Subfactorial

Notations

Traditional name

Subfactorial

Traditional notation

$\text{Subfactorial}(n)$

Mathematica StandardForm notation

`Subfactorial[n]`

Primary definition

06.42.02.0001.01

$$\text{Subfactorial}(n) = \frac{\Gamma(z+1, -1)}{e}$$

The numbers $\text{Subfactorial}(n)$ represent the number of complete permutations (i.e., derangements) of n objects (that is the number of a permutation in which no object appears in its "natural" (i.e., ordered) place).

Examples: Subfactorial(3) == 2 because the only derangements of {1, 2, 3} are {2, 3, 1} and {3, 1, 2}.
Subfactorial(4) == 9 because the only derangements of {1, 2, 3, 4} are {2, 1, 4, 3}, {2, 3, 4, 1}, {2, 4, 1, 3}, {3, 1, 4, 2}, {3, 4, 1, 2}, {3, 4, 2, 1}, {4, 1, 2, 3}, {4, 3, 1, 2}, and {4, 3, 2, 1}.

Specific values

Specialized values

06.42.03.0001.01

$$\text{Subfactorial}(n) = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} \;/;\; n \in \mathbb{N}$$

06.42.03.0002.01

$$\text{Subfactorial}(n) = \left\lfloor \frac{n!}{e} \right\rceil \;/;\; n \in \mathbb{N}^+$$

06.42.03.0003.01

$$\text{Subfactorial}(n) = \left\lfloor \frac{n!+1}{e} \right\rfloor \;/;\; n \in \mathbb{N}^+$$

06.42.03.0004.01

$$\text{Subfactorial}(n) = \left\lfloor \left(e + \frac{1}{e}\right) n! \right\rfloor - \lfloor e\, n! \rfloor \;/;\; (n \in \mathbb{Z} \land n \geq 2) \lor n = 0$$

06.42.03.0005.01

$$\text{Subfactorial}(-n) = \frac{(-1)^n}{e\,(n-1)!}\,(i\,\pi + \text{Ei}(1)) - \sum_{k=1}^{n-1} \frac{(-1)^{k-n}}{(1-n)_k} \;/;\; n \in \mathbb{N}^+$$

06.42.03.0006.01

$$\text{Subfactorial}(n) = \frac{(-1)^n}{e\,(-n-1)!}\,(i\,\pi + \text{Ei}(1)) + \sum_{k=0}^{n} \frac{(-1)^k}{(n+1)_{k-n}} - \sum_{k=n+1}^{-1} \frac{(-1)^k}{(n+1)_{k-n}} \;/;\; n \in \mathbb{Z}$$

06.42.03.0007.01

$$\text{Subfactorial}\left(n - \frac{1}{2}\right) = \frac{1}{e}\,\text{erfc}(i)\,\Gamma\left(n + \frac{1}{2}\right) + (-1)^{n-1}\, i \sum_{k=0}^{n-1} \left(\frac{1}{2} - n\right)_{n-k-1} \;/;\; n \in \mathbb{N}$$

06.42.03.0008.01

$$\text{Subfactorial}\left(-n - \frac{1}{2}\right) = \frac{(-1)^n \sqrt{\pi}\;\text{erfc}(i)}{e \left(\frac{1}{2}\right)_n} - i\,(-1)^n \sum_{k=0}^{n-1} \frac{(-1)^k}{\left(\frac{1}{2} - n\right)_{k+1}} \;/;\; n \in \mathbb{N}$$

06.42.03.0009.01

$$\text{Subfactorial}\left(n - \frac{1}{2}\right) = \frac{\text{erfc}(i)\,\Gamma\left(n + \frac{1}{2}\right)}{e} + i \sum_{k=0}^{n-1} \frac{(-1)^k}{\left(n + \frac{1}{2}\right)_{k-n+1}} - i \sum_{k=n}^{-1} \frac{(-1)^k}{\left(n + \frac{1}{2}\right)_{k-n+1}} \;/;\; n \in \mathbb{Z}$$

Values at fixed points

06.42.03.0010.01

$$\text{Subfactorial}(-10) = \frac{i\,\pi + \text{Ei}(1)}{362\,880\,e} - \frac{23\,117}{181\,440}$$

06.42.03.0011.01

$$\text{Subfactorial}\left(-\frac{19}{2}\right) = -\frac{47\,458\,i}{348\,075} - \frac{512\sqrt{\pi}\;\text{erfc}(i)}{34\,459\,425\,e}$$

06.42.03.0012.01

$$\text{Subfactorial}(-9) = \frac{2957}{20\,160} - \frac{i\,\pi + \text{Ei}(1)}{40\,320\,e}$$

