Log2

Notations

Traditional name

Logarithm

Traditional notation

$\log_a(z)$

Mathematica StandardForm notation

`Log[a, z]`

Primary definition

01.05.02.0001.01

$$\log_a(z) = \frac{\log(z)}{\log(a)}$$

Specific values

Specialized values

For fixed *a*

01.05.03.0001.01

$$\log_a(e) = \frac{1}{\log(a)}$$

01.05.03.0002.01

$$\log_a(e^n) = \frac{n}{\log(a)} \;/;\; n \in \mathbb{Z}$$

01.05.03.0003.01

$$\log_a(0) = \frac{-\infty}{\log(a)}$$

01.05.03.0004.01

$$\log_a(1) = 0$$

01.05.03.0005.01

$$\log_a(-1) = \frac{i\,\pi}{\log(a)}$$

For fixed z

01.05.03.0006.01

$$\log_e(z) == \log(z)$$

01.05.03.0007.01

$$\log_0(z) == 0$$

01.05.03.0008.01

$$\log_1(z) == \tilde{\infty}$$

01.05.03.0009.01

$$\log_{-1}(z) == -\frac{i \log(z)}{\pi}$$

Values at fixed points

01.05.03.0010.01

$$\log_1(1) == ¿$$

General characteristics

Domain and analyticity

$\log_a(z)$ is an analytical function of a and z which is defined over $\mathbb{C}^2$.

01.05.04.0001.01

$$(a * z) \longrightarrow \log_a(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

01.05.04.0002.01

$$\log_{\bar{a}}(\bar{z}) == \overline{\log_a(z)} \;/;\; a \notin (-\infty, 0) \bigwedge z \notin (-\infty, 0)$$

Quasi-permutation symmetry

01.05.04.0003.01

$$\log_a(z) \;==\; \frac{1}{\log_z(a)}$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed a, the function $\log_a(z)$ does not have poles and essential singularities.

01.05.04.0004.01

$$\mathcal{S}ing_z(\log_a(z)) == \{\}$$

With respect to a

For fixed z, the function $\log_a(z)$has one simple pole at point $a = 1$with residue $\log(z)$.

01.05.04.0005.01

$$\mathcal{Sing}_a(\log_a(z)) = \{1\}$$

01.05.04.0006.01

$$\operatorname{res}_a(\log_a(z))(\pi k) = \log(z)$$

Branch points

With respect to z

For fixed a, the function $\log_a(z)$ has two branch points: $z = 0$, $z = \tilde{\infty}$.

01.05.04.0007.01

$$\mathcal{BP}_z(\log_a(z)) = \{0, \tilde{\infty}\}$$

01.05.04.0008.01

$$\mathcal{R}_z(\log_a(z), 0) = \log$$

01.05.04.0009.01

$$\mathcal{R}_z(\log_a(z), \tilde{\infty}) = \log$$

With respect to a

For fixed z, the function $\log_a(z)$ has two branch points: $a = 0$, $a = \tilde{\infty}$.

01.05.04.0010.01

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01.05.04.0011.01

$$\mathcal{R}_a(\log_a(z), 0) = \log$$

01.05.04.0012.01

$$\mathcal{R}_a(\log_a(z), \tilde{\infty}) = \log$$

Branch cuts

With respect to z

For fixed a, the function $\log_a(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$ where it is continuous from above.

01.05.04.0013.01

$$\mathcal{BC}_z(\log_a(z)) = \{\{(-\infty, 0), -i\}\}$$

01.05.04.0014.01

$$\lim_{\epsilon \to +0} \log_a(x + i\epsilon) = \log_a(x) \;/;\; x < 0$$

01.05.04.0015.01

$$\lim_{\epsilon \to +0} \log_a(x - i\epsilon) = \log_a(-x) - \frac{1}{\log_{-1}(a)} \;/;\; x < 0$$

01.05.04.0016.01

$$\lim_{\epsilon\to+0} \log_a(x - i\,\epsilon) == \log_a(x) - \frac{2\,i\,\pi}{\log(a)} \;/;\; x < 0$$

With respect to *a*

For fixed z, the function $\log_a(z)$ is a single-valued function on the a-plane cut along the interval $(-\infty,\ 0)$ where it is continuous from above.

