Re

Notations

Traditional name

Real part

Traditional notation

$\operatorname{Re}(z)$

Mathematica StandardForm notation

`Re[z]`

Primary definition

$\operatorname{Re}(z)$ gives the real part of the number z.

Specific values

Specialized values

12.03.03.0001.01

$\operatorname{Re}(x) = x\ /;\ x \in \mathbb{R}$

12.03.03.0002.01

$\operatorname{Re}(i\,x) = 0\ /;\ x \in \mathbb{R}$

12.03.03.0003.01

$\operatorname{Re}(x + i\,y) = x\ /;\ x \in \mathbb{R} \bigwedge y \in \mathbb{R}$

Values at fixed points

12.03.03.0004.01

$\operatorname{Re}(0) = 0$

12.03.03.0005.01

$\operatorname{Re}(1) = 1$

12.03.03.0006.01

$\operatorname{Re}(-1) = -1$

12.03.03.0007.01

$\operatorname{Re}(i) = 0$

12.03.03.0008.01
$\mathrm{Re}(-i) == 0$

12.03.03.0020.01
$\mathrm{Re}(1+i) == 1$

12.03.03.0021.01
$\mathrm{Re}(-1+i) == -1$

12.03.03.0022.01
$\mathrm{Re}(-1-i) == -1$

12.03.03.0023.01
$\mathrm{Re}(1-i) == 1$

12.03.03.0024.01
$\mathrm{Re}\left(\sqrt{3}+i\right) == \sqrt{3}$

12.03.03.0025.01
$\mathrm{Re}\left(1+i\sqrt{3}\right) == 1$

12.03.03.0026.01
$\mathrm{Re}\left(-1+i\sqrt{3}\right) == -1$

12.03.03.0027.01
$\mathrm{Re}\left(-\sqrt{3}+i\right) == -\sqrt{3}$

12.03.03.0028.01
$\mathrm{Re}\left(-\sqrt{3}-i\right) == -\sqrt{3}$

12.03.03.0029.01
$\mathrm{Re}\left(-1-i\sqrt{3}\right) == -1$

12.03.03.0030.01
$\mathrm{Re}\left(1-i\sqrt{3}\right) == 1$

12.03.03.0031.01
$\mathrm{Re}\left(\sqrt{3}-i\right) == \sqrt{3}$

12.03.03.0009.01
$\mathrm{Re}(2) == 2$

12.03.03.0010.01
$\mathrm{Re}(-2) == -2$

12.03.03.0011.01
$\mathrm{Re}(\pi) == \pi$

12.03.03.0012.01
$\mathrm{Re}(3\,i) == 0$

12.03.03.0013.01
$\mathrm{Re}(-2\,i) == 0$

12.03.03.0014.01
$\mathrm{Re}(2+i) == 2$

Values at infinities

12.03.03.0015.01

$\mathrm{Re}(\infty) = \infty$

12.03.03.0016.01

$\mathrm{Re}(-\infty) = -\infty$

12.03.03.0017.01

$\mathrm{Re}(i\,\infty) = 0$

12.03.03.0018.01

$\mathrm{Re}(-i\,\infty) = 0$

12.03.03.0019.01

$\mathrm{Re}(\tilde{\infty}) = ¿$

General characteristics

Domain and analyticity

Re(z) is a nonanalytical function; it is a real-analytic function of the variable z.

12.03.04.0001.01

$z \longrightarrow \mathrm{Re}(z) :: \mathbb{C} \longrightarrow \mathbb{R}$

Symmetries and periodicities

Parity

Re(z) is an odd function.

12.03.04.0002.01

$\mathrm{Re}(-z) = -\mathrm{Re}(z)$

Mirror symmetry

12.03.04.0003.01

$\mathrm{Re}(\bar{z}) = \overline{\mathrm{Re}(z)}$

Periodicity

No periodicity

Homogeneity

12.03.04.0004.01

$\mathrm{Re}(a\,z) = a\,\mathrm{Re}(z)\ /;\ a \in \mathbb{R}$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

12.03.16.0001.01

$$\operatorname{Re}(-z) = -\operatorname{Re}(z)$$

12.03.16.0002.01

$$\operatorname{Re}(a z) = a \operatorname{Re}(z) \;/;\; a \in \mathbb{R}$$

12.03.16.0003.01

$$\operatorname{Re}(i x) = 0 \;/;\; x \in \mathbb{R}$$

12.03.16.0004.01

$$\operatorname{Re}(i z) = -\operatorname{Im}(z)$$

12.03.16.0005.01

$$\operatorname{Re}(-i z) = \operatorname{Im}(z)$$

12.03.16.0006.01

$$\operatorname{Re}\left(\frac{1}{z}\right) = \frac{\operatorname{Re}(z)}{|z|^2}$$

