SinIntegral

Notations

Traditional name

Sine integral

Traditional notation

$\mathrm{Si}(z)$

Mathematica StandardForm notation

`SinIntegral[z]`

Primary definition

06.37.02.0001.01

$$\mathrm{Si}(z) = \int_0^z \frac{\sin(t)}{t}\, dt$$

Specific values

Values at fixed points

06.37.03.0001.01

$$\mathrm{Si}(0) = 0$$

Values at infinities

06.37.03.0002.01

$$\mathrm{Si}(\infty) = \frac{\pi}{2}$$

06.37.03.0003.01

$$\mathrm{Si}(-\infty) = -\frac{\pi}{2}$$

06.37.03.0004.01

$$\mathrm{Si}(i\,\infty) = i\,\infty$$

06.37.03.0005.01

$$\mathrm{Si}(-i\,\infty) = -i\,\infty$$

06.37.03.0006.01

$$\mathrm{Si}(\tilde{\infty}) = ¿$$

General characteristics

Domain and analyticity

Si(z) is an entire function of z which is defined in the whole complex z-plane.

06.37.04.0001.01

$$z \longrightarrow \mathrm{Si}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

Si(z) is an odd function.

06.37.04.0002.01

$$\mathrm{Si}(-z) = -\mathrm{Si}(z)$$

Mirror symmetry

06.37.04.0003.01

$$\mathrm{Si}(\bar{z}) = \overline{\mathrm{Si}(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function Si(z) has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

06.37.04.0004.01

$$\mathcal{S}ing_z(\mathrm{Si}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function Si(z) does not have branch points.

06.37.04.0005.01

$$\mathcal{BP}_z(\mathrm{Si}(z)) = \{\}$$

Branch cuts

The function Si(z) does not have branch cuts.

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$$\mathcal{BC}_z(\mathrm{Si}(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

06.37.06.0010.01

$$\mathrm{Si}(z) \propto \mathrm{Si}(z_0) + \frac{\sin(z_0)}{z_0}(z - z_0) + \frac{z_0 \cos(z_0) - \sin(z_0)}{2 z_0^2}(z - z_0)^2 + \ldots /; (z \to z_0)$$

06.37.06.0011.01

$$\mathrm{Si}(z) \propto \mathrm{Si}(z_0) + \frac{\sin(z_0)}{z_0}(z - z_0) + \frac{z_0 \cos(z_0) - \sin(z_0)}{2 z_0^2}(z - z_0)^2 + O\left((z - z_0)^3\right)$$

06.37.06.0012.01

$$\mathrm{Si}(z) = \mathrm{Si}(z_0) - \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^{k-j} z_0^{j-k}}{k\, j!} \sin\left(\frac{\pi j}{2} + z_0\right)(z - z_0)^k$$

06.37.06.0013.01

$$\mathrm{Si}(z) = \pi \sum_{k=0}^{\infty} \frac{2^{k-2} z_0^{1-k}}{k!} \, {}_2\tilde{F}_3\left(\frac{1}{2}, 1; \frac{3}{2}, 1 - \frac{k}{2}, \frac{3-k}{2}; -\frac{z_0^2}{4}\right)(z - z_0)^k$$

06.37.06.0014.01

$$\mathrm{Si}(z) \propto \mathrm{Si}(z_0)\,(1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

06.37.06.0001.02

$$\mathrm{Si}(z) \propto z\left(1 - \frac{z^2}{18} + \frac{z^4}{600} - \ldots\right) /; (z \to 0)$$

06.37.06.0015.01

$$\mathrm{Si}(z) \propto z\left(1 - \frac{z^2}{18} + \frac{z^4}{600} - O\left(z^6\right)\right)$$

06.37.06.0016.01

$$\mathrm{Si}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{(2k+1)\,(2k+1)!}$$

Andrea Piccolroaz

06.37.06.0002.02

$$\mathrm{Si}(z) = z \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k}}{(1+2k)^2\,(2k)!}$$

06.37.06.0003.01

$$\mathrm{Si}(z) = z\, {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{z^2}{4}\right)$$

06.37.06.0004.02

$$\mathrm{Si}(z) \propto z\left(1 + O\left(z^2\right)\right)$$

06.37.06.0017.01

$$\operatorname{Si}(z) = F_\infty(z) /; \left(\left(F_n(z) = z \sum_{k=0}^{n} \frac{(-1)^k z^{2k}}{(2k+1)^2 (2k)!} = \operatorname{Si}(z) + \frac{(-1)^n z^{2n+3}}{(2n+3)^2 (2n+2)!} \, {}_2F_3\left(1, n+\frac{3}{2}; n+2, n+\frac{5}{2}, n+\frac{5}{2}; -\frac{z^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

06.37.06.0005.01

$$\operatorname{Si}(z) \propto \frac{\pi \sqrt{z^2}}{2z} - \frac{\cos(z)}{z} \, {}_3F_0\left(\frac{1}{2}, 1, 1; ; -\frac{4}{z^2}\right) - \frac{\sin(z)}{z^2} \, {}_3F_0\left(1, 1, \frac{3}{2}; ; -\frac{4}{z^2}\right) /; (|z| \to \infty)$$

06.37.06.0006.01

$$\operatorname{Si}(z) \propto \frac{\pi \sqrt{z^2}}{2z} - \frac{\cos(z)}{z} \left(1 + O\left(\frac{1}{z^2}\right)\right) - \frac{\sin(z)}{z^2} \left(1 + O\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

Residue representations

06.37.06.0007.01

$$\operatorname{Si}(z) = \frac{\sqrt{\pi}}{4} z \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma\left(\frac{1}{2} - s\right) \left(\frac{z^2}{4}\right)^{-s}}{\Gamma\left(\frac{3}{2} - s\right)^2} \Gamma(s)\right)(-j)$$

06.37.06.0008.02

$$\operatorname{Si}(z) = \frac{\sqrt{\pi}}{2} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(-s) \left(\frac{z}{2}\right)^{-2s}}{\Gamma(1-s)^2} \Gamma\left(s + \frac{1}{2}\right)\right)\left(-\frac{1}{2} - j\right)$$

Other series representations

06.37.06.0009.01

$$\operatorname{Si}(z) = \pi \sum_{k=0}^{\infty} J_{k+\frac{1}{2}}\left(\frac{z}{2}\right)^2$$

Integral representations

On the real axis

Of the direct function

06.37.07.0001.01

$$\operatorname{Si}(z) = \int_0^z \frac{\sin(t)}{t} \, dt$$

06.37.07.0002.01

$$\operatorname{Si}(z) = \frac{\pi}{2} - \int_z^\infty \frac{\sin(t)}{t} \, dt$$

Contour integral representations

06.37.07.0003.01

$$\operatorname{Si}(z) = \frac{\sqrt{\pi}\, z}{8 \pi i} \int_{\mathcal{L}} \frac{\Gamma(s)\, \Gamma\left(\frac{1}{2} - s\right)}{\Gamma\left(\frac{3}{2} - s\right)^2} \left(\frac{z^2}{4}\right)^{-s} ds$$

06.37.07.0004.01

$$\operatorname{Si}(x) = \frac{\sqrt{\pi}}{4 \pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\Gamma\left(s + \frac{1}{2}\right) \Gamma(-s)}{\Gamma(1 - s)^2} \left(\frac{x}{2}\right)^{-2s} ds \,/;\, -\frac{1}{2} < \gamma < 0 \bigwedge x > 0$$

Limit representations

06.37.09.0001.01

$$\operatorname{Si}(x) = \frac{\pi}{2} - \lim_{n \to \infty} \left(\sum_{k=n+1}^{\infty} \frac{1}{k} \sin\left(\frac{k x}{n}\right) \right) /;\, x \in \mathbb{R} \land x > 0$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.37.13.0001.01

$$z\, w^{(3)}(z) + 2\, w''(z) + z\, w'(z) = 0 \,/;\, w(z) = c_1 \operatorname{Si}(z) + c_2 \operatorname{Ci}(z) + c_3$$

06.37.13.0004.01

$$W_z(1, \operatorname{Si}(z), \operatorname{Ci}(z)) = -\frac{1}{z^2}$$

06.37.13.0002.01

$$z\, w^{(3)}(z) + 2\, w''(z) + z\, w'(z) = 0 \,/;\, w(z) = c_1 \operatorname{Si}(z) + c_2 \operatorname{Ei}(i z) + c_3$$

06.37.13.0005.01

$$W_z(1, \operatorname{Si}(z), \operatorname{Ei}(i z)) = -\frac{1}{z^2}$$

06.37.13.0003.01

$$z\, w^{(3)}(z) + 2\, w''(z) + z\, w'(z) = 0 \,/;\, w(z) = c_1 \operatorname{Si}(z) + c_2 \operatorname{Ei}(-i z) + c_3$$

06.37.13.0006.01

$$W_z(1, \operatorname{Si}(z), \operatorname{Ei}(-i z)) = -\frac{1}{z^2}$$

06.37.13.0007.01

$$w^{(3)}(z) + \left(\frac{2 g'(z)}{g(z)} - \frac{3 g''(z)}{g'(z)}\right) w''(z) + \left(g'(z)^2 + \frac{3 g''(z)^2}{g'(z)^2} - \frac{2 g''(z)}{g(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) = 0 \,/;\, w(z) = c_1 \operatorname{Si}(g(z)) + c_2 \operatorname{Ci}(g(z)) + c_3$$

06.37.13.0008.01

$$W_z(\operatorname{Si}(g(z)), \operatorname{Ci}(g(z)), 1) = -\frac{g'(z)^3}{g(z)^2}$$

