SpheroidalPSPrime

Notations

Traditional name

Derivative of the angular spheroidal function of the first kind

Traditional notation

$PS_{\nu,\mu}'(\gamma, z)$

Mathematica StandardForm notation

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SpheroidalPSPrime[ν, μ, γ, z]
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Primary definition

11.12.02.0001.01

$$PS_{\nu,\mu}'(\gamma, z) = \frac{\partial PS_{\nu,\mu}(\gamma, z)}{\partial z}$$

Specific values

Specialized values

For fixed ν, μ, z

11.12.03.0001.01

$$PS_{\nu,\mu}'(0, z) = \frac{-z\,(\nu+1)\,P_\nu^\mu(z) + (-\mu+\nu+1)\,P_{\nu+1}^\mu(z)}{z^2-1}$$

For fixed ν, γ, z

11.12.03.0002.01

$$PS_{\nu,\frac{1}{2}}'(\gamma, z) = \frac{1}{\sqrt{2\pi}\,(1-z^2)^{5/4}}\left(z\,\mathrm{Ce}\left(a_{\nu+\frac{1}{2}}\left(\frac{\gamma^2}{4}\right), \frac{\gamma^2}{4}, \cos^{-1}(z)\right) - 2\sqrt{1-z^2}\;\mathrm{Ce}_{\cos^{-1}(z)}\left(a_{\nu+\frac{1}{2}}\left(\frac{\gamma^2}{4}\right), \frac{\gamma^2}{4}, \cos^{-1}(z)\right)\right)$$

For fixed ν, z

11.12.03.0003.01

$$PS_{\nu,0}'(0, z) = \frac{(\nu+1)\,P_{\nu+1}(z) - z\,(\nu+1)\,P_\nu(z)}{z^2-1}$$

11.12.03.0004.01

$$PS_{\nu,\frac{1}{2}}{}'(0, z) = \frac{z\cos\left(\left(\nu+\frac{1}{2}\right)\cos^{-1}(z)\right) + \sqrt{1-z^2}\,(2\nu+1)\sin\left(\left(\nu+\frac{1}{2}\right)\cos^{-1}(z)\right)}{\sqrt{2\pi}\,\left(1-z^2\right)^{5/4}}$$

General characteristics

Domain and analyticity

$PS_{\nu,\mu}{}'(\gamma, z)$ is an analytical function of ν, μ, γ, z which is defined in $\mathbb{C}^4$.

11.12.04.0001.01

$$(\nu * \mu * \gamma * z) \longrightarrow PS_{\nu,\mu}{}'(\gamma, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$PS_{\nu,\mu}{}'(\gamma, z)$ is an even function with respect to γ.

11.12.04.0002.01

$$PS_{\nu,\mu}{}'(-\gamma, z) = PS_{\nu,\mu}{}'(\gamma, z)$$

11.12.04.0003.01

$$PS_{-\nu,\mu}{}'(\gamma, z) = PS_{\nu-1,\mu}{}'(\gamma, z)$$

Mirror symmetry

11.12.04.0004.01

$$PS_{\overline{\nu},\overline{\mu}}{}'(\overline{\gamma}, \overline{z}) == \overline{PS_{\nu,\mu}{}'(\gamma, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at $\gamma == 0$