Addition formulas

12.03.16.0007.01

$$\operatorname{Re}(x + i y) = x \;/;\; x \in \mathbb{R} \land y \in \mathbb{R}$$

12.03.16.0008.01

$$\operatorname{Re}\left(\sum_{k=1}^{n} z_k\right) = \sum_{k=1}^{n} \operatorname{Re}(z_k)$$

12.03.16.0009.01

$$\operatorname{Re}(z_1 + z_2) = \operatorname{Re}(z_1) + \operatorname{Re}(z_2)$$

Multiple arguments

12.03.16.0010.01

$$\operatorname{Re}(a z) = a \operatorname{Re}(z) \;/;\; a \in \mathbb{R}$$

12.03.16.0011.01

$$\operatorname{Re}(i x) = 0 \;/;\; x \in \mathbb{R}$$

12.03.16.0012.01

$$\operatorname{Re}(i z) = -\operatorname{Im}(z)$$

12.03.16.0013.01

$$\operatorname{Re}(-i z) = \operatorname{Im}(z)$$

12.03.16.0014.01

$$\operatorname{Re}(z_1 z_2) = \operatorname{Re}(z_1) \operatorname{Re}(z_2) - \operatorname{Im}(z_2) \operatorname{Im}(z_1)$$

Ratio of arguments

12.03.16.0024.01

$$\operatorname{Re}\left(\frac{z_1}{z_2}\right) = \frac{\operatorname{Re}(z_1) \operatorname{Re}(z_2) + \operatorname{Im}(z_1) \operatorname{Im}(z_2)}{|z_2|^2}$$

Power of arguments

12.03.16.0015.01

$$\operatorname{Re}(x^a) = x^{\operatorname{Re}(a)} \cos(\operatorname{Im}(a) \log(x)) \;/;\; x \in \mathbb{R} \land x > 0$$

12.03.16.0016.01

$$\operatorname{Re}(z^a) = |z|^a \cos\left(a \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right) \;/;\; a \in \mathbb{R}$$

12.03.16.0017.01

$$\operatorname{Re}(z^a) = |z|^a \cos(a \arg(z)) \;/;\; a \in \mathbb{R}$$

12.03.16.0018.01

$$\operatorname{Re}(z^a) = \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-a)_{j+l}}{(l-j)!\, j!\, \left(\frac{1}{2}\right)_j} (1 - \operatorname{Re}(z))^l \left(\frac{\operatorname{Im}(z)^2}{4\,(\operatorname{Re}(z) - 1)}\right)^j \;/;\; a \in \mathbb{R}$$

12.03.16.0019.01

$$\operatorname{Re}(z^a) = F_{0\times 1\times 1}^{1\times 0\times 0}\left(\begin{matrix} -a;;; \\ ;\frac{1}{2};\frac{1}{2}; \end{matrix} \frac{1}{2}\left(1 - \operatorname{Re}(z) + \sqrt{\operatorname{Im}(z)^2 + (1 - \operatorname{Re}(z))^2}\right), \frac{1}{2}\left(1 - \operatorname{Re}(z) - \sqrt{\operatorname{Im}(z)^2 + (1 - \operatorname{Re}(z))^2}\right)\right) \;/;\; a \in \mathbb{R}$$

12.03.16.0020.01

$$\operatorname{Re}(z^n) = \sum_{j=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^j \binom{n}{2j} \operatorname{Im}(z)^{2j} \operatorname{Re}(z)^{n-2j} \;/;\; n \in \mathbb{N}^+$$

12.03.16.0021.01

$$\operatorname{Re}(z^a) = |z|^{\operatorname{Re}(a)}\, e^{-\operatorname{Im}(a) \arg(z)} \cos(\operatorname{Im}(a) \log(|z|) + \arg(z) \operatorname{Re}(a))$$

12.03.16.0022.01

$$\operatorname{Re}(z^a) = \exp\left(-\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) \operatorname{Im}(a)\right) |z|^{\operatorname{Re}(a)} \cos\left(\operatorname{Im}(a) \log(|z|) + \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) \operatorname{Re}(a)\right)$$

Exponent of arguments

12.03.16.0025.01

$$\operatorname{Re}\left(e^{x+i\,y}\right) = e^x \cos(y)$$

12.03.16.0026.01

$$\operatorname{Re}(e^z) = e^{\operatorname{Re}(z)} \cos(\operatorname{Im}(z))$$

12.03.16.0027.01

$$\operatorname{Re}\left(e^{i\,z}\right) = e^{-\operatorname{Im}(z)} \cos(\operatorname{Re}(z))$$