06.37.13.0009.01

$$w^{(3)}(z)+\left(\frac{2\,g'(z)}{g(z)}-\frac{3\,h'(z)}{h(z)}-\frac{3\,g''(z)}{g'(z)}\right)w''(z)+\left(g'(z)^2-\frac{4\,h'(z)\,g'(z)}{g(z)\,h(z)}+\frac{6\,h'(z)^2}{h(z)^2}+\frac{3\,g''(z)^2}{g'(z)^2}+\frac{6\,h'(z)\,g''(z)}{h(z)\,g'(z)}-\frac{2\,g''(z)}{g(z)}-\frac{3\,h''(z)}{h(z)}-\frac{g^{(3)}(z)}{g'(z)}\right)w'(z)+\left(-\frac{6\,h'(z)^3}{h(z)^3}+\frac{4\,g'(z)\,h'(z)^2}{g(z)\,h(z)^2}-\frac{6\,g''(z)\,h'(z)^2}{h(z)^2\,g'(z)}+\frac{6\,h''(z)\,h'(z)}{h(z)^2}-\frac{3\,g''(z)^2\,h'(z)}{h(z)\,g'(z)^2}+\frac{2\,h'(z)\,g''(z)-2\,g'(z)\,h''(z)}{h(z)\,g(z)}+\frac{3\,g''(z)\,h''(z)+h'(z)\,g^{(3)}(z)}{h(z)\,g'(z)}-\frac{h'(z)\,g'(z)^2+h^{(3)}(z)}{h(z)}\right)w(z)=0\;/;\;w(z)=c_1\,h(z)\,\mathrm{Si}(g(z))+c_2\,h(z)\,\mathrm{Ci}(g(z))+c_3\,h(z)$$

06.37.13.0010.01

$$W_z(h(z)\,\mathrm{Si}(g(z)),\,h(z)\,\mathrm{Ci}(g(z)),\,h(z))=-\frac{h(z)^3\,g'(z)^3}{g(z)^2}$$

06.37.13.0011.01

$$z^3\,w^{(3)}(z)-(r+3\,s-3)\,z^2\,w''(z)+\left(a^2\,r^2\,z^{2\,r}+3\,(s-1)\,s+r\,(2\,s-1)+1\right)z\,w'(z)-s\left(a^2\,r^2\,z^{2\,r}+s\,(r+s)\right)w(z)=0\;/;\;w(z)=c_1\,z^s\,\mathrm{Si}(a\,z^r)+c_2\,z^s\,\mathrm{Ci}(a\,z^r)+c_3\,z^s$$

06.37.13.0012.01

$$W_z(z^s\,\mathrm{Si}(a\,z^r),\,z^s\,\mathrm{Ci}(a\,z^r),\,z^s)=-a\,r^3\,z^{r+3\,s-3}$$

06.37.13.0013.01

$$w^{(3)}(z)-(\log(r)+3\log(s))\,w''(z)+\left(a^2\log^2(r)\,r^{2\,z}+3\log^2(s)+2\log(r)\log(s)\right)w'(z)-\log(s)\left(a^2\log^2(r)\,r^{2\,z}+\log(s)\,(\log(r)+\log(s))\right)w(z)=0\;/;\;w(z)=c_1\,s^z\,\mathrm{Si}(a\,r^z)+c_2\,s^z\,\mathrm{Ci}(a\,r^z)+c_3\,s^z$$

06.37.13.0014.01

$$W_z(s^z\,\mathrm{Si}(a\,r^z),\,s^z\,\mathrm{Ci}(a\,r^z),\,s^z)=-a\,r^z\,s^{3\,z}\log^3(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.37.16.0001.01

$$\mathrm{Si}(-z)=-\mathrm{Si}(z)$$

06.37.16.0002.01

$$\mathrm{Si}(i\,z)=i\,\mathrm{Shi}(z)$$

06.37.16.0003.01

$$\mathrm{Si}(-i\,z)=-i\,\mathrm{Shi}(z)$$

06.37.16.0004.01

$$\mathrm{Si}(a\,(b\,z^c)^m)=\frac{(b\,z^c)^m}{b^m\,z^{m\,c}}\,\mathrm{Si}(a\,b^m\,z^{m\,c})\;/;\;2\,m\in\mathbb{Z}$$

06.37.16.0005.01

$$\mathrm{Si}\left(\sqrt{z^2}\right)=\frac{\sqrt{z^2}}{z}\,\mathrm{Si}(z)$$

Complex characteristics

Real part

06.37.19.0001.01

$$\operatorname{Re}(\operatorname{Si}(x+i\,y)) = x\sum_{j=0}^{\infty}\frac{y^{2k}}{(2k+1)!}\,{}_1F_2\!\left(k+\frac{1}{2};\frac{3}{2},k+\frac{3}{2};-\frac{x^2}{4}\right)$$

06.37.19.0002.01

$$\operatorname{Re}(\operatorname{Si}(x+i\,y)) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^j\,y^{2k-2j}\,x^{2j+1}}{(2k+1)\,(2k-2j)!\,(2j+1)!}$$

06.37.19.0003.01

$$\operatorname{Re}(\operatorname{Si}(x+i\,y)) = \frac{1}{2}\left(\operatorname{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

06.37.19.0004.01

$$\operatorname{Im}(\operatorname{Si}(x+i\,y)) = \sum_{j=0}^{\infty}\frac{y^{2k+1}}{(2k+1)!\,(2k+1)}\,{}_1F_2\!\left(k+\frac{1}{2};\frac{1}{2},k+\frac{3}{2};-\frac{x^2}{4}\right)$$

06.37.19.0005.01

$$\operatorname{Im}(\operatorname{Si}(x+i\,y)) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^j\,y^{2k-2j+1}\,x^{2j}}{(2k+1)\,(2j)!\,(2k-2j+1)!}$$

06.37.19.0006.01

$$\operatorname{Im}(\operatorname{Si}(x+i\,y)) = \frac{x}{2y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

06.37.19.0007.01

$$|\operatorname{Si}(x+i\,y)| = \sqrt{\operatorname{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

06.37.19.0008.01

$$\arg(\operatorname{Si}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\operatorname{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right),\ \frac{x}{2y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

06.37.19.0009.01

$$\overline{\text{Si}(x+i\,y)} = \frac{1}{2}\left(\text{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\text{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)-\frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\text{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\text{Si}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

06.37.19.0010.01

$$\text{sgn}(\text{Si}(x+i\,y)) = \left(\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\;x\left(\text{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\text{Si}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)+\text{Si}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+\text{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)\Big/\left(2\sqrt{\text{Si}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\text{Si}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}\right)$$

Differentiation

Low-order differentiation

06.37.20.0001.01

$$\frac{\partial\,\text{Si}(z)}{\partial z} = \frac{\sin(z)}{z}$$

06.37.20.0002.01

$$\frac{\partial^2\,\text{Si}(z)}{\partial z^2} = \frac{\cos(z)}{z}-\frac{\sin(z)}{z^2}$$

Symbolic differentiation

06.37.20.0006.01

$$\frac{\partial^n\,\text{Si}(z)}{\partial z^n} = \delta_n\,\text{Si}(z)-\sum_{k=0}^{n-1}\frac{(-1)^{n-k}\,z^{k-n}\,(n-1)!}{k!}\sin\left(\frac{\pi\,k}{2}+z\right)\text{/; } n\in\mathbb{N}$$

06.37.20.0003.01

$$\frac{\partial^n\,\text{Si}(z)}{\partial z^n} = \delta_n\,\text{Si}(z)+\text{Boole}\left(n\neq 0,\,(n-1)!\sum_{k=0}^{n-1}\frac{(-1)^{n-k-1}\,z^{k-n}}{k!}\sin\left(z+\frac{\pi\,k}{2}\right)\right)\text{/; } n\in\mathbb{N}$$

06.37.20.0004.02

$$\frac{\partial^n\,\text{Si}(z)}{\partial z^n} = 2^{n-2}\,\pi\,z^{1-n}\,{}_2\tilde{F}_3\left(\frac{1}{2},\,1;\,\frac{3}{2},\,1-\frac{n}{2},\,\frac{3-n}{2};\,-\frac{z^2}{4}\right)\text{/; } n\in\mathbb{N}$$

Fractional integro-differentiation

06.37.20.0005.01

$$\frac{\partial^\alpha\,\text{Si}(z)}{\partial z^\alpha} = 2^{\alpha-2}\,\pi\,z^{1-\alpha}\,{}_2\tilde{F}_3\left(\frac{1}{2},\,1;\,\frac{3}{2},\,1-\frac{\alpha}{2},\,\frac{3-\alpha}{2};\,-\frac{z^2}{4}\right)$$

Integration

Indefinite integration

Involving only one direct function

06.37.21.0001.01

$$\int \operatorname{Si}(b + a z)\, dz = \frac{\cos(b + a z) + (b + a z) \operatorname{Si}(b + a z)}{a}$$

06.37.21.0002.01

$$\int \operatorname{Si}(a z)\, dz = \frac{\cos(a z)}{a} + z \operatorname{Si}(a z)$$

06.37.21.0003.01

$$\int \operatorname{Si}(z)\, dz = \cos(z) + z \operatorname{Si}(z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

06.37.21.0004.01

$$\int z^{\alpha-1} \operatorname{Si}(a z)\, dz = \frac{1}{2 \alpha} \left(z^\alpha \left(-i\, \Gamma(\alpha, -i a z) (-i a z)^{-\alpha} + i (i a z)^{-\alpha} \Gamma(\alpha, i a z) + 2 \operatorname{Si}(a z)\right)\right)$$

06.37.21.0005.01

$$\int z^{\alpha-1} \operatorname{Si}(z)\, dz = \frac{z^\alpha}{\alpha} \operatorname{Si}(z) + \frac{i}{2 \alpha} \left((i z)^{-\alpha} z^\alpha\, \Gamma(\alpha, i z) - (-i z)^{-\alpha} z^\alpha\, \Gamma(\alpha, -i z)\right)$$