01.05.04.0017.01

$$\mathcal{BC}_a\left(\log_a(z)\right) == \{\{(-\infty,\,0),\,-i\}\}$$

01.05.04.0018.01

$$\lim_{\epsilon\to+0} \log_{c+i\,\epsilon}(z) == \log_c(z) \;/;\; c < 0$$

01.05.04.0019.01

$$\lim_{\epsilon\to+0} \log_{c-i\,\epsilon}(z) == \frac{\log_{-1}(z)}{\log_z(-c)\,\log_{-1}(z) - 1} \;/;\; c < 0$$

01.05.04.0020.01

$$\lim_{\epsilon\to+0} \log_{c-i\,\epsilon}(z) == \frac{\log_{-1}(z)}{\log_z(c)\,\log_{-1}(z) - 2} \;/;\; c < 0$$

Analytic continuations

$\log_a(z)$ is chosen to be the principal branch of the general logarithmic function $\tilde{\log}_a(z)$ which has double infinitely many sheets: $\tilde{\log}_a(z) = (\log(z) + 2\,\pi\,i\,k)\,/\,(\log(a) + 2\,\pi\,i\,l),\ \ k,\,l \in \mathbb{Z}$.

Series representations

Generalized power series

Expansions at *a* == 1

For the function itself

01.05.06.0001.02

$$\log_a(z) \propto \frac{\log(z)}{a-1} + \frac{\log(z)}{2} - \frac{\log(z)}{12}\,(a-1)\left(1 - \frac{a-1}{2} + \frac{19}{60}\,(a-1)^2 - \ldots\right) /;\; (a \to 1)$$

01.05.06.0014.01

$$\log_a(z) \propto \frac{\log(z)}{a-1} + \frac{\log(z)}{2} - \frac{\log(z)}{12}\,(a-1)\left(1 - \frac{a-1}{2} + \frac{19}{60}\,(a-1)^2 - O\left((a-1)^3\right)\right)$$

01.05.06.0002.01

$$\log_a(z) \;==\; \frac{\log(z)}{2} + \frac{\log(z)}{a-1} - \sum_{k=0}^{\infty} G(k)\,(a-1)^k \log(z) \;/;\; |a-1| < 1 \bigwedge G(0) == 0 \bigwedge G(k) == \frac{(-1)^{k+1}\,k}{2\,(k+1)\,(k+2)} + \sum_{j=1}^{k} \frac{(-1)^{j+1}\,G(k-j)}{j+1}$$

01.05.06.0003.02

$$\log_a(z) \propto \frac{\log(z)}{a-1} + \frac{\log(z)}{2} + O(a-1)$$

Expansions at $z == 1$

For the function itself

01.05.06.0008.02

$$\log_a(z) \propto \frac{z-1}{\log(a)} \left(1 - \frac{z-1}{2} + \frac{(z-1)^2}{3} + \ldots\right) /; (z \to 1)$$

01.05.06.0015.01

$$\log_a(z) \propto \frac{z-1}{\log(a)} \left(1 - \frac{z-1}{2} + \frac{(z-1)^2}{3} + O\left((z-1)^3\right)\right)$$

01.05.06.0009.01

$$\log_a(z) == \frac{1}{\log(a)} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} (z-1)^k}{k} /; |z-1| < 1$$

01.05.06.0010.01

$$\log_a(z) == \frac{z-1}{\log(a)} \, {}_2F_1(1, 1; 2; 1-z)$$

01.05.06.0011.02

$$\log_a(z) \propto \frac{z-1}{\log(a)} (1 + O(z-1))$$

01.05.06.0016.01

$$\log_a(z) == F_\infty(z, a) /; \left(\left(F_n(z, a) == \frac{1}{\log(a)} \sum_{k=1}^{n} \frac{(-1)^{k-1} (z-1)^k}{k} == \frac{\mathrm{B}_{1-z}(n+1, 0) + \log(z)}{\log(a)}\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions of $\log_a(1+z)$ at $z == 0$

For the function itself

01.05.06.0004.02

$$\log_a(1+z) \propto \frac{1}{\log(a)} \left(z - \frac{1}{2} z^2 + \frac{1}{3} z^3 + \ldots\right) /; (z \to 0)$$

01.05.06.0017.01

$$\log_a(1+z) \propto \frac{1}{\log(a)} \left(z - \frac{1}{2} z^2 + \frac{1}{3} z^3 + O\left(z^4\right)\right)$$

01.05.06.0005.01

$$\log_a(1+z) == \frac{1}{\log(a)} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} z^k}{k} /; |z| < 1$$

