Min

Notations

Traditional name

Minimum

Traditional notation

\( \min(x_1, x_2, \ldots) \)

Mathematica StandardForm notation

\( \text{Min}\left[ x_1, x_2, \ldots \right] \)

Primary definition

\[ 01.35.02.0001.01 \]
\[ \min(x_1, x_2) = \frac{1}{2} \left( x_1 + x_2 - \sqrt{(x_1 - x_2)^2} \right); \quad x_1, x_2 \in \mathbb{R} \]

\[ 01.35.02.0002.01 \]
\[ \min(x_1, x_2, \ldots, x_n) = \min(\min(x_1, x_2), x_3, \ldots, x_n) \]

\( \min(x_1, x_2, \ldots) \) is the numerically smallest of the real numbers \( x_k \).

\( \min(z_1, z_2, \ldots) \) is not defined for complex numbers \( z_k \).

Specific values

Specialized values

\[ 01.35.03.0001.01 \]
\[ \min(x) = x \]

\[ 01.35.03.0002.01 \]
\[ \min(x_1, x_1, \ldots, x_1) = x_1 \]

\[ 01.35.03.0003.01 \]
\[ \min(x_1, x_2, x_3) = x_1 /; \quad x_1, x_2, x_3 \in \mathbb{R} \land x_1 \leq x_2 \land x_1 \leq x_3 \]

Values at fixed points

\[ 01.35.03.0004.01 \]
\[ \min() = \infty \]
01.35.03.0005.01
\[\min(-1, 3) = -1\]

01.35.03.0006.01
\[\min(2, 6, 1, \pi, 8, 12, 8, 2, 6, 4, 6, 9) = 1\]

**Values at infinities**

01.35.03.0007.01
\[\min(\infty, x_2, \ldots, x_n) = \min(x_2, \ldots, x_n)\]

01.35.03.0008.01
\[\min(-\infty, x_2, \ldots, x_n) = -\infty\]

01.35.03.0009.01
\[\min(-\infty, \infty) = -\infty\]

**General characteristics**

**Domain and analyticity**

\(\min\) is real valued function of an arbitrary number of real variables. In \(\mathbb{R}^n\) it is a piecewise linear function. The derivative of \(\min(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n)\) is discontinuous at \(x_j = x_k\) for all \(j, k\).

01.35.04.0001.01
\[(x_1 \ast x_2 \ast \ldots \ast x_n) \mapsto \min(x_1, x_2, \ldots, x_n) : \mathbb{R}^n \rightarrow \mathbb{R}\]

**Symmetries and periodicities**

**Permutation symmetry**

01.35.04.0002.01
\(\min(x_1, x_2) = \min(x_2, x_1)\)

01.35.04.0003.01
\(\min(x_1, x_2, \ldots, x_1, \ldots, x_j, \ldots, x_n) = \min(x_1, x_2, \ldots, x_j, \ldots, x_k, \ldots, x_n)\)

**Periodicity**

No periodicity

**Sets of discontinuity**

The function \(\min(x_1, x_2, \ldots, x_n)\) is continuous function in \(\mathbb{R}^n\).

01.35.04.0004.01
\(\mathcal{D}S_{x_j}(\min(x_1, x_2, \ldots, x_n)) = \{\} \); \(1 \leq k \leq n\)

**Limit representations**

01.35.09.0001.01
\[\min(x_1, x_2) = -\left(\lim_{\varepsilon \to 0} \varepsilon \log \left(\frac{e^{-\varepsilon} + e^{-\varepsilon}}{\varepsilon} \right)\right)\]
Transformations and argument simplifications

\[ \min(-x_1, -x_2, \ldots, -x_n) = -\max(x_1, x_2, \ldots, x_n) \]

\[ \min(|x|, -|x|) = -|x| \]

Functional identities

\[ \min(x_1, x_2) = \min(x_2, x_1) \]

\[ \min(x_1, x_2, x_3, \ldots) = \min(x_1, \min(x_2, x_3, \ldots)) \]