11.12.06.0001.01

$$PS_{\nu,\mu}'(\gamma, z) \propto \frac{(-\mu+\nu+1) P_{\nu+1}^{\mu}(z) - z(\nu+1) P_{\nu}^{\mu}(z)}{z^2-1} + \frac{1}{2(z^2-1)(2\nu+1)} \left(\frac{(\mu+\nu-1)(\mu+\nu)\left(z(\nu-1) P_{\nu-2}^{\mu}(z) + (\mu-\nu+1) P_{\nu-1}^{\mu}(z)\right)}{(1-2\nu)^2} + \frac{(-\mu+\nu+1)(-\mu+\nu+2)\left((-\mu+\nu+3) P_{\nu+3}^{\mu}(z) - z(\nu+3) P_{\nu+2}^{\mu}(z)\right)}{(2\nu+3)^2} \right) \gamma^2 - \frac{1}{8(z^2-1)} \left(\frac{(\mu+\nu-3)(\mu+\nu-2)(\mu+\nu-1)(\mu+\nu)\left(z(\nu-3) P_{\nu-4}^{\mu}(z) + (\mu-\nu+3) P_{\nu-3}^{\mu}(z)\right)}{\left(4\nu^2-8\nu-5\right)\left(4\nu^2-8\nu+3\right)^2} + \frac{8\left(4\mu^2-1\right)\left(\mu^2+(2\nu-1)\mu+(\nu-1)\nu\right)\left(z(\nu-1) P_{\nu-2}^{\mu}(z) + (\mu-\nu+1) P_{\nu-1}^{\mu}(z)\right)}{(1-2\nu)^4\left(8\nu^3-4\nu^2-34\nu-15\right)} + \frac{1}{2\nu+1} \left(\frac{(\mu-\nu)(\mu-\nu+1)(\mu+\nu-1)(\mu+\nu)}{(1-2\nu)^4(2\nu-3)} + \frac{(-\mu+\nu+1)(-\mu+\nu+2)(\mu+\nu+1)(\mu+\nu+2)}{(2\nu+3)^4(2\nu+5)} \right) \left((-\mu+\nu+1) P_{\nu+1}^{\mu}(z) - z(\nu+1) P_{\nu}^{\mu}(z)\right) + \frac{(\mu-\nu-4)(\mu-\nu-3)(\mu-\nu-2)(\mu-\nu-1)\left(z(\nu+5) P_{\nu+4}^{\mu}(z) + (\mu-\nu-5) P_{\nu+5}^{\mu}(z)\right)}{\left(4\nu^2+16\nu+7\right)\left(4\nu^2+16\nu+15\right)^2} - \frac{8\left(4\mu^2-1\right)\left(\mu^2-(2\nu+3)\mu+\nu^2+3\nu+2\right)\left(z(\nu+3) P_{\nu+2}^{\mu}(z) + (\mu-\nu-3) P_{\nu+3}^{\mu}(z)\right)}{(2\nu+3)^4\left(8\nu^3+28\nu^2-2\nu-7\right)} \right) \gamma^4 + \ldots /; (\gamma \to 0)$$

11.12.06.0002.01

$$PS_{\nu,\mu}'(\gamma, z) \propto \frac{(-\mu+\nu+1) P_{\nu+1}^{\mu}(z) - z(\nu+1) P_{\nu}^{\mu}(z)}{z^2-1} \left(1 + O\left(\gamma^2\right)\right)$$

Expansions at generic point $z == z_0$

11.12.06.0003.01

$$PS_{\nu,\mu}'(\gamma, z) \propto PS_{\nu,\mu}'(\gamma, z_0) + \frac{1}{1-z_0^2} \left(2 PS_{\nu,\mu}'(\gamma, z_0) z_0 + PS_{\nu,\mu}(\gamma, z_0) \left(\left(z_0^2-1\right)\gamma^2 - \lambda_{\nu,\mu}(\gamma) + \frac{\mu^2}{1-z_0^2} \right) \right) (z-z_0) - \frac{1}{2\left(z_0^2-1\right)^3} \left(PS_{\nu,\mu}'(\gamma, z_0) \left(z_0^2-1\right) \left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right) z_0^2 + \gamma^2 - \mu^2 - \lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 2\right) - 2 PS_{\nu,\mu}(\gamma, z_0) z_0 \left(-3\mu^2 + \gamma^2\left(z_0^2-1\right)^2 - 2\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right)\right) \right) (z-z_0)^2 + \frac{1}{6\left(z_0^2-1\right)^4} \left(4 PS_{\nu,\mu}'(\gamma, z_0) z_0 \left(z_0^2-1\right) \left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right) z_0^2 + \gamma^2 - 3\mu^2 - 2\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 6\right) + PS_{\nu,\mu}(\gamma, z_0) \left(\gamma^4 \left(z_0^2-1\right)^4 + \lambda_{\nu,\mu}(\gamma)^2 \left(z_0^2-1\right)^2 - 2\gamma^2\left(\mu^2+4z_0^2+2\right)\left(z_0^2-1\right)^2 - 2\lambda_{\nu,\mu}(\gamma)\left(\gamma^2 z_0^4 - \left(2\gamma^2+9\right) z_0^2 + \gamma^2 - \mu^2 - 3\right)\left(z_0^2-1\right) + \mu^2\left(\mu^2+36z_0^2+8\right) \right) \right) (z-z_0)^3 + \ldots /; (z \to z_0)$$

11.12.06.0004.01

$$PS_{\nu,\mu}'(\gamma, z) \propto PS_{\nu,\mu}'(\gamma, z_0) (1 + O(z-z_0))$$

Expansions at $z == 0$

11.12.06.0005.01

$$PS_{\nu,\mu}'(\gamma, z) \propto PS_{\nu,\mu}'(\gamma, 0) - \left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)\right) PS_{\nu,\mu}(\gamma, 0)\, z - \frac{1}{2} \left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma) - 2\right) PS_{\nu,\mu}'(\gamma, 0)\, z^2 + \frac{1}{6} \left(\gamma^4 - 2\left(\mu^2 + 2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma)^2 + \mu^2\left(\mu^2 + 8\right) + 2\left(\gamma^2 - \mu^2 - 3\right)\lambda_{\nu,\mu}(\gamma)\right) PS_{\nu,\mu}(\gamma, 0)\, z^3 + \ldots \;/;\; (z \to 0)$$