01.05.06.0006.01

$$\log_a(1+z) = \frac{z}{\log(a)}\, {}_2F_1(1, 1; 2; -z)$$

01.05.06.0007.02

$$\log_a(1+z) \propto \frac{z}{\log(a)} + O\left(z^2\right)$$

01.05.06.0018.01

$$\log_a(1+z) = F_\infty(z, a) \;/;\; \left(\left(F_n(z, a) = \frac{1}{\log(a)} \sum_{k=1}^{n} \frac{(-1)^{k-1} z^k}{k} = \frac{\mathrm{B}_{-z}(n+1, 0) + \log(1+z)}{\log(a)}\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

01.05.06.0012.01

$$\log_a(z) \propto \frac{\log(z)}{\log(a)} \;/;\; (z \to 0 \,|\, \infty)$$

01.05.06.0013.01

$$\log_a(z) \propto \frac{\log(z)}{\log(a)} \;/;\; (a \to 0 \,|\, \infty)$$

Differential equations

Ordinary linear differential equations and wronskians

With respect to *z*

01.05.13.0002.01

$$z \log(a)\, w'(z) - 1 = 0 \;/;\; w(z) = \log_a(z) \bigwedge w(1) = 0$$

01.05.13.0003.01

$$z\, w''(z) + w'(z) = 0 \;/;\; w(z) = c_1 + c_2 \log_a(z)$$

01.05.13.0004.01

$$W_z\left(1, \log_a(z)\right) = \frac{1}{z \log(a)}$$

Ordinary nonlinear differential equations

With respect to *a*

01.05.13.0005.01

$$a\, w(a)\, w''(a) - 2\, a\, w'(a)^2 + w(a)\, w'(a) = 0 \;/;\; w(a) = \log_a(z)$$

Partial differential equations

01.05.13.0001.01

$$a \frac{\partial w(a, z)}{\partial a} + z \frac{\partial w(a, z)}{\partial z} w(a, z) = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0006.01

$$a\, w^{(0,2)}(a, z)\, w^{(1,0)}(a, z) - w(a, z)\, w^{(0,1)}(a, z)^2 = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0007.01

$$z\, w^{(0,1)}(a, z)^2 + a\, w^{(1,1)}(a, z) = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0008.01

$$w(a, z)\, w^{(1,1)}(a, z) - w^{(0,1)}(a, z)\, w^{(1,0)}(a, z) = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0009.01

$$a\, w^{(0,2)}(a, z)\, w^{(1,1)}(a, z) - w^{(0,1)}(a, z)^3 = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0010.01

$$2\, a\, w^{(1,0)}(a, z)\, w^{(1,1)}(a, z) - w^{(0,1)}(a, z)\left(w^{(1,0)}(a, z) + a\, w^{(2,0)}(a, z)\right) = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0011.01

$$\left(2\, z\, w^{(0,1)}(a, z) + 1\right) w^{(1,0)}(a, z)^2 - z\, w(a, z)\, w^{(0,1)}(a, z)\, w^{(2,0)}(a, z) = 0 \;/;\; w(a, z) = \log_a(z)$$

01.05.13.0012.01

$$\left(2\, z\, w^{(0,1)}(a, z) + 1\right) w^{(1,0)}(a, z)\, w^{(1,1)}(a, z) - z\, w^{(0,1)}(a, z)^2\, w^{(2,0)}(a, z) = 0 \;/;\; w(a, z) = \log_a(z)$$

Transformations

Multiple arguments

01.05.16.0001.01

$$\log_a(c\, z) == \log_a(c) + \log_a(z) \;/;\; c > 0$$

01.05.16.0002.01

$$\log_a(z_1\, z_2) == \log_a(z_1) + \log_a(z_2) \;/;\; z_1 + z_2 \geq 0$$

01.05.16.0003.01

$$\log_a\left(\frac{z_1}{z_2}\right) == \log_a(z_1) - \log_a(z_2) \;/;\; z_2 - z_1 \geq 0$$

01.05.16.0004.01

$$\log_a\left(\frac{z_1}{z_2}\right) == \log_a(-z_1) - \log_a(-z_2) \;/;\; z_2 - z_1 < 0$$

01.05.16.0005.01

$$\log_a\left(\frac{z_1}{z_2}\right) == \log_a(z_1) + \log_a\left(\frac{1}{z_2}\right) \;/;\; z_1 + z_2 \geq 0$$

