

SpheroidalPSPPrime

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Notations

Traditional name

Derivative of the angular spheroidal function of the first kind

Traditional notation

$$PS_{\nu,\mu}'(\gamma, z)$$

Mathematica StandardForm notation

SpheroidalPSPPrime[ν, μ, γ, z]

Primary definition

11.12.02.0001.01

$$PS_{\nu,\mu}'(\gamma, z) = \frac{\partial PS_{\nu,\mu}(\gamma, z)}{\partial z}$$

Specific values

Specialized values

For fixed ν, μ, z

11.12.03.0001.01

$$PS_{\nu,\mu}'(0, z) = \frac{-z(\nu+1)P_{\nu}^{\mu}(z) + (-\mu + \nu + 1)P_{\nu+1}^{\mu}(z)}{z^2 - 1}$$

For fixed ν, γ, z

11.12.03.0002.01

$$PS_{\nu,\frac{1}{2}}'(\gamma, z) = \frac{1}{\sqrt{2\pi}(1-z^2)^{5/4}} \left(z \operatorname{Ce} \left(a_{\nu+\frac{1}{2}} \left(\frac{\gamma^2}{4} \right), \frac{\gamma^2}{4}, \cos^{-1}(z) \right) - 2\sqrt{1-z^2} \operatorname{Ce}_{\cos^{-1}(z)} \left(a_{\nu+\frac{1}{2}} \left(\frac{\gamma^2}{4} \right), \frac{\gamma^2}{4}, \cos^{-1}(z) \right) \right)$$

For fixed ν, z

11.12.03.0003.01

$$PS_{\nu,0}'(0, z) = \frac{(\nu+1)P_{\nu+1}(z) - z(\nu+1)P_{\nu}(z)}{z^2 - 1}$$

11.12.03.0004.01

$$PS_{\nu, \frac{1}{2}}'(0, z) = \frac{z \cos\left(\left(\nu + \frac{1}{2}\right) \cos^{-1}(z)\right) + \sqrt{1 - z^2} (2\nu + 1) \sin\left(\left(\nu + \frac{1}{2}\right) \cos^{-1}(z)\right)}{\sqrt{2\pi} (1 - z^2)^{5/4}}$$

General characteristics

Domain and analyticity

$PS_{\nu, \mu}'(\gamma, z)$ is an analytical function of ν, μ, γ, z which is defined in \mathbb{C}^4 .

11.12.04.0001.01

$$(\nu * \mu * \gamma * z) \rightarrow PS_{\nu, \mu}'(\gamma, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$PS_{\nu, \mu}'(\gamma, z)$ is an even function with respect to γ .

11.12.04.0002.01

$$PS_{\nu, \mu}'(-\gamma, z) = PS_{\nu, \mu}'(\gamma, z)$$

11.12.04.0003.01

$$PS_{-\nu, \mu}'(\gamma, z) = PS_{\nu-1, \mu}'(\gamma, z)$$

Mirror symmetry

11.12.04.0004.01

$$PS_{\bar{\nu}, \bar{\mu}}'(\bar{\gamma}, \bar{z}) = \overline{PS_{\nu, \mu}'(\gamma, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at $\gamma = 0$

11.12.06.0001.01

$$PS_{v,\mu}'(\gamma, z) \propto \frac{(-\mu + v + 1) P_{v+1}^\mu(z) - z(v + 1) P_v^\mu(z)}{z^2 - 1} + \frac{1}{2(z^2 - 1)(2v + 1)} \left(\frac{(\mu + v - 1)(\mu + v)(z(v - 1) P_{v-2}^\mu(z) + (\mu - v + 1) P_{v-1}^\mu(z))}{(1 - 2v)^2} + \frac{(-\mu + v + 1)(-\mu + v + 2)((-\mu + v + 3) P_{v+3}^\mu(z) - z(v + 3) P_{v+2}^\mu(z))}{(2v + 3)^2} \right) \gamma^2 - \frac{1}{8(z^2 - 1)} \left(\frac{(\mu + v - 3)(\mu + v - 2)(\mu + v - 1)(\mu + v)(z(v - 3) P_{v-4}^\mu(z) + (\mu - v + 3) P_{v-3}^\mu(z))}{(4v^2 - 8v - 5)(4v^2 - 8v + 3)^2} + \frac{8(4\mu^2 - 1)(\mu^2 + (2v - 1)\mu + (v - 1)v)(z(v - 1) P_{v-2}^\mu(z) + (\mu - v + 1) P_{v-1}^\mu(z))}{(1 - 2v)^4(8v^3 - 4v^2 - 34v - 15)} + \frac{1}{2v + 1} \left(\frac{(\mu - v)(\mu - v + 1)(\mu + v - 1)(\mu + v)}{(1 - 2v)^4(2v - 3)} + \frac{(-\mu + v + 1)(-\mu + v + 2)(\mu + v + 1)(\mu + v + 2)}{(2v + 3)^4(2v + 5)} \right) + \frac{((-\mu + v + 1) P_{v+1}^\mu(z) - z(v + 1) P_v^\mu(z)) + (\mu - v - 4)(\mu - v - 3)(\mu - v - 2)(\mu - v - 1)(z(v + 5) P_{v+4}^\mu(z) + (\mu - v - 5) P_{v+5}^\mu(z))}{(4v^2 + 16v + 7)(4v^2 + 16v + 15)^2} \right) \gamma^4 + \dots /; (\gamma \rightarrow 0)$$

