KelvinKer

Notations

Traditional name

Kelvin function of the second kind

Traditional notation

$\operatorname{ker}(z)$

Mathematica StandardForm notation

`KelvinKer[z]`

Primary definition

03.16.02.0001.01

$$\operatorname{ker}(z) = \operatorname{ker}_0(z)$$

Specific values

Values at fixed points

03.16.03.0001.01

$$\operatorname{ker}(0) = \tilde{\infty}$$

Values at infinities

03.16.03.0002.01

$$\lim_{x\to\infty} \operatorname{ker}(x) = 0$$

03.16.03.0003.01

$$\lim_{x\to-\infty} \operatorname{ker}(x) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$\operatorname{ker}(z)$ is an analytical function of z, which is defined over the whole complex z-plane.

03.16.04.0001.01

$$z \longrightarrow \operatorname{ker}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

03.16.04.0002.01

$\text{ker}(\bar{z}) = \overline{\text{ker}(z)} \;/;\; z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

The function ker(z) has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point.

03.16.04.0003.01

$\mathcal{Sing}_z(\text{ker}(z)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

The function ker(z) has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

03.16.04.0004.01

$\mathcal{BP}_z(\text{ker}(z)) = \{0, \tilde{\infty}\}$

03.16.04.0005.01

$\mathcal{R}_z(\text{ker}(z), 0) = \log$

03.16.04.0006.01

$\mathcal{R}_z(\text{ker}(z), \tilde{\infty}) = \log$

Branch cuts

The function ker(z) is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$ where it is continuous from above.

03.16.04.0007.01

$\mathcal{BC}_z(\text{ker}(z)) = \{\{(-\infty, 0), -i\}\}$

03.16.04.0008.01

$\lim_{\epsilon \to +0} \text{ker}(x + i\,\epsilon) = \text{ker}(x) \;/;\; x \in \mathbb{R} \land x < 0$

03.16.04.0009.01

$\lim_{\epsilon \to +0} \text{ker}(x - i\,\epsilon) = \text{ker}(x) + 2\,i\,\pi\,\text{ber}(x) \;/;\; x \in \mathbb{R} \land x < 0$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

03.16.06.0001.01

$$\operatorname{ker}(z) \propto \operatorname{ker}(z_0) - 2\,i\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor \operatorname{ber}(z_0) - \frac{2\,i\,\pi \left\lfloor \frac{\arg(z-z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\,\pi} \right\rfloor (\operatorname{bei}_1(z_0) + \operatorname{ber}_1(z_0)) - \operatorname{kei}_1(z_0) - \operatorname{ker}_1(z_0)}{\sqrt{2}} (z - z_0) - \frac{1}{4} \left(-2\,i\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor (\operatorname{bei}(z_0) - \operatorname{bei}_2(z_0)) + \operatorname{kei}(z_0) - \operatorname{kei}_2(z_0) \right) (z - z_0)^2 + \ldots \;/;\; (z \to z_0)$$

03.16.06.0002.01

$$\operatorname{ker}(z) = \sum_{k=0}^{\infty} \frac{\operatorname{ker}^{(k)}(z_0)\,(z - z_0)^k}{k!} \;/;\; |\arg(z_0)| < \pi$$

03.16.06.0003.01

$$\operatorname{ker}(z) = \frac{1}{4} \sum_{k=0}^{\infty} \frac{1}{k!}\, G_{3,7}^{3,3}\!\left(\frac{z_0}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{k}{4}, \frac{1-k}{4}, \frac{3-k}{4} \\ -\frac{k}{4}, -\frac{k}{4}, \frac{2-k}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix} \right) (z - z_0)^k \;/;\; |\arg(z_0)| < \pi$$

03.16.06.0004.01

$$\operatorname{ker}(z) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1+i)^k\, 2^{-\frac{3k}{2}}}{k!} \left(\sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2\,j} \left(i\,(1 - i^k) \left(\operatorname{kei}_{4\,j-k}(z_0) - 2\,i\,(-1)^k\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor \operatorname{bei}_{k-4\,j}(z_0) \right) + (1 + i^k) \left(\operatorname{ker}_{4\,j-k}(z_0) - 2\,i\,(-1)^k\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor \operatorname{ber}_{k-4\,j}(z_0) \right) \right) - \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2\,j+1} \left(i\,(1 - i^k) \left(\operatorname{kei}_{4\,j-k+2}(z_0) - 2\,i\,(-1)^k\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor \operatorname{bei}_{-4\,j+k-2}(z_0) \right) + (1 + i^k) \left(\operatorname{ker}_{4\,j-k+2}(z_0) - 2\,i\,(-1)^k\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor \operatorname{ber}_{-4\,j+k-2}(z_0) \right) \right) \right) (z - z_0)^k$$

03.16.06.0005.01

$$\operatorname{ker}(z) \propto \left(\operatorname{ker}(z_0) - 2\,i\,\pi \left\lfloor \frac{\arg(z - z_0)}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\,\pi} \right\rfloor \operatorname{ber}(z_0) \right) (1 + O(z - z_0))$$

Expansions on branch cuts

03.16.06.0006.01

$$\operatorname{ker}(z) \propto \operatorname{ker}(x) - 2\,i\,\pi \left\lfloor \frac{\arg(z - x)}{2\,\pi} \right\rfloor \operatorname{ber}(x) - \frac{\left(-2\,i\,\pi \left\lfloor \frac{\arg(z-x)}{2\,\pi} \right\rfloor (\operatorname{bei}_1(x) + \operatorname{ber}_1(x)) + \operatorname{kei}_1(x) + \operatorname{ker}_1(x) \right)}{\sqrt{2}} (x - z) + \frac{1}{4} \left(2\,i\,\pi \left\lfloor \frac{\arg(z - x)}{2\,\pi} \right\rfloor (\operatorname{bei}(x) - \operatorname{bei}_2(x)) - \operatorname{kei}(x) + \operatorname{kei}_2(x) \right) (x - z)^2 + \ldots \;/;\; (z \to x) \land x \in \mathbb{R} \land x < 0$$

03.16.06.0007.01

$$\operatorname{ker}(z)=\frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1+i)^k\,2^{-\frac{3k}{2}}}{k!}\left(\sum_{j=0}^{\left\lfloor\frac{k}{2}\right\rfloor}\binom{k}{2j}\left(i\left(1-i^k\right)\left(\operatorname{kei}_{4j-k}(x)-2\,i\,(-1)^k\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\operatorname{bei}_{k-4j}(x)\right)+\left(1+i^k\right)\left(\operatorname{ker}_{4j-k}(x)-2\,i\,(-1)^k\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\operatorname{ber}_{k-4j}(x)\right)\right)-\sum_{j=0}^{\left\lfloor\frac{k-1}{2}\right\rfloor}\binom{k}{2j+1}\left(i\left(1-i^k\right)\left(\operatorname{kei}_{4j-k+2}(x)-2\,i\,(-1)^k\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\operatorname{bei}_{-4j+k-2}(x)\right)+\left(1+i^k\right)\left(\operatorname{ker}_{4j-k+2}(x)-2\,i\,(-1)^k\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\operatorname{ber}_{-4j+k-2}(x)\right)\right)\right)(z-x)^k\;/;\;x\in\mathbb{R}\land x<0$$

03.16.06.0008.01

$$\operatorname{ker}(z)\propto\left(\operatorname{ker}(x)-2\,i\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\operatorname{ber}(x)\right)(1+O(z-x))\;/;\;x\in\mathbb{R}\land x<0$$

Expansions at *z* == 0

For the function itself

03.16.06.0009.01

$$\operatorname{ker}(z)\propto-\log\left(\frac{z}{2}\right)\left(1-\frac{z^4}{64}+\frac{z^8}{147456}+\ldots\right)+\left(-\gamma+\frac{2\gamma-3}{128}\,z^4-\frac{12\gamma-25}{1769472}\,z^8+\ldots\right)+\frac{\pi z^2}{16}\left(1-\frac{z^4}{576}+\frac{z^8}{3686400}+\ldots\right)/;\;(z\to 0)$$

03.16.06.0010.01

$$\operatorname{ker}(z)=\sum_{k=0}^{\infty}\frac{(-1)^k\,\psi(2k+1)}{((2k)!)^2}\left(\frac{z}{2}\right)^{4k}+\frac{\pi z^2}{16}\sum_{k=0}^{\infty}\frac{(-1)^k}{((2k+1)!)^2}\left(\frac{z}{2}\right)^{4k}-\log\left(\frac{z}{2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k}{((2k)!)^2}\left(\frac{z}{2}\right)^{4k}$$

03.16.06.0011.01

$$\operatorname{ker}(z)=\frac{\pi z^2}{16}\,{}_0F_3\left(;1,\frac{3}{2},\frac{3}{2};-\frac{z^4}{256}\right)-\log\left(\frac{z}{2}\right){}_0F_3\left(;\frac{1}{2},\frac{1}{2},1;-\frac{z^4}{256}\right)+\sum_{k=0}^{\infty}\frac{(-1)^k\,\psi(2k+1)}{((2k)!)^2}\left(\frac{z}{2}\right)^{4k}$$

