SpheroidalS1Prime

Notations

Traditional name

Derivative of the radial spheroidal function of the first kind

Traditional notation

$S_{\nu,\mu}^{(1)\prime}(\gamma, z)$

Mathematica StandardForm notation

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SpheroidalS1Prime[ν, μ, γ, z]
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Primary definition

11.14.02.0001.01

$$S_{\nu,\mu}^{(1)\prime}(\gamma, z) = \frac{\partial S_{\nu,\mu}^{(1)}(\gamma, z)}{\partial z}$$

Specific values

General characteristics

Domain and analyticity

$S_{\nu,\mu}^{(1)\prime}(\gamma, z)$ is an analytical function of ν, μ, γ, z which is defined in $\mathbb{C}^4$.

11.14.04.0001.01

$$(\nu * \mu * \gamma * z) \longrightarrow S_{\nu,\mu}^{(1)\prime}(\gamma, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$S_{\nu,\mu}^{(1)\prime}(\gamma, z)$ is an even function with respect to μ.

11.14.04.0002.01

$$S_{\nu,-\mu}^{(1)\prime}(\gamma, z) = S_{\nu,\mu}^{(1)\prime}(\gamma, z)$$

Mirror symmetry

11.14.04.0003.01

$$S_{\overline{\nu},\overline{\mu}}^{(1)\prime}(\overline{\gamma}, \overline{z}) = \overline{S_{\nu,\mu}^{(1)\prime}(\gamma, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z = z_0$

11.14.06.0001.01

$$S_{\nu,\mu}^{(1)\prime}(\gamma, z) \propto S_{\nu,\mu}^{(1)\prime}(\gamma, z_0) + \frac{1}{1-z_0^2}\left(2\, S_{\nu,\mu}^{(1)\prime}(\gamma, z_0)\, z_0 + S_{\nu,\mu}^{(1)}(\gamma, z_0)\left(\left(z_0^2-1\right)\gamma^2 - \lambda_{\nu,\mu}(\gamma) + \frac{\mu^2}{1-z_0^2}\right)\right)(z-z_0) - \frac{1}{2\left(z_0^2-1\right)^3}\left(S_{\nu,\mu}^{(1)\prime}(\gamma, z_0)\left(z_0^2-1\right)\left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right)z_0^2 + \gamma^2 - \mu^2 - \lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 2\right) - 2\, S_{\nu,\mu}^{(1)}(\gamma, z_0)\, z_0\left(-3\,\mu^2 + \gamma^2\left(z_0^2-1\right)^2 - 2\,\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right)\right)\right)(z-z_0)^2 + \frac{1}{6\left(z_0^2-1\right)^4}\left(4\, S_{\nu,\mu}^{(1)\prime}(\gamma, z_0)\, z_0\left(z_0^2-1\right)\left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right)z_0^2 + \gamma^2 - 3\,\mu^2 - 2\,\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 6\right) + S_{\nu,\mu}^{(1)}(\gamma, z_0)\left(\gamma^4\left(z_0^2-1\right)^4 + \lambda_{\nu,\mu}(\gamma)^2\left(z_0^2-1\right)^2 - 2\,\gamma^2\left(\mu^2 + 4\, z_0^2 + 2\right)\left(z_0^2-1\right)^2 - 2\,\lambda_{\nu,\mu}(\gamma)\left(\gamma^2 z_0^4 - \left(2\,\gamma^2+9\right)z_0^2 + \gamma^2 - \mu^2 - 3\right)\left(z_0^2-1\right) + \mu^2\left(\mu^2 + 36\, z_0^2 + 8\right)\right)\right)(z-z_0)^3 + \ldots \;/;\; (z \to z_0)$$

11.14.06.0002.01

$$S_{\nu,\mu}^{(1)\prime}(\gamma, z) \propto S_{\nu,\mu}^{(1)\prime}(\gamma, z_0)\,(1 + O(z-z_0))$$

Expansions at $z = 0$

11.14.06.0003.01

$$S_{\nu,\mu}^{(1)\prime}(\gamma, z) \propto S_{\nu,\mu}^{(1)\prime}(\gamma, 0) - \left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)\right) S_{\nu,\mu}^{(1)}(\gamma, 0)\, z - \frac{1}{2}\left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma) - 2\right) S_{\nu,\mu}^{(1)\prime}(\gamma, 0)\, z^2 + \frac{1}{6}\left(2\,\gamma^2 + 2\,\mu^2 + \left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)\right)^2 - 6\left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)\right)\right) S_{\nu,\mu}^{(1)}(\gamma, 0)\, z^3 + \ldots \;/;\; (z \to 0)$$

