SiegelTheta3

Notations

Traditional name

Siegel theta function

Traditional notation

$$\Theta\left[\begin{smallmatrix}\{u_1,\,\ldots,\,u_r\}\\ \{v_1,\,\ldots,\,v_r\}\end{smallmatrix}\right]\left(\begin{pmatrix} m_{1,1} & \ldots & m_{1,r}\\ \ldots & \ldots & \ldots\\ m_{r,1} & \ldots & m_{r,r}\end{pmatrix}, \{s_1,\,\ldots,\,s_r\}\right)$$

Mathematica StandardForm notation

```
SiegelTheta[{{u1, .., ur}, {v1, .., vr}}, {{m1,1, .., m1,r}, .., {mr,1, .., mr,r}}, {s1, .., sr}]
```

Primary definition

09.59.02.0001.01

$$\Theta\left[\begin{smallmatrix}\{u_1,\,\ldots,\,u_r\}\\ \{v_1,\,\ldots,\,v_r\}\end{smallmatrix}\right]\left(\begin{pmatrix} m_{1,1} & \ldots & m_{1,r}\\ \ldots & \ldots & \ldots\\ m_{r,1} & \ldots & m_{r,r}\end{pmatrix}, \{s_1,\,\ldots,\,s_r\}\right) = \sum_{n_1=-\infty}^{\infty} \ldots \sum_{n_r=-\infty}^{\infty} e^{i\pi((n+u)\cdot\Omega\cdot(n+u)+2(n+u)\cdot(s+v))} /;\, u = \{u_1,\,\ldots,\,u_r\} \bigwedge v = \{v_1,\,\ldots,\,v_r\} \bigwedge \Omega = \{\{m_{1,1},\,\ldots,\,m_{1,r}\},\,\ldots,\,\{m_{r,1},\,\ldots,\,m_{r,r}\}\} \bigwedge s = \{s_1,\,\ldots,\,s_r\} \bigwedge n = \{n_1,\,\ldots,\,n_r\} \bigwedge n+u = \{n_1+u_1,\,\ldots,\,n_r+u_r\} \bigwedge s+v = \{s_1+v_1,\,\ldots,\,s_r+v_r\}$$

The Siegel theta function $\Theta\left[\begin{smallmatrix}u\\ v\end{smallmatrix}\right](\Omega,\,s)$ with characteristic $\begin{pmatrix}u\\ v\end{pmatrix} /;\, u == \{u_1,\,\ldots,\,u_r\} \bigwedge v == \{v_1,\,\ldots,\,v_r\}$, symmetric Riemann modular matrix $\Omega = \{\{m_{1,1},\,\ldots,\,m_{1,r}\},\,\ldots,\,\{m_{r,1},\,\ldots,\,m_{r,r}\}\}$ with positive definite imaginary part and vector $s = \{s_1,\,\ldots,\,s_r\}$ is defined through $\sum_{n_1=-\infty}^{\infty} \ldots \sum_{n_r=-\infty}^{\infty} e^{i\pi\left((n+u)\cdot\Omega^T\cdot(n+u)+2(n+u)\cdot(s+v)\right)}$, where Ω^T means transposed to Ω matrix (or vector) and n ranges over all possible vectors in the r-dimensional integer lattice.

Copyright