03.16.06.0012.01

$$\operatorname{ker}(z)=\sum_{k=0}^{\infty}\frac{(-1)^k\,\psi(2k+1)}{((2k)!)^2}\left(\frac{z}{2}\right)^{4k}-\frac{i\pi}{8}\left(I_0\left(\sqrt[4]{-1}\,z\right)-J_0\left(\sqrt[4]{-1}\,z\right)\right)-\frac{1}{2}\log\left(\frac{z}{2}\right)\left(I_0\left(\sqrt[4]{-1}\,z\right)+J_0\left(\sqrt[4]{-1}\,z\right)\right)$$

03.16.06.0013.01

$$\operatorname{ker}(z)\propto-\log(z)\left(1+O\left(z^4\right)\right)+(\log(2)-\gamma)\left(1+O\left(z^2\right)\right)$$

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z\to\infty$

In exponential form ||| In exponential form

03.16.06.0014.01

$$\operatorname{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}} \Bigg(e^{-\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(5 i \pi)+\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} - e^{\frac{5 i \pi}{8}+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) - e^{\frac{z}{\sqrt{2}}} \left(e^{\frac{i \pi}{8}-\frac{i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} + e^{-\frac{1}{8}(i \pi)+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) +$$

$$\frac{1-4\nu^2}{8 z} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(5 i \pi)-\frac{i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} + e^{\frac{5 i \pi}{8}+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) + e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i \pi}{8}+\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} - e^{-\frac{1}{8}(i \pi)+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) \right) +$$

$$\frac{i\left(16\nu^4-40\nu^2+9\right)}{128 z^2} \left(e^{-\frac{z}{\sqrt{2}}} \left(-e^{-\frac{1}{8}(5 i \pi)+\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} - e^{\frac{5 i \pi}{8}+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) - e^{\frac{z}{\sqrt{2}}} \left(e^{\frac{i \pi}{8}-\frac{i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} - e^{-\frac{1}{8}(i \pi)+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) \right) -$$

$$\frac{i\left(64\nu^6-560\nu^4+1036\nu^2-225\right)}{3072 z^3} \Bigg(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(5 i \pi)-\frac{i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} - e^{\frac{5 i \pi}{8}+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) +$$

$$e^{-\frac{z}{\sqrt{2}}} \left(-e^{\frac{i \pi}{8}+\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} - e^{-\frac{1}{8}(i \pi)+\frac{i \pi \nu}{2}+\frac{i z}{\sqrt{2}}} \right) \Bigg) + \ldots \Bigg) /;\ -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.16.06.0015.01

$$\operatorname{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}} \Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{4 z^2}\right)^k$$

$$\left(e^{-\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{3 i \pi \nu}{2}-\frac{5 \pi i}{8}-\frac{i z}{\sqrt{2}}} - e^{\frac{i \pi \nu}{2}+\frac{5 \pi i}{8}+\frac{i z}{\sqrt{2}}} \right) - e^{\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{i \pi \nu}{2}-\frac{\pi i}{8}+\frac{i z}{\sqrt{2}}} + e^{-\frac{1}{2}(i \pi \nu)+\frac{\pi i}{8}-\frac{i z}{\sqrt{2}}} \right) \right) +$$

$$\frac{1}{2 z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{4 z^2}\right)^k \Bigg(e^{-\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{3 i \pi \nu}{2}+\frac{\pi i}{8}-\frac{i z}{\sqrt{2}}} - e^{\frac{i \pi \nu}{2}-\frac{\pi i}{8}+\frac{i z}{\sqrt{2}}} \right) +$$

$$e^{\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{i \pi \nu}{2}+\frac{5 \pi i}{8}+\frac{i z}{\sqrt{2}}} + e^{-\frac{1}{2}(i \pi \nu)-\frac{5 \pi i}{8}-\frac{i z}{\sqrt{2}}} \right) \Bigg) + \ldots \Bigg) /;\ -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.16.06.0016.01

$$\operatorname{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}}\left(e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)-e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)\right)-e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{1}{2}(i\pi\nu)+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+\frac{1-4\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{1}{2}(i\pi\nu)-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)-e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)\right)\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.16.06.0017.01

$$\operatorname{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}}\left(-e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{-\frac{i\pi\nu}{2}+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+\frac{1-4\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{-\frac{i\pi\nu}{2}-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

In trigonometric form ||| In trigonometric form

03.16.06.0018.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}\,e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}}\left(\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)-\frac{1-4\nu^2}{8z}\sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)+\frac{16\nu^4-40\nu^2+9}{128z^2}\sin\left(\frac{1}{8}\left(-4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(1-4\nu)\right)\right)+\ldots\right)/;\ (|z|\to\infty)$$

03.16.06.0019.01

$$\operatorname{ker}_{\nu}(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}} \left(\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{1}{4z^2}\right)^k \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right) - \frac{1}{2z} \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(-\frac{1}{4z^2}\right)^k \sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right) + \ldots \right) /;\ (|z| \to \infty) \land n \in \mathbb{N}$$

03.16.06.0020.01

$$\operatorname{ker}_{\nu}(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}} \Bigg(\cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right) {}_8F_3\left(\frac{1}{8}(1-2\nu), \frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(2\nu+1), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7); \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{16}{z^4}\right) - \frac{1-4\nu^2}{8z} \sin\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right) {}_8F_3\left(\frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9); \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -\frac{16}{z^4}\right) - \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right) {}_8F_3\left(\frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11); \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{16}{z^4}\right) + \frac{-64\nu^6 + 560\nu^4 - 1036\nu^2 + 225}{3072 z^3} \cos\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right) {}_8F_3\left(\frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(13-2\nu), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11), \frac{1}{8}(2\nu+13); \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{16}{z^4}\right)\Bigg) /;\ (|z| \to \infty)$$

03.16.06.0021.01

$$\operatorname{ker}_{\nu}(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}} \left(\cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right) - \frac{1-4\nu^2}{8z} \sin\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right) - \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right) + \frac{-64\nu^6 + 560\nu^4 - 1036\nu^2 + 225}{3072 z^3} \cos\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right)\right) /;\ (|z| \to \infty)$$

Expansions containing $z \to -\infty$

In exponential form ||| In exponential form

03.16.06.0022.01

$$\text{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\,\sqrt{-z}} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} + e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + e^{\frac{z}{\sqrt{2}}} \left(-e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} + e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) - \right.$$

$$\frac{1-4\nu^2}{8z} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} + e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} - e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) \right) +$$

$$\frac{i\left(16\nu^4-40\nu^2+9\right)}{128z^2} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} - e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + e^{\frac{z}{\sqrt{2}}} \left(-e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) \right) +$$

$$\frac{i\left(64\nu^6-560\nu^4+1036\nu^2-225\right)}{3072z^3} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} - e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + \right.$$

$$\left. \left. e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) \right) + \ldots \right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.16.06.0023.01

$$\text{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\,\sqrt{-z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k} \left(\frac{i}{4z^2}\right)^k}{(2k)!} \right.$$

$$\left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi\nu}{2}+\frac{\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} + (-1)^k e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) + e^{\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{5i\pi\nu}{2}+\frac{3\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} - e^{\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) \right) -$$

$$\frac{1}{2z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1} \left(\frac{i}{4z^2}\right)^k}{(2k+1)!} \left(e^{\frac{z}{\sqrt{2}}} \left(-(-1)^k e^{\frac{5i\pi\nu}{2}+\frac{\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} + e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) + \right.$$

$$\left. \left. e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi\nu}{2}+\frac{3\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} + (-1)^k e^{\frac{3i\pi\nu}{2}-\frac{3\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) \right) + \ldots \right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.16.06.0024.01

$$\text{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{5i\pi\nu}{2}+\frac{3\pi i}{8}}e^{\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)-e^{\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}}e^{-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}}e^{\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+e^{-\frac{iz}{\sqrt{2}}}e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)-\frac{1-\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}e^{-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)-e^{\frac{5i\pi\nu}{2}+\frac{\pi i}{8}}e^{\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}}e^{\frac{i\pi\nu}{2}+\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+e^{-\frac{iz}{\sqrt{2}}}e^{\frac{3i\pi\nu}{2}-\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)\right)\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.16.06.0025.01

$$\text{ber}_\nu(z) \propto \frac{(-1)^{3/8}\,e^{\frac{i\pi\nu}{2}}}{2\sqrt{2\pi}\sqrt{-z}}\left(-e^{-\frac{z}{\sqrt{2}}}\left((-1)^{3/4}e^{\frac{iz}{\sqrt{2}}}+i\,e^{i\pi(k+\nu)-\frac{iz}{\sqrt{2}}}\right)\left(1+O\left(\frac{1}{z}\right)\right)+e^{\frac{z}{\sqrt{2}}}\left(\sqrt[4]{-1}\,e^{i\pi\nu-\frac{iz}{\sqrt{2}}}+e^{\frac{iz}{\sqrt{2}}+i\pi(k+2\nu)}\right)\left(1+O\left(\frac{1}{z}\right)\right)\right)\;/;\;(z\to-\infty)$$

03.16.06.0026.01

$$\text{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{5i\pi\nu}{2}+\frac{3\pi i}{8}}e^{\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}}e^{-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}}e^{\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{-\frac{iz}{\sqrt{2}}}e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)-\frac{1-\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}e^{-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{5i\pi\nu}{2}+\frac{\pi i}{8}}e^{\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}}e^{\frac{i\pi\nu}{2}+\frac{3\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{-\frac{iz}{\sqrt{2}}}e^{\frac{3i\pi\nu}{2}-\frac{3\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right)\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

