrac{\nu}{2}\right)+\frac{1}{16}\left(\sqrt{\pi}\,e^{-\frac{3i\pi\nu}{4}}z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}(1-i\cot(\pi\nu))\right)G_{0,4}^{3,0}\left(-\frac{z^4}{64}\,\middle|\,\tfrac{1}{2},0,\tfrac{\nu}{2},-\tfrac{\nu}{2}\right)+\frac{i\pi^{5/2}e^{-\frac{3i\pi\nu}{4}}z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\csc(\pi\nu)\csc\left(\pi\left(\nu+\frac{1}{4}\right)\right)}{4\sqrt{2}}G_{3,5}^{2,1}\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},-\nu-\frac{1}{4}\\0,-\nu,\nu,\frac{1}{4},-\nu-\frac{1}{4}\end{matrix}\right)-\frac{e^{\frac{3i\pi\nu}{4}}\pi^{5/2}z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}(i\cot(\pi\nu)+1)\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)}{4\sqrt{2}}G_{3,5}^{2,1}\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},\nu-\frac{1}{4}\\0,\nu,\frac{1}{4},-\nu,\nu-\frac{1}{4}\end{matrix}\right)\,/;\,\nu\notin\mathbb{Z}\bigwedge-\frac{\pi}{2}<\arg(z)\leq 0$$

Classical cases involving ${}_0F_1$

03.19.26.0045.01

$${}_0F_1\left(;\nu+1;\frac{i\sqrt{z}}{4}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)=2^{\nu-3}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\,\Gamma(\nu+1)\left(i\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{\pi\sqrt{2}}\left(i\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\right)\right)$$

03.19.26.0046.01

$${}_0F_1\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)=2^{-\nu-3}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{\nu}\,\Gamma(1-\nu)\left(e^{i\pi\nu}\left(i\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)-\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(i\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right)$$

03.19.26.0047.01

$${}_0F_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{kei}_\nu(z)=2^{\nu-3}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^{-\nu}\,\Gamma(\nu+1)\left(i\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{\pi\sqrt{2}}\left(i\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\right)\right)\,/;\,-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

03.19.26.0048.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\text{kei}_\nu(z) = 2^{-\nu-3} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^\nu\, \Gamma(1-\nu)\left(e^{i\pi\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)-\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

Classical cases involving ${}_0\tilde{F}_1$

03.19.26.0049.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i\sqrt{z}}{4}\right)\text{kei}_\nu\left(\sqrt[4]{z}\right) = 2^{\nu-3} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^{-\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{\pi\sqrt{2}}\left(i\, G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\right)\right) $$

03.19.26.0050.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\text{kei}_\nu\left(\sqrt[4]{z}\right) = 2^{-\nu-3} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^\nu\left(e^{i\pi\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)-\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(i\, G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right) $$

03.19.26.0051.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\text{kei}_\nu(z) = 2^{\nu-3} e^{\frac{1}{4}(-3) i \pi \nu} \sqrt{\pi}\, z^{-\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{\pi\sqrt{2}}\left(i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

03.19.26.0052.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\text{kei}_\nu(z) = 2^{-\nu-3} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^\nu\left(e^{i\pi\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)-G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)-\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

Generalized cases for the direct function itself

03.19.26.0053.01

$$\operatorname{kei}_\nu(z) == -\frac{1}{4} G_{1,5}^{4,0}\left(\frac{z}{4}, \frac{1}{4} \middle| \begin{array}{c} \frac{\nu}{2} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{2-\nu}{4}, \frac{\nu+2}{4}, \frac{\nu}{2} \end{array}\right)$$

03.19.26.0054.01

$$\operatorname{kei}_{-\nu}(z) + \operatorname{kei}_\nu(z) = -\frac{1}{2} \cos\left(\frac{\pi \nu}{2}\right) G_{1,5}^{4,0}\left(\frac{z}{4}, \frac{1}{4} \middle| \begin{array}{c} 0 \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{2-\nu}{4}, \frac{\nu+2}{4}, 0 \end{array}\right)$$