06.37.21.0006.01

$$\int z \operatorname{Si}(a z)\, dz = \frac{a^2 \operatorname{Si}(a z) z^2 + a \cos(a z) z - \sin(a z)}{2 a^2}$$

06.37.21.0007.01

$$\int \frac{\operatorname{Si}(a z)}{z}\, dz = \frac{1}{2} \Big(a z\, {}_3F_3(1, 1, 1; 2, 2, 2; -i a z) + a z\, {}_3F_3(1, 1, 1; 2, 2, 2; i a z) - i \log(z) \left(\Gamma(0, -i a z) - \Gamma(0, i a z) + \log(-i a z) - \log(i a z) + 2 i \operatorname{Si}(a z)\right)\Big)$$

06.37.21.0008.01

$$\int \frac{\operatorname{Si}(a z)}{z^2}\, dz = -\frac{-a z \operatorname{Ci}(a z) + \sin(a z) + \operatorname{Si}(a z)}{z}$$

06.37.21.0009.01

$$\int \frac{\operatorname{Si}(b + a z)}{z^2}\, dz = \frac{a z \operatorname{Ci}(a z) \sin(b) + a z \cos(b) \operatorname{Si}(a z) - (b + a z) \operatorname{Si}(b + a z)}{b z}$$

Power arguments

06.37.21.0010.01

$$\int z^{\alpha-1}\,\mathrm{Si}(a\,z^r)\,dz = \frac{z^\alpha}{2\,\alpha}\left(-i\,\Gamma\!\left(\frac{\alpha}{r},\,-i\,a\,z^r\right)(-i\,a\,z^r)^{-\frac{\alpha}{r}} + i\,(i\,a\,z^r)^{-\frac{\alpha}{r}}\,\Gamma\!\left(\frac{\alpha}{r},\,i\,a\,z^r\right) + 2\,\mathrm{Si}(a\,z^r)\right)$$

Involving exponential function

Involving exp

06.37.21.0011.01

$$\int e^{b\,z}\,\mathrm{Si}(a\,z)\,dz = \frac{2\,e^{b\,z}\,\mathrm{Si}(a\,z) - i\,(\mathrm{Ei}((b-i\,a)\,z) - \mathrm{Ei}((b+a\,i)\,z))}{2\,b}$$

06.37.21.0012.01

$$\int e^{i\,a\,z}\,\mathrm{Si}(a\,z)\,dz = \frac{\mathrm{Ei}(2\,i\,a\,z) - \log(a\,z) - 2\,i\,e^{i\,a\,z}\,\mathrm{Si}(a\,z)}{2\,a}$$

06.37.21.0013.01

$$\int e^{-i\,a\,z}\,\mathrm{Si}(a\,z)\,dz = \frac{\mathrm{Ei}(-2\,i\,a\,z) - \log(a\,z) + 2\,e^{-i\,a\,z}\,i\,\mathrm{Si}(a\,z)}{2\,a}$$

Involving exponential function and a power function

Involving exp and power

06.37.21.0014.01

$$\int z^n\,e^{b\,z}\,\mathrm{Si}(a\,z)\,dz = \frac{1}{2}\,n!\,(-b)^{-n-1}\left(i\,\mathrm{Ei}((b-i\,a)\,z) - i\,\mathrm{Ei}((b+a\,i)\,z) - 2\,e^{b\,z}\,\mathrm{Si}(a\,z)\sum_{k=0}^{n}\frac{(-b\,z)^k}{k!} + e^{(b+a\,i)\,z}\,i\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b+a\,i}\right)^m\sum_{k=0}^{m-1}\frac{(-b-i\,a)^k\,z^k}{k!} - i\,e^{(b-i\,a)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-i\,a}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a-b)^k\,z^k}{k!}\right)$$

06.37.21.0015.01

$$\int z^n\,e^{i\,a\,z}\,\mathrm{Si}(a\,z)\,dz = \frac{1}{2\,a}\left((-i\,a)^{-n}\left(n!\left(\mathrm{Ei}(2\,i\,a\,z) - \log(z) - 2\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(-i\,a\,z)^k}{2\,k} + 2^{-k-1}\,\Gamma(k,\,-2\,i\,a\,z)\right)\right) - 2\,i\,\Gamma(n+1,\,-i\,a\,z)\,\mathrm{Si}(a\,z)\right)\right)\,/;\, n\in\mathbb{N}$$

06.37.21.0016.01

$$\int z^n\,e^{-i\,a\,z}\,\mathrm{Si}(a\,z)\,dz = \frac{(i\,a)^{-n}}{2\,a}\left(2\,i\,\Gamma(n+1,\,i\,a\,z)\,\mathrm{Si}(a\,z) + n!\left(\mathrm{Ei}(-2\,i\,a\,z) - \log(z) - 2\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(i\,a\,z)^k}{2\,k} + 2^{-k-1}\,\Gamma(k,\,2\,i\,a\,z)\right)\right)\right)\,/;\, n\in\mathbb{N}$$

06.37.21.0017.01

$$\int z\, e^{b z}\, \mathrm{Si}(a z)\, dz = \frac{1}{2 b^2 \left(a^2+b^2\right)} \left(-i\, \mathrm{Ei}((b+a i) z)\, a^2 + 2 b\, e^{b z} \cos(a z)\, a + \left(a^2+b^2\right) i\, \mathrm{Ei}((b-i a) z) - i\, b^2\, \mathrm{Ei}((b+a i) z) - 2 b^2\, e^{b z} \sin(a z) + 2 \left(a^2+b^2\right) e^{b z} (b z-1)\, \mathrm{Si}(a z)\right)$$

06.37.21.0018.01

$$\int z^2\, e^{b z}\, \mathrm{Si}(a z)\, dz = \frac{1}{b^3} \left(i\, (\mathrm{Ei}((b+a i) z) - \mathrm{Ei}((b-i a) z)) - \frac{1}{\left(a^2+b^2\right)^2} \left(b\, e^{b z} \left(b \left((b z-1) a^2 + b^2 (b z-3)\right) \sin(a z) - a \left((b z-2) a^2 + b^2 (b z-4)\right) \cos(a z)\right)\right) + e^{b z} \left(b^2 z^2 - 2 b z + 2\right) \mathrm{Si}(a z)\right)$$

06.37.21.0019.01

$$\int z^3\, e^{b z}\, \mathrm{Si}(a z)\, dz = \frac{1}{b^4} \left(3 i\, (\mathrm{Ei}((b-i a) z) - \mathrm{Ei}((b+a i) z)) - \frac{1}{\left(a^2+b^2\right)^3} \left(b\, e^{b z} \left(b \left(\left(b^2 z^2 - b z + 3\right) a^4 + 2 b^2 \left(b^2 z^2 - 3 b z + 3\right) a^2 + b^4 \left(b^2 z^2 - 5 b z + 11\right)\right) \sin(a z) - a \left(\left(b^2 z^2 - 3 b z + 6\right) a^4 + 2 b^2 \left(b^2 z^2 - 5 b z + 8\right) a^2 + b^4 \left(b^2 z^2 - 7 b z + 18\right)\right) \cos(a z)\right)\right) + e^{b z} \left(b^3 z^3 - 3 b^2 z^2 + 6 b z - 6\right) \mathrm{Si}(a z)\right)$$

Involving trigonometric functions

Involving sin

06.37.21.0020.01

$$\int \sin(b z)\, \mathrm{Si}(a z)\, dz = \frac{1}{4 b} \left(i\, (\mathrm{Ei}(-i (a-b) z) - \mathrm{Ei}(i (a-b) z) + \mathrm{Ei}(-i (a+b) z) - \mathrm{Ei}(i (a+b) z) + 4 i \cos(b z)\, \mathrm{Si}(a z))\right)$$

06.37.21.0021.01

$$\int \sin(a z)\, \mathrm{Si}(a z)\, dz = \frac{\mathrm{Si}(2 a z) - 2 \cos(a z)\, \mathrm{Si}(a z)}{2 a}$$

Involving cos

06.37.21.0022.01

$$\int \cos(b z)\, \mathrm{Si}(a z)\, dz = \frac{1}{4 b} \left(-\mathrm{Ei}(-i (a-b) z) - \mathrm{Ei}(i (a-b) z) + \mathrm{Ei}(-i (a+b) z) + \mathrm{Ei}(i (a+b) z) + 4 \sin(b z)\, \mathrm{Si}(a z)\right)$$

06.37.21.0023.01

$$\int \cos(a z)\, \mathrm{Si}(a z)\, dz = \frac{\mathrm{Ci}(2 a z) - \log(a z) + 2 \sin(a z)\, \mathrm{Si}(a z)}{2 a}$$

Involving trigonometric functions and a power function

Involving sin and power

06.37.21.0024.01

$$\int z^n \sin(b z)\,\mathrm{Si}(a z)\,dz = -\frac{i}{4}(i b)^{-n-2}\, b\, n! \Bigg((-1)^n \mathrm{Ei}(-i(a-b)z) - \mathrm{Ei}(i(a-b)z) + \mathrm{Ei}(-i(a+b)z) - (-1)^{-n}\mathrm{Ei}(i(a+b)z) + \frac{1}{\Gamma(n+2)}(2 i (n+1)((-1)^n \Gamma(n+1, -i b z) + \Gamma(n+1, i b z))\,\mathrm{Si}(a z)) - e^{-i(a+b)z} \sum_{m=1}^{n} \frac{\left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(i a + i b)^k z^k}{k!}}{m} + e^{i(a-b)z} \sum_{m=1}^{n} \frac{\left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(-(i a) + i b)^k z^k}{k!}}{m} - (-1)^n e^{-i(a-b)z} \sum_{m=1}^{n} \frac{\left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(i a - i b)^k z^k}{k!}}{m} + (-1)^n e^{i(a+b)z} \sum_{m=1}^{n} \frac{\left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(-(i a) - i b)^k z^k}{k!}}{m}\Bigg) /;\ n \in \mathbb{N}$$

