

StieltjesGamma

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Notations

Traditional name

Stieltjes constant

Traditional notation

γ_n

Mathematica StandardForm notation

StieltjesGamma[n]

Primary definition

10.05.02.0001.01

$$\gamma_n = (-1)^n n! \left([(s-1)^n] \left(\zeta(s) - \frac{1}{s-1} \right) \right) /; n \in \mathbb{N}$$

Specific values

Values at fixed points

10.05.03.0001.01

$$\gamma_0 = \gamma$$

10.05.03.0002.01

$$\gamma_n = (-1)^n [(s-1)^n] \zeta(s) /; n \in \mathbb{N}$$

General characteristics

Domain and analyticity

γ_n is a nonanalytical function which is defined only for nonnegative integer n .

10.05.04.0001.01

$$n \rightarrow \gamma_n :: \mathbb{Z} \rightarrow \mathbb{R}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Other series representations

10.05.06.0001.01

$$\gamma_n = \frac{\log^n(2)}{n+1} \sum_{k=1}^{\infty} \frac{(-1)^k}{k} B_{n+1}\left(\frac{\log(k)}{\log(2)}\right)$$

10.05.06.0002.01

$$\gamma_n = n! \log^n(2) \sum_{m=1}^{n+1} \frac{(-1)^{m-1}}{m!} \sum_{k=1}^{\infty} \frac{(-1)^k [\log_2(k)]^m}{k} B_{n-m+1}\left(\frac{\log(k)}{\log(2)}\right)$$

10.05.06.0003.01

$$\gamma_1 = \frac{\log(2)}{2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k} (2 \log_2(k) - \lfloor \log_2(2k) \rfloor) \lfloor \log_2(k) \rfloor$$

10.05.06.0004.01

$$\gamma_n = (-1)^n n! \sum_{k_0=1}^{n+1} \sum_{k_1=1}^{n+1} \dots \sum_{k_{n+1}=1}^{n+1} \delta_{n+1, \sum_{j=0}^{n+1} (j+1)k_j} \prod_{j=0}^{n+1} \frac{\left(-\frac{\eta_j}{j+1}\right)^{k_j}}{k_j!} ; n \in \mathbb{N}^+ \wedge \eta_n = [s^n] \left(-\frac{\zeta'(s+1)}{\zeta(s+1)} \right)$$

(StieltjesGamma[n_] /; n ∈ Integers ∧ n ≥ 0) :>

```
Module[{k}, Expand[(-1)^n (n)!
Sum[KroneckerDelta[n+1, Sum[(1+j) k_j, j=0, n+1]]]
Product[1/k_j!, j=0, n+1] (-eta_j/(j+1))^k_j,
Evaluate[Sequence@@Table[{k_j, 0, n+1}, {j, 0, n+1}]]]] /.
eta_k_ :> SeriesTerm[-Zeta'[1+s]/Zeta[1+s], {s, 0, k}] ]
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K. Maslanka: An Explicit Formula Relating Stieltjes Numbers and Li's Numbers math.NT/0406312 ({}[[1,1]]) {}[[1,1]]

Integral representations

Contour integral representations

10.05.07.0001.01

$$\gamma_n = \frac{(-1)^n n!}{2\pi i} \int_{|s-1|=1} \frac{1}{(s-1)^{n+1}} \left(\zeta(s) - \frac{1}{s-1} \right) ds \quad ; n \in \mathbb{N}$$

Limit representations

10.05.09.0001.01

$$\gamma_n = \lim_{m \rightarrow \infty} \left(\sum_{k=1}^m \frac{\log^n(k)}{k} - \frac{\log^{n+1}(m)}{n+1} \right)$$

10.05.09.0002.01

$$\gamma_n = \lim_{m \rightarrow \infty} \left(\sum_{k=1}^m \frac{\log^n(k)}{k} - \int_1^m \frac{\log^n(t)}{t} dt \right)$$

Generating functions

10.05.11.0001.01

$$\eta_n = (-1)^n n! \left([z^n] \left(\zeta(s) - \frac{1}{s-1} \right) \right) \quad ; n \in \mathbb{N}$$

Identities

Relation to Li's numbers

10.05.17.0001.01

$$\eta_n = (n+1) \sum_{k_0=1}^{n+1} \sum_{k_1=1}^{n+1} \dots \sum_{k_{n+1}=1}^{n+1} \left(\Gamma \left(\sum_{j=0}^{n+1} k_j \right) \delta_{n+1, \sum_{j=0}^{n+1} (j+1)k_j} \right) \prod_{j=0}^{n+1} \frac{\left(\frac{(-1)^{k_j} \gamma_j}{j!} \right)^{k_j}}{k_j!} \quad ; n \in \mathbb{Z} \wedge v \geq 0 \wedge \eta_n = [s^n] \left(-\frac{\zeta'(s+1)}{\zeta(s+1)} \right)$$

(SeriesTerm[-Zeta'[1+s_]/Zeta[1+s_], {s_, 0, n_}] / ;
n ∈ Integers ∧ n ≥ 0) :>

Module[{k}, Expand[(n+1) Sum[If[n+1 == Sum[j=0 to n+1] (1+j) k_j, Gamma[Sum[j=0 to n+1] k_j], 0]

$$\prod_{j=0}^{n+1} \frac{\left(-(-1)^{k_j} \text{StieltjesGamma}[j] / j! \right)^{k_j}}{k_j!},$$

Evaluate[Sequence@@Table[{k_j, 0, n+1}, {j, 0, n+1}]]]

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{[[1,1]]}

Inequalities

$$|\gamma_n| < \frac{2(n-1)!}{\pi^n}$$

Theorems

Riemann hypothesis

The Riemann hypothesis is equivalent to (Li 1997) $\lambda_n > 0$ for all integer $n > 0$. The

$$\lambda_n = \frac{1}{(n-1)!} \frac{\partial^n}{\partial s^n} (s^{n-1} \log(\zeta(s))) \Big|_{s=1}$$

contain derivatives of the Riemann zeta function at $s = 1$ and can be expressed through Stieltjes constants.

History

–E. Cahen (1894)

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