03.19.26.0055.01

$$\operatorname{kei}_\nu(z) - \operatorname{kei}_{-\nu}(z) = -\frac{1}{2} \sin\left(\frac{\pi \nu}{2}\right) G_{1,5}^{4,0}\left(\frac{z}{4}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{2} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{2-\nu}{4}, \frac{1}{2} \end{array}\right)$$

Generalized cases for powers of kei

03.19.26.0056.01

$$\operatorname{kei}_\nu(z)^2 = \frac{1}{16\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right) - \frac{\sqrt{\pi}}{2^{7/2}} G_{3,7}^{6,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{4}, \frac{3}{4}, \nu+\frac{1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \nu+\frac{1}{2} \end{array}\right)$$

Brychkov Yu.A. (2006)

Generalized cases for products of kei

03.19.26.0057.01

$$\operatorname{kei}_{-\nu}(z) \operatorname{kei}_\nu(z) = \frac{\cos(\pi \nu)}{16\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right) - \frac{1}{8}\sqrt{\frac{\pi}{2}} G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2} \end{array}\right)$$

Generalized cases involving bei

03.19.26.0058.01

$$\operatorname{bei}_\nu(z) \operatorname{kei}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{array}\right) - \frac{1}{2^{7/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{array}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0059.01

$$\operatorname{bei}_{-\nu}(z) \operatorname{kei}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{2,6}^{3,1}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} -\frac{\nu}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{array}\right) - \frac{1}{2^{7/2}\sqrt{\pi}} G_{3,7}^{4,2}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{4}, \frac{3}{4}, \nu+\frac{1}{2} \\ 0, \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \nu+\frac{1}{2}, \frac{\nu+1}{2} \end{array}\right)$$

Generalized cases involving ber

03.19.26.0060.01

$$\operatorname{ber}_\nu(z) \operatorname{kei}_\nu(z) = -\frac{\sqrt{\pi}}{8} G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{array}\right) - \frac{1}{2^{7/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, 0, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{array}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0061.01

$$\operatorname{ber}_{-\nu}(z) \operatorname{kei}_\nu(z) = -\frac{\sqrt{\pi}}{8} G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}\right) - \frac{1}{8\sqrt{2\pi}} G_{3,7}^{4,2}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1}{4}, \frac{3}{4}, \nu \\ 0, \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \nu, \frac{\nu+1}{2} \end{array}\right)$$

Generalized cases involving powers of ker

03.19.26.0062.01

$$\operatorname{kei}_\nu(z)^2 + \operatorname{ker}_\nu(z)^2 = \frac{1}{8\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0063.01

$$\operatorname{kei}_\nu(z)^2 - \operatorname{ker}_\nu(z)^2 = -\frac{\sqrt{\pi}}{2^{5/2}} G_{3,7}^{6,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, \nu+\frac{1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \nu+\frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ker

03.19.26.0064.01

$$\operatorname{ker}_\nu(z)\operatorname{kei}_\nu(z) = -\frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{3,7}^{6,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, \nu \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, \nu \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0065.01

$$\operatorname{kei}_\nu(z)\operatorname{ker}_{-\nu}(z) = -\frac{\sin(\pi\nu)}{16\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right) - \frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, 0 \end{matrix}\right)$$

03.19.26.0066.01

$$\operatorname{ker}_\nu(z)\operatorname{kei}_\nu(z) + \operatorname{ker}_{-\nu}(z)\operatorname{kei}_{-\nu}(z) = -2^{-\frac{5}{2}}\sqrt{\pi}\cos(\pi\nu)\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, 0 \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0067.01

$$\operatorname{ker}_\nu(z)\operatorname{kei}_\nu(z) - \operatorname{ker}_{-\nu}(z)\operatorname{kei}_{-\nu}(z) = -2^{-\frac{5}{2}}\sqrt{\pi}\sin(\pi\nu)\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ber, bei and ker

03.19.26.0068.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z) + \operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z) = \frac{1}{4}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0069.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z) - \operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z) = -\frac{1}{2^{5/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0070.01

$$\mathrm{ber}_\nu(z)\,\mathrm{kei}_\nu(z)+\mathrm{bei}_\nu(z)\,\mathrm{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.19.26.0071.01

$$\mathrm{bei}_\nu(z)\,\mathrm{ker}_\nu(z)-\mathrm{ber}_\nu(z)\,\mathrm{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving Bessel *J*