06.37.21.0025.01

$$\int z^n \sin(a z)\,\mathrm{Si}(a z)\,dz = \frac{(-i a)^{-n}}{4 a}\left(i\, n! \left((-1)^n \mathrm{Ei}(-2 i a z) - \mathrm{Ei}(2 i a z) + (1-(-1)^n)\log(z) + 2\sum_{k=1}^{n} \frac{1}{k!}\left(\frac{(-i a z)^k}{2k} + 2^{-k-1}\Gamma(k, -2 i a z)\right) - 2(-1)^n \sum_{k=1}^{n} \frac{1}{k!}\left(\frac{(i a z)^k}{2k} + 2^{-k-1}\Gamma(k, 2 i a z)\right)\right) - 2(\Gamma(n+1, -i a z) + (-1)^n \Gamma(n+1, i a z))\,\mathrm{Si}(a z)\right) /;\ n \in \mathbb{N}$$

06.37.21.0026.01

$$\int z \sin(b z)\,\mathrm{Si}(a z)\,dz = \frac{1}{4 b^2}\left(\frac{2 b \cos((a-b)z)}{b-a} - \mathrm{Ei}(-i(a-b)z) - \mathrm{Ei}(i(a-b)z) + \mathrm{Ei}(-i(a+b)z) + \mathrm{Ei}(i(a+b)z) - 2 i(\Gamma(2, -i b z) - \Gamma(2, i b z))\,\mathrm{Si}(a z) - \frac{2 b \cos((a+b)z)}{a+b}\right)$$

06.37.21.0027.01

$$\int z^2 \sin(b z)\,\mathrm{Si}(a z)\,dz = -\frac{1}{2 b^3}\Bigg(i\Bigg(\mathrm{Ei}(-i(a-b)z) + \frac{1}{(a-b)^2 (a+b)^2}\Big(2 i \cos(b z)\left(a\left(b^2-a^2\right) z \cos(a z) - \left(a^2 - 3 b^2\right)\sin(a z)\right) b^2 + 2 i\left(\left(b^2-a^2\right) z \sin(a z)\, b^2 + 2 a\left(a^2 - 2 b^2\right)\cos(a z)\right)\sin(b z)\, b - \left(a^2-b^2\right)^2 (\mathrm{Ei}(i(a-b)z) - \mathrm{Ei}(-i(a+b)z) + \mathrm{Ei}(i(a+b)z))\Big) + i(\Gamma(3, -i b z) + \Gamma(3, i b z))\,\mathrm{Si}(a z)\Bigg)\Bigg)$$

06.37.21.0028.01

$$\int z^3 \sin(b\, z)\, \mathrm{Si}(a\, z)\, dz = -\frac{1}{4\, b^4} \left(\frac{1}{(a-b)^3\, (a+b)^3} \left(-6\, (\mathrm{Ei}(-i\, (a-b)\, z) + \mathrm{Ei}(i\, (a-b)\, z) - \mathrm{Ei}(-i\, (a+b)\, z) - \mathrm{Ei}(i\, (a+b)\, z)) \left(a^2 - b^2\right)^3 + 4\, b \cos(b\, z) \left(\left(a^4 - 6\, b^2\, a^2 + 5\, b^4\right) z \sin(a\, z)\, b^2 + a \left(b^2 \left(a^2 - b^2\right)^2 z^2 - 2 \left(3\, a^4 - 8\, b^2\, a^2 + 9\, b^4\right)\right) \cos(a\, z)\right) + 4\, b^2 \left(a \left(-3\, a^4 + 10\, b^2\, a^2 - 7\, b^4\right) z \cos(a\, z) + \left(-3\, a^4 + 6\, b^2\, a^2 - 11\, b^4 + b^2 \left(a^2 - b^2\right)^2 z^2\right) \sin(a\, z)\right) \sin(b\, z)\right) - 2\, i\, (\Gamma(4, -i\, b\, z) - \Gamma(4, i\, b\, z))\, \mathrm{Si}(a\, z)\right)$$

Involving cos and power

06.37.21.0029.01

$$\int z^n \cos(b\, z)\, \mathrm{Si}(a\, z)\, dz = \frac{1}{4} (-i)\, (i\, b)^{-n-1}\, n! \left((-1)^n\, \mathrm{Ei}(-i\, (a-b)\, z) + \mathrm{Ei}(i\, (a-b)\, z) - \mathrm{Ei}(-i\, (a+b)\, z) - (-1)^{-n}\, \mathrm{Ei}(i\, (a+b)\, z) + \frac{1}{\Gamma(n+2)} (2\, i\, (n+1)\, ((-1)^n\, \Gamma(n+1, -i\, b\, z) - \Gamma(n+1, i\, b\, z))\, \mathrm{Si}(a\, z)) + e^{-i\, (a+b)\, z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b+a}\right)^m \sum_{k=0}^{m-1} \frac{(i\, a + i\, b)^k\, z^k}{k!} - e^{i\, (a-b)\, z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(-(i\, a) + i\, b)^k\, z^k}{k!} - (-1)^n\, e^{-i\, (a-b)\, z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(i\, a - i\, b)^k\, z^k}{k!} + (-1)^n\, e^{i\, (a+b)\, z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b+a}\right)^m \sum_{k=0}^{m-1} \frac{(-(i\, a) - i\, b)^k\, z^k}{k!} \right) /;\, n \in \mathbb{N}$$

06.37.21.0030.01

$$\int z^n \cos(a\, z)\, \mathrm{Si}(a\, z)\, dz = \frac{1}{4}\, i^n\, a^{-n-1} \left(n! \left((-1)^n\, \mathrm{Ei}(-2\, i\, a\, z) + \mathrm{Ei}(2\, i\, a\, z) - (1 + (-1)^n) \log(z) - 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-i\, a\, z)^k}{2\, k} + 2^{-k-1}\, \Gamma(k, -2\, i\, a\, z)\right) - 2\, (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(i\, a\, z)^k}{2\, k} + 2^{-k-1}\, \Gamma(k, 2\, i\, a\, z)\right)\right) - 2\, i\, (\Gamma(n+1, -i\, a\, z) - (-1)^n\, \Gamma(n+1, i\, a\, z))\, \mathrm{Si}(a\, z)\right) /;\, n \in \mathbb{N}$$

06.37.21.0031.01

$$\int z \cos(b\, z)\, \mathrm{Si}(a\, z)\, dz = \frac{1}{4\, b^2} \left(-i\, \mathrm{Ei}(-i\, (a-b)\, z) + i\, \mathrm{Ei}(i\, (a-b)\, z) - i\, \mathrm{Ei}(-i\, (a+b)\, z) + i\, \mathrm{Ei}(i\, (a+b)\, z) + \frac{2\, b \sin((a-b)\, z)}{b-a} + \frac{2\, b \sin((a+b)\, z)}{a+b} + 2\, (\Gamma(2, -i\, b\, z) + \Gamma(2, i\, b\, z))\, \mathrm{Si}(a\, z)\right)$$

06.37.21.0032.01

$$\int z^2 \cos(b\, z)\, \mathrm{Si}(a\, z)\, dz = \frac{1}{2\, b^3} \left(\frac{1}{(a-b)^2\, (a+b)^2} \left(-2 \left(a \left(b^2 - a^2\right) z \cos(a\, z) - \left(a^2 - 3\, b^2\right) \sin(a\, z)\right) \sin(b\, z)\, b^2 + \left(a^2 - b^2\right)^2 (\mathrm{Ei}(-i\, (a-b)\, z) + \mathrm{Ei}(i\, (a-b)\, z) - \mathrm{Ei}(-i\, (a+b)\, z) - \mathrm{Ei}(i\, (a+b)\, z)) + 2 \cos(b\, z) \left(\left(b^2 - a^2\right) z \sin(a\, z)\, b^3 + 2\, a \left(a^2 - 2\, b^2\right) \cos(a\, z)\, b\right)\right) + i\, (\Gamma(3, -i\, b\, z) - \Gamma(3, i\, b\, z))\, \mathrm{Si}(a\, z)\right)$$

06.37.21.0033.01

$$\int z^3 \cos(b z) \operatorname{Si}(a z) \, dz = -\frac{1}{4 b^4} \left(i \left(\frac{1}{(a-b)^3 (a+b)^3} \left(-6 \left(\operatorname{Ei}(-i (a-b) z) - \operatorname{Ei}(i (a-b) z) + \operatorname{Ei}(-i (a+b) z) - \operatorname{Ei}(i (a+b) z) \right) \left(a^2-b^2\right)^3 + 4 b^2 i \cos(b z) \left(a \left(3 a^4 - 10 b^2 a^2 + 7 b^4\right) z \cos(a z) - \left(-3 a^4 + 6 b^2 a^2 - 11 b^4 + b^2 \left(a^2-b^2\right)^2 z^2\right) \sin(a z) \right) + 4 b i \left(\left(a^4 - 6 b^2 a^2 + 5 b^4\right) z \sin(a z) b^2 + a \left(b^2 \left(a^2-b^2\right)^2 z^2 - 2 \left(3 a^4 - 8 b^2 a^2 + 9 b^4\right) \right) \cos(a z) \right) \sin(b z) \right) - 2 i \left(\Gamma(4, -i b z) + \Gamma(4, i b z) \right) \operatorname{Si}(a z) \right) \right)$$

Involving hyperbolic functions

Involving sinh

06.37.21.0034.01

$$\int \sinh(b z) \operatorname{Si}(a z) \, dz = -\frac{-2 \cosh(b z) \operatorname{Si}(a z) + \operatorname{Si}((a + b i) z) + \operatorname{Si}((a - i b) z)}{2 b}$$