In trigonometric form ||| In trigonometric form

03.16.06.0027.01

$$\operatorname{ker}_{\nu}(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt{-z}} \left(2\,e^{\frac{z}{\sqrt{2}}} \cos(\pi\,\nu) \cos\left(\frac{1}{8}\left(-4\sqrt{2}\,z + 4\,\pi\,\nu + \pi\right)\right) + e^{-\frac{z}{\sqrt{2}}}\,i \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z + \pi\,(4\,\nu - 3)\right)\right) + \frac{1 - 4\,\nu^2}{8\,z} \left(2\,e^{\frac{z}{\sqrt{2}}} \cos(\pi\,\nu) \cos\left(\frac{1}{8}\left(\pi\,(4\,\nu + 3) - 4\sqrt{2}\,z\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z + \pi\,(4\,\nu - 1)\right)\right)\right) + \frac{16\,\nu^4 - 40\,\nu^2 + 9}{128\,z^2} \left(e^{-\frac{z}{\sqrt{2}}}\,i \cos\left(\frac{1}{8}\left(-4\sqrt{2}\,z - \pi\,(4\,\nu - 3)\right)\right) + 2\,e^{\frac{z}{\sqrt{2}}} \cos(\pi\,\nu) \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z - 4\,\pi\,\nu - \pi\right)\right)\right) + \frac{-64\,\nu^6 + 560\,\nu^4 - 1036\,\nu^2 + 225}{3072\,z^3} \left(2\,e^{\frac{z}{\sqrt{2}}} \cos(\pi\,\nu) \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z - \pi\,(4\,\nu + 3)\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(-4\sqrt{2}\,z - \pi\,(4\,\nu - 1)\right)\right)\right) + \ldots\right) /; (z \to -\infty)$$

03.16.06.0028.01

$$\operatorname{ker}_{\nu}(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt{-z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2} - \nu\right)_{2k} \left(\nu + \frac{1}{2}\right)_{2k} \left(\frac{1}{4\,z^2}\right)^k}{(2\,k)!} \left(2\,e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi\,k}{2} + \frac{1}{8}\left(-4\sqrt{2}\,z + 4\,\pi\,\nu + \pi\right)\right) \cos(\pi\,\nu) + e^{-\frac{z}{\sqrt{2}}}\,i \sin\left(\frac{\pi\,k}{2} + \frac{1}{8}\left(4\sqrt{2}\,z + \pi\,(4\,\nu - 3)\right)\right)\right) + \frac{1}{2\,z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2} - \nu\right)_{2k+1} \left(\nu + \frac{1}{2}\right)_{2k+1} \left(\frac{1}{4\,z^2}\right)^k}{(2\,k+1)!} \left(2 \cos(\pi\,\nu)\,e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi\,k}{2} + \frac{1}{8}\left(\pi\,(4\,\nu + 3) - 4\sqrt{2}\,z\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{\pi\,k}{2} + \frac{1}{8}\left(4\sqrt{2}\,z + \pi\,(4\,\nu - 1)\right)\right)\right) + \ldots\right) /; (z \to -\infty) \land n \in \mathbb{N}$$

03.16.06.0029.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt{-z}} \Bigg(\bigg(2\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\left(-4\sqrt{2}\,z+4\pi\nu+\pi\right)\Big)\cos(\pi\nu)+e^{-\frac{z}{\sqrt{2}}}\,i\sin\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-3)\right)\Big)\bigg)\,{}_8F_3\Big(\frac{1}{8}(1-2\nu),\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(2\nu+1),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7);\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\Big)+\frac{1-4\nu^2}{8z}\bigg(2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\Big)-i\,e^{-\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-1)\right)\Big)\bigg)\,{}_8F_3\Big(\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9);\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\Big)+\frac{16\nu^4-40\nu^2+9}{128z^2}\bigg(i\,e^{-\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-3)\right)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\left(-4\sqrt{2}\,z+4\pi\nu+\pi\right)\Big)\bigg)\,{}_8F_3\Big(\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11);\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\Big)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\bigg(e^{-\frac{z}{\sqrt{2}}}(-i)\cos\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-1)\right)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\Big)\bigg)\,{}_8F_3\Big(\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(13-2\nu),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11),\frac{1}{8}(2\nu+13);\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\Big)+\ldots\Bigg)\,/;\,(z\to-\infty)$$

03.16.06.0030.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt{-z}} \Bigg(\bigg(2\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\left(-4\sqrt{2}\,z+4\pi\nu+\pi\right)\Big)\cos(\pi\nu)+e^{-\frac{z}{\sqrt{2}}}\,i\sin\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-3)\right)\Big)\bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+\frac{1-4\nu^2}{8z}\bigg(2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\Big)-i\,e^{-\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-1)\right)\Big)\bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+\frac{16\nu^4-40\nu^2+9}{128z^2}\bigg(i\,e^{-\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-3)\right)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\left(-4\sqrt{2}\,z+4\pi\nu+\pi\right)\Big)\bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\bigg(e^{-\frac{z}{\sqrt{2}}}(-i)\cos\Big(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu-1)\right)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\Big)\bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+\ldots\Bigg)\,/;\,(z\to-\infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-free arguments Ker

03.16.06.0031.01

$$
\begin{aligned}
\operatorname{ker}(z) \propto & -\frac{e^{-\frac{(1+i) z}{\sqrt{2}}}}{8 \sqrt{2 \pi} \sqrt{-\sqrt[4]{-1}\, z}\, \left((-1)^{3/4} z\right)^{3/2}} \Bigg(\Bigg(\sqrt{(-1)^{3/4} z}\, \Bigg(4 \left(e^{i \sqrt{2} z} z-(-1)^{3/4} e^{\sqrt{2} z} \sqrt{-i z^2} \right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z) \right) - \\
& \frac{\pi \left(\sqrt{2}\, e^{i \sqrt{2} z} i z+e^{\sqrt{2} z} (1+i) \left(\sqrt{2}\, (-2+2 i) z+\sqrt{-i z^2} \right) \right)}{\sqrt{2}} \Bigg) + \sqrt{-\sqrt[4]{-1}\, z} \\
& \left(\pi \left(\frac{\sqrt{i z^2}\, (3-3 i)}{\sqrt{2}}-3 i e^{(1+i) \sqrt{2} z} z+4 z \right) + 4 \left(e^{(1+i) \sqrt{2} z} z+\sqrt[4]{-1} \sqrt{i z^2} \right) \left(\log\left((-1)^{3/4} z\right)-\log(z) \right) \right) \Bigg) \\
& \left(1+O\left(\frac{1}{z^4}\right) \right) - \frac{(-1)^{3/4}}{8 z} \Bigg(\sqrt{(-1)^{3/4} z}\, \Bigg(\frac{1}{2} \left((1+i) e^{\sqrt{2} z} \pi \left((4+4 i) z-i \sqrt{2} \sqrt{-i z^2} \right) -2 e^{i \sqrt{2} z} \pi z \right) + \\
& 4 \left(\sqrt[4]{-1}\, e^{\sqrt{2} z} \sqrt{-i z^2}-i e^{i \sqrt{2} z} z \right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z) \right) \Bigg) + \sqrt{-\sqrt[4]{-1}\, z} \\
& \left(\pi \left(-3 i e^{2 \sqrt[4]{-1} z} z-4 z+3 (-1)^{3/4} \sqrt{i z^2} \right) + 4 \left(e^{(1+i) \sqrt{2} z} z-\sqrt[4]{-1} \sqrt{i z^2} \right) \left(\log\left((-1)^{3/4} z\right)-\log(z) \right) \right) \Bigg) \\
& \left(1+O\left(\frac{1}{z^4}\right) \right) + \frac{9 i}{128 z^2} \Bigg(\sqrt{(-1)^{3/4} z}\, \Bigg(4 \left(e^{i \sqrt{2} z} z-(-1)^{3/4} e^{\sqrt{2} z} \sqrt{-i z^2} \right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z) \right) - \\
& \frac{\pi \left(\sqrt{2}\, e^{i \sqrt{2} z} i z+e^{\sqrt{2} z} (1+i) \left(\sqrt{2}\, (-2+2 i) z+\sqrt{-i z^2} \right) \right)}{\sqrt{2}} \Bigg) - \sqrt{-\sqrt[4]{-1}\, z} \\
& \left(\pi \left(\frac{\sqrt{i z^2}\, (3-3 i)}{\sqrt{2}}-3 i e^{(1+i) \sqrt{2} z} z+4 z \right) + 4 \left(e^{(1+i) \sqrt{2} z} z+\sqrt[4]{-1} \sqrt{i z^2} \right) \left(\log\left((-1)^{3/4} z\right)-\log(z) \right) \right) \Bigg) \\
& \left(1+O\left(\frac{1}{z^4}\right) \right) + \frac{75 \sqrt[4]{-1}}{1024 z^3} \Bigg(\sqrt{(-1)^{3/4} z}\, \Bigg(\frac{1}{2} \left((1+i) e^{\sqrt{2} z} \pi \left((4+4 i) z-i \sqrt{2} \sqrt{-i z^2} \right) -2 e^{i \sqrt{2} z} \pi z \right) + \\
& 4 \left(\sqrt[4]{-1}\, e^{\sqrt{2} z} \sqrt{-i z^2}-i e^{i \sqrt{2} z} z \right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z) \right) \Bigg) - \\
& \sqrt{-\sqrt[4]{-1}\, z}\, \Bigg(\pi \left(-3 i e^{2 \sqrt[4]{-1} z} z-4 z+3 (-1)^{3/4} \sqrt{i z^2} \right) + 4 \left(e^{(1+i) \sqrt{2} z} z-\sqrt[4]{-1} \sqrt{i z^2} \right) \\
& \left(\log\left((-1)^{3/4} z\right)-\log(z) \right) \Bigg) \Bigg) \left(1+O\left(\frac{1}{z^4}\right) \right) \Bigg) \;/;\; (|z| \to \infty)
\end{aligned}
$$