03.19.26.0072.01

$$J_\nu\!\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=$$
$$\frac{1}{8}\,e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(-i\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\right.$$
$$\left.\frac{1}{\pi\sqrt{2}}\left(G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)\right)\right)$$

03.19.26.0073.01

$$J_{-\nu}\!\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=$$
$$\frac{1}{8}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^{\nu}\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{e^{i\pi\nu}}{\sqrt{2}\,\pi}\left(i\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)-G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)-\right.$$
$$\left.e^{-i\pi\nu}\left(G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)+i\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)\right)\right)$$

Generalized cases involving Bessel *I*

03.19.26.0074.01

$$I_\nu\!\left(\sqrt[4]{-1}\,z\right)\mathrm{kei}_\nu(z)=$$
$$\frac{1}{8}\,e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(i\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\right.$$
$$\left.\frac{1}{\pi\sqrt{2}}\left(i\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\right)\right)$$

03.19.26.0075.01

$$I_{-\nu}\left(\sqrt[4]{-1}\; z\right) \operatorname{kei}_\nu(z) = \frac{1}{8} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\; z^\nu \left(\sqrt[4]{-1}\; z\right)^{-\nu} \left(e^{i \pi \nu} \left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}(1-3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1}{2}(1-3\nu), \frac{\nu}{2} \end{array}\right) - G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} -\frac{1}{2}(3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{1}{2}(3\nu), \frac{\nu}{2} \end{array}\right)\right) - \frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi} \left(i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right) + G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, 0, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right)\right)\right)$$

Generalized cases involving Bessel K

03.19.26.0076.01

$$K_\nu\left(\sqrt[4]{-1}\; z\right) \operatorname{kei}_\nu(z) = \frac{1}{16}\left(i \sqrt{\pi}\; e^{\frac{3 i \pi \nu}{4}} z^{-\nu} \left(\sqrt[4]{-1}\; z\right)^\nu \csc(\pi\nu)\right) G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\; z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}\right) + \frac{1}{16}\sqrt{\pi}\; e^{\frac{1}{4}(-3) i \pi \nu} z^\nu \left(\sqrt[4]{-1}\; z\right)^{-\nu} (1 - i\cot(\pi\nu))\, G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\; z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \frac{1}{2}, 0, \frac{\nu}{2}, -\frac{\nu}{2}\right) + \frac{i \pi^{5/2} e^{-\frac{3 i \pi \nu}{4}} z^{-\nu} \left(\sqrt[4]{-1}\; z\right)^\nu \csc(\pi\nu) \csc\left(\pi\left(\nu + \frac{1}{4}\right)\right)}{4\sqrt{2}} G_{3,5}^{2,1}\left(\sqrt[4]{-1}\; z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2}, \frac{1}{4}, -\nu - \frac{1}{4} \\ 0, -\nu, \nu, \frac{1}{4}, -\nu - \frac{1}{4} \end{array}\right) - \frac{e^{\frac{3 i \pi \nu}{4}} \pi^{5/2} z^\nu \left(\sqrt[4]{-1}\; z\right)^{-\nu} (i\cot(\pi\nu) + 1) \csc\left(\pi\left(\nu + \frac{3}{4}\right)\right)}{4\sqrt{2}} G_{3,5}^{2,1}\left(\sqrt[4]{-1}\; z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2}, \frac{1}{4}, \nu - \frac{1}{4} \\ 0, \nu, \frac{1}{4}, -\nu, \nu - \frac{1}{4} \end{array}\right) /;\ \nu \notin \mathbb{Z}$$

Generalized cases involving ${}_0F_1$

03.19.26.0077.01

$${}_0F_1\left(;\nu+1; \frac{i z^2}{4}\right) \operatorname{kei}_\nu(z) = 2^{\nu-3} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\; z^{-\nu}\, \Gamma(\nu+1) \left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{array}\right) - G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{array}\right) - \frac{1}{\pi\sqrt{2}} \left(i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{array}\right) + G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 0 \end{array}\right)\right)\right)$$

