

# SpheroidalS1Prime

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## Notations

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### Traditional name

Derivative of the radial spheroidal function of the first kind

### Traditional notation

$$S_{\nu,\mu}^{(1)'}(\gamma, z)$$

### Mathematica StandardForm notation

SpheroidalS1Prime[ $\nu$ ,  $\mu$ ,  $\gamma$ ,  $z$ ]

## Primary definition

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11.14.02.0001.01

$$S_{\nu,\mu}^{(1)'}(\gamma, z) = \frac{\partial S_{\nu,\mu}^{(1)}(\gamma, z)}{\partial z}$$

## Specific values

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### General characteristics

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#### Domain and analyticity

$S_{\nu,\mu}^{(1)'}(\gamma, z)$  is an analytical function of  $\nu, \mu, \gamma, z$  which is defined in  $\mathbb{C}^4$ .

11.14.04.0001.01

$$(\nu * \mu * \gamma * z) \rightarrow S_{\nu,\mu}^{(1)'}(\gamma, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Parity

$S_{\nu,\mu}^{(1)'}(\gamma, z)$  is an even function with respect to  $\mu$ .

11.14.04.0002.01

$$S_{\nu,-\mu}^{(1)'}(\gamma, z) = S_{\nu,\mu}^{(1)'}(\gamma, z)$$

#### Mirror symmetry

11.14.04.0003.01

$$S_{\nu,\mu}^{(1)'}(\bar{\gamma}, \bar{z}) = S_{\nu,\mu}^{(1)'}(\gamma, z)$$

**Periodicity**

No periodicity

**Series representations**

**Generalized power series**

**Expansions at generic point  $z = z_0$**

11.14.06.0001.01

$$S_{\nu,\mu}^{(1)'}(\gamma, z) \propto S_{\nu,\mu}^{(1)'}(\gamma, z_0) + \frac{1}{1-z_0^2} \left( 2 S_{\nu,\mu}^{(1)'}(\gamma, z_0) z_0 + S_{\nu,\mu}^{(1)}(\gamma, z_0) \left( (z_0^2 - 1) \gamma^2 - \lambda_{\nu,\mu}(\gamma) + \frac{\mu^2}{1-z_0^2} \right) \right) (z - z_0) - \frac{1}{2(z_0^2 - 1)^3} \left( S_{\nu,\mu}^{(1)'}(\gamma, z_0) (z_0^2 - 1) (\gamma^2 z_0^4 - 2(\gamma^2 + 3) z_0^2 + \gamma^2 - \mu^2 - \lambda_{\nu,\mu}(\gamma) (z_0^2 - 1) - 2) - 2 S_{\nu,\mu}^{(1)}(\gamma, z_0) z_0 (-3\mu^2 + \gamma^2 (z_0^2 - 1)^2 - 2\lambda_{\nu,\mu}(\gamma) (z_0^2 - 1)) \right) (z - z_0)^2 + \frac{1}{6(z_0^2 - 1)^4} \left( 4 S_{\nu,\mu}^{(1)'}(\gamma, z_0) z_0 (z_0^2 - 1) (\gamma^2 z_0^4 - 2(\gamma^2 + 3) z_0^2 + \gamma^2 - 3\mu^2 - 2\lambda_{\nu,\mu}(\gamma) (z_0^2 - 1) - 6) + S_{\nu,\mu}^{(1)}(\gamma, z_0) (\gamma^4 (z_0^2 - 1)^4 + \lambda_{\nu,\mu}(\gamma)^2 (z_0^2 - 1)^2 - 2\gamma^2 (\mu^2 + 4z_0^2 + 2) (z_0^2 - 1)^2 - 2\lambda_{\nu,\mu}(\gamma) (\gamma^2 z_0^4 - (2\gamma^2 + 9) z_0^2 + \gamma^2 - \mu^2 - 3) (z_0^2 - 1) + \mu^2 (\mu^2 + 36z_0^2 + 8)) \right) (z - z_0)^3 + \dots /; (z \rightarrow z_0)$$

11.14.06.0002.01

$$S_{\nu,\mu}^{(1)'}(\gamma, z) \propto S_{\nu,\mu}^{(1)'}(\gamma, z_0) (1 + O(z - z_0))$$