Complex characteristics

Real part

\[ \text{Re}(\min(x_1, x_2, \ldots, x_n)) = \min(x_1, x_2, \ldots, x_n) \]

Imaginary part

\[ \text{Im}(\min(x_1, x_2, \ldots, x_n)) = 0 \]

Absolute value

\[ |\min(x_1, x_2, \ldots, x_n)| = \sqrt{\min(x_1, x_2, \ldots, x_n)^2} \]

Argument

\[ \arg(\min(x_1, x_2, \ldots, x_n)) = \tan^{-1}(\min(x_1, x_2, \ldots, x_n), 0) \]

Conjugate value

\[ \min(x_1, x_2, \ldots, x_n) = \min(x_1, x_2, \ldots, x_n) \]
Summation

\[
\sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \min \left( \frac{k}{m}, \frac{l}{n} \right) = \frac{m n}{3} - \frac{m + n - 1}{4} - \frac{m^2 + n^2 - \gcd(m, n)^2}{12 m n} ; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+
\]

\[
\sum_{j=0}^{m-1} \sum_{k=0}^{n-1} \sum_{l=0}^{o-1} \min \left( \frac{j}{m}, \frac{k}{n}, \frac{l}{o} \right) = \frac{m n o}{4} + \frac{1}{8} (m + n + o - 1) - \frac{1}{6} (\gcd(m, n) o + m) + \frac{m + o - 2 o m}{24 n} + \frac{n + o - 2 o n}{24 m} + \frac{m + n - 2 n m}{24 o} + \frac{(o - 1) \gcd(m, n)^2}{24 m n} + \frac{(n - 1) \gcd(m, o)^2}{24 m o} + \frac{(m - 1) \gcd(n, o)^2}{24 n o} ; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land o \in \mathbb{N}^+
\]

Integral transforms

Fourier exp transforms

\[
\mathcal{F}_{1,2} [\min(t_1, t_2)] (z_1, z_2) = \frac{\delta(z_1 + z_2)}{z_1^{2}} - i \pi (\delta(z_2) \delta'(z_1) + \delta(z_1) \delta'(z_2))
\]

Laplace transforms

\[
\mathcal{L}_{1,2} [\min(t_1, t_2)] (z_1, z_2) = \frac{1}{z_1 z_2 (z_1 + z_2)}
\]

Representations through more general functions

Through other functions

\[
\min(x_1, x_2) = \frac{1}{2} \left( x_1 + x_2 - \sqrt{(x_1 - x_2)^2} \right) ; x_1 \in \mathbb{R} \land x_2 \in \mathbb{R}
\]

Representations through equivalent functions

\[
\min(x_1, x_2) = x_2 + (x_1 - x_2) \theta(x_2 - x_1)
\]

\[
\min(x_1, x_2, x_3) = x_2 + (x_1 - x_2) \theta(x_2 - x_1) + (x_3 - x_2 - (x_1 - x_2) \theta(x_2 - x_1)) (\theta(x_1 - x_2) + \theta(x_2 - x_1) \theta(x_1 - x_3)) \theta(x_2 - x_3)
\]

Inequalities

\[
\min(x_1, x_2, \ldots, x_n) \leq \frac{1}{n} \sum_{k=1}^{n} x_k^{1/k} \leq \max(x_1, x_2, \ldots, x_n) ; x_k > 0 \land 1 \leq k \leq n
\]
\[
\min(x_1, x_2, \ldots, x_n) \leq \left( \prod_{k=1}^{n} x_k \right)^{1/n} \leq \max(x_1, x_2, \ldots, x_n) ;
\]
\[x_k > 0 \land 1 \leq k \leq n\]

**Theorems**

**Operations in Boolean algebra**

The identification negation \(x \rightarrow \neg x\), conjunction \((x, y) \rightarrow \min(x, y)\) and disjunction \((x, y) \rightarrow \max(x, y)\) is a complete system of \(R\)-operations representing the Boolean algebra.

**History**

The function \(\min\) is encountered often in mathematics and the natural sciences.
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