Products, sums, and powers of the direct function

Sums of the direct function

12.03.16.0023.01

$$\operatorname{Re}(z_1) + \operatorname{Re}(z_2) = \operatorname{Re}(z_1 + z_2)$$

Complex characteristics

Real part

12.03.19.0001.01

$$\operatorname{Re}(\operatorname{Re}(x + i\,y)) = x$$

12.03.19.0002.01

$\operatorname{Re}(\operatorname{Re}(z)) == \operatorname{Re}(z)$

Imaginary part

12.03.19.0003.01

$\operatorname{Im}(\operatorname{Re}(x + i\, y)) == 0$

12.03.19.0004.01

$\operatorname{Im}(\operatorname{Re}(z)) == 0$

Absolute value

12.03.19.0005.01

$|\operatorname{Re}(x + i\, y)| == \sqrt{x^2}$

12.03.19.0009.01

$|\operatorname{Re}(z)| == \sqrt{\operatorname{Re}(z)^2}$

Argument

12.03.19.0006.01

$\arg(\operatorname{Re}(x + i\, y)) == \tan^{-1}(x, 0)$

12.03.19.0010.01

$\arg(\operatorname{Re}(x + i\, y)) == (1 - \theta(x))\, \pi$

12.03.19.0011.01

$\arg(\operatorname{Re}(z)) == \tan^{-1}(\operatorname{Re}(z), 0)$

12.03.19.0012.01

$\arg(\operatorname{Re}(z)) == (1 - \theta(\operatorname{Re}(z)))\, \pi$

Conjugate value

12.03.19.0007.01

$\overline{\operatorname{Re}(x + i\, y)} == x$

12.03.19.0008.01

$\overline{\operatorname{Re}(z)} == \operatorname{Re}(z)$

Signum value

12.03.19.0013.01

$\operatorname{sgn}(\operatorname{Re}(x + i\, y)) == \operatorname{sgn}(x)$

12.03.19.0014.01

$$\operatorname{sgn}(\operatorname{Re}(z)) == \frac{\operatorname{Re}(z)}{\sqrt{\operatorname{Re}(z)^2}}$$

Differentiation

Low-order differentiation

In a distributional sense for $x \in \mathbb{R}$.

12.03.20.0001.01

$$\frac{\partial \operatorname{Re}(x)}{\partial x} = 1$$

Fractional integro-differentiation

12.03.20.0002.01

$$\frac{\partial^{\alpha} \operatorname{Re}(x)}{\partial x^{\alpha}} = \frac{x^{1-\alpha}}{\Gamma(2-\alpha)} \; /; \; x \in \mathbb{R}$$

Representations through equivalent functions

With related functions

With Im

12.03.27.0002.01

$$\operatorname{Re}(z) = \operatorname{Im}(i\, z)$$

12.03.27.0007.01

$$\operatorname{Re}(z) = z - i \operatorname{Im}(z)$$

12.03.27.0003.01

$$\operatorname{Re}(i\, z) = -\operatorname{Im}(z)$$

With Abs

12.03.27.0008.01

$$\operatorname{Re}(z) = \frac{z^2 + |z|^2}{2\, z}$$

With Arg

12.03.27.0009.01

$$\operatorname{Re}(z) = \frac{1}{2} e^{-2\, i \arg(z)} \left(1 + e^{2\, i \arg(z)}\right) z$$

12.03.27.0001.01

$$\operatorname{Re}(z) = |z| \cos(\arg(z))$$

With Conjugate

12.03.27.0004.01

$$\operatorname{Re}(z) = \frac{z + \bar{z}}{2}$$

12.03.27.0006.01

$$\operatorname{Re}(z) = \bar{z} + i \operatorname{Im}(z)$$

With Sign

12.03.27.0010.01

$$\operatorname{Re}(z) = z\, \frac{\operatorname{sgn}(z)^2 + 1}{2 \operatorname{sgn}(z)^2}$$

12.03.27.0005.01

$$\mathrm{Re}(z) == \frac{z \cos(\arg(z))}{\mathrm{sgn}(z)} \;/;\; z \neq 0$$

Inequalities

12.03.29.0001.01

$$|\mathrm{Re}(z)| \leq |z|$$

Zeros

12.03.30.0001.01

$$\mathrm{Re}(z) == 0 \;/;\; i\, z \in \mathbb{R}$$

History

The function Re is encountered often in mathematics and the natural sciences.

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