Involving cosh

06.37.21.0035.01

$$\int \cosh(b z) \operatorname{Si}(a z) \, dz = \frac{i \left(-\operatorname{Ci}((a + b i) z) + \operatorname{Ci}((a - i b) z) - 2 i \sinh(b z) \operatorname{Si}(a z) \right)}{2 b}$$

Involving hyperbolic functions and a power function

Involving sinh and power

06.37.21.0036.01

$$\int z^n \sinh(b z) \operatorname{Si}(a z) \, dz = -\frac{i}{4} b^{-n-1} n! \left((-1)^n \operatorname{Ei}((b - i a) z) - (-1)^{-n} \operatorname{Ei}((b + a i) z) + \operatorname{Ei}(-b z + a (-i) z) - \operatorname{Ei}(i a z - b z) + \frac{1}{\Gamma(n+2)} \left(2 i (n+1) \left((-1)^n \Gamma(n+1, -b z) + \Gamma(n+1, b z) \right) \operatorname{Si}(a z) \right) - (-1)^n e^{(b - i a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b - i a} \right)^m \sum_{k=0}^{m-1} \frac{(i a - b)^k z^k}{k!} + (-1)^n e^{(b + a i) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b + a i} \right)^m \sum_{k=0}^{m-1} \frac{(-b - i a)^k z^k}{k!} - e^{-(b + a i) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b + a i} \right)^m \sum_{k=0}^{m-1} \frac{(b + a i)^k z^k}{k!} + e^{i a z - b z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b - i a} \right)^m \sum_{k=0}^{m-1} \frac{(b - i a)^k z^k}{k!} \right) /; n \in \mathbb{N}$$

06.37.21.0037.01

$$\int z \sinh(b z) \operatorname{Si}(a z) \, dz = \frac{1}{4 b^2 \left(a^2 + b^2\right)} \left(-4 \sin(a z) \sinh(b z) b^2 + 4 a \cos(a z) \cosh(b z) b + \left(a^2 + b^2\right) \left(i \left(\operatorname{Ei}((b - i a) z) - \operatorname{Ei}((b + a i) z) - \operatorname{Ei}(-b z + a (-i) z) + \operatorname{Ei}(i a z - b z) \right) + 2 \left(\Gamma(2, b z) - \Gamma(2, -b z) \right) \operatorname{Si}(a z) \right) \right)$$

06.37.21.0038.01

$$\int z^2 \sinh(b z)\,\mathrm{Si}(a z)\,dz = -\frac{1}{4 b^3}\left(i\left(\frac{1}{\left(a^2+b^2\right)^2}\left(4\,i \sin(a z)\left(\left(a^2+3 b^2\right)\cosh(b z)-b\left(a^2+b^2\right) z \sinh(b z)\right) b^2+\right.\right.\right.$$
$$4\,a\,i \cos(a z)\left(b\left(a^2+b^2\right) z\cosh(b z)-2\left(a^2+2 b^2\right)\sinh(b z)\right) b+2\left(a^2+b^2\right)^2$$
$$\left.\left.\left.(\mathrm{Ei}((b-i a) z)-\mathrm{Ei}((b+a i) z)+\mathrm{Ei}(-b z+a(-i) z)-\mathrm{Ei}(i a z-b z))\right)+2\,i\,(\Gamma(3,-b z)+\Gamma(3,b z))\,\mathrm{Si}(a z)\right)\right)$$

06.37.21.0039.01

$$\int z^3 \sinh(b z)\,\mathrm{Chi}(a z)\,dz = -\frac{1}{2 b^4}\left(i\left(i\left(2\,\mathrm{Chi}(a z)\left(b z\left(b^2 z^2+6\right)\cosh(b z)-3\left(b^2 z^2+2\right)\sinh(b z)\right)+\right.\right.\right.$$
$$\frac{1}{\left(a^2-b^2\right)^3}\left(3\,(-\mathrm{Ei}((a-b) z)+\mathrm{Ei}((b-a) z)-\mathrm{Ei}(-(a+b) z)+\mathrm{Ei}((a+b) z))\left(a^2-b^2\right)^3+\right.$$
$$2\,a\,b \sinh(a z)\left(\left(-b^2\left(a^2-b^2\right)^2 z^2-2\left(3 a^4-8 b^2 a^2+9 b^4\right)\right)\cosh(b z)+b\left(b^2-a^2\right)\left(7 b^2-3 a^2\right) z\sinh(b z)\right)-$$
$$\left.\left.\left.\left.2 b^2\cosh(a z)\left(b\left(a^2-b^2\right)\left(a^2-5 b^2\right) z\cosh(b z)+\left(-3 a^4+6 b^2 a^2-11 b^4-b^2\left(a^2-b^2\right)^2 z^2\right)\sinh(b z)\right)\right)\right)\right)\right)$$

Involving cosh and power

06.37.21.0040.01

$$\int z^n \cosh(b z)\,\mathrm{Si}(a z)\,dz = -\frac{i}{4}\, b^{-n-1}\, n!\left((-1)^n\,\mathrm{Ei}((b-i a) z)-(-1)^{-n}\,\mathrm{Ei}((b+a i) z)-\right.$$
$$\mathrm{Ei}(-b z+a(-i) z)+\mathrm{Ei}(i a z-b z)+\frac{1}{\Gamma(n+2)}\left(2\,i\,(n+1)\left((-1)^n\,\Gamma(n+1,-b z)-\Gamma(n+1,b z)\right)\mathrm{Si}(a z)\right)-$$
$$(-1)^n e^{(b-i a) z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-i a}\right)^m\sum_{k=0}^{m-1}\frac{(i a-b)^k z^k}{k!}+(-1)^n e^{(b+a i) z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b+a i}\right)^m\sum_{k=0}^{m-1}\frac{(-b-i a)^k z^k}{k!}+$$
$$\left.e^{-(b+a i) z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b+a i}\right)^m\sum_{k=0}^{m-1}\frac{(b+a i)^k z^k}{k!}-e^{i a z-b z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-i a}\right)^m\sum_{k=0}^{m-1}\frac{(b-i a)^k z^k}{k!}\right)/;\ n\in\mathbb{N}$$

06.37.21.0041.01

$$\int z^n \cosh(a z)\,\mathrm{Chi}(a z)\,dz =$$
$$\frac{(-1)^n}{4}\, a^{-n-1}\left(2\,\mathrm{Chi}(a z)\left(\Gamma(n+1,-a z)-(-1)^n\,\Gamma(n+1,a z)\right)+n!\left((-1)^n\,\mathrm{Ei}(-2 a z)-\mathrm{Ei}(2 a z)+\left((-1)^n-1\right)\log(z)-\right.\right.$$
$$\left.\left.2\sum_{k=1}^{n}\frac{(-a)^k}{k!}\left(\frac{z^k}{2 k}-2^{-k-1}(-a)^{-k}\,\Gamma(k,-2 a z)\right)+2\,(-1)^n\sum_{k=1}^{n}\frac{a^k}{k!}\left(\frac{z^k}{2 k}-2^{-k-1} a^{-k}\,\Gamma(k,2 a z)\right)\right)\right)/;\ n\in\mathbb{N}$$

06.37.21.0042.01

$$\int z\cosh(b z)\,\mathrm{Si}(a z)\,dz = \frac{1}{4 b^2\left(a^2+b^2\right)}\left(-4\cosh(b z)\sin(a z) b^2+4 a\cos(a z)\sinh(b z) b+\right.$$
$$\left.\left(a^2+b^2\right) i\,(\mathrm{Ei}((b-i a) z)-\mathrm{Ei}((b+a i) z)+\mathrm{Ei}(-b z+a(-i) z)-\mathrm{Ei}(i a z-b z)+2\,i\,(\Gamma(2,-b z)+\Gamma(2,b z))\,\mathrm{Si}(a z))\right)$$

06.37.21.0043.01

$$\int z^2 \cosh(b z)\,\mathrm{Si}(a z)\,dz = -\frac{1}{4 b^3}\left(i\left(\frac{1}{\left(a^2+b^2\right)^2}\left(-4 i \sin(a z)\left(b\left(a^2+b^2\right) z \cosh(b z)-\left(a^2+3 b^2\right)\sinh(b z)\right) b^2 + 4 a i \cos(a z)\left(b\left(a^2+b^2\right) z \sinh(b z)-2\left(a^2+2 b^2\right)\cosh(b z)\right) b + 2\left(a^2+b^2\right)^2 (\mathrm{Ei}((b-i a) z)-\mathrm{Ei}((b+a i) z)-\mathrm{Ei}(-b z+a(-i) z)+\mathrm{Ei}(i a z-b z))\right) + 2 i (\Gamma(3,-b z)-\Gamma(3, b z))\,\mathrm{Si}(a z)\right)\right)$$

06.37.21.0044.01

$$\int z^3 \cosh(b z)\,\mathrm{Si}(a z)\,dz = -\frac{1}{4 b^4}\left(i\left(-\frac{1}{\left(a^2+b^2\right)^3}\left(6\left(\mathrm{Ei}((b-i a) z)-\mathrm{Ei}((b+a i) z)+\mathrm{Ei}(-b z+a(-i) z)-\mathrm{Ei}(i a z-b z)\right)\left(a^2+b^2\right)^3 + 4 b^2 i \sin(a z)\left(\left(3 a^4+6 b^2 a^2+11 b^4+b^2\left(a^2+b^2\right)^2 z^2\right)\cosh(b z)-b\left(a^2+b^2\right)\left(a^2+5 b^2\right) z \sinh(b z)\right) + 4 b i \cos(a z)\left(a b\left(a^2+b^2\right)\left(3 a^2+7 b^2\right) z \cosh(b z) - a\left(b^2\left(a^2+b^2\right)^2 z^2+2\left(3 a^4+8 b^2 a^2+9 b^4\right)\right)\sinh(b z)\right)\right) - 2 i (\Gamma(4,-b z)+\Gamma(4, b z))\,\mathrm{Si}(a z)\right)\right)$$