03.16.06.0032.01

$$\ker(z) \propto -\frac{e^{-\frac{(1+i)z}{\sqrt{2}}}}{8\sqrt{2\pi}\sqrt{-\sqrt[4]{-1}\,z}\,((-1)^{3/4}z)^{3/2}} \left(\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{1}{(2k)!}\left(\frac{1}{2}\right)_{2k}^{2}\left(\frac{i}{4z^2}\right)^k \left(\frac{\pi}{\sqrt{2}}\left((-1)^{k+\frac{3}{4}}\sqrt{2}\left(4-3\,i\,e^{(1+i)\sqrt{2}\,z}\right)\left(-\sqrt[4]{-1}\,z\right)^{3/2}+3(-1)^k(1-i)\sqrt{i z^2}\sqrt{-\sqrt[4]{-1}\,z}+\right.\right.\right.$$
$$\left.\left.\sqrt{(-1)^{3/4}z}\left(\sqrt{2}\,e^{i\sqrt{2}\,z}(-i)z-(1+i)e^{\sqrt{2}\,z}\left(2\sqrt{2}\,(-1+i)z+\sqrt{-i z^2}\right)\right)\right)+\right.$$
$$4\sqrt{(-1)^{3/4}z}\left(e^{i\sqrt{2}\,z}z-(-1)^{3/4}e^{\sqrt{2}\,z}\sqrt{-i z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)+$$
$$\left.4(-1)^k\sqrt{-\sqrt[4]{-1}\,z}\left(e^{(1+i)\sqrt{2}\,z}z+\sqrt[4]{-1}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4}z\right)-\log(z)\right)\right)-$$
$$\frac{(-1)^{3/4}}{2z}\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor}\frac{1}{(2k+1)!}\left(\frac{1}{2}\right)_{2k+1}^{2}\left(\frac{i}{4z^2}\right)^k\left(\frac{(1+i)\pi}{2}\left((-1)^{k+\frac{3}{4}}\left(4+3\,i\,e^{2\sqrt[4]{-1}\,z}\right)(-1+i)\left(-\sqrt[4]{-1}\,z\right)^{3/2}+3(-1)^{k+\frac{3}{4}}\right.\right.$$
$$\left.\left.(1-i)\sqrt{i z^2}\sqrt{-\sqrt[4]{-1}\,z}+\sqrt{(-1)^{3/4}z}\left(e^{i\sqrt{2}\,z}(-1+i)z+e^{\sqrt{2}\,z}\left(4(1+i)z-i\sqrt{2}\sqrt{-i z^2}\right)\right)\right)+$$
$$4\sqrt{(-1)^{3/4}z}\left(\sqrt[4]{-1}\,e^{\sqrt{2}\,z}\sqrt{-i z^2}-i\,e^{i\sqrt{2}\,z}z\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)+$$
$$\left.\left.4(-1)^k\sqrt{-\sqrt[4]{-1}\,z}\left(e^{(1+i)\sqrt{2}\,z}z-\sqrt[4]{-1}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4}z\right)-\log(z)\right)\right)+\ldots\right)\;/;\;(|z|\to\infty)\land n\in\mathbb{N}$$

03.16.06.0033.01

$$\ker(z) \propto -\frac{e^{-\frac{(1+i)z}{\sqrt{2}}}}{8\sqrt{2\pi}\sqrt{-\sqrt[4]{-1}\,z}\,((-1)^{3/4}z)^{3/2}}\left(\left(\sqrt{(-1)^{3/4}z}\left(4\left(e^{i\sqrt{2}\,z}z-(-1)^{3/4}e^{\sqrt{2}\,z}\sqrt{-i z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)-\right.\right.\right.$$
$$\left.\frac{\pi\left(\sqrt{2}\,e^{i\sqrt{2}\,z}\,i\,z+e^{\sqrt{2}\,z}(1+i)\left(\sqrt{2}\,(-2+2i)z+\sqrt{-i z^2}\right)\right)}{\sqrt{2}}\right)+\sqrt{-\sqrt[4]{-1}\,z}$$
$$\left.\left(\pi\left(\frac{\sqrt{i z^2}\,(3-3i)}{\sqrt{2}}-3\,i\,e^{(1+i)\sqrt{2}\,z}z+4z\right)+4\left(e^{(1+i)\sqrt{2}\,z}z+\sqrt[4]{-1}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4}z\right)-\log(z)\right)\right)\right)$$
$${}_8F_3\left(\frac{1}{8},\frac{1}{8},\frac{3}{8},\frac{3}{8},\frac{5}{8},\frac{5}{8},\frac{7}{8},\frac{7}{8};\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\right)-\frac{(-1)^{3/4}}{8z}$$
$$\left(\sqrt{(-1)^{3/4}z}\left(\frac{1}{2}\left((1+i)e^{\sqrt{2}\,z}\pi\left((4+4i)z-i\sqrt{2}\sqrt{-i z^2}\right)-2\,e^{i\sqrt{2}\,z}\pi z\right)+\right.\right.$$
$$4\left(\sqrt[4]{-1}\,e^{\sqrt{2}\,z}\sqrt{-i z^2}-i\,e^{i\sqrt{2}\,z}z\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)\Bigg)+\sqrt{-\sqrt[4]{-1}\,z}$$

$$\left(\pi\left(-3\,i\,e^{2\sqrt[4]{-1}\,z}\,z-4\,z+3\,(-1)^{3/4}\sqrt{i\,z^2}\right)+4\left(e^{(1+i)\sqrt{2}\,z}\,z-\sqrt[4]{-1}\,\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)$$

$${}_8F_3\left(\frac{3}{8},\frac{3}{8},\frac{5}{8},\frac{5}{8},\frac{7}{8},\frac{7}{8},\frac{9}{8},\frac{9}{8};\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\right)+\frac{9\,i}{128\,z^2}$$

$$\left(\sqrt{(-1)^{3/4}\,z}\,\left(4\left(e^{i\sqrt{2}\,z}\,z-(-1)^{3/4}\,e^{\sqrt{2}\,z}\sqrt{-i\,z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)-\right.\right.$$

$$\left.\frac{\pi\left(\sqrt{2}\,e^{i\sqrt{2}\,z}\,i\,z+e^{\sqrt{2}\,z}\,(1+i)\left(\sqrt{2}\,(-2+2\,i)\,z+\sqrt{-i\,z^2}\right)\right)}{\sqrt{2}}\right)-\sqrt{-\sqrt[4]{-1}\,z}$$

$$\left(\pi\left(\frac{\sqrt{i\,z^2}\,(3-3\,i)}{\sqrt{2}}-3\,i\,e^{(1+i)\sqrt{2}\,z}\,z+4\,z\right)+4\left(e^{(1+i)\sqrt{2}\,z}\,z+\sqrt[4]{-1}\,\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)$$

$${}_8F_3\left(\frac{5}{8},\frac{5}{8},\frac{7}{8},\frac{7}{8},\frac{9}{8},\frac{9}{8},\frac{11}{8},\frac{11}{8};\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\right)+\frac{75\sqrt[4]{-1}}{1024\,z^3}$$

$$\left(\sqrt{(-1)^{3/4}\,z}\,\left(\frac{1}{2}\left((1+i)\,e^{\sqrt{2}\,z}\,\pi\left((4+4\,i)\,z-i\sqrt{2}\,\sqrt{-i\,z^2}\right)-2\,e^{i\sqrt{2}\,z}\,\pi\,z\right)+\right.\right.$$

$$\left.4\left(\sqrt[4]{-1}\,e^{\sqrt{2}\,z}\sqrt{-i\,z^2}-i\,e^{i\sqrt{2}\,z}\,z\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)\right)-\sqrt{-\sqrt[4]{-1}\,z}$$

$$\left(\pi\left(-3\,i\,e^{2\sqrt[4]{-1}\,z}\,z-4\,z+3\,(-1)^{3/4}\sqrt{i\,z^2}\right)+4\left(e^{(1+i)\sqrt{2}\,z}\,z-\sqrt[4]{-1}\,\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)$$

$$\left.\left.{}_8F_3\left(\frac{7}{8},\frac{7}{8},\frac{9}{8},\frac{9}{8},\frac{11}{8},\frac{11}{8},\frac{13}{8},\frac{13}{8};\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\right)\right)\right)\ /;\ (|z|\to\infty)$$