**Expansions at  $z = 0$**

11.14.06.0003.01

$$S_{\nu,\mu}^{(1)'}(\gamma, z) \propto S_{\nu,\mu}^{(1)'}(\gamma, 0) - (\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)) S_{\nu,\mu}^{(1)}(\gamma, 0) z - \frac{1}{2} (\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma) - 2) S_{\nu,\mu}^{(1)'}(\gamma, 0) z^2 + \frac{1}{6} (2\gamma^2 + 2\mu^2 + (\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma))^2 - 6(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma))) S_{\nu,\mu}^{(1)}(\gamma, 0) z^3 + \dots /; (z \rightarrow 0)$$

11.14.06.0004.01

$$S_{\nu,\mu}^{(1)'}(\gamma, z) \propto S_{\nu,\mu}^{(1)'}(\gamma, 0) (1 + O(z))$$

**Differential equations**

**Ordinary linear differential equations and wronskians**

**For the direct function itself**

11.14.13.0001.01

$$(1 - z^2) w''(z) - 2z \left( \frac{(1 - z^2)^2 \gamma^2 + \mu^2}{\mu^2 - (1 - z^2)((1 - z^2)\gamma^2 + \lambda_{\nu,\mu}(\gamma))} + 2 \right) w'(z) + \left( \frac{4((1 - z^2)^2 \gamma^2 + \mu^2) z^2}{(1 - z^2)(-(1 - z^2)^2 \gamma^2 + \mu^2 - (1 - z^2)\lambda_{\nu,\mu}(\gamma))} + (1 - z^2)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1 - z^2} - 2 \right) w(z) = 0 /; w(z) = c_1 S_{\nu,\mu}^{(1)'}(\gamma, z) + c_2 S_{\nu,\mu}^{(2)'}(\gamma, z)$$

11.14.13.0002.01

$$W_z(S_{\nu,\mu}^{(1)'}(\gamma, z), S_{\nu,\mu}^{(2)'}(\gamma, z)) = \left( -\frac{\gamma^2}{1 - z^2} + \frac{\mu^2}{(1 - z^2)^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{(1 - z^2)^2} \right) (S_{\nu,\mu}^{(1)'}(\gamma, 0) S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0))$$

11.14.13.0003.01

$$(1 - g(z)^2) w''(z) + \left( (g(z)^2 - 1) \frac{g''(z)}{g'(z)} - 2g(z)g'(z) \right) w'(z) + \left( \frac{(\gamma g(z)^2 + \mu - \gamma)(-\gamma g(z)^2 + \mu + \gamma)g'(z)^2}{g(z)^2 - 1} + \lambda_{\nu,\mu}(\gamma)g'(z)^2 \right) w(z) = 0 /; w(z) = c_1 S_{\nu,\mu}^{(1)'}(\gamma, g(z)) + c_2 S_{\nu,\mu}^{(2)'}(\gamma, g(z))$$

11.14.13.0004.01

$$W_z(S_{\nu,\mu}^{(1)'}(\gamma, g(z)), S_{\nu,\mu}^{(2)'}(\gamma, g(z))) = g'(z) \left( -\frac{\gamma^2}{1 - g(z)^2} - \frac{\lambda_{\nu,\mu}(\gamma)}{(1 - g(z)^2)^2} + \frac{\mu^2}{(1 - g(z)^2)^3} \right) (S_{\nu,\mu}^{(1)'}(\gamma, 0) S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0))$$

11.14.13.0005.01

$$(1 - g(z)^2) w''(z) + \left( (g(z)^2 - 1) \left( \frac{2h'(z)}{h(z)} + \frac{g''(z)}{g'(z)} \right) - 2g(z)g'(z) \right) w'(z) + \left( \frac{(\gamma g(z)^2 + \mu - \gamma)(-\gamma g(z)^2 + \mu + \gamma)g'(z)^2}{g(z)^2 - 1} + \lambda_{\nu,\mu}(\gamma)g'(z)^2 + \frac{2g(z)h'(z)g'(z)}{h(z)} + \frac{(g(z)^2 - 1)(h(z)h''(z) - 2h'(z)^2)}{h(z)^2} - \frac{(g(z)^2 - 1)h'(z)g''(z)}{h(z)g'(z)} \right) w(z) = 0 /; w(z) = c_1 h(z) S_{\nu,\mu}^{(1)'}(\gamma, g(z)) + c_2 h(z) S_{\nu,\mu}^{(2)'}(\gamma, g(z))$$

11.14.13.0006.01

$$W_z(h(z) S_{\nu,\mu}^{(1)'}(\gamma, g(z)), h(z) S_{\nu,\mu}^{(2)'}(\gamma, g(z))) = h(z)^2 g'(z) \left( -\frac{\gamma^2}{1 - g(z)^2} - \frac{\lambda_{\nu,\mu}(\gamma)}{(1 - g(z)^2)^2} + \frac{\mu^2}{(1 - g(z)^2)^3} \right) (S_{\nu,\mu}^{(1)'}(\gamma, 0) S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0))$$