Involving logarithm

Involving log

06.37.21.0045.01

$$\int \log(b z)\,\mathrm{Shi}(a z)\,dz = \frac{(\log(b z)-1)(\cos(a z)+a z\,\mathrm{Si}(a z))-\mathrm{Ci}(a z)}{a}$$

Involving logarithm and a power function

Involving log and power

06.37.21.0046.01

$$\int z^{\alpha-1}\log(b z)\,\mathrm{Si}(a z)\,dz = \frac{1}{2\alpha^3}\left(z^\alpha\left(a^2 z^2\right)^{-\alpha}\left(i\,{}_2F_2(\alpha,\alpha;\alpha+1,\alpha+1;-i a z)\left(a^2 z^2\right)^\alpha - i\,{}_2F_2(\alpha,\alpha;\alpha+1,\alpha+1;i a z)\left(a^2 z^2\right)^\alpha + \alpha\left(2\left(a^2 z^2\right)^\alpha(\alpha\log(b z)-1)\,\mathrm{Si}(a z) - i\left(-\Gamma(\alpha, i a z)(\alpha\log(b z)-1)(-i a z)^\alpha + \left((-i a z)^\alpha-(i a z)^\alpha\right)\Gamma(\alpha+1)\log(z) + (i a z)^\alpha\,\Gamma(\alpha,-i a z)(\alpha\log(b z)-1)\right)\right)\right)\right)$$

06.37.21.0047.01

$$\int z\log(b z)\,\mathrm{Si}(a z)\,dz = \frac{1}{4 a^2}\left(a z\cos(a z)(2\log(b z)-1)-(2\log(b z)+1)\sin(a z)+\left(-a^2 z^2+2 a^2\log(b z) z^2+2\right)\mathrm{Si}(a z)\right)$$

06.37.21.0048.01

$$\int z^2 \log(b\,z)\,\mathrm{Si}(a\,z)\,dz = -\frac{1}{9\,a^3}\left(a^3\,(1-3\log(b\,z))\,\mathrm{Si}(a\,z)\,z^3 + a^2\cos(a\,z)\,z^2 - 3\,a^2\cos(a\,z)\log(b\,z)\,z^2 + a\sin(a\,z)\,z + 6\,a\log(b\,z)\sin(a\,z)\,z + 7\cos(a\,z) - 6\,\mathrm{Ci}(a\,z) + 6\cos(a\,z)\log(b\,z)\right)$$

06.37.21.0049.01

$$\int z^3 \log(b\,z)\,\mathrm{Si}(a\,z)\,dz = \frac{1}{16\,a^4}\left(a\,z\cos(a\,z)\left(-a^2\,z^2 + 4\left(a^2\,z^2-6\right)\log(b\,z) - 14\right) + \left(-a^2\,z^2 - 12\left(a^2\,z^2-2\right)\log(b\,z) + 38\right)\sin(a\,z) + \left(-a^4\,z^4 + 4\,a^4\log(b\,z)\,z^4 - 24\right)\mathrm{Si}(a\,z)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

06.37.21.0050.01

$$\int \mathrm{Si}(a\,z)^2\,dz = \frac{a\,z\,\mathrm{Si}(a\,z)^2 + 2\cos(a\,z)\,\mathrm{Si}(a\,z) - \mathrm{Si}(2\,a\,z)}{a}$$

Involving products of the direct function

06.37.21.0051.01

$$\int \mathrm{Si}(a\,z)\,\mathrm{Si}(b\,z)\,dz = \frac{1}{2\,a\,b}\left((b-a)\,\mathrm{Si}((a-b)\,z) + 2\,b\cos(a\,z)\,\mathrm{Si}(b\,z) + 2\,a\,\mathrm{Si}(a\,z)\,(\cos(b\,z) + b\,z\,\mathrm{Si}(b\,z)) - a\,\mathrm{Si}((a+b)\,z) - b\,\mathrm{Si}((a+b)\,z)\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

06.37.21.0052.01

$$\int z^n\,\mathrm{Si}(a\,z)^2\,dz = -\frac{1}{2\,(n+1)}\left((-i\,a)^{-n-1}\left(2\,i\,(\Gamma(n+1,-i\,a\,z) + (-1)^n\,\Gamma(n+1,i\,a\,z))\,\mathrm{Si}(a\,z) + n!\left((-1)^n\,\mathrm{Ei}(-2\,i\,a\,z) - \mathrm{Ei}(2\,i\,a\,z) + (1-(-1)^n)\log(z) + 2\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(-i\,a\,z)^k}{2\,k} + 2^{-k-1}\,\Gamma(k,-2\,i\,a\,z)\right) - 2\,(-1)^n\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(i\,a\,z)^k}{2\,k} + 2^{-k-1}\,\Gamma(k,2\,i\,a\,z)\right)\right)\right) - 2\,z^{n+1}\,\mathrm{Si}(a\,z)^2\right)\text{ /; } n\in\mathbb{N}$$

06.37.21.0053.01

$$\int z\,\mathrm{Si}(a\,z)^2\,dz = \frac{1}{4\,a^2}\left(2\,a^2\,z^2\,\mathrm{Si}(a\,z)^2 + 4\,(a\,z\cos(a\,z) - \sin(a\,z))\,\mathrm{Si}(a\,z) + \cos(2\,a\,z) - 2\,\mathrm{Ci}(2\,a\,z) + 2\log(z)\right)$$

06.37.21.0054.01

$$\int z^2\,\text{Si}(a\,z)^2\,dz = \frac{1}{12\,a^3}\left(4\,a^3\,\text{Si}(a\,z)^2\,z^3 + 8\,a\,z + 2\,a\cos(2\,a\,z)\,z - 5\sin(2\,a\,z) + 8\left(\left(a^2\,z^2-2\right)\cos(a\,z) - 2\,a\,z\sin(a\,z)\right)\text{Si}(a\,z) + 8\,\text{Si}(2\,a\,z)\right)$$

06.37.21.0055.01

$$\int z^3\,\text{Si}(a\,z)^2\,dz = \frac{1}{8\,a^4}\left(2\,a^4\,\text{Si}(a\,z)^2\,z^4 + 3\,a^2\,z^2 + a^2\cos(2\,a\,z)\,z^2 - 4\,a\sin(2\,a\,z)\,z - 8\cos(2\,a\,z) + 12\,\text{Ci}(2\,a\,z) - 12\log(z) + 4\left(a\,z\left(a^2\,z^2-6\right)\cos(a\,z) - 3\left(a^2\,z^2-2\right)\sin(a\,z)\right)\text{Si}(a\,z)\right)$$

Involving products of the direct function and a power function

06.37.21.0056.01

$$\int z^n\,\text{Si}(a\,z)\,\text{Si}(b\,z)\,dz = \frac{i\,(-i\,a)^{-n}}{4\,a\,(n+1)}\Bigg(n!\Bigg((-1)^n\,\text{Ei}(i\,(b-a)\,z) - \text{Ei}(i\,(a-b)\,z) - (-1)^n\,\text{Ei}(-i\,(a+b)\,z) + \text{Ei}(i\,(a+b)\,z) + \sum_{k=1}^{n}\frac{1}{k!}\left(a^k\left((a-b)^{-k}\,\Gamma(k,\,i\,(b-a)\,z) - (a+b)^{-k}\,\Gamma(k,\,-i\,(a+b)\,z)\right)\right) + (-1)^n\sum_{k=1}^{n}\frac{1}{k!}\left(a^k\left((a+b)^{-k}\,\Gamma(k,\,i\,(a+b)\,z) - (a-b)^{-k}\,\Gamma(k,\,i\,(a-b)\,z)\right)\right)\Bigg) - 2\,i\,(2\,a\,z\,\text{Si}(a\,z)\,(-i\,a\,z)^n + \Gamma(n+1,\,-i\,a\,z) + (-1)^n\,\Gamma(n+1,\,i\,a\,z))\,\text{Si}(b\,z)\Bigg) - \frac{(i\,b)^{-n}\,n!}{b}\Bigg((-1)^n\,\text{Ei}(-i\,(a-b)\,z) - \text{Ei}(i\,(a-b)\,z) + \text{Ei}(-i\,(a+b)\,z) - (-1)^{-n}\,\text{Ei}(i\,(a+b)\,z) + \frac{1}{(n+1)!}(2\,i\,(n+1)\,((-1)^n\,\Gamma(n+1,\,-i\,b\,z) + \Gamma(n+1,\,i\,b\,z))\,\text{Si}(a\,z)) + e^{i\,(a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\left(\frac{b}{b-a}\right)^m\sum_{k=0}^{m-1}\frac{(-i\,a+i\,b)^k\,z^k}{k!}\right) - e^{-i\,(a+b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\left(\frac{b}{a+b}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a+i\,b)^k\,z^k}{k!}\right) + (-1)^n\,e^{i\,(a+b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\left(\frac{b}{a+b}\right)^m\sum_{k=0}^{m-1}\frac{(-i\,a-i\,b)^k\,z^k}{k!}\right) - (-1)^n\,e^{-i\,(a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\left(\frac{b}{b-a}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a-i\,b)^k\,z^k}{k!}\right)\Bigg)\;/;\;n\in\mathbb{N}$$