03.19.26.0078.01

$${}_0F_1\left(;1-\nu; \frac{i z^2}{4}\right) \operatorname{kei}_\nu(z) = 2^{-\nu-3} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\; z^{\nu}\, \Gamma(1-\nu) \left(e^{i\pi\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}(1-3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1}{2}(1-3\nu), \frac{\nu}{2} \end{array}\right) - G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} -\frac{1}{2}(3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{1}{2}(3\nu), \frac{\nu}{2} \end{array}\right)\right) - \frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi} \left(i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right) + G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, 0, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right)\right)\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.19.26.0079.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{kei}_\nu(z) = 2^{\nu-3} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^{-\nu} \left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{array}\right) - G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{array}\right) - \frac{1}{\pi\sqrt{2}}\left(i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{array}\right) + G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 0 \end{array}\right)\right)\right) $$

03.19.26.0080.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{kei}_\nu(z) = 2^{-\nu-3} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^{\nu}\, \Gamma(1-\nu) \left(e^{i\pi\nu}\left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}(1-3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1}{2}(1-3\nu), \frac{\nu}{2} \end{array}\right) - G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} -\frac{1}{2}(3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{1}{2}(3\nu), \frac{\nu}{2} \end{array}\right)\right) - \frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right) + G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, 0, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right)\right)\right) $$

Representations through equivalent functions

With related functions

03.19.27.0001.01

$$ \operatorname{kei}_\nu(z) = \frac{1}{2}\pi\left(\csc(\pi\nu)\operatorname{bei}_{-\nu}(z) - \cot(\pi\nu)\operatorname{bei}_\nu(z) + \operatorname{ber}_\nu(z)\right) /; \nu \notin \mathbb{Z} $$

03.19.27.0002.01

$$ \operatorname{kei}_\nu(z) = \cot(\pi\nu)\operatorname{ker}_\nu(z) - \csc(\pi\nu)\operatorname{ker}_{-\nu}(z) /; \nu \notin \mathbb{Z} $$

03.19.27.0003.01

$$ \operatorname{kei}_\nu(z) = -\frac{i}{4}\left(2 K_\nu\left(\sqrt[4]{-1}\, z\right)(-i)^\nu + (-1)^\nu \pi\, Y_\nu\left(\sqrt[4]{-1}\, z\right) - i\left(\log(4) + 4\log(z) - 4\log((1+i)z)\right)\operatorname{bei}_\nu(z) - i\pi\operatorname{ber}_\nu(z)\right) /; \nu \in \mathbb{Z} $$

03.19.27.0004.01

$$ \operatorname{kei}_\nu(z) = \frac{1}{4}\pi z^{-\nu}\left(-z^4\right)^{\frac{1}{4}(-\nu-2)}\csc(\pi\nu) \left(-\left(I_{-\nu}\left(\sqrt[4]{-z^4}\right)\left(\sqrt{-z^4}\sin\left(\frac{3\pi\nu}{4}\right) - z^2\cos\left(\frac{3\pi\nu}{4}\right)\right) + J_{-\nu}\left(\sqrt[4]{-z^4}\right)\left(\cos\left(\frac{3\pi\nu}{4}\right)z^2 + \sqrt{-z^4}\sin\left(\frac{3\pi\nu}{4}\right)\right)\right)\left(-z^4\right)^{\nu/2} + z^{2\nu} I_\nu\left(\sqrt[4]{-z^4}\right)\left(\sqrt{-z^4}\sin\left(\frac{\pi\nu}{4}\right) - z^2\cos\left(\frac{\pi\nu}{4}\right)\right) + z^{2\nu} J_\nu\left(\sqrt[4]{-z^4}\right)\left(\cos\left(\frac{\pi\nu}{4}\right)z^2 + \sqrt{-z^4}\sin\left(\frac{\pi\nu}{4}\right)\right)\right) /; \nu \notin \mathbb{Z} $$