11.12.06.0002.01

$$PS_{v,\mu}'(\gamma, z) \propto \frac{(-\mu + v + 1) P_{v+1}^\mu(z) - z(v + 1) P_v^\mu(z)}{z^2 - 1} (1 + O(\gamma^2))$$

Expansions at generic point $z = z_0$

11.12.06.0003.01

$$PS_{v,\mu}'(\gamma, z) \propto PS_{v,\mu}'(\gamma, z_0) + \frac{1}{1 - z_0^2} \left(2 PS_{v,\mu}'(\gamma, z_0) z_0 + PS_{v,\mu}(\gamma, z_0) \left((z_0^2 - 1) \gamma^2 - \lambda_{v,\mu}(\gamma) + \frac{\mu^2}{1 - z_0^2} \right) \right) (z - z_0) - \frac{1}{2(z_0^2 - 1)^3} \left(PS_{v,\mu}'(\gamma, z_0) (z_0^2 - 1) (\gamma^2 z_0^4 - 2(\gamma^2 + 3) z_0^2 + \gamma^2 - \mu^2 - \lambda_{v,\mu}(\gamma) (z_0^2 - 1) - 2) - 2 PS_{v,\mu}(\gamma, z_0) z_0 (-3\mu^2 + \gamma^2 (z_0^2 - 1)^2 - 2\lambda_{v,\mu}(\gamma) (z_0^2 - 1)) \right) (z - z_0)^2 + \frac{1}{6(z_0^2 - 1)^4} \left(4 PS_{v,\mu}'(\gamma, z_0) z_0 (z_0^2 - 1) (\gamma^2 z_0^4 - 2(\gamma^2 + 3) z_0^2 + \gamma^2 - 3\mu^2 - 2\lambda_{v,\mu}(\gamma) (z_0^2 - 1) - 6) + PS_{v,\mu}(\gamma, z_0) (\gamma^4 (z_0^2 - 1)^4 + \lambda_{v,\mu}(\gamma)^2 (z_0^2 - 1)^2 - 2\gamma^2 (\mu^2 + 4z_0^2 + 2) (z_0^2 - 1)^2 - 2\lambda_{v,\mu}(\gamma) (\gamma^2 z_0^4 - (2\gamma^2 + 9) z_0^2 + \gamma^2 - \mu^2 - 3) (z_0^2 - 1) + \mu^2 (\mu^2 + 36z_0^2 + 8)) \right) (z - z_0)^3 + \dots /; (z \rightarrow z_0)$$

11.12.06.0004.01

$$PS_{v,\mu}'(\gamma, z) \propto PS_{v,\mu}'(\gamma, z_0) (1 + O(z - z_0))$$

Expansions at $z = 0$

11.12.06.0005.01

$$PS_{v,\mu}'(\gamma, z) \propto PS_{v,\mu}'(\gamma, 0) - (\gamma^2 - \mu^2 + \lambda_{v,\mu}(\gamma)) PS_{v,\mu}(\gamma, 0) z - \frac{1}{2} (\gamma^2 - \mu^2 + \lambda_{v,\mu}(\gamma) - 2) PS_{v,\mu}'(\gamma, 0) z^2 + \frac{1}{6} (\gamma^4 - 2(\mu^2 + 2)\gamma^2 + \lambda_{v,\mu}(\gamma)^2 + \mu^2(\mu^2 + 8) + 2(\gamma^2 - \mu^2 - 3)\lambda_{v,\mu}(\gamma)) PS_{v,\mu}(\gamma, 0) z^3 + \dots /; (z \rightarrow 0)$$