Power of arguments

01.05.16.0006.01

$$\log_a\left(z^2\right) == \log_a(-i\, z) + \log_a(i\, z)$$

01.05.16.0007.01

$$\log_a\left(\sqrt{z}\right) == \frac{\log_a(z)}{2}$$

01.05.16.0008.01

$$\log_a(z^c) == c \log_a(z) \;/;\; z > 0 \land c \in \mathbb{R}$$

Products, sums, and powers of the direct function

Products of the direct function

01.05.16.0009.01

$$\log_a(z)\log_z(w) = \log_a(w)$$

Sums of the direct function

01.05.16.0010.01

$$\log_a(z_1) + \log_a(z_2) = \log_a(z_1 z_2) \;/;\; z_1 + z_2 \geq 0$$

01.05.16.0011.01

$$\log_a(z_1) - \log_a(z_2) = \log_a\left(\frac{z_1}{z_2}\right) \;/;\; z_2 - z_1 \geq 0$$

Identities

Functional identities

01.05.17.0001.01

$$\log_a(z) = \log_a(b)\log_b(z)$$

01.05.17.0002.01

$$w(z_1 z_2) = w(z_1) + w(z_2) \;/;\; w(z) = \log_a(z) \bigwedge z_1 + z_2 \geq 0$$

Complex characteristics

Real part

01.05.19.0001.01

$$\operatorname{Re}(\log_a(z)) = \frac{\tan^{-1}(\operatorname{Re}(a), \operatorname{Im}(a))\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) + \log(|a|)\log(|z|)}{\tan^{-1}(\operatorname{Re}(a), \operatorname{Im}(a))^2 + \log^2(|a|)}$$

Imaginary part

01.05.19.0002.01

$$\operatorname{Im}(\log_a(z)) = \frac{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\log(|a|) - \tan^{-1}(\operatorname{Re}(a), \operatorname{Im}(a))\log(|z|)}{\tan^{-1}(\operatorname{Re}(a), \operatorname{Im}(a))^2 + \log^2(|a|)}$$

Absolute value

01.05.19.0003.01

$$|\log_a(z)| = \frac{\sqrt{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))^2 + \log^2(|z|)}}{\sqrt{\tan^{-1}(\operatorname{Re}(a), \operatorname{Im}(a))^2 + \log^2(|a|)}}$$

Argument

01.05.19.0004.01

$$\arg(\log_a(z)) = \tan^{-1}\left(\frac{\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))\tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z)) + \log(|a|)\log(|z|)}{\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))^2 + \log^2(|a|)}, \frac{\tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))\log(|a|) - \tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))\log(|z|)}{\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))^2 + \log^2(|a|)}\right)$$

Conjugate value

01.05.19.0005.01

$$\overline{\log_a(z)} = \left(\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))\tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z)) + \log(|a|)\log(|z|) + i\left(\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))\log(|z|) - \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))\log(|a|)\right)\right) \Big/ \left(\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))^2 + \log^2(|a|)\right)$$

Signum value

01.05.19.0006.01

$$\mathrm{sgn}(\log_a(z)) = \left(\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))\tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z)) + \log(|a|)\log(|z|) - i\left(\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))\log(|z|) - \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))\log(|a|)\right)\right) \Big/ \left(\sqrt{\tan^{-1}(\mathrm{Re}(a), \mathrm{Im}(a))^2 + \log^2(|a|)}\sqrt{\tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))^2 + \log^2(|z|)}\right)$$

Differentiation

Low-order differentiation

With respect to *a*

01.05.20.0001.01

$$\frac{\partial \log_a(z)}{\partial a} = -\frac{\log(z)}{a\log^2(a)}$$

01.05.20.0002.01

$$\frac{\partial^2 \log_a(z)}{\partial a^2} = \frac{(\log(a) + 2)\log(z)}{a^2\log^3(a)}$$

With respect to *z*

01.05.20.0003.01

$$\frac{\partial \log_a(z)}{\partial z} = \frac{1}{z\log(a)}$$

01.05.20.0004.01

$$\frac{\partial^2 \log_a(z)}{\partial z^2} = -\frac{1}{z^2\log(a)}$$

Symbolic differentiation

With respect to *a*

01.05.20.0013.01

$$\frac{\partial^n \log_a(z)}{\partial a^n} = a^{-n} \log(z) \log^{-n-1}(a) \sum_{k=0}^{n} (-1)^k k! \, S_n^{(k)} \log^{n-k}(a) \, /; \, n \in \mathbb{N}$$