06.37.21.0057.01

$$\int z\,\text{Si}(a\,z)\,\text{Si}(b\,z)\,dz = \frac{1}{8\,a^2\,b^2}\left(4\,\text{Si}(a\,z)\left(b^2\,\text{Si}(b\,z)\,z^2 + b\cos(b\,z)\,z - \sin(b\,z)\right)a^2 + 4\,b\cos(a\,z)\cos(b\,z)\,a + \left(a^2+b^2\right)(\text{Ei}(-i\,(a-b)\,z) + \text{Ei}(i\,(a-b)\,z) - \text{Ei}(-i\,(a+b)\,z) - \text{Ei}(i\,(a+b)\,z)) + 4\,b^2\,(a\,z\cos(a\,z) - \sin(a\,z))\,\text{Si}(b\,z)\right)$$

06.37.21.0058.01

$$\int z^2\,\text{Si}(a\,z)\,\text{Si}(b\,z)\,dz = \frac{1}{3}\left(\frac{\left(\left(b^2 z^2-2\right)\cos(b\,z)-2\,b\,z\sin(b\,z)\right)\text{Si}(a\,z)}{b^3}+\frac{1}{a^3}\left(\left(a^3\,\text{Si}(a\,z)\,z^3-2\,a\sin(a\,z)\,z+\left(a^2 z^2-2\right)\cos(a\,z)\right)\text{Si}(b\,z)\right)+\frac{1}{a^3\,(a-b)\,b^3\,(a+b)}\left(a\,b\left(b\left(a^2+2\,b^2\right)\cos(b\,z)\sin(a\,z)+a\cos(a\,z)\left(b\left(a^2-b^2\right)z\cos(b\,z)-\left(2\,a^2+b^2\right)\sin(b\,z)\right)\right)+\left(a^5-b^2\,a^3-b^3\,a^2+b^5\right)\text{Si}((a-b)\,z)+\left(a^5-b^2\,a^3+b^3\,a^2-b^5\right)\text{Si}((a+b)\,z)\right)\right)$$

06.37.21.0059.01

$$\int z^3\,\text{Si}(a\,z)\,\text{Si}(b\,z)\,dz = \frac{1}{8}\left(\frac{2\left(b\,z\left(b^2 z^2-6\right)\cos(b\,z)-3\left(b^2 z^2-2\right)\sin(b\,z)\right)\text{Si}(a\,z)}{b^4}+\frac{1}{a^4}\left(2\left(a^4\,\text{Si}(a\,z)\,z^4+a\left(a^2 z^2-6\right)\cos(a\,z)\,z-3\left(a^2 z^2-2\right)\sin(a\,z)\right)\text{Si}(b\,z)\right)-\left(2\left(3\left(a^4+b^4\right)\text{Ci}((a-b)\,z)\left(a^2-b^2\right)^2-3\left(a^4+b^4\right)\text{Ci}((a+b)\,z)\left(a^2-b^2\right)^2-a\,b\left(a\,b\sin(a\,z)\left(b\left(a^4+2\,b^2\,a^2-3\,b^4\right)z\cos(b\,z)+\left(-3\,a^4+14\,b^2\,a^2-3\,b^4\right)\sin(b\,z)\right)+\cos(a\,z)\left(b\left(-3\,a^4+2\,b^2\,a^2+b^4\right)z\sin(b\,z)\,a^2+\left(\left(b^2 z^2-6\right)a^6-2\,b^2\left(b^2 z^2-5\right)a^4+b^4\left(b^2 z^2+10\right)a^2-6\,b^6\right)\cos(b\,z)\right)\right)\right)\Big/\left(a^4\,(a-b)^2\,b^4\,(a+b)^2\right)\right)\right)$$

Involving direct function and Gamma-, Beta-, Erf-type functions

Involving exponential integral-type functions

Involving Ei

06.37.21.0060.01

$$\int \text{Ei}(b\,z)\,\text{Si}(a\,z)\,dz = \frac{1}{2\,a\,b}\left(2\,b\cos(a\,z)\,\text{Ei}(b\,z)+2\,a\,b\,z\,\text{Si}(a\,z)\,\text{Ei}(b\,z)-b\,\text{Ei}((b-i\,a)\,z)+a\,i\,\text{Ei}((b-i\,a)\,z)-i\,a\,\text{Ei}((b+a\,i)\,z)-b\,\text{Ei}((b+a\,i)\,z)-2\,a\,e^{b\,z}\,\text{Si}(a\,z)\right)$$

Involving exponential integral-type functions and a power function

Involving Ei and power

06.37.21.0061.01

$$\int z^n \operatorname{Ei}(b z) \operatorname{Si}(a z) \, dz = \frac{\left(\Gamma(n+1, -b z) (-b)^{-n-1} + z^{n+1} \operatorname{Ei}(b z)\right) \operatorname{Si}(a z)}{n+1} + \frac{i}{n+1} \left(\frac{(i a)^{-n-1}}{2} \left(-\operatorname{Ei}((b - i a) z) \, n! + n! \sum_{k=1}^{n} \frac{a^k (a + b i)^{-k} \Gamma(k, (i a - b) z)}{k!} + \operatorname{Ei}(b z) \Gamma(n+1, i a z) + (-1)^n \left(-\operatorname{Ei}((b + a i) z) \, n! + n! \sum_{k=1}^{n} \frac{a^k (a - i b)^{-k} \Gamma(k, -(b + a i) z)}{k!} + \operatorname{Ei}(b z) \Gamma(n+1, -i a z) \right) \right) + \frac{1}{2} (-b)^{-n-1} n! \left(-\operatorname{Ei}(b z - i a z) + \operatorname{Ei}(b z + a i z) + \sum_{k=1}^{n} \frac{1}{k!} \left(b^k \left((b - i a)^{-k} \Gamma(k, (i a - b) z) - (b + a i)^{-k} \Gamma(k, -(b + a i) z) \right) \right) \right) \right) /; n \in \mathbb{N}$$

06.37.21.0062.01

$$\int z \operatorname{Ei}(b z) \operatorname{Si}(a z) \, dz = -\frac{1}{4 a^2 b^2} \left(-i \operatorname{Ei}((b + a i) z) a^2 - 2 \left(b^2 \operatorname{Ei}(b z) z^2 + \Gamma(2, -b z) \right) \operatorname{Si}(a z) a^2 + b e^{(b - i a) z} a + b e^{(b + a i) z} a + \left(a^2 - b^2 \right) i \operatorname{Ei}((b - i a) z) + b^2 i \operatorname{Ei}((b + a i) z) - i b^2 \operatorname{Ei}(b z) \Gamma(2, -i a z) + b^2 i \operatorname{Ei}(b z) \Gamma(2, i a z) \right)$$

06.37.21.0063.01

$$\int z^2 \operatorname{Ei}(b z) \operatorname{Si}(a z) \, dz = \frac{1}{3} \left(-\frac{1}{b^3} \left(\frac{i b}{2} \left(\frac{2 i e^{(b - i a) z}}{a + b i} - \frac{2 e^{(b + a i) z}}{b + a i} + \frac{b \Gamma(2, -b z + a (-i) z)}{(a - i b)^2} + \frac{b \Gamma(2, i a z - b z)}{(b - i a)^2} \right) - \operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z) \right) - \frac{1}{2 a^3} \left(\frac{\Gamma(2, -b z + a (-i) z) a^2}{(a - i b)^2} + \frac{\Gamma(2, i a z - b z) a^2}{(a + b i)^2} + \frac{2 e^{(b - i a) z} a}{a + b i} + \frac{2 e^{(b + a i) z} a}{a - i b} - 2 \operatorname{Ei}((b - i a) z) - 2 \operatorname{Ei}((b + a i) z) + \operatorname{Ei}(b z) \Gamma(3, -i a z) + \operatorname{Ei}(b z) \Gamma(3, i a z) \right) + \left(z^3 \operatorname{Ei}(b z) - \frac{\Gamma(3, -b z)}{b^3} \right) \operatorname{Si}(a z) \right)$$

06.37.21.0064.01

$$\int z^3 \operatorname{Ei}(b z) \operatorname{Si}(a z) \, dz = -\frac{i}{4} \left(-\frac{1}{2 b^3} \left(\frac{\Gamma(3, i a z - b z) b^2}{(b - i a)^3} - \frac{\Gamma(3, -b z + a (-i) z) b^2}{(b + a i)^3} - \frac{3 \Gamma(2, -b z + a (-i) z) b}{(b + a i)^2} - \frac{3 \Gamma(2, i a z - b z) b}{(a + b i)^2} + \frac{6 e^{(b - i a) z}}{b - i a} - \frac{6 e^{(b + a i) z}}{b + a i} \right) + \frac{3 (\operatorname{Ei}((b - i a) z) - \operatorname{Ei}((b + a i) z))}{b^4} - \frac{1}{2 a^4} \left(\frac{\Gamma(3, i a z - b z) a^3}{(a + b i)^3} - \frac{\Gamma(3, -b z + a (-i) z) a^3}{(a - i b)^3} + \frac{3 \Gamma(2, -b z + a (-i) z) a^2}{(b + a i)^2} + \frac{3 \Gamma(2, i a z - b z) a^2}{(a + b i)^2} + \frac{6 e^{(b - i a) z} a}{a + b i} - \frac{6 e^{(b + a i) z} a}{a - i b} - 6 \operatorname{Ei}((b - i a) z) + 6 \operatorname{Ei}((b + a i) z) - \operatorname{Ei}(b z) \Gamma(4, -i a z) + \operatorname{Ei}(b z) \Gamma(4, i a z) \right) + \left(\operatorname{Ei}(b z) z^4 + \frac{\Gamma(4, -b z)}{b^4} \right) i \operatorname{Si}(a z) \right)$$

Definite integration

For the direct function itself

06.37.21.0065.01

$$\int_0^\infty t^{\alpha-1}\,\mathrm{Si}(t)\,d = -\frac{\Gamma(\alpha)}{\alpha}\sin\left(\frac{\pi\,\alpha}{2}\right)\,/;\,-1<\mathrm{Re}(\alpha)<0$$