11.14.06.0004.01

$$S_{\nu,\mu}^{(1)\prime}(\gamma, z) \propto S_{\nu,\mu}^{(1)\prime}(\gamma, 0)\,(1 + O(z))$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.14.13.0001.01

$$\left(1-z^2\right) w''(z) - 2 z \left(\frac{\left(1-z^2\right)^2 \gamma^2 + \mu^2}{\mu^2 - \left(1-z^2\right)\left(\left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma)\right)} + 2\right) w'(z) + \left(\frac{4\left(\left(1-z^2\right)^2 \gamma^2 + \mu^2\right) z^2}{\left(1-z^2\right)\left(-\left(1-z^2\right)^2 \gamma^2 + \mu^2 - \left(1-z^2\right)\lambda_{\nu,\mu}(\gamma)\right)} + \left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2} - 2\right) w(z) = 0 \;/;\; w(z) = c_1\, S_{\nu,\mu}^{(1)\prime}(\gamma, z) + c_2\, S_{\nu,\mu}^{(2)\prime}(\gamma, z)$$

11.14.13.0002.01

$$W_z\left(S_{\nu,\mu}^{(1)\prime}(\gamma, z), S_{\nu,\mu}^{(2)\prime}(\gamma, z)\right) = \left(-\frac{\gamma^2}{1-z^2} + \frac{\mu^2}{\left(1-z^2\right)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-z^2\right)^2}\right)\left(S_{\nu,\mu}^{(1)\prime}(\gamma, 0)\, S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0)\, S_{\nu,\mu}^{(2)\prime}(\gamma, 0)\right)$$

11.14.13.0003.01

$$\left(1-g(z)^2\right) w''(z) + \left(\left(g(z)^2-1\right)\frac{g''(z)}{g'(z)} - 2\, g(z)\, g'(z)\right) w'(z) + \left(\frac{\left(\gamma\, g(z)^2 + \mu - \gamma\right)\left(-\gamma\, g(z)^2 + \mu + \gamma\right) g'(z)^2}{g(z)^2 - 1} + \lambda_{\nu,\mu}(\gamma)\, g'(z)^2\right) w(z) = 0 \;/;\; w(z) = c_1\, S_{\nu,\mu}^{(1)\prime}(\gamma, g(z)) + c_2\, S_{\nu,\mu}^{(2)\prime}(\gamma, g(z))$$

11.14.13.0004.01

$$W_z\left(S_{\nu,\mu}^{(1)\prime}(\gamma, g(z)), S_{\nu,\mu}^{(2)\prime}(\gamma, g(z))\right) = g'(z)\left(-\frac{\gamma^2}{1-g(z)^2} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2} + \frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left(S_{\nu,\mu}^{(1)\prime}(\gamma, 0)\, S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0)\, S_{\nu,\mu}^{(2)\prime}(\gamma, 0)\right)$$

11.14.13.0005.01

$$\left(1-g(z)^2\right) w''(z) + \left(\left(g(z)^2-1\right)\left(\frac{2\, h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) - 2\, g(z)\, g'(z)\right) w'(z) + \left(\frac{\left(\gamma\, g(z)^2 + \mu - \gamma\right)\left(-\gamma\, g(z)^2 + \mu + \gamma\right) g'(z)^2}{g(z)^2 - 1} + \lambda_{\nu,\mu}(\gamma)\, g'(z)^2 + \frac{2\, g(z)\, h'(z)\, g'(z)}{h(z)} + \frac{\left(g(z)^2-1\right)\left(h(z)\, h''(z) - 2\, h'(z)^2\right)}{h(z)^2} - \frac{\left(g(z)^2-1\right) h'(z)\, g''(z)}{h(z)\, g'(z)}\right) w(z) = 0 \;/;\; w(z) = c_1\, h(z)\, S_{\nu,\mu}^{(1)\prime}(\gamma, g(z)) + c_2\, h(z)\, S_{\nu,\mu}^{(2)\prime}(\gamma, g(z))$$

11.14.13.0006.01

$$W_z\left(h(z)\, S_{\nu,\mu}^{(1)\prime}(\gamma, g(z)), h(z)\, S_{\nu,\mu}^{(2)\prime}(\gamma, g(z))\right) = h(z)^2\, g'(z)\left(-\frac{\gamma^2}{1-g(z)^2} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2} + \frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left(S_{\nu,\mu}^{(1)\prime}(\gamma, 0)\, S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0)\, S_{\nu,\mu}^{(2)\prime}(\gamma, 0)\right)$$