11.12.06.0006.01

$$PS_{v,\mu}'(\gamma, z) \propto PS_{v,\mu}'(\gamma, 0) (1 + O(z))$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.12.13.0001.01

$$(1 - z^2) w''(z) - 2z \left(\frac{(1 - z^2)^2 \gamma^2 + \mu^2}{\mu^2 - (1 - z^2)((1 - z^2)\gamma^2 + \lambda_{v,\mu}(\gamma))} + 2 \right) w'(z) + \left(\frac{4((1 - z^2)^2 \gamma^2 + \mu^2) z^2}{(1 - z^2)(-(1 - z^2)^2 \gamma^2 + \mu^2 - (1 - z^2)\lambda_{v,\mu}(\gamma))} + (1 - z^2)\gamma^2 + \lambda_{v,\mu}(\gamma) - \frac{\mu^2}{1 - z^2} - 2 \right) w(z) = 0 /; w(z) = c_1 PS_{v,\mu}'(\gamma, z) + c_2 QS_{v,\mu}'(\gamma, z)$$

11.12.13.0002.01

$$W_z(PS_{v,\mu}'(\gamma, z), QS_{v,\mu}'(\gamma, z)) = \left(-\frac{\gamma^2}{1 - z^2} + \frac{\mu^2}{(1 - z^2)^3} - \frac{\lambda_{v,\mu}(\gamma)}{(1 - z^2)^2} \right) (PS_{v,\mu}'(\gamma, 0) QS_{v,\mu}(\gamma, 0) - PS_{v,\mu}(\gamma, 0) QS_{v,\mu}'(\gamma, 0))$$

11.12.13.0003.01

$$(1 - g(z)^2) w''(z) + \left((g(z)^2 - 1) \frac{g''(z)}{g'(z)} - 2g(z)g'(z) \right) w'(z) + \left(\frac{(\gamma g(z)^2 + \mu - \gamma)(-\gamma g(z)^2 + \mu + \gamma)g'(z)^2}{g(z)^2 - 1} + \lambda_{v,\mu}(\gamma)g'(z)^2 \right) w(z) = 0 /; w(z) = c_1 PS_{v,\mu}'(\gamma, g(z)) + c_2 QS_{v,\mu}'(\gamma, g(z))$$

11.12.13.0004.01

$$W_z(PS_{v,\mu}'(\gamma, g(z)), QS_{v,\mu}'(\gamma, g(z))) = g'(z) \left(-\frac{\gamma^2}{1 - g(z)^2} - \frac{\lambda_{v,\mu}(\gamma)}{(1 - g(z)^2)^2} + \frac{\mu^2}{(1 - g(z)^2)^3} \right) (PS_{v,\mu}'(\gamma, 0) QS_{v,\mu}(\gamma, 0) - PS_{v,\mu}(\gamma, 0) QS_{v,\mu}'(\gamma, 0))$$

11.12.13.0005.01

$$(1 - g(z)^2) w''(z) + \left((g(z)^2 - 1) \left(\frac{2h'(z)}{h(z)} + \frac{g''(z)}{g'(z)} \right) - 2g(z)g'(z) \right) w'(z) + \left(\frac{(\gamma g(z)^2 + \mu - \gamma)(-\gamma g(z)^2 + \mu + \gamma)g'(z)^2}{g(z)^2 - 1} + \lambda_{v,\mu}(\gamma)g'(z)^2 + \frac{2g(z)h'(z)g'(z)}{h(z)} + \frac{(g(z)^2 - 1)(h(z)h''(z) - 2h'(z)^2)}{h(z)^2} - \frac{(g(z)^2 - 1)h'(z)g''(z)}{h(z)g'(z)} \right) w(z) = 0 /; w(z) = c_1 h(z) PS_{v,\mu}'(\gamma, g(z)) + c_2 h(z) QS_{v,\mu}'(\gamma, g(z))$$

11.12.13.0006.01

$$W_z(h(z) PS_{v,\mu}'(\gamma, g(z)), h(z) QS_{v,\mu}'(\gamma, g(z))) =$$

$$h(z)^2 g'(z) \left(-\frac{\gamma^2}{1-g(z)^2} - \frac{\lambda_{v,\mu}(\gamma)}{(1-g(z))^2} + \frac{\mu^2}{(1-g(z))^3} \right) (PS_{v,\mu}'(\gamma, 0) QS_{v,\mu}(\gamma, 0) - PS_{v,\mu}(\gamma, 0) QS_{v,\mu}'(\gamma, 0))$$

