Max

Notations

Traditional name

Maximum

Traditional notation

\( \max(x_1, x_2, \ldots) \)

Mathematica StandardForm notation

\[ \text{Max} \{ x_1, x_2, \ldots \} \]

Primary definition

\[
\begin{align*}
\max(x_1, x_2) &= \frac{1}{2} \left( x_1 + x_2 + \sqrt{(x_1 - x_2)^2} \right); \quad x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R} \\
\max(x_1, x_2, \ldots, x_n) &= \max(\max(x_1, x_2), x_3, \ldots, x_n)
\end{align*}
\]

\( \max(x_1, x_2, \ldots) \) is the numerically largest of the real numbers \( x_k \).

\( \max(z_1, z_2, \ldots) \) is not defined for complex numbers \( z_k \).

Specific values

Specialized values

\[
\begin{align*}
\max(x) &= x \\
\max(x_1, x_1, \ldots, x_1) &= x_1 \\
\max(x_1, x_2, x_3) &= x_1; \quad x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R} \wedge x_3 \in \mathbb{R} \wedge x_1 \geq x_2 \wedge x_1 \geq x_3
\end{align*}
\]

Values at fixed points

\[
\begin{align*}
\max() &= -\infty
\end{align*}
\]
Values at infinities

\[
\max(\infty, x_2, \ldots, x_n) = \infty
\]
\[
\max(-\infty, x_2, \ldots, x_n) = \max(x_2, \ldots, x_n)
\]
\[
\max(-\infty, -\infty) = \infty
\]

General characteristics

Domain and analyticity

max is real valued function of an arbitrary number of real variables. In \(\mathbb{R}^n\) it is a piecewise linear function. The derivative of \(\max(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n)\) is discontinuous at \(x_j = x_k\) for all \(j, k\).

Symmetries and periodicities

Permutation symmetry

\[
\max(x_1, x_2) = \max(x_2, x_1)
\]
\[
\max(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n) = \max(x_1, x_2, \ldots, x_j, \ldots, x_k, \ldots, x_n)
\]

Periodicity

No periodicity

Sets of discontinuity

The function \(\max(x_1, x_2, \ldots, x_n)\) is continuous function in \(\mathbb{R}^n\).

Limit representations

\[
\max(x_1, x_2) = \lim_\varepsilon \log \left( e^{\varepsilon x_1} + e^{\varepsilon x_2} \right)
\]
max\( (x_1, x_2, \ldots, x_n) = \lim_{z \to \infty} \left( \frac{1}{n} \sum_{k=1}^{n} x_k^z \right)^{1/z} \)

### Transformations

#### Transformations and argument simplifications

\[ \max(-x_1, -x_2, \ldots, -x_n) = -\min(x_1, x_2, \ldots, x_n) \]

\[ \max(|x|, -|x|) = |x| \]

### Identities

#### Functional identities

\[ \max(x_1, x_2) = \max(x_2, x_1) \]

\[ \max(x_1, x_2, x_3, \ldots) = \max(x_1, \max(x_2, x_3, \ldots)) \]

### Complex characteristics

#### Real part

\[ \Re(\max(x_1, x_2, \ldots, x_n)) = \max(x_1, x_2, \ldots, x_n) \]

#### Imaginary part

\[ \Im(\max(x_1, x_2, \ldots, x_n)) = 0 \]

#### Absolute value

\[ |\max(x_1, x_2, \ldots, x_n)| = \sqrt{\max(x_1, x_2, \ldots, x_n)^2} \]

#### Argument

\[ \arg(\max(x_1, x_2, \ldots, x_n)) = \tan^{-1}(\max(x_1, x_2, \ldots, x_n), 0) \]

#### Conjugate value

\[ \max(x_1, x_2, \ldots, x_n) = \max(x_1, x_2, \ldots, x_n) \]
Summation

\[
\sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \max \left( \frac{k}{m}, \frac{l}{n} \right) = \frac{2mn}{3} - \frac{m+n+1}{4} - \frac{m^2 + n^2 - \gcd(m, n)^2}{12mn} \quad /; \; m \in \mathbb{N}^* \land n \in \mathbb{N}^*
\]

Integral transforms

Fourier exp transforms

\[
\mathcal{F}_{(t_1, t_2)}[\max(t_1, t_2)] (z_1, z_2) = -i \pi (\delta(z_2) \delta'(z_1) + \delta(z_1) \delta'(z_2)) \frac{\delta(z_1 + z_2)}{z_1^2}
\]

Laplace transforms

\[
\mathcal{L}_{(t_1, t_2)}[\max(t_1, t_2)] (z_1, z_2) = \frac{z_1^2 + z_2 z_1 + z_2^2}{z_1^2 z_2^2 (z_1 + z_2)}
\]

Representations through more general functions

Through other functions

\[
\max(x_1, x_2) = \frac{1}{2} \left( x_1 + x_2 + \sqrt{(x_1 - x_2)^2} \right)
\]

Representations through equivalent functions

\[
\max(x_1, x_2, x_3) = x_2 + (x_1 - x_3) \theta(x_1 - x_2)
\]

\[
\max(x_1, x_3) = x_2 + (x_1 - x_3) \theta(x_1 - x_2) + \theta(x_3 - x_2) (\theta(x_2 - x_1) + \theta(x_3 - x_1) \theta(x_1 - x_2)) (x_3 - x_2 - (x_1 - x_2) \theta(x_1 - x_2))
\]
The function $\max$ is encountered often in mathematics and the natural sciences.
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