01.05.20.0005.02

$$\frac{\partial^n \log_a(z)}{\partial a^n} = a^{-n} \log(z) \sum_{k=0}^{n} (-1)^k k! \, S_n^{(k)} \log^{-k-1}(a) \, /; \, n \in \mathbb{N}$$

With respect to *z*

01.05.20.0014.01

$$\frac{\partial^n \log_a(z)}{\partial z^n} = \delta_n \log_a(z) + \frac{S_n^{(1)} z^{-n}}{\log(a)} \, /; \, n \in \mathbb{N}$$

01.05.20.0006.01

$$\frac{\partial^n \log_a(z)}{\partial z^n} = \frac{(-1)^{n-1} (n-1)! \, z^{-n}}{\log(a)} \, /; \, n \in \mathbb{N}^+$$

01.05.20.0015.01

$$\frac{\partial^n \log_a(z)}{\partial z^n} = \frac{(z-1)^{1-n}}{\log(a)} \, {}_2\tilde{F}_1(1, 1; 2-n; 1-z) \, /; \, n \in \mathbb{N}$$

01.05.20.0007.02

$$\frac{\partial^n \log_a^w(z)}{\partial z^n} = z^{-n} \sum_{k=0}^{n} (w-k+1)_k \, S_n^{(k)} \log^{-k}(a) \log_a^{w-k}(z) \, /; \, n \in \mathbb{N}$$

01.05.20.0008.01

$$\frac{\partial^n \log_a(f(z))}{\partial z^n} = \frac{1}{\log(a)} \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k \, f(z)^k} \binom{n}{k} \frac{\partial^n f(z)^k}{\partial z^n} \, /; \, n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *a*

01.05.20.0009.01

$$\frac{\partial^\alpha \log_a(z)}{\partial a^\alpha} = \frac{\theta(\mathrm{Re}(-\alpha)) \, a^{-\alpha-1} \log(z)}{\Gamma(-\alpha)} \sum_{k=0}^{\infty} \frac{(\alpha+1)_k \, \mathrm{Ei}((k+1)\log(a)) \, a^{-k}}{k!} +$$
$$\frac{\theta(-\mathrm{Re}(-\alpha)) \, a^{-\alpha-1} \log_a(z)}{\Gamma(1-\alpha+\lfloor\alpha\rfloor)} \sum_{k=0}^{\infty} \frac{(\alpha-\lfloor\alpha\rfloor)_k \, a^{-k}}{k!} \sum_{m=0}^{\lfloor\alpha\rfloor+1} \binom{\lfloor\alpha\rfloor+1}{1-m+\lfloor\alpha\rfloor} (m-k-\alpha)_{1-m+\lfloor\alpha\rfloor} \sum_{p=0}^{m-1} \frac{1}{p!} \big((-1)^p \, p! + (k+1)\log(a)$$
$${}_2\tilde{F}_2(1, 1; 2, 1-p; (k+1)\log(a))\big) \left((-1)^{m+p+1} (m-1)! + \sum_{h=0}^{m} \log^{1-h}(a) \, S_m^{(h)} \, p! \sum_{j=0}^{p-1} \frac{(-1)^j}{j! \, (1-j+p-h)!}\right)$$

With respect to *z*

01.05.20.0010.01

$$\frac{\partial^\alpha \log_a(1+z)}{\partial z^\alpha} = \frac{z^{1-\alpha}}{\log(a)} \, {}_2\tilde{F}_1(1, 1; 2-\alpha; -z)$$

01.05.20.0011.01

$$\frac{\partial^{\alpha} \log_a(z)}{\partial z^{\alpha}} = \frac{\mathcal{F}C_{\log}^{(\alpha)}(z)}{\log(a)} z^{-\alpha}$$

01.05.20.0012.01

$$\frac{\partial^{\alpha} \left(z^b \log_a(z)\right)}{\partial z^{\alpha}} = \frac{\mathcal{F}C_{\log}^{(\alpha)}(z, b)}{\log(a)} z^{b-\alpha}$$

Integration

Indefinite integration

Involving only one direct function

01.05.21.0001.01

$$\int \log_a(z) \, dz = \frac{z \log(z) - z}{\log(a)}$$

01.05.21.0002.01

$$\int z^{\alpha-1} \log_a(z) \, dz = \frac{z^{\alpha} (\alpha \log(z) - 1)}{\alpha^2 \log(a)}$$