03.19.27.0005.01

$$ \operatorname{kei}_\nu(z) = -\frac{i}{2} e^{-\frac{3 i\pi\nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu K_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{\pi i}{4} e^{\frac{3 i\pi\nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu Y_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{\pi i}{4}\left(e^{\frac{3 i\pi\nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu \cot(\pi\nu) - e^{-\frac{3 i\pi\nu}{4}} z^\nu\left(\sqrt[4]{-1}\, z\right)^{-\nu}(i + \cot(\pi\nu))\right) J_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{\pi i}{4}\left(e^{\frac{3 i\pi\nu}{4}} z^\nu\left(\sqrt[4]{-1}\, z\right)^{-\nu}(-i + \cot(\pi\nu)) - e^{-\frac{3 i\pi\nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu \csc(\pi\nu)\right) I_\nu\left(\sqrt[4]{-1}\, z\right) /; \nu \notin \mathbb{Z} $$

03.19.27.0006.01

$$\operatorname{kei}_\nu(z) = -\frac{1}{8}\, i^\nu \left(-4\, i \log(z) + 4\, i \log\left(\sqrt[4]{-1}\; z\right) + \pi\right) I_\nu\left(\sqrt[4]{-1}\; z\right) - \frac{1}{8}\, (-1)^\nu \left(4\, i \log(z) - 4\, i \log\left(\sqrt[4]{-1}\; z\right) + \pi\right) J_\nu\left(\sqrt[4]{-1}\; z\right) + \frac{1}{2}\, (-i)^{\nu+1}\, K_\nu\left(\sqrt[4]{-1}\; z\right) + \frac{1}{4}\left(\pi\, i\, (-1)^{\nu-1}\right) Y_\nu\left(\sqrt[4]{-1}\; z\right) /;\, \nu \in \mathbb{Z}$$

03.19.27.0007.01

$$\operatorname{kei}_\nu(z) = \begin{cases} -\frac{\pi i}{4}\left(e^{-i\pi\nu}\, Y_\nu\left(\sqrt[4]{-1}\; z\right) + (3\, i \cos(\pi\nu) - \sin(\pi\nu))\, J_\nu\left(\sqrt[4]{-1}\; z\right)\right) - e^{-\frac{i\pi\nu}{2}}\, \pi \cos(\pi\nu)\, I_\nu\left(\sqrt[4]{-1}\; z\right) - \frac{1}{2}\, i\, e^{-\frac{5 i\pi\nu}{2}}\, K_\nu\left(\sqrt[4]{-1}\; z\right) & \frac{3\pi}{4} < \text{a} \\ -\frac{\pi}{4}\, e^{i\pi\nu}\left(J_\nu\left(\sqrt[4]{-1}\; z\right) + i\, Y_\nu\left(\sqrt[4]{-1}\; z\right)\right) - \frac{1}{2}\, i\, e^{-\frac{i\pi\nu}{2}}\, K_\nu\left(\sqrt[4]{-1}\; z\right) & \text{T} \end{cases}$$

$$/;\ \nu \in \mathbb{Z}$$

03.19.27.0008.01

$$\operatorname{kei}_\nu(z) = \frac{\pi i}{4}\csc(\pi\nu)\, z^{-\nu}\left(e^{\frac{i\pi\nu}{4}}\, Y_{-\nu}\left(\sqrt[4]{-1}\; z\right)\left(e^{\frac{i\pi\nu}{2}}\left(\sqrt[4]{-1}\; z\right)^{2\nu}\cot(\pi\nu) - z^{2\nu}\csc(\pi\nu)\right)\left(\sqrt[4]{-1}\; z\right)^{-\nu} + e^{\frac{i\pi\nu}{4}}\, Y_\nu\left(\sqrt[4]{-1}\; z\right)\left(z^{2\nu}\cot(\pi\nu) - e^{\frac{i\pi\nu}{2}}\left(\sqrt[4]{-1}\; z\right)^{2\nu}\csc(\pi\nu)\right)\left(\sqrt[4]{-1}\; z\right)^{-\nu} + e^{\frac{1}{4}(-3)\, i\pi\nu}\left((-1)^{3/4} z\right)^{-\nu} Y_{-\nu}\left((-1)^{3/4} z\right)\left(e^{\frac{i\pi\nu}{2}}\, z^{2\nu}\csc(\pi\nu) - \left((-1)^{3/4} z\right)^{2\nu}\cot(\pi\nu)\right) + e^{\frac{1}{4}(-3)\, i\pi\nu}\left((-1)^{3/4} z\right)^{-\nu} Y_\nu\left((-1)^{3/4} z\right)\left(\left((-1)^{3/4} z\right)^{2\nu}\csc(\pi\nu) - e^{\frac{i\pi\nu}{2}}\, z^{2\nu}\cot(\pi\nu)\right)\right) /;\, \nu \notin \mathbb{Z}$$