11.14.13.0007.01

$$(1 - a^2 z^{2r}) w''(z) - \frac{a^2(r - 2s + 1)z^{2r} + r + 2s - 1}{z} w'(z) + \left( a^2 r^2 \lambda_{\nu,\mu}(\gamma) z^{2r-2} + \frac{-a^2 r^2 ((a^2 z^{2r} - 1)^2 \gamma^2 - \mu^2) z^{2r} - s^2 (a^2 z^{2r} - 1)^2 + r s (a^4 z^{4r} - 1)}{z^2 (a^2 z^{2r} - 1)} \right) w(z) = 0 /; w(z) = c_1 z^s S_{\nu,\mu}^{(1)'}(\gamma, a z^r) + c_2 z^s S_{\nu,\mu}^{(2)'}(\gamma, a z^r)$$

11.14.13.0008.01

$$W_z(z^s S_{\nu,\mu}^{(1)'}(\gamma, a z^r), z^s S_{\nu,\mu}^{(2)'}(\gamma, a z^r)) = a r z^{r+2s-1} \left( -\frac{\gamma^2}{1-a^2 z^{2r}} + \frac{\mu^2}{(1-a^2 z^{2r})^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{(1-a^2 z^{2r})^2} \right) (S_{\nu,\mu}^{(1)'}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0))$$

11.14.13.0009.01

$$(1-a^2 r^{2z}) w''(z) + (-a^2 \log(r) r^{2z} + 2 a^2 \log(s) r^{2z} - \log(r) - 2 \log(s)) w'(z) + \left( a^2 \log^2(r) \lambda_{\nu,\mu}(\gamma) r^{2z} + \frac{1}{a^2 r^{2z} - 1} (-a^2 ((a^2 r^{2z} - 1)^2 \gamma^2 - \mu^2) \log^2(r) r^{2z} - (a^2 r^{2z} - 1)^2 \log^2(s) + (a^4 r^{4z} - 1) \log(r) \log(s)) \right) w(z) = 0; w(z) = c_1 s^z S_{\nu,\mu}^{(1)'}(\gamma, a r^z) + c_2 s^z S_{\nu,\mu}^{(2)'}(\gamma, a r^z)$$

11.14.13.0010.01

$$W_z(s^z S_{\nu,\mu}^{(1)'}(\gamma, a r^z), s^z S_{\nu,\mu}^{(2)'}(\gamma, a r^z)) = a s^{2z} r^z \log(r) \left( -\frac{\gamma^2}{1-a^2 r^{2z}} + \frac{\mu^2}{(1-a^2 r^{2z})^3} - \frac{\lambda_{\nu,\mu}(\gamma)}{(1-a^2 r^{2z})^2} \right) (S_{\nu,\mu}^{(1)'}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)'}(\gamma, 0))$$

## Differentiation

### Low-order differentiation

With respect to z

11.14.20.0001.01

$$\frac{\partial S_{\nu,\mu}^{(1)'}(\gamma, z)}{\partial z} = \frac{1}{1-z^2} \left( 2 z S_{\nu,\mu}^{(1)'}(\gamma, z) - \left( (1-z^2) \gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2} \right) S_{\nu,\mu}^{(1)'}(\gamma, z) \right)$$

11.14.20.0002.01

$$\frac{\partial^2 S_{\nu,\mu}^{(1)'}(\gamma, z)}{\partial z^2} = \frac{1}{(z^2-1)^3} \left( 2 z \left( (z^2-1)^2 \gamma^2 - 3 \mu^2 - 2 (z^2-1) \lambda_{\nu,\mu}(\gamma) \right) S_{\nu,\mu}^{(1)'}(\gamma, z) - (z^2-1) (\gamma^2 z^4 - 2 (\gamma^2 + 3) z^2 + \gamma^2 - \mu^2 - (z^2-1) \lambda_{\nu,\mu}(\gamma) - 2) S_{\nu,\mu}^{(1)'}(\gamma, z) \right)$$

## Integration

### Indefinite integration

Involving only one direct function

11.14.21.0001.01

$$\int S_{\nu,\mu}^{(1)'}(\gamma, z) dz = S_{\nu,\mu}^{(1)}(\gamma, z)$$

## Operations

### Representations through equivalent functions

### Theorems

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## History

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