Involving only one direct function with respect to *a*

01.05.21.0003.01

$$\int \log_a(z) \, da = \log(z) \operatorname{li}(a)$$

Involving one direct function and elementary functions with respect to *a*

Involving power function

01.05.21.0004.01

$$\int a^{\alpha-1} \log_a(z) \, da = \operatorname{Ei}(\alpha \log(a)) \log(z)$$

Representations through more general functions

Through Meijer *G*

Classical cases for the direct function itself

01.05.26.0001.01

$$\log_a(z) = \frac{1}{\log(a)} G_{2,2}^{1,2}\left(z - 1 \,\middle|\, \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right)$$

01.05.26.0002.01

$$\log_a(z + 1) = \frac{1}{\log(a)} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right)$$

Representations through equivalent functions

With inverse function

01.05.27.0003.01

$$\log_a(a^z) == z + \frac{2\,i\,\pi}{\log(a)} \left\lfloor \frac{\pi - \operatorname{Im}(z \log(a))}{2\,\pi} \right\rfloor$$

The left side of above formula corresponds to composition $f\left(f^{(-1)}(z)\right)$ /; $f(z) == \log_a(z)$, which generically does not equal to z.

01.05.27.0002.01

$$\log_a(a^z) == z \;/;\; a \in \mathbb{R} \land z > 0 \bigvee a > 0 \bigwedge -\pi < \operatorname{Im}(z \log(a)) \leq \pi$$

The left side of above formula corresponds to composition $f\left(f^{(-1)}(z)\right)$ /; $f(z) == \log_a(z)$, which equal to z under restriction $-\pi < \operatorname{Im}(z \log(a)) \leq \pi$.

01.05.27.0001.01

$$a^{\log_a(z)} == z$$

The left side of above formula corresponds to composition $f^{(-1)}(f(z))$ /; $f(z) == \log_a(z)$, which generically equal to z.

01.05.27.0006.01

$$\log_{z^{1/a}}(z) = \frac{a \log(z)}{2\,a\,i\,\pi \left\lfloor \frac{\pi - \operatorname{Im}\left(\frac{\log(z)}{a}\right)}{2\,\pi} \right\rfloor + \log(z)}$$

The left side of above formula corresponds to composition $f\left(f^{(-1)}(a)\right)$ /; $f(a) == \log_a(z)$, which generically does not equal to a.

01.05.27.0007.01

$$\log_{z^{1/a}}(z) = a \;/;\; -\pi < \operatorname{Im}\left(\frac{\log(z)}{a}\right) \leq \pi$$

The left side of above formula corresponds to composition $f\left(f^{(-1)}(a)\right)$ /; $f(a) == \log_a(z)$, which equal to a under restriction $-\pi < \operatorname{Im}\left(\frac{\log(z)}{a}\right) \leq \pi$.

01.05.27.0008.01

$$z^{\frac{1}{\log_a(z)}} = a$$

The left side of above formula corresponds to composition $f^{(-1)}(f(a))$ /; $f(a) == \log_a(z)$, which generically equal to a.

With related functions

01.05.27.0004.01

$$\log_a(z) == \frac{\log(z)}{\log(a)}$$

01.05.27.0005.01

$z^{\log_a(b)} == b^{\log_a(z)}$

Zeros

01.05.30.0001.01

$\log_a(z) == 0 /; z == 1$

Theorems

Benford's law

The probability p_n that the first digit in a large set of data is n ($n = 1, 2, \ldots, 9$) is given by $p_n == \log_{10}\left(\frac{n+1}{n}\right)$.

The probability for the number *n* to occur in the normal continued fraction expansion

For almost all real numbers x, the probability $p_n$of occurrence of the integer n in the normal continued fraction expansion is given by $p_n == -\log_2\left(1 - \frac{1}{(n+1)^2}\right)$.

History

– J. Napier (1614) published the first tables and used word Log
– H. Briggs (1617) published the first tables in base 10 and found logarithms of the first 25 primes
– J. Burgi (1620)
– J. Kepler (1624)
– B. Cavalieri (1632)
– J. Gregory (1668) found series expansion for log
– N. Mercator (1668) used "Log naturalis"
– J. N. Lambert (1770) and J.-L. Lagrange (1776) found continued fraction representations for Log
– L. Euler (1749) found that log was multivalued

The function log is encountered often in mathematics and the natural sciences.

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