03.16.06.0034.01

$$\operatorname{ker}(z)\propto-\frac{e^{-\frac{(1+i)\,z}{\sqrt{2}}}}{8\sqrt{2\pi}\,\sqrt{-\sqrt[4]{-1}\,z}\,\left((-1)^{3/4}\,z\right)^{3/2}}\left(\sqrt{(-1)^{3/4}\,z}\,\left(4\left(e^{i\sqrt{2}\,z}\,z-(-1)^{3/4}\,e^{\sqrt{2}\,z}\sqrt{-i\,z^2}\right)\left(\log\left(-\sqrt[4]{-1}\,z\right)-\log(z)\right)-\right.\right.$$

$$\left.\frac{\pi\left(\sqrt{2}\,e^{i\sqrt{2}\,z}\,i\,z+e^{\sqrt{2}\,z}\,(1+i)\left(\sqrt{2}\,(-2+2\,i)\,z+\sqrt{-i\,z^2}\right)\right)}{\sqrt{2}}\right)+$$

$$\left.\sqrt{-\sqrt[4]{-1}\,z}\,\left(\pi\left(\frac{\sqrt{i\,z^2}\,(3-3\,i)}{\sqrt{2}}-3\,i\,e^{(1+i)\sqrt{2}\,z}\,z+4\,z\right)+4\left(e^{(1+i)\sqrt{2}\,z}\,z+\sqrt[4]{-1}\,\sqrt{i\,z^2}\right)\left(\log\left((-1)^{3/4}\,z\right)-\log(z)\right)\right)\right)$$

$$\left(1+O\left(\frac{1}{z^4}\right)\right)\ /;\ (|z|\to\infty)$$

03.16.06.0035.01

$$\ker(z) \propto \begin{cases} \frac{\sqrt[8]{-1}\left((1-i)+\sqrt{2}\, e^{i\sqrt{2}\, z}\right)\sqrt{\pi}}{4\, e^{\sqrt[4]{-1}\, z}\sqrt{z}} & 4\arg(z) \le \pi \\ \sqrt{\frac{\pi}{2}}\, \frac{\sqrt[8]{-1}}{2\sqrt{z}}\left(-(-1)^{3/4}\, e^{-\sqrt[4]{-1}\, z} + 2\sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z} + e^{(-1)^{3/4} z}\right) & 4\arg(z) \le 3\pi \\ \frac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z}\sqrt{\pi}}{4\sqrt{z}}\left((1-i)+\sqrt{2}\, e^{i\sqrt{2}\, z} + 2\sqrt{2}\, e^{\sqrt{2}\, z}\, i + e^{2\sqrt[4]{-1}\, z}(2+2i)\right) & \text{True} \end{cases} /;\ (|z| \to \infty)$$

Residue representations

03.16.06.0036.01

$$\ker(z) = \frac{1}{4}\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)^2 \left(\frac{z}{4}\right)^{-4s}}{\Gamma\left(\frac{1}{2}-s\right)}\, \Gamma\left(s+\frac{1}{2}\right)\right)\left(-j-\frac{1}{2}\right) + \frac{1}{4}\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma\left(s+\frac{1}{2}\right)\left(\frac{z}{4}\right)^{-4s}}{\Gamma\left(\frac{1}{2}-s\right)}\, \Gamma(s)^2\right)(-j)$$

Integral representations

On the real axis

Contour integral representations

03.16.07.0001.01

$$\ker(z) = \frac{1}{8\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)^2\, \Gamma\left(s+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}-s\right)} \left(\frac{z}{4}\right)^{-4s} ds$$

Limit representations

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.16.13.0001.01

$$w^{(4)}(z)\, z^4 + 2\, w^{(3)}(z)\, z^3 - w''(z)\, z^2 + w'(z)\, z + z^4\, w(z) = 0 /;\ w(z) = c_1 \operatorname{ber}(z) + c_2 \operatorname{bei}(z) + c_3 \ker(z) + c_4 \operatorname{kei}(z)$$

03.16.13.0002.01

$$W_z(\operatorname{ber}(z), \operatorname{bei}(z), \ker(z), \operatorname{kei}(z)) = -\frac{1}{z^2}$$

03.16.13.0003.01

$$g(z)^4\, g'(z)^3\, w^{(4)}(z) + 2\, g(z)^3 \left(g'(z)^2 - 3\, g(z)\, g''(z)\right) g'(z)^2\, w^{(3)}(z) - g(z)^2 \left(g'(z)^4 + 6\, g(z)\, g''(z)\, g'(z)^2 + 4\, g(z)^2\, g^{(3)}(z)\, g'(z) - 15\, g(z)^2\, g''(z)^2\right) g'(z)\, w''(z) + g(z)\left(g'(z)^6 + g(z)\, g''(z)\, g'(z)^4 - 2\, g(z)^2\, g^{(3)}(z)\, g'(z)^3 + g(z)^2\left(6\, g''(z)^2 - g(z)\, g^{(4)}(z)\right) g'(z)^2 + 10\, g(z)^3\, g''(z)\, g^{(3)}(z)\, g'(z) - 15\, g(z)^3\, g''(z)^3\right) w'(z) + g(z)^4\, g'(z)^7\, w(z) = 0 /;\ w(z) = c_1 \operatorname{ber}(g(z)) + c_2 \operatorname{bei}(g(z)) + c_3 \ker(g(z)) + c_4 \operatorname{kei}(g(z))$$

03.16.13.0004.01

$$W_z(\operatorname{ber}(g(z)), \operatorname{bei}(g(z)), \ker(g(z)), \operatorname{kei}(g(z))) = -\frac{g'(z)^6}{g(z)^2}$$

03.16.13.0005.01

$$g(z)^4 g'(z)^3 h(z)^4 w^{(4)}(z) + 2 g(z)^3 g'(z)^2 \left(h(z)\left(g'(z)^2 - 3 g(z) g''(z)\right) - 2 g(z) g'(z) h'(z)\right) h(z)^3 w^{(3)}(z) + g(z)^2 g'(z) \left(-\left(g'(z)^4 + 6 g(z) g''(z) g'(z)^2 + 4 g(z)^2 g^{(3)}(z) g'(z) - 15 g(z)^2 g''(z)^2\right) h(z)^2 - 6 g(z) g'(z) \left(h'(z) g'(z)^2 + g(z) h''(z) g'(z) - 3 g(z) h'(z) g''(z)\right) h(z) + 12 g(z)^2 g'(z)^2 h'(z)^2\right) h(z)^2 w''(z) + g(z) \left(\left(g'(z)^6 + g(z) g''(z) g'(z)^4 - 2 g(z)^2 g^{(3)}(z) g'(z)^3 + g(z)^2 \left(6 g''(z)^2 - g(z) g^{(4)}(z)\right) g'(z)^2 + 10 g(z)^3 g''(z) g^{(3)}(z) g'(z) - 15 g(z)^3 g''(z)^3\right) h(z)^3 + 2 g(z) g'(z) \left(h'(z) g'(z)^4 - 3 g(z) h''(z) g'(z)^3 - 2 g(z) \left(g(z) h^{(3)}(z) - 3 h'(z) g''(z)\right) g'(z)^2 + g(z)^2 \left(9 g''(z) h''(z) + 4 h'(z) g^{(3)}(z)\right) g'(z) - 15 g(z)^2 h'(z) g''(z)^2\right) h(z)^2 + 12 g(z)^2 g'(z)^2 h'(z) \left(h'(z) g'(z)^2 + 2 g(z) h''(z) g'(z) - 3 g(z) h'(z) g''(z)\right) h(z) - 24 g(z)^3 g'(z)^3 h'(z)^3\right) h(z) w'(z) + \left(g(z)^4 h(z)^4 g'(z)^7 + g(z)^4 \left(24 h'(z)^4 - 36 h(z) h''(z) h'(z)^2 + 8 h(z)^2 h^{(3)}(z) h'(z) + h(z)^2 \left(6 h''(z)^2 - h(z) h^{(4)}(z)\right)\right) g'(z)^3 - 2 g(z)^3 h(z) \left(g'(z)^2 - 3 g(z) g''(z)\right) \left(6 h'(z)^3 - 6 h(z) h''(z) h'(z) + h(z)^2 h^{(3)}(z)\right) g'(z)^2 + g(z)^2 h(z)^2 \left(h(z) h''(z) - 2 h'(z)^2\right) \left(g'(z)^4 + 6 g(z) g''(z) g'(z)^2 + 4 g(z)^2 g^{(3)}(z) g'(z) - 15 g(z)^2 g''(z)^2\right) g'(z) - g(z) h(z)^3 h'(z) \left(g'(z)^6 + g(z) g''(z) g'(z)^4 - 2 g(z)^2 g^{(3)}(z) g'(z)^3 + g(z)^2 \left(6 g''(z)^2 - g(z) g^{(4)}(z)\right) g'(z)^2 + 10 g(z)^3 g''(z) g^{(3)}(z) g'(z) - 15 g(z)^3 g''(z)^3\right)\right) w(z) = 0 \;/;$$
$$w(z) = c_1 h(z) \operatorname{ber}(g(z)) + c_2 h(z) \operatorname{bei}(g(z)) + c_3 h(z) \operatorname{ker}(g(z)) + c_4 h(z) \operatorname{kei}(g(z))$$

03.16.13.0006.01

$$W_z(h(z) \operatorname{ber}(g(z)), h(z) \operatorname{bei}(g(z)), h(z) \operatorname{ker}(g(z)), h(z) \operatorname{kei}(g(z))) = -\frac{h(z)^4 g'(z)^6}{g(z)^2}$$