11.14.13.0007.01

$$\left(1 - a^2 z^{2r}\right) w''(z) - \frac{a^2 (r - 2s + 1) z^{2r} + r + 2s - 1}{z} w'(z) + \left(a^2 r^2 \lambda_{\nu,\mu}(\gamma)\, z^{2r-2} + \frac{-a^2 r^2 \left(\left(a^2 z^{2r} - 1\right)^2 \gamma^2 - \mu^2\right) z^{2r} - s^2 \left(a^2 z^{2r} - 1\right)^2 + r s \left(a^4 z^{4r} - 1\right)}{z^2 \left(a^2 z^{2r} - 1\right)}\right) w(z) = 0 \;/;\; w(z) = c_1\, z^s\, S_{\nu,\mu}^{(1)\prime}(\gamma, a z^r) + c_2\, z^s\, S_{\nu,\mu}^{(2)\prime}(\gamma, a z^r)$$

11.14.13.0008.01

$$W_z\left(z^s\, {S_{\nu,\mu}^{(1)}}'(\gamma,\, a\, z^r),\, z^s\, {S_{\nu,\mu}^{(2)}}'(\gamma,\, a\, z^r)\right) = a\, r\, z^{r+2s-1} \left(-\frac{\gamma^2}{1-a^2\, z^{2r}} + \frac{\mu^2}{\left(1-a^2\, z^{2r}\right)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2\, z^{2r}\right)^2}\right) \left({S_{\nu,\mu}^{(1)}}'(\gamma,\, 0)\, S_{\nu,\mu}^{(2)}(\gamma,\, 0) - S_{\nu,\mu}^{(1)}(\gamma,\, 0)\, {S_{\nu,\mu}^{(2)}}'(\gamma,\, 0)\right)$$

11.14.13.0009.01

$$\left(1-a^2\, r^{2z}\right) w''(z) + \left(-a^2 \log(r)\, r^{2z} + 2\, a^2 \log(s)\, r^{2z} - \log(r) - 2\log(s)\right) w'(z) + \left(a^2 \log^2(r)\, \lambda_{\nu,\mu}(\gamma)\, r^{2z} + \frac{1}{a^2\, r^{2z}-1} \left(-a^2 \left(\left(a^2\, r^{2z}-1\right)^2 \gamma^2 - \mu^2\right) \log^2(r)\, r^{2z} - \left(a^2\, r^{2z}-1\right)^2 \log^2(s) + \left(a^4\, r^{4z}-1\right) \log(r) \log(s)\right)\right) w(z) = 0 \;/;\; w(z) = c_1\, s^z\, {S_{\nu,\mu}^{(1)}}'(\gamma,\, a\, r^z) + c_2\, s^z\, {S_{\nu,\mu}^{(2)}}'(\gamma,\, a\, r^z)$$

11.14.13.0010.01

$$W_z\left(s^z\, {S_{\nu,\mu}^{(1)}}'(\gamma,\, a\, r^z),\, s^z\, {S_{\nu,\mu}^{(2)}}'(\gamma,\, a\, r^z)\right) = a\, s^{2z}\, r^z \log(r) \left(-\frac{\gamma^2}{1-a^2\, r^{2z}} + \frac{\mu^2}{\left(1-a^2\, r^{2z}\right)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2\, r^{2z}\right)^2}\right) \left({S_{\nu,\mu}^{(1)}}'(\gamma,\, 0)\, S_{\nu,\mu}^{(2)}(\gamma,\, 0) - S_{\nu,\mu}^{(1)}(\gamma,\, 0)\, {S_{\nu,\mu}^{(2)}}'(\gamma,\, 0)\right)$$

Differentiation

Low-order differentiation

With respect to z

11.14.20.0001.01

$$\frac{\partial {S_{\nu,\mu}^{(1)}}'(\gamma,\, z)}{\partial z} = \frac{1}{1-z^2} \left(2\, z\, {S_{\nu,\mu}^{(1)}}'(\gamma,\, z) - \left(\left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2}\right) S_{\nu,\mu}^{(1)}(\gamma,\, z)\right)$$

11.14.20.0002.01

$$\frac{\partial^2 {S_{\nu,\mu}^{(1)}}'(\gamma,\, z)}{\partial z^2} = \frac{1}{\left(z^2-1\right)^3} \left(2\, z \left(\left(z^2-1\right)^2 \gamma^2 - 3\, \mu^2 - 2\left(z^2-1\right) \lambda_{\nu,\mu}(\gamma)\right) S_{\nu,\mu}^{(1)}(\gamma,\, z) - \left(z^2-1\right)\left(\gamma^2\, z^4 - 2\left(\gamma^2+3\right) z^2 + \gamma^2 - \mu^2 - \left(z^2-1\right) \lambda_{\nu,\mu}(\gamma) - 2\right) {S_{\nu,\mu}^{(1)}}'(\gamma,\, z)\right)$$

Integration

Indefinite integration

Involving only one direct function

11.14.21.0001.01

$$\int {S_{\nu,\mu}^{(1)}}'(\gamma,\, z)\, dz = S_{\nu,\mu}^{(1)}(\gamma,\, z)$$

Operations

Representations through equivalent functions

Theorems

History

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