Involving the direct function

06.37.21.0066.01

$$\int_0^\infty t^{\alpha-1}\,e^{-z\,t}\,\mathrm{Si}(t)\,dt = z^{-\alpha-1}\,\Gamma(\alpha+1)\,{}_3F_2\left(\frac{1}{2},\,\frac{\alpha+1}{2},\,\frac{\alpha}{2}+1;\,\frac{3}{2},\,\frac{3}{2};\,-\frac{1}{z^2}\right)/;\,\mathrm{Re}(z)>0\land\mathrm{Re}(\alpha)>-1$$

06.37.21.0067.01

$$\int_0^\infty \sin(t)\left(\mathrm{Si}(t)-\frac{\pi}{2}\right)dt = -\frac{\pi}{4}$$

06.37.21.0068.01

$$\int_0^\infty \left(\mathrm{Si}(t)-\frac{\pi}{2}\right)^2 dt = \frac{\pi}{2}$$

Involving related functions

06.37.21.0069.01

$$\int_0^\infty \left(\mathrm{Si}(t)-\frac{\pi}{2}\right)\mathrm{Ci}(t)\,dt = \log(2)$$

Integral transforms

Laplace transforms

06.37.22.0001.01

$$\mathcal{L}_t[\mathrm{Si}(t)]\,(z) = \frac{1}{z}\tan^{-1}\left(\frac{1}{z}\right)/;\,\mathrm{Re}(z)>0$$

Mellin transforms

06.37.22.0002.01

$$\mathcal{M}_t[\mathrm{Si}(t)]\,(z) = -\frac{\Gamma(z)}{z}\sin\left(\frac{\pi\,z}{2}\right)/;\,-1<\mathrm{Re}(z)<0$$

Operations

Limit operation

06.37.25.0001.01

$$\lim_{x\to\infty}\mathrm{Si}(a+b\,x) = \begin{cases} \frac{\pi}{2} & \arg(b)=0 \\ -\frac{\pi}{2} & \arg(b)=\pi \\ i\,\infty & |\arg(a)|=\frac{\pi}{2}\bigwedge\arg(b)=\frac{\pi}{2} \\ -i\,\infty & |\arg(a)|=\frac{\pi}{2}\bigwedge\arg(b)=-\frac{\pi}{2} \\ \tilde{\infty} & \text{True}\end{cases}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

06.37.26.0001.01

$$\text{Si}(z) = z\, {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{z^2}{4}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

06.37.26.0002.01

$$\text{Si}(z) = \frac{\sqrt{\pi}}{4}\, z\, G_{1,3}^{1,1}\left(\frac{z^2}{4} \middle| \begin{matrix} \frac{1}{2} \\ 0, -\frac{1}{2}, -\frac{1}{2} \end{matrix}\right)$$

06.37.26.0003.01

$$\text{Si}(z) = \frac{\sqrt{\pi z^2}}{2z}\, G_{1,3}^{1,1}\left(\frac{z^2}{4} \middle| \begin{matrix} 1 \\ \frac{1}{2}, 0, 0 \end{matrix}\right)$$

06.37.26.0004.01

$$\text{Si}(z) = \frac{\sqrt{\pi}}{2}\, G_{1,3}^{1,1}\left(\frac{z^2}{4} \middle| \begin{matrix} 1 \\ \frac{1}{2}, 0, 0 \end{matrix}\right) /; \text{Re}(z) > 0$$

06.37.26.0005.01

$$\text{Si}\left(\sqrt{z}\right) = \frac{\pi}{2} - \frac{\sqrt{\pi}}{2}\, G_{1,3}^{2,0}\left(\frac{z}{4} \middle| \begin{matrix} 1 \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

Classical cases for powers of Ci and Si

06.37.26.0019.01

$$\text{Ci}\left(\sqrt{z}\right)^2 + \text{Si}\left(\sqrt{z}\right)^2 = \frac{1}{2}\pi^{3/2}\, G_{3,5}^{3,1}\left(\frac{z}{4} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) + \frac{\pi^2}{4}$$

06.37.26.0006.01

$$\left(\text{Si}\left(\sqrt{z}\right) - \frac{\pi}{2}\right)^2 + \text{Ci}\left(\sqrt{z}\right)^2 = \frac{1}{2\sqrt{\pi}}\, G_{2,4}^{4,1}\left(\frac{z}{4} \middle| \begin{matrix} 0, 1 \\ 0, 0, 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving cos, sin, Ci

06.37.26.0007.01

$$\cos\left(\sqrt{z}\right) \text{Ci}\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right) \text{Si}\left(\sqrt{z}\right) = -\frac{1}{2}\pi^{3/2}\, G_{2,4}^{2,1}\left(\frac{z}{4} \middle| \begin{matrix} 0, \frac{1}{2} \\ 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

06.37.26.0008.01

$$\cos\left(\sqrt{z}\right) \text{Ci}\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right) \left(\text{Si}\left(\sqrt{z}\right) - \frac{\pi}{2}\right) = -\frac{1}{2\sqrt{\pi}}\, G_{1,3}^{3,1}\left(\frac{z}{4} \middle| \begin{matrix} 0 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

06.37.26.0009.01

$$\sin\left(\sqrt{z}\right)\operatorname{Ci}\left(\sqrt{z}\right)-\cos\left(\sqrt{z}\right)\operatorname{Si}\left(\sqrt{z}\right)=-\frac{1}{2}\pi^{3/2}G_{2,4}^{2,1}\left(\frac{z}{4}\,\middle|\,\begin{matrix}\frac{1}{2},1\\ \frac{1}{2},\frac{1}{2},1,0\end{matrix}\right)$$

06.37.26.0010.01

$$\sin\left(\sqrt{z}\right)\operatorname{Ci}\left(\sqrt{z}\right)-\cos\left(\sqrt{z}\right)\left(\operatorname{Si}\left(\sqrt{z}\right)-\frac{\pi}{2}\right)=\frac{1}{2\sqrt{\pi}}G_{1,3}^{3,1}\left(\frac{z}{4}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

Generalized cases for the direct function itself

06.37.26.0011.01

$$\operatorname{Si}(z)=\frac{\sqrt{\pi}}{2}G_{1,3}^{1,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}1\\ \frac{1}{2},0,0\end{matrix}\right)$$

Generalized cases for powers of Ci and Si

06.37.26.0020.01

$$\operatorname{Ci}(z)^2+\operatorname{Si}(z)^2=\frac{1}{2}\pi^{3/2}G_{3,5}^{3,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2},1\\ 0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)+\frac{\pi^2}{4}$$

06.37.26.0012.01

$$\left(\operatorname{Si}(z)-\frac{\pi}{2}\right)^2+\operatorname{Ci}(z)^2=\frac{1}{2\sqrt{\pi}}G_{2,4}^{4,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}0,1\\ 0,0,0,\frac{1}{2}\end{matrix}\right)$$

Generalized cases involving cos, sin, Ci

06.37.26.0013.01

$$\cos(z)\operatorname{Ci}(z)+\sin(z)\operatorname{Si}(z)=-\frac{1}{2}\pi^{3/2}G_{2,4}^{2,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2}\\ 0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

06.37.26.0014.01

$$\cos(z)\operatorname{Ci}(z)+\sin(z)\left(\operatorname{Si}(z)-\frac{\pi}{2}\right)=-\frac{1}{2\sqrt{\pi}}G_{1,3}^{3,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}0\\ 0,0,\frac{1}{2}\end{matrix}\right)$$

06.37.26.0015.01

$$\sin(z)\operatorname{Ci}(z)-\cos(z)\operatorname{Si}(z)=-\frac{1}{2}\pi^{3/2}G_{2,4}^{2,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},1\\ \frac{1}{2},\frac{1}{2},1,0\end{matrix}\right)$$

06.37.26.0016.01

$$\sin(z)\operatorname{Ci}(z)-\cos(z)\left(\operatorname{Si}(z)-\frac{\pi}{2}\right)=\frac{1}{2\sqrt{\pi}}G_{1,3}^{3,1}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

Through other functions

06.37.26.0017.01

$$\operatorname{Si}(z)=\frac{i}{2}\left(\Gamma(0,-iz)-\Gamma(0,iz)+\log(-iz)-\log(iz)\right)$$

06.37.26.0018.01

$$\operatorname{Si}(z) = \frac{i}{2}\left(E_1(-i z) - E_1(i z) + \log(-i z) - \log(i z)\right)$$

Representations through equivalent functions

With related functions

06.37.27.0001.01

$$\operatorname{Si}(z) = -i \operatorname{Shi}(i z)$$

06.37.27.0002.01

$$\operatorname{Si}(z) = \frac{i}{4}\left(2\left(\operatorname{Ei}(-i z) - \operatorname{Ei}(i z)\right) + \log\left(\frac{i}{z}\right) - \log\left(-\frac{i}{z}\right) - \log(-i z) + \log(i z)\right)$$

06.37.27.0003.01

$$\operatorname{Si}(z) = \frac{1}{2 i}\left(\operatorname{li}\left(e^{i z}\right) - \operatorname{li}\left(e^{-i z}\right)\right) - \frac{\pi}{2}\operatorname{sgn}(\operatorname{Re}(z)) \;/;\; |\operatorname{Re}(z)| < \pi$$

Zeros

06.37.30.0001.01

$$\operatorname{Si}(z) = 0 \;/;\; z = 0$$

Theorems

The Hartree and exchange energy of a metal surface within the semi-infinite jellium model

The Hartree and exchange energy of a metal surface within the semi-infinite jellium model can be expressed using the sine integral.

History

– L. Mascheroni (1790, 1819)
– F. W. Bessel (1812)
– C. A. Bretschneider (1843)
– O. Schlomilch (1846)
– F. Arndt (1847)
– J. W. L. Glaisher (1870) introduced the notations Si and Ci
– N. Nielsen (1904) used notations Si and Ci

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