11.12.13.0007.01

$$(1 - a^2 z^{2r}) w''(z) - \frac{a^2 (r - 2s + 1) z^{2r} + r + 2s - 1}{z} w'(z) +$$

$$\left(a^2 r^2 \lambda_{v,\mu}(\gamma) z^{2r-2} + \frac{-a^2 r^2 ((a^2 z^{2r} - 1)^2 \gamma^2 - \mu^2) z^{2r} - s^2 (a^2 z^{2r} - 1)^2 + r s (a^4 z^{4r} - 1)}{z^2 (a^2 z^{2r} - 1)} \right) w(z) = 0 /;$$

$$w(z) = c_1 z^s PS_{v,\mu}'(\gamma, a z^r) + c_2 z^s QS_{v,\mu}'(\gamma, a z^r)$$

11.12.13.0008.01

$$W_z(z^s PS_{v,\mu}'(\gamma, a z^r), z^s QS_{v,\mu}'(\gamma, a z^r)) =$$

$$a r z^{r+2s-1} \left(-\frac{\gamma^2}{1-a^2 z^{2r}} + \frac{\mu^2}{(1-a^2 z^{2r})^3} - \frac{\lambda_{v,\mu}(\gamma)}{(1-a^2 z^{2r})^2} \right) (PS_{v,\mu}'(\gamma, 0) QS_{v,\mu}(\gamma, 0) - PS_{v,\mu}(\gamma, 0) QS_{v,\mu}'(\gamma, 0))$$

11.12.13.0009.01

$$(1 - a^2 r^{2z}) w''(z) + (-a^2 \log(r) r^{2z} + 2 a^2 \log(s) r^{2z} - \log(r) - 2 \log(s)) w'(z) +$$

$$\left(a^2 \log^2(r) \lambda_{v,\mu}(\gamma) r^{2z} + \frac{1}{a^2 r^{2z} - 1} (-a^2 ((a^2 r^{2z} - 1)^2 \gamma^2 - \mu^2) \log^2(r) r^{2z} - (a^2 r^{2z} - 1)^2 \log^2(s) + (a^4 r^{4z} - 1) \log(r) \log(s)) \right)$$

$$w(z) = 0 /; w(z) = c_1 s^z PS_{v,\mu}'(\gamma, a r^z) + c_2 s^z QS_{v,\mu}'(\gamma, a r^z)$$

11.12.13.0010.01

$$W_z(s^z PS_{v,\mu}'(\gamma, a r^z), s^z QS_{v,\mu}'(\gamma, a r^z)) =$$

$$a s^{2z} r^z \log(r) \left(-\frac{\gamma^2}{1-a^2 r^{2z}} + \frac{\mu^2}{(1-a^2 r^{2z})^3} - \frac{\lambda_{v,\mu}(\gamma)}{(1-a^2 r^{2z})^2} \right) (PS_{v,\mu}'(\gamma, 0) QS_{v,\mu}(\gamma, 0) - PS_{v,\mu}(\gamma, 0) QS_{v,\mu}'(\gamma, 0))$$

Differentiation

Low-order differentiation

With respect to z

11.12.20.0001.01

$$\frac{\partial PS_{v,\mu}'(\gamma, z)}{\partial z} = \frac{1}{1-z^2} \left(2z PS_{v,\mu}'(\gamma, z) - \left((1-z^2) \gamma^2 + \lambda_{v,\mu}(\gamma) - \frac{\mu^2}{1-z^2} \right) PS_{v,\mu}(\gamma, z) \right)$$

11.12.20.0002.01

$$\frac{\partial^2 PS_{v,\mu}'(\gamma, z)}{\partial z^2} = \frac{1}{(z^2-1)^3} \left(2z \left((z^2-1)^2 \gamma^2 - 3\mu^2 - 2(z^2-1) \lambda_{v,\mu}(\gamma) \right) PS_{v,\mu}(\gamma, z) - \right.$$

$$\left. (z^2-1) (\gamma^2 z^4 - 2(\gamma^2+3) z^2 + \gamma^2 - \mu^2 - (z^2-1) \lambda_{v,\mu}(\gamma) - 2) PS_{v,\mu}'(\gamma, z) \right)$$

Integration

Indefinite integration

Involving only one direct function

11.12.21.0001.01

$$\int PS_{\nu,\mu}'(\gamma, z) dz = PS_{\nu,\mu}(\gamma, z)$$

Operations

Representations through equivalent functions

Theorems

History

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