03.16.13.0007.01

$$z^4 w^{(4)}(z) + (6 - 4 r - 4 s) z^3 w^{(3)}(z) + \left(4 r^2 + 12 (s-1) r + 6 (s-2) s + 7\right) z^2 w''(z) + (2 r + 2 s - 1)(-2 (s-1) s + r (2 - 4 s) - 1) z w'(z) + \left(a^4 r^4 z^{4 r} + s^4 + 4 r s^3 + 4 r^2 s^2\right) w(z) = 0 \;/;$$
$$w(z) = c_1 z^s \operatorname{ber}(a z^r) + c_2 z^s \operatorname{bei}(a z^r) + c_3 z^s \operatorname{ker}(a z^r) + c_4 z^s \operatorname{kei}(a z^r)$$

03.16.13.0008.01

$$W_z(z^s \operatorname{ber}(a z^r), z^s \operatorname{bei}(a z^r), z^s \operatorname{ker}(a z^r), z^s \operatorname{kei}(a z^r)) = -a^4 r^6 z^{4 r + 4 s - 6}$$

03.16.13.0009.01

$$w^{(4)}(z) - 4 (\log(r) + \log(s)) w^{(3)}(z) + 2 \left(2 \log^2(r) + 6 \log(s) \log(r) + 3 \log^2(s)\right) w''(z) + 4 (\log(r) + \log(s)) \left(-\log^2(s) - 2 \log(r) \log(s)\right) w'(z) + \left(a^4 \log^4(r) r^{4 z} + \log^4(s) + 4 \log(r) \log^3(s) + 4 \log^2(r) \log^2(s)\right) w(z) = 0 \;/;\; w(z) = c_1 s^z \operatorname{ber}(a r^z) + c_2 s^z \operatorname{bei}(a r^z) + c_3 s^z \operatorname{ker}(a r^z) + c_4 s^z \operatorname{kei}(a r^z)$$

03.16.13.0010.01

$$W_z(s^z \operatorname{ber}(a r^z), s^z \operatorname{bei}(a r^z), s^z \operatorname{ker}(a r^z), s^z \operatorname{kei}(a r^z)) = -a^4 r^{4 z} s^{4 z} \log^6(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.16.16.0001.01

$$\operatorname{ker}(-z) = \operatorname{ker}(z) + \operatorname{ber}(z) (\log(z) - \log(-z))$$

03.16.16.0002.01

$$\operatorname{ker}(i z) = \operatorname{ker}(z) - \frac{1}{2} \pi \operatorname{bei}(z) - (\log(i z) - \log(z)) \operatorname{ber}(z)$$

03.16.16.0003.01

$$\operatorname{ker}(-i z) = \operatorname{ker}(z) - \frac{1}{2}\pi \operatorname{bei}(z) - (\log(-i z) - \log(z)) \operatorname{ber}(z)$$

03.16.16.0004.01

$$\operatorname{ker}\left(\frac{1}{\sqrt[4]{-1}} z\right) = \operatorname{ker}\left(\sqrt[4]{-1}\ z\right) - \frac{1}{2}\pi \operatorname{bei}\left(\sqrt[4]{-1}\ z\right) - \left(\log\left(-(-1)^{3/4} z\right) - \log\left(\sqrt[4]{-1}\ z\right)\right) \operatorname{ber}\left(\sqrt[4]{-1}\ z\right)$$

03.16.16.0005.01

$$\operatorname{ker}\left((-1)^{-3/4} z\right) = \operatorname{ker}\left(\sqrt[4]{-1}\ z\right) + \left(\log\left(\sqrt[4]{-1}\ z\right) - \log\left(-\sqrt[4]{-1}\ z\right)\right) \operatorname{ber}\left(\sqrt[4]{-1}\ z\right)$$

03.16.16.0006.01

$$\operatorname{ker}\left((-1)^{3/4} z\right) = \operatorname{ker}\left(\sqrt[4]{-1}\ z\right) - \frac{1}{2}\pi \operatorname{bei}\left(\sqrt[4]{-1}\ z\right) - \left(\log\left((-1)^{3/4} z\right) - \log\left(\sqrt[4]{-1}\ z\right)\right) \operatorname{ber}\left(\sqrt[4]{-1}\ z\right)$$

03.16.16.0007.01

$$\operatorname{ker}\left(\sqrt[4]{z^4}\right) = \operatorname{ker}(z) + \frac{\pi\left(2\sqrt{z^4} - 2 z^2\right)}{8 z^2} \operatorname{bei}(z) + \frac{1}{4}\left(4\log(z) - \log\left(z^4\right)\right) \operatorname{ber}(z)$$

Addition formulas

03.16.16.0008.01

$$\operatorname{ker}(z_1 - z_2) = \sum_{k=-\infty}^{\infty} (\operatorname{ber}_k(z_2) \operatorname{ker}_k(z_1) - \operatorname{bei}_k(z_2) \operatorname{kei}_k(z_1)) /; \left|\frac{z_2}{z_1}\right| < 1$$

03.16.16.0009.01

$$\operatorname{ker}(z_1 + z_2) = \sum_{k=-\infty}^{\infty} (\operatorname{ber}_k(z_2) \operatorname{ker}_{-k}(z_1) - \operatorname{bei}_k(z_2) \operatorname{kei}_{-k}(z_1)) /; \left|\frac{z_2}{z_1}\right| < 1$$

Multiple arguments

03.16.16.0010.01

$$\operatorname{ker}(z_1 z_2) = \sum_{k=0}^{\infty} \frac{\left(1 - z_1^2\right)^k \left(\frac{z_2}{2}\right)^k}{k!} \left(\cos\left(\frac{3 k \pi}{4}\right) \operatorname{ker}_k(z_2) - \sin\left(\frac{3 k \pi}{4}\right) \operatorname{kei}_k(z_2)\right) /; \left|z_1^2 - 1\right| < 1$$

Related transformations

Involving kei(z)

03.16.16.0011.01

$$\operatorname{ker}(z) + i \operatorname{kei}(z) = K_0\left(\sqrt[4]{-1}\ z\right) + I_0\left(\sqrt[4]{-1}\ z\right)\left(-\frac{1}{4}(\pi i) - \log(z) + \log\left(\sqrt[4]{-1}\ z\right)\right)$$

03.16.16.0012.01

$$\operatorname{ker}(z) - i \operatorname{kei}(z) = \left(\frac{i \pi}{4} - \log(z) + \log\left(\sqrt[4]{-1}\ z\right)\right) J_0\left(\sqrt[4]{-1}\ z\right) - \frac{1}{2}\pi Y_0\left(\sqrt[4]{-1}\ z\right)$$

Differentiation

Low-order differentiation

03.16.20.0001.01

$$\frac{\partial \operatorname{ker}(z)}{\partial z} = \frac{\operatorname{kei}_1(z) + \operatorname{ker}_1(z)}{\sqrt{2}}$$

03.16.20.0002.01

$$\frac{\partial^2 \operatorname{ker}(z)}{\partial z^2} = \frac{1}{2} (\operatorname{kei}_2(z) - \operatorname{kei}(z))$$

Symbolic differentiation

03.16.20.0003.01

$$\frac{\partial^n \operatorname{ker}(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{2k} (i(1-i^n) \operatorname{kei}_{4k-n}(z) + (1+i^n) \operatorname{ker}_{4k-n}(z)) + \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2k+1} (-i(1-i^n) \operatorname{kei}_{4k-n+2}(z) - (1+i^n) \operatorname{ker}_{4k-n+2}(z)) \right) /; n \in \mathbb{N}$$

03.16.20.0004.01

$$\frac{\partial^n \operatorname{ker}(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \left(\frac{n+1}{2k+1} \binom{n}{2k} \left(\left(i - i^{n+1}\right) \operatorname{kei}_{4k-n}(z) + (1+i^n) \operatorname{ker}_{4k-n}(z) \right) - \frac{(1+i)\sqrt{2}\,(4k-n+1)}{z} \binom{n}{2k+1} \left((-i+i^n) \operatorname{kei}_{4k-n+1}(z) + \left(-1+i^{n+1}\right) \operatorname{ker}_{4k-n+1}(z) \right) \right) /; n \in \mathbb{N}$$

03.16.20.0005.01

$$\frac{\partial^n \operatorname{ker}(z)}{\partial z^n} = \frac{1}{4} G_{3,7}^{3,3}\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{n}{4}, \frac{1-n}{4}, \frac{3-n}{4} \\ -\frac{n}{4}, -\frac{n}{4}, \frac{2-n}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix} \right) /; n \in \mathbb{N}$$

Fractional integro-differentiation

03.16.20.0006.01

$$\frac{\partial^\alpha \operatorname{ker}(z)}{\partial z^\alpha} = \frac{\pi z^{2-\alpha}}{16} \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-4k} (4k+2)!}{((2k+1)!)^2 \Gamma(4k-\alpha+3)} z^{4k} + z^{-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-4k} (4k)! (\log(2) + \psi(2k+1))}{((2k)!)^2 \Gamma(4k-\alpha+1)} z^{4k} - z^{-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-4k} \mathcal{FC}_{\log}^{(\alpha)}(z, 4k)}{((2k)!)^2} z^{4k}$$