11.12.06.0006.01

$$PS_{\nu,\mu}'(\gamma, z) \propto PS_{\nu,\mu}'(\gamma, 0)\,(1 + O(z))$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.12.13.0001.01

$$\left(1 - z^2\right) w''(z) - 2\, z \left(\frac{\left(1 - z^2\right)^2 \gamma^2 + \mu^2}{\mu^2 - \left(1 - z^2\right)\left(\left(1 - z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma)\right)} + 2\right) w'(z) + \left(\frac{4\left(\left(1 - z^2\right)^2 \gamma^2 + \mu^2\right) z^2}{\left(1 - z^2\right)\left(-\left(1 - z^2\right)^2 \gamma^2 + \mu^2 - \left(1 - z^2\right)\lambda_{\nu,\mu}(\gamma)\right)} + \left(1 - z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1 - z^2} - 2\right) w(z) = 0 \;/;\; w(z) = c_1\, PS_{\nu,\mu}'(\gamma, z) + c_2\, QS_{\nu,\mu}'(\gamma, z)$$

11.12.13.0002.01

$$W_z\left(PS_{\nu,\mu}'(\gamma, z), QS_{\nu,\mu}'(\gamma, z)\right) = \left(-\frac{\gamma^2}{1 - z^2} + \frac{\mu^2}{\left(1 - z^2\right)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1 - z^2\right)^2}\right)\left(PS_{\nu,\mu}'(\gamma, 0)\, QS_{\nu,\mu}(\gamma, 0) - PS_{\nu,\mu}(\gamma, 0)\, QS_{\nu,\mu}'(\gamma, 0)\right)$$

11.12.13.0003.01

$$\left(1 - g(z)^2\right) w''(z) + \left(\left(g(z)^2 - 1\right)\frac{g''(z)}{g'(z)} - 2\, g(z)\, g'(z)\right) w'(z) + \left(\frac{\left(\gamma\, g(z)^2 + \mu - \gamma\right)\left(-\gamma\, g(z)^2 + \mu + \gamma\right) g'(z)^2}{g(z)^2 - 1} + \lambda_{\nu,\mu}(\gamma)\, g'(z)^2\right) w(z) = 0 \;/;\; w(z) = c_1\, PS_{\nu,\mu}'(\gamma, g(z)) + c_2\, QS_{\nu,\mu}'(\gamma, g(z))$$

11.12.13.0004.01

$$W_z\left(PS_{\nu,\mu}'(\gamma, g(z)), QS_{\nu,\mu}'(\gamma, g(z))\right) = g'(z)\left(-\frac{\gamma^2}{1 - g(z)^2} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1 - g(z)^2\right)^2} + \frac{\mu^2}{\left(1 - g(z)^2\right)^3}\right)\left(PS_{\nu,\mu}'(\gamma, 0)\, QS_{\nu,\mu}(\gamma, 0) - PS_{\nu,\mu}(\gamma, 0)\, QS_{\nu,\mu}'(\gamma, 0)\right)$$

11.12.13.0005.01

$$\left(1 - g(z)^2\right) w''(z) + \left(\left(g(z)^2 - 1\right)\left(\frac{2\, h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) - 2\, g(z)\, g'(z)\right) w'(z) + \left(\frac{\left(\gamma\, g(z)^2 + \mu - \gamma\right)\left(-\gamma\, g(z)^2 + \mu + \gamma\right) g'(z)^2}{g(z)^2 - 1} + \lambda_{\nu,\mu}(\gamma)\, g'(z)^2 + \frac{2\, g(z)\, h'(z)\, g'(z)}{h(z)} + \frac{\left(g(z)^2 - 1\right)\left(h(z)\, h''(z) - 2\, h'(z)^2\right)}{h(z)^2} - \frac{\left(g(z)^2 - 1\right) h'(z)\, g''(z)}{h(z)\, g'(z)}\right) w(z) = 0 \;/;\; w(z) = c_1\, h(z)\, PS_{\nu,\mu}'(\gamma, g(z)) + c_2\, h(z)\, QS_{\nu,\mu}'(\gamma, g(z))$$

11.12.13.0006.01

$$W_z\left(h(z)\,PS_{\nu,\mu}'(\gamma, g(z)),\, h(z)\,QS_{\nu,\mu}'(\gamma, g(z))\right) = h(z)^2\, g'(z) \left(-\frac{\gamma^2}{1-g(z)^2} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2} + \frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left(PS_{\nu,\mu}'(\gamma, 0)\, QS_{\nu,\mu}(\gamma, 0) - PS_{\nu,\mu}(\gamma, 0)\, QS_{\nu,\mu}'(\gamma, 0)\right)$$

