

RamanujanTau

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Notations

Traditional name

Ramanujan tau function

Traditional notation

$\tau(n)$

Mathematica StandardForm notation

RamanujanTau[n]

Primary definition

13.13.02.0001.01

$$\text{RamanujanTau}(n) = \frac{1}{2} \int_{i\gamma-1}^{i\gamma+1} e^{-2\pi i n z} \eta(z)^{24} dz ; n \in \mathbb{Z} \wedge n \geq 0 \wedge \text{Re}(\gamma) > 0$$

Specific values

Values at fixed points

13.13.03.0001.01

$$\tau(0) = 0$$

13.13.03.0002.01

$$\tau(1) = 1$$

13.13.03.0003.01

$$\tau(2) = -24$$

13.13.03.0004.01

$$\tau(3) = 252$$

13.13.03.0005.01

$$\tau(4) = -1472$$

13.13.03.0006.01

$$\tau(5) = 4830$$

13.13.03.0007.01

$$\tau(6) = -6048$$

13.13.03.0008.01

$$\tau(7) = -16744$$

13.13.03.0009.01

$$\tau(8) = 84480$$

13.13.03.0010.01

$$\tau(9) = -113643$$

13.13.03.0011.01

$$\tau(10) = -115920$$

General characteristics

Domain and analyticity

$\tau(n)$ is a nonanalytical function which is defined for nonnegative integer n .

13.13.04.0001.01

$$n \rightarrow \tau(n) :: \mathbb{Z} \rightarrow \mathbb{Z}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

13.13.06.0001.01

$$\tau(n) = n^4 \sigma_1(n) - 24 \sum_{k=1}^{n-1} (35k^4 - 52nk^3 + 18n^2k^2) \sigma_1(k) \sigma_1(n-k) ; n > 0$$

13.13.06.0002.01

$$\tau(n) = \frac{691}{756} \sigma_5(n) + \frac{65}{756} \sigma_{11}(n) - \frac{691}{3} \sum_{k=1}^{n-1} \sigma_5(k) \sigma_5(n-k) ; n > 0$$

13.13.06.0003.01

$$\tau(n) = \frac{691}{1800} \sigma_3(n) + \frac{691}{900} \sigma_7(n) - \frac{91}{600} \sigma_{11}(n) + \frac{2764}{15} \sum_{k=1}^{n-1} \sigma_3(k) \sigma_7(n-k) ; n > 0$$

Asymptotic series expansions

13.13.06.0004.01

$$\tau(n) \propto O(n^6) ; (n \rightarrow \infty)$$

Transformations

Multiple arguments

13.13.16.0001.01

$$\tau(nm) = \tau(n)\tau(m) \text{ ; } \gcd(n, m) = 1$$

Identities

Functional identities

13.13.17.0001.01

$$(n-1)\tau(n) = \sum_{k=1}^{\left\lfloor \frac{1}{2}(\sqrt{8n+1}-1) \right\rfloor} (-1)^{k-1} (2k+1) \left(n-1 - \frac{9k(k+1)}{2} \right) \tau\left(n - \frac{k(k+1)}{2} \right)$$

13.13.17.0002.01

$$\tau(p_j^n) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k \binom{n-k}{n-2k} p_j^{11k} \tau(p_j)^{n-2k} \text{ ; } p_j \in \mathbb{P}$$

13.13.17.0003.01

$$\tau(p_j^{n+1}) = \tau(p_j)\tau(p_j^n) - p_j^{11} \tau(p_j^{n-1}) \text{ ; } p_j \in \mathbb{P} \wedge n > 0$$

13.13.17.0004.01

$$\tau(n p_j^m) = \tau(p_j)\tau(n p_j^{m-1}) - p_j^{11} \tau(n p_j^{m-2}) \text{ ; } n \in \mathbb{N}^+ \wedge m \in \mathbb{Z} \wedge m \geq 2 \wedge \gcd(n, p_j) = 1 \wedge p_j \in \mathbb{P}$$

Summation

Infinite summation

13.13.23.0001.01

$$\sum_{k=0}^{\infty} \text{RamanujanTau}(k) z^k = \eta\left(-\frac{i \log(z)}{2\pi}\right)^{24} \text{ ; } |z| < 1$$

13.13.23.0002.01

$$\sum_{k=1}^{\infty} \text{RamanujanTau}(k) z^k = z \prod_{k=1}^{\infty} (1-z^k)^{24} \text{ ; } |z| < 1$$

13.13.23.0003.01

$$\sum_{k=1}^{\infty} \text{RamanujanTau}(k) z^k = z \left(\sum_{k=0}^{\infty} (-1)^k (2k+1) z^{\frac{1}{2}k(k+1)} \right)^8 \text{ ; } |z| < 1$$

13.13.23.0004.01

$$\sum_{k=1}^{\infty} \text{RamanujanTau}(k) e^{2\pi i k z} = \eta(z)^{24}$$

Representations through equivalent functions

With related functions

Involving other related functions

13.13.27.0001.01

$$\tau(2n+1) = \frac{691}{33152} r_{24}(2n+1) - \frac{1}{2072} \sigma_{11}(2n+1)$$

Inequalities

13.13.29.0001.01

$$\tau(n) \neq 0 \ ; \ n > 0 \wedge n < 214928639999$$

13.13.29.0002.01

$$|\tau(n)| \leq \sigma_k(n) n^{11/2} \ ; \ n > 0$$

13.13.29.0003.01

$$|\tau(n)| \leq (2\chi^2 + \chi) n^5 \log^2(\log(n)) \ ; \ \chi = e^\gamma + \frac{6482}{\log^2(\log(3))} \bigwedge n > 0$$

History

- S. Ramanujan (1916)
- L.J. Mordell (1917) proved that $\tau(n)$ is a multiplicative function
- Hardy (1927)
- Wilton (1929)
- G.N. Watson (1935)
- Walfisz (1938)
- Bambah and Chowla (1947)
- Sengupta (1948)
- van der Blij (1948)

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