03.16.20.0007.01

$$\frac{\partial^\alpha \operatorname{ker}(z)}{\partial z^\alpha} = 2^{2\alpha-\frac{15}{2}} \pi^3 z^{2-\alpha} {}_2\tilde{F}_5\left(\frac{3}{4}, \frac{5}{4}; \frac{3}{2}, \frac{3-\alpha}{4}, 1-\frac{\alpha}{4}, \frac{5-\alpha}{4}, \frac{6-\alpha}{4}; -\frac{z^4}{256} \right) + 2^{2\alpha+\frac{1}{2}} \pi^2 \log(2) z^{-\alpha} {}_2\tilde{F}_5\left(\frac{1}{4}, \frac{3}{4}; \frac{1}{2}, \frac{1-\alpha}{4}, \frac{2-\alpha}{4}, \frac{3-\alpha}{4}, 1-\frac{\alpha}{4}; -\frac{z^4}{256} \right) + z^{-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-4k} (4k)!\, \psi(2k+1)}{((2k)!)^2 \Gamma(4k-\alpha+1)} z^{4k} - z^{-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-4k} \mathcal{FC}_{\log}^{(\alpha)}(z, 4k)}{((2k)!)^2} z^{4k}$$

Integration

Indefinite integration

03.16.21.0001.01

$$\int \operatorname{ker}(a z)\, dz = \frac{1}{16} z\, G_{1,5}^{3,1}\left(\frac{a z}{4}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{3}{4} \\ 0, 0, \frac{1}{2}, -\frac{1}{4}, \frac{1}{2} \end{array}\right)$$

Definite integration

03.16.21.0002.01

$$\int_0^\infty t^{\alpha-1} e^{-p t} \operatorname{ker}(t)\, dt = \frac{1}{3} 2^{\alpha-3} \left(3 \left(2 \cos\left(\frac{\pi \alpha}{4}\right) {}_4F_3\left(\frac{\alpha}{4} + \frac{1}{2}, \frac{\alpha}{4} + \frac{1}{2}, \frac{\alpha}{4}, \frac{\alpha}{4}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -p^4\right) - p^2 \alpha^2 \sin\left(\frac{\pi \alpha}{4}\right) {}_4F_3\left(\frac{\alpha}{4} + \frac{1}{2}, \frac{\alpha}{4} + \frac{1}{2}, \frac{\alpha}{4} + 1, \frac{\alpha}{4} + 1; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -p^4\right)\right) \Gamma\left(\frac{\alpha}{2}\right)^2 + 2 p\, \Gamma\left(\frac{\alpha+1}{2}\right)^2 \left(p^2 (\alpha+1)^2 \cos\left(\frac{1}{4} (\pi - \pi \alpha)\right) {}_4F_3\left(\frac{\alpha}{4} + \frac{3}{4}, \frac{\alpha}{4} + \frac{3}{4}, \frac{\alpha}{4} + \frac{5}{4}, \frac{\alpha}{4} + \frac{5}{4}; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -p^4\right) - 6 \cos\left(\frac{1}{4} \pi (\alpha+1)\right) {}_4F_3\left(\frac{\alpha}{4} + \frac{1}{4}, \frac{\alpha}{4} + \frac{1}{4}, \frac{\alpha}{4} + \frac{3}{4}, \frac{\alpha}{4} + \frac{3}{4}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -p^4\right)\right)\right) /; \operatorname{Re}(\alpha) > 0 \bigwedge \operatorname{Re}(p) > -\frac{1}{\sqrt{2}}$$

Integral transforms

Laplace transforms

03.16.22.0001.01

$$\mathcal{L}_t[\operatorname{ker}(t)](z) = \frac{1}{12 \sqrt[4]{z^4+1}} \left(8 z^3 {}_3F_2\left(1, 1, \frac{3}{2}; \frac{5}{4}, \frac{7}{4}; -z^4\right) \sqrt[4]{z^4+1} + 3 \sqrt{2}\, \pi \left(\cos\left(\frac{1}{2} \tan^{-1}(z^2)\right) - \sin\left(\frac{1}{2} \tan^{-1}(z^2)\right)\right)\right) /; \operatorname{Re}(z) > -\frac{1}{\sqrt{2}}$$

Mellin transforms

03.16.22.0002.01

$$\mathcal{M}_t[\operatorname{ker}(t)](z) = 2^{z-2} \cos\left(\frac{\pi z}{4}\right) \Gamma\left(\frac{z}{2}\right)^2 /; \operatorname{Re}(z) > 0$$

Representations through more general functions

Through hypergeometric functions

Involving hypergeometric *U*

03.16.26.0001.01

$$\operatorname{ker}(z)=\frac{1}{2}\,e^{-\sqrt[4]{-1}\,z}\sqrt{\pi}\,U\left(\frac{1}{2},1,2\sqrt[4]{-1}\,z\right)+\frac{1}{2}\,e^{-(-1)^{3/4}z}\sqrt{\pi}\,U\left(\frac{1}{2},1,2\,(-1)^{3/4}z\right)+\frac{1}{8}\left(-i\,\pi-4\log(z)+4\log\left(\sqrt[4]{-1}\,z\right)\right){}_0F_1\left(;1;\frac{i\,z^2}{4}\right)+\frac{1}{8}\left(i\,\pi-4\log(z)+4\log\left((-1)^{3/4}z\right)\right){}_0F_1\left(;1;-\frac{i\,z^2}{4}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

03.16.26.0002.01

$$\operatorname{ker}(z)=\frac{1}{4}\,G_{0,4}^{3,0}\left(\frac{z^4}{256}\,\middle|\,0,0,\tfrac{1}{2},\tfrac{1}{2}\right)/;-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{4}$$

Classical cases for powers of ker

03.16.26.0003.01

$$\operatorname{ker}\left(\sqrt[4]{z}\right)^2=\frac{1}{16\sqrt{\pi}}\,G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,0,0,\tfrac{1}{2}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{5,0}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\tfrac{1}{4},\tfrac{3}{4}\\0,0,0,\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0004.01

$$\operatorname{ker}(z)^2=\frac{1}{16\sqrt{\pi}}\,G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,0,0,\tfrac{1}{2}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\tfrac{1}{4},\tfrac{3}{4}\\0,0,0,\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving bei

03.16.26.0005.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\,0,\tfrac{1}{2},0,0\right)-\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\tfrac{1}{4},\tfrac{3}{4}\\0,\tfrac{1}{2},\tfrac{1}{2},0,0,\tfrac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0006.01

$$\operatorname{bei}(z)\operatorname{ker}(z)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\,0,\tfrac{1}{2},0,0\right)-\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\tfrac{1}{4},\tfrac{3}{4}\\0,\tfrac{1}{2},\tfrac{1}{2},0,0,\tfrac{1}{2}\end{matrix}\right)/;0\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ber

03.16.26.0007.01

$$\operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\,0,0,0,\tfrac{1}{2}\right)+\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\tfrac{1}{4},\tfrac{3}{4}\\0,0,\tfrac{1}{2},0,\tfrac{1}{2},\tfrac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0008.01

$$\operatorname{ber}(z)\operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z^4}{64} \,\middle|\, 0, 0, 0, \frac{1}{2}\right) + \frac{1}{8\sqrt{2\pi}}\, G_{2,6}^{3,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) /;\ 0 \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving powers of kei

03.16.26.0009.01

$$\operatorname{kei}\left(\sqrt[4]{z}\right)^2 + \operatorname{ker}\left(\sqrt[4]{z}\right)^2 = \frac{1}{8\sqrt{\pi}}\, G_{0,4}^{4,0}\left(\frac{z}{64} \,\middle|\, 0, 0, 0, \frac{1}{2}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0010.01

$$\operatorname{kei}\left(\sqrt[4]{z}\right)^2 - \operatorname{ker}\left(\sqrt[4]{z}\right)^2 = -\frac{1}{4}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0011.01

$$\operatorname{kei}(z)^2 + \operatorname{ker}(z)^2 = \frac{1}{8\sqrt{\pi}}\, G_{0,4}^{4,0}\left(\frac{z^4}{64} \,\middle|\, 0, 0, 0, \frac{1}{2}\right) /;\ -\frac{\pi}{4} < \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.16.26.0012.01

$$\operatorname{kei}(z)^2 - \operatorname{ker}(z)^2 = -\frac{1}{4}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) /;\ -\frac{\pi}{4} < \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving kei

03.16.26.0013.01

$$\operatorname{kei}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right) = -\frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0014.01

$$\operatorname{kei}(z)\operatorname{ker}(z) = -\frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right) /;\ -\frac{\pi}{4} < \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ber, bei and kei

03.16.26.0015.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right) + \operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{64} \,\middle|\, 0, 0, 0, \frac{1}{2}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0016.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right)-\operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0017.01

$$\operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right)+\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0018.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right)-\operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right)=\frac{1}{4}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},0,0\right)$$

Brychkov Yu.A. (2006)

03.16.26.0019.01

$$\operatorname{bei}(z)\operatorname{kei}(z)+\operatorname{ber}(z)\operatorname{ker}(z)=\frac{1}{4}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\,0,0,0,\frac{1}{2}\right)\,/;\,-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.16.26.0020.01

$$\operatorname{bei}(z)\operatorname{kei}(z)-\operatorname{ber}(z)\operatorname{ker}(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{array}\right)\,/;\,-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.16.26.0021.01

$$\operatorname{ber}(z)\operatorname{kei}(z)+\operatorname{bei}(z)\operatorname{ker}(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{array}\right)\,/;\,-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.16.26.0022.01

$$\operatorname{bei}(z)\operatorname{ker}(z)-\operatorname{ber}(z)\operatorname{kei}(z)=\frac{1}{4}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},0,0\right)\,/;\,-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{4}\bigvee\frac{3\pi}{4}<\arg(z)\leq\pi\bigvee-\pi<\arg(z)\leq-\frac{3\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving Bessel *J*