11.12.13.0007.01

$$\left(1-a^2 z^{2r}\right) w''(z) - \frac{a^2 (r-2s+1) z^{2r} + r + 2s - 1}{z} w'(z) + \left(a^2 r^2 \lambda_{\nu,\mu}(\gamma) z^{2r-2} + \frac{-a^2 r^2 \left(\left(a^2 z^{2r}-1\right)^2 \gamma^2 - \mu^2\right) z^{2r} - s^2 \left(a^2 z^{2r}-1\right)^2 + r s \left(a^4 z^{4r}-1\right)}{z^2 \left(a^2 z^{2r}-1\right)}\right) w(z) = 0 \,/;\, w(z) = c_1 z^s PS_{\nu,\mu}'(\gamma, a z^r) + c_2 z^s QS_{\nu,\mu}'(\gamma, a z^r)$$

11.12.13.0008.01

$$W_z\left(z^s PS_{\nu,\mu}'(\gamma, a z^r),\, z^s QS_{\nu,\mu}'(\gamma, a z^r)\right) = a\, r\, z^{r+2s-1} \left(-\frac{\gamma^2}{1-a^2 z^{2r}} + \frac{\mu^2}{\left(1-a^2 z^{2r}\right)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2 z^{2r}\right)^2}\right)\left(PS_{\nu,\mu}'(\gamma, 0)\, QS_{\nu,\mu}(\gamma, 0) - PS_{\nu,\mu}(\gamma, 0)\, QS_{\nu,\mu}'(\gamma, 0)\right)$$

11.12.13.0009.01

$$\left(1-a^2 r^{2z}\right) w''(z) + \left(-a^2 \log(r)\, r^{2z} + 2 a^2 \log(s)\, r^{2z} - \log(r) - 2\log(s)\right) w'(z) + \left(a^2 \log^2(r)\, \lambda_{\nu,\mu}(\gamma)\, r^{2z} + \frac{1}{a^2 r^{2z}-1}\left(-a^2 \left(\left(a^2 r^{2z}-1\right)^2 \gamma^2 - \mu^2\right) \log^2(r)\, r^{2z} - \left(a^2 r^{2z}-1\right)^2 \log^2(s) + \left(a^4 r^{4z}-1\right) \log(r) \log(s)\right)\right) w(z) = 0 \,/;\, w(z) = c_1 s^z PS_{\nu,\mu}'(\gamma, a r^z) + c_2 s^z QS_{\nu,\mu}'(\gamma, a r^z)$$

11.12.13.0010.01

$$W_z\left(s^z PS_{\nu,\mu}'(\gamma, a r^z),\, s^z QS_{\nu,\mu}'(\gamma, a r^z)\right) = a\, s^{2z} r^z \log(r) \left(-\frac{\gamma^2}{1-a^2 r^{2z}} + \frac{\mu^2}{\left(1-a^2 r^{2z}\right)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2 r^{2z}\right)^2}\right)\left(PS_{\nu,\mu}'(\gamma, 0)\, QS_{\nu,\mu}(\gamma, 0) - PS_{\nu,\mu}(\gamma, 0)\, QS_{\nu,\mu}'(\gamma, 0)\right)$$

Differentiation

Low-order differentiation

With respect to *z*

11.12.20.0001.01

$$\frac{\partial PS_{\nu,\mu}'(\gamma, z)}{\partial z} = \frac{1}{1-z^2}\left(2 z\, PS_{\nu,\mu}'(\gamma, z) - \left(\left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2}\right) PS_{\nu,\mu}(\gamma, z)\right)$$

11.12.20.0002.01

$$\frac{\partial^2 PS_{\nu,\mu}'(\gamma, z)}{\partial z^2} = \frac{1}{\left(z^2-1\right)^3}\left(2 z \left(\left(z^2-1\right)^2 \gamma^2 - 3\mu^2 - 2\left(z^2-1\right) \lambda_{\nu,\mu}(\gamma)\right) PS_{\nu,\mu}(\gamma, z) - \left(z^2-1\right)\left(\gamma^2 z^4 - 2\left(\gamma^2+3\right) z^2 + \gamma^2 - \mu^2 - \left(z^2-1\right)\lambda_{\nu,\mu}(\gamma) - 2\right) PS_{\nu,\mu}'(\gamma, z)\right)$$

Integration

Indefinite integration

Involving only one direct function

11.12.21.0001.01

$$\int PS_{\nu,\mu}'(\gamma, z)\, dz = PS_{\nu,\mu}(\gamma, z)$$

Operations

Representations through equivalent functions

Theorems

History

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