KroneckerDelta2

Notations

Traditional name

Multivariate Kronecker delta function

Traditional notation

$\delta_{n_1,n_2,\ldots}$

Mathematica StandardForm notation

`KroneckerDelta[`n_1`, `n_2`, …]`

Primary definition

04.20.02.0001.01

$\delta_{n_1,n_2,\ldots} = 1 \;/;\; n_1 = n_2 = \ldots \bigwedge n_1 \in \mathbb{Q} \bigwedge n_2 \in \mathbb{Q} \bigwedge \ldots$

04.20.02.0002.01

$\delta_{n_1,n_2,\ldots} = 0 \;/;\; \neg\, n_1 = n_2 = \ldots$

Kronecker delta function equal to 1 if all its arguments are equal, and 0 otherwise.

Specific values

Specialized values

04.20.03.0001.01

$\delta_{n,0} = \delta_n$

04.20.03.0002.01

$\delta_{0,n} = \delta_n$

Values at fixed points

04.20.03.0003.01

$\delta_0 = 1$

04.20.03.0004.01

$\delta_1 = 0$

04.20.03.0005.01

$\delta_{1,1} = 1$

04.20.03.0006.01

$\delta_{1,2} = 0$

04.20.03.0007.01

$\delta_{1,1,1} = 1$

04.20.03.0008.01

$\delta_{1,1,2} = 0$

04.20.03.0009.01

$\delta_{1,2,1} = 0$

04.20.03.0010.01

$\delta_{2,1,1} = 0$

04.20.03.0011.01

$\delta_{1,1,1,1} = 1$

04.20.03.0012.01

$\delta_{1,1,1,2} = 0$

04.20.03.0013.01

$\delta_{1,1,2,1} = 0$

Values at infinities

04.20.03.0014.01

$\delta_{\infty} = 0$

04.20.03.0015.01

$\delta_{-\infty} = 0$

04.20.03.0016.01

$\delta_{\infty,-\infty} = 0$

04.20.03.0017.01

$\delta_{-\infty,\infty} = 0$

General characteristics

Domain and analyticity

$\delta_{n_1,n_2,\ldots,n_m}$ is a nonanalytical function defined over $\mathbb{Q}^m$. Its possible values are 0 and 1.

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$(n_1 * n_2 * \ldots * n_m) \longrightarrow \delta_{n_1,n_2,\ldots,n_m} :: \mathbb{Q}^m \longrightarrow \{0, 1\}$

Symmetries and periodicities

Parity

$\delta_{n_1,n_2,\ldots,n_m}$ is an even function.

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$\delta_{-n_1,-n_2,\ldots,-n_m} = \delta_{n_1,n_2,\ldots,n_m}$

Permutation symmetry

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$\delta_{m,n} = \delta_{n,m}$

04.20.04.0004.01

$\delta_{n_1,n_2,\ldots,n_k,\ldots,n_j,\ldots,n_m} = \delta_{n_1,n_2,\ldots,n_j,\ldots,n_k,\ldots,n_m} \,/;\, n_k \neq n_j \bigwedge k \neq j$

Periodicity

No periodicity

Integral representations

On the real axis

04.20.07.0001.01

$$\delta_{n,m} = \frac{1}{2\pi} \int_0^{2\pi} e^{i\,t\,(n-m)}\, dt$$

Contour integral representations

04.20.07.0002.01

$$\delta_{n,m} = \frac{1}{2\pi i} \int_{|z|=1} z^{n-m-1}\, dz$$

Transformations

Transformations and argument simplifications

04.20.16.0001.01

$\delta_{-n_1,-n_2,\ldots,-n_m} = \delta_{n_1,n_2,\ldots,n_m}$

04.20.16.0002.01

$\delta_{-n,m} = \delta_{n,m} - \theta(|n| - |m|)\,\mathrm{sgn}(n\,m) \,/;\, n \in \mathbb{Z} \land m \in \mathbb{Z}$

Products, sums, and powers of the direct function

04.20.16.0003.01

$\delta_{n_1,n_2,\ldots,n_m}\, \delta_{n_{m+1},n_{m+2},\ldots,n_{m+r}} = \delta_{n_1,n_2,\ldots,n_m,n_{m+1},n_{m+2},\ldots,n_{m+r}} \,/;\, m \geq 2 \land r \geq 2$

Identities

Functional identities

04.20.17.0001.01

$\delta_{n_1,n_2,\ldots,n_m}\, \delta_{n_{m+1},n_{m+2},\ldots,n_{m+r}} = \delta_{n_1,n_2,\ldots,n_m,n_{m+1},n_{m+2},\ldots,n_{m+r}} \,/;\, m \geq 2 \land r \geq 2$

Complex characteristics

Real part

04.20.19.0001.01

$$\mathrm{Re}\left(\delta_{n_1,n_2,\ldots,n_m}\right) = \delta_{n_1,n_2,\ldots,n_m}$$

Imaginary part

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$$\mathrm{Im}\left(\delta_{n_1,n_2,\ldots,n_m}\right) = 0$$

Absolute value

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$$\left|\delta_{n_1,n_2,\ldots,n_m}\right| = \delta_{n_1,n_2,\ldots,n_m}$$

Argument

04.20.19.0004.01

$$\arg\left(\delta_{n_1,n_2,\ldots,n_m}\right) = \tan^{-1}\left(\delta_{n_1,n_2,\ldots,n_m},\, 0\right)$$

Conjugate value

04.20.19.0005.01

$$\overline{\delta_{n_1,n_2,\ldots,n_m}} = \delta_{n_1,n_2,\ldots,n_m}$$

Summation

Infinite summation

04.20.23.0001.01

$$\sum_{k=-\infty}^{\infty} \delta_{k,n}\, a_k = a_n$$

Above relation represents the sifting property of Kronecker delta function.

Representations through equivalent functions

04.20.27.0001.01

$$\delta_{n_1,n_2} = \delta_{n_1-n_2}$$

04.20.27.0002.01

$$\delta_{n_1,n_2,\ldots,n_m} = \delta_{n_1-n_m,n_2-n_m,\ldots,n_{m-1}-n_m}$$

04.20.27.0003.01

$$\delta_{n_1,n_2,\ldots,n_m,0} = \delta(n_1,\, n_2,\, \ldots,\, n_m)$$

History

– L. Kronecker (1866, 1903)

The function $\delta(n_1,\, n_2,\, \ldots,\, n_m)$ is encountered often in mathematics and the natural sciences.

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