03.16.26.0023.01

$$J_0\left(\sqrt[4]{-1}\, z\right) \operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) - i\, G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\, 0, \tfrac{1}{2}, 0, 0\right) + \frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2}\end{matrix}\right) + i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2}\end{matrix}\right)\right)\right) /; -\frac{\pi}{4} < \arg(z) \leq \frac{\pi}{4}$$

Classical cases involving Bessel I

03.16.26.0024.01

$$I_0\left(\sqrt[4]{-1}\, z\right) \operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) + i\, G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\, 0, \tfrac{1}{2}, 0, 0\right) + \frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2}\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2}\end{matrix}\right)\right)\right) /; -\frac{\pi}{4} < \arg(z) \leq \frac{\pi}{4}$$

Classical cases involving Bessel K

03.16.26.0025.01

$$K_0\left(\sqrt[4]{-z}\right) \operatorname{ker}\left(\sqrt[4]{z}\right) = \frac{1}{16\sqrt{\pi}}\, G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) + \frac{1}{8\sqrt{2\pi}}\, G_{2,6}^{6,0}\left(-\frac{z}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\end{matrix}\right)$$

Classical cases involving ${}_0F_1$

03.16.26.0026.01

$${}_0F_1\left(; 1; \frac{i\sqrt{z}}{4}\right) \operatorname{ker}\left(\sqrt[4]{z}\right) = \frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) + i\, G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\, 0, \tfrac{1}{2}, 0, 0\right) + \frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2}\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2}\end{matrix}\right)\right)\right)$$

03.16.26.0027.01

$${}_0F_1\left(; 1; \frac{i\, z^2}{4}\right) \operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) + i\, G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\, 0, \tfrac{1}{2}, 0, 0\right) + \frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2}\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2}\end{matrix}\right)\right)\right) /; -\frac{\pi}{4} < \arg(z) \leq \frac{\pi}{4}$$

Generalized cases for the direct function itself

03.16.26.0028.01

$$\operatorname{ker}(z) = \frac{1}{4}\, G_{0,4}^{3,0}\left(\frac{z}{4}, \frac{1}{4}\,\middle|\, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

Generalized cases for powers of ker

03.16.26.0029.01

$$\operatorname{ker}(z)^2 = \frac{1}{16\sqrt{\pi}}\, G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4}\,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) + \frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4}\,\middle|\, \begin{matrix}\tfrac{1}{4}, \tfrac{3}{4}\\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving bei

03.16.26.0030.01

$$\operatorname{bei}(z)\operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \frac{1}{2}, 0, 0\right) - \frac{1}{8\sqrt{2\pi}}\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{1}{2}, 0, 0, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ber

03.16.26.0031.01

$$\operatorname{ber}(z)\operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \frac{1}{2}\right) + \frac{1}{8\sqrt{2\pi}}\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving powers of kei

03.16.26.0032.01

$$\operatorname{kei}(z)^2 + \operatorname{ker}(z)^2 = \frac{1}{8\sqrt{\pi}}\, G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \frac{1}{2}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0033.01

$$\operatorname{kei}(z)^2 - \operatorname{ker}(z)^2 = -\frac{1}{4}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving kei

03.16.26.0034.01

$$\operatorname{kei}(z)\operatorname{ker}(z) = -\frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ber, bei and kei

03.16.26.0035.01

$$\operatorname{bei}(z)\operatorname{kei}(z) + \operatorname{ber}(z)\operatorname{ker}(z) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \frac{1}{2}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0036.01

$$\operatorname{bei}(z)\operatorname{kei}(z) - \operatorname{ber}(z)\operatorname{ker}(z) = -\frac{1}{4\sqrt{2\pi}}\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0037.01

$$\operatorname{bei}(z)\operatorname{ker}(z)+\operatorname{ber}(z)\operatorname{kei}(z)=-\frac{1}{4\sqrt{2\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.16.26.0038.01

$$\operatorname{bei}(z)\operatorname{ker}(z)-\operatorname{ber}(z)\operatorname{kei}(z)=\frac{1}{4}\sqrt{\pi}\,G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},0,0\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving Bessel J

03.16.26.0039.01

$$J_0\!\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}(z)=\frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,0,0,\frac{1}{2}\right)-\right.$$
$$\left.i\,G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},0,0\right)+\frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)+i\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{matrix}\right)\right)\right)$$

Generalized cases involving Bessel I

03.16.26.0040.01

$$I_0\!\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}(z)=\frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,0,0,\frac{1}{2}\right)+\right.$$
$$\left.i\,G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},0,0\right)+\frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{matrix}\right)\right)\right)$$

Generalized cases involving Bessel K

03.16.26.0041.01

$$K_0\!\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}(z)=\frac{1}{16\sqrt{\pi}}\,G_{0,4}^{4,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,0,0,\frac{1}{2}\right)+\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{6,0}\!\left(\frac{1}{2}\sqrt[4]{-1}\,z,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2}\end{matrix}\right)\,/;$$
$$-\pi<\arg(z)\leq\frac{3\pi}{4}$$

Generalized cases involving ${}_0F_1$

03.16.26.0042.01

$${}_0F_1\!\left(;1;\frac{i\,z^2}{4}\right)\operatorname{ker}(z)=\frac{1}{8}\sqrt{\pi}\left(G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,0,0,\frac{1}{2}\right)+\right.$$
$$\left.i\,G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},0,0\right)+\frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{matrix}\right)\right)\right)$$

Representations through equivalent functions

With related functions

03.16.27.0001.01

$$\operatorname{ker}(z)=\frac{1}{4}\left(2K_0\left(\sqrt[4]{-1}\,z\right)-\pi Y_0\left(\sqrt[4]{-1}\,z\right)+\pi\operatorname{bei}(z)-4\left(\log(z)-\log\left(\sqrt[4]{-1}\,z\right)\right)\operatorname{ber}(z)\right)$$

03.16.27.0002.01

$$\operatorname{ker}(z)=\frac{1}{8}\left(4K_0\left(\sqrt[4]{-1}\,z\right)-2\pi Y_0\left(\sqrt[4]{-1}\,z\right)+\left(-i\pi-4\left(\log(z)-\log\left(\sqrt[4]{-1}\,z\right)\right)\right)I_0\left(\sqrt[4]{-1}\,z\right)+\left(i\pi-4\left(\log(z)-\log\left(\sqrt[4]{-1}\,z\right)\right)\right)J_0\left(\sqrt[4]{-1}\,z\right)\right)$$

03.16.27.0003.01

$$\operatorname{ker}(z)=\begin{cases}-i\pi I_0\left(\sqrt[4]{-1}\,z\right)+\frac{1}{2}K_0\left(\sqrt[4]{-1}\,z\right)-\frac{1}{4}\pi\left(3iJ_0\left(\sqrt[4]{-1}\,z\right)+Y_0\left(\sqrt[4]{-1}\,z\right)\right) & \frac{3\pi}{4}<\arg(z)\le\pi\\ \frac{1}{2}K_0\left(\sqrt[4]{-1}\,z\right)-\frac{1}{4}\pi\left(Y_0\left(\sqrt[4]{-1}\,z\right)-iJ_0\left(\sqrt[4]{-1}\,z\right)\right) & \text{True}\end{cases}$$

03.16.27.0004.01

$$\operatorname{ker}(z)+i\operatorname{kei}(z)=K_0\left(\sqrt[4]{-1}\,z\right)+\frac{1}{4}I_0\left(\sqrt[4]{-1}\,z\right)\left(-i\pi-4\log(z)+4\log\left(\sqrt[4]{-1}\,z\right)\right)$$

03.16.27.0005.01

$$\operatorname{ker}(z)+i\operatorname{kei}(z)=\begin{cases}K_0\left(\sqrt[4]{-1}\,z\right)-2i\pi I_0\left(\sqrt[4]{-1}\,z\right) & \frac{3\pi}{4}<\arg(z)\le\pi\\ K_0\left(\sqrt[4]{-1}\,z\right) & \text{True}\end{cases}$$

03.16.27.0006.01

$$\operatorname{ker}(z)-i\operatorname{kei}(z)=\frac{1}{4}\left(\left(i\pi-4\log(z)+4\log\left(\sqrt[4]{-1}\,z\right)\right)J_0\left(\sqrt[4]{-1}\,z\right)-2\pi Y_0\left(\sqrt[4]{-1}\,z\right)\right)$$

03.16.27.0007.01

$$\operatorname{ker}(z)-i\operatorname{kei}(z)=\begin{cases}-\frac{1}{2}\pi\left(3iJ_0\left(\sqrt[4]{-1}\,z\right)+Y_0\left(\sqrt[4]{-1}\,z\right)\right) & \frac{3\pi}{4}<\arg(z)\le\pi\\ -\frac{1}{2}\pi\left(Y_0\left(\sqrt[4]{-1}\,z\right)-iJ_0\left(\sqrt[4]{-1}\,z\right)\right) & \text{True}\end{cases}$$

Theorems

History

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