03.19.27.0009.01

$$\operatorname{kei}_\nu(z) + i \operatorname{ker}_\nu(z) = -\frac{\pi i}{2}\left(e^{\frac{3 i\pi\nu}{4}}\, z^{-\nu}\left(\sqrt[4]{-1}\; z\right)^{\nu} Y_\nu\left(\sqrt[4]{-1}\; z\right) + \left(e^{-\frac{3 i\pi\nu}{4}}\, z^{\nu}\left(\sqrt[4]{-1}\; z\right)^{-\nu}(i + \cot(\pi\nu)) - e^{\frac{3 i\pi\nu}{4}}\, z^{-\nu}\left(\sqrt[4]{-1}\; z\right)^{\nu}\cot(\pi\nu)\right) J_\nu\left(\sqrt[4]{-1}\; z\right)\right) /;\, \nu \notin \mathbb{Z}$$

03.19.27.0010.01

$$\operatorname{kei}_\nu(z) + i \operatorname{ker}_\nu(z) = \begin{cases} \frac{1}{2}\,(-(\pi\, i))\left(e^{-i\pi\nu}\, Y_\nu\left(\sqrt[4]{-1}\; z\right) + (3\, i\cos(\pi\nu) - \sin(\pi\nu))\, J_\nu\left(\sqrt[4]{-1}\; z\right)\right) & \frac{3\pi}{4} < \arg(z) \le \pi \\ \frac{1}{2}\left(-\left(\pi\, i\, e^{i\pi\nu}\right)\right)\left(Y_\nu\left(\sqrt[4]{-1}\; z\right) - i\, J_\nu\left(\sqrt[4]{-1}\; z\right)\right) & \text{True} \end{cases} /;\, \nu \in \mathbb{Z}$$

03.19.27.0011.01

$$\operatorname{kei}_\nu(z) - i \operatorname{ker}_\nu(z) = -i\, e^{-\frac{3 i\pi\nu}{4}}\, z^{-\nu}\left(\sqrt[4]{-1}\; z\right)^{\nu} K_\nu\left(\sqrt[4]{-1}\; z\right) - \frac{\pi i}{2}\left(e^{-\frac{3 i\pi\nu}{4}}\, z^{-\nu}\left(\sqrt[4]{-1}\; z\right)^{\nu}\csc(\pi\nu) + e^{\frac{3 i\pi\nu}{4}}\, z^{\nu}\left(\sqrt[4]{-1}\; z\right)^{-\nu}(i - \cot(\pi\nu))\right) I_\nu\left(\sqrt[4]{-1}\; z\right) /;\, \nu \notin \mathbb{Z}$$

03.19.27.0012.01

$$\operatorname{kei}_\nu(z) - i \operatorname{ker}_\nu(z) = \begin{cases} -i\, e^{-\frac{5 i\pi\nu}{2}}\, K_\nu\left(\sqrt[4]{-1}\; z\right) - 2\pi\, e^{-\frac{1}{2} i\pi\nu}\cos(\pi\nu)\, I_\nu\left(\sqrt[4]{-1}\; z\right) & \frac{3\pi}{4} < \arg(z) \le \pi \\ -i\, e^{-\frac{1}{2} i\pi\nu}\, K_\nu\left(\sqrt[4]{-1}\; z\right) & \text{True} \end{cases} /;\, \nu \in \mathbb{Z}$$

Theorems

History

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