06.42.03.0013.01

$$\text{Subfactorial}\left(-\frac{17}{2}\right) = \frac{3254\,i}{20\,475} + \frac{256\sqrt{\pi}\;\text{erfc}(i)}{2\,027\,025\,e}$$

06.42.03.0014.01

$$\text{Subfactorial}(-8) = \frac{i\,\pi + \text{Ei}(1)}{5040\,e} - \frac{437}{2520}$$

06.42.03.0015.01

$$\text{Subfactorial}\left(-\frac{15}{2}\right) = -\frac{262\,i}{1365} - \frac{128\sqrt{\pi}\;\text{erfc}(i)}{135\,135\,e}$$

06.42.03.0016.01

$$\text{Subfactorial}(-7) = \frac{77}{360} - \frac{i\pi + \text{Ei}(1)}{720\, e}$$

06.42.03.0017.01

$$\text{Subfactorial}\left(-\frac{13}{2}\right) = \frac{26\, i}{105} + \frac{64\sqrt{\pi}\ \text{erfc}(i)}{10\,395\, e}$$

06.42.03.0018.01

$$\text{Subfactorial}(-6) = \frac{i\pi + \text{Ei}(1)}{120\, e} - \frac{17}{60}$$

06.42.03.0019.01

$$\text{Subfactorial}\left(-\frac{11}{2}\right) = -\frac{1}{105}(38\, i) - \frac{32\sqrt{\pi}\ \text{erfc}(i)}{945\, e}$$

06.42.03.0020.01

$$\text{Subfactorial}(-5) = \frac{5}{12} - \frac{i\pi + \text{Ei}(1)}{24\, e}$$

06.42.03.0021.01

$$\text{Subfactorial}\left(-\frac{9}{2}\right) = \frac{22\, i}{35} + \frac{16\sqrt{\pi}\ \text{erfc}(i)}{105\, e}$$

06.42.03.0022.01

$$\text{Subfactorial}(-4) = \frac{i\pi + \text{Ei}(1)}{6\, e} - \frac{2}{3}$$

06.42.03.0023.01

$$\text{Subfactorial}\left(-\frac{7}{2}\right) = -\frac{1}{5}(6\, i) - \frac{8\sqrt{\pi}\ \text{erfc}(i)}{15\, e}$$

06.42.03.0024.01

$$\text{Subfactorial}(-3) = 1 - \frac{i\pi + \text{Ei}(1)}{2\, e}$$

06.42.03.0025.01

$$\text{Subfactorial}\left(-\frac{5}{2}\right) = 2\, i + \frac{4\sqrt{\pi}\ \text{erfc}(i)}{3\, e}$$

06.42.03.0026.01

$$\text{Subfactorial}(-2) = \frac{i\pi + \text{Ei}(1)}{e} - 1$$

06.42.03.0027.01

$$\text{Subfactorial}\left(-\frac{3}{2}\right) = -2\, i - \frac{2\sqrt{\pi}\ \text{erfc}(i)}{e}$$

06.42.03.0028.01

$$\text{Subfactorial}(-1) = -\frac{i\pi + \text{Ei}(1)}{e}$$

06.42.03.0029.01

$$\text{Subfactorial}\left(-\frac{1}{2}\right) = \frac{\sqrt{\pi}\ \text{erfc}(i)}{e}$$

06.42.03.0030.01

$$\text{Subfactorial}(0) = 1$$

06.42.03.0031.01

$$\text{Subfactorial}\left(\frac{1}{2}\right) = i + \frac{\sqrt{\pi}\ \text{erfc}(i)}{2\,e}$$

06.42.03.0032.01

$$\text{Subfactorial}(1) = 0$$

06.42.03.0033.01

$$\text{Subfactorial}\left(\frac{3}{2}\right) = \frac{i}{2} + \frac{3\sqrt{\pi}\ \text{erfc}(i)}{4\,e}$$

06.42.03.0034.01

$$\text{Subfactorial}(2) = 1$$

06.42.03.0035.01

$$\text{Subfactorial}\left(\frac{5}{2}\right) = \frac{9\,i}{4} + \frac{15\sqrt{\pi}\ \text{erfc}(i)}{8\,e}$$

06.42.03.0036.01

$$\text{Subfactorial}(3) = 2$$

06.42.03.0037.01

$$\text{Subfactorial}\left(\frac{7}{2}\right) = \frac{55\,i}{8} + \frac{105\sqrt{\pi}\ \text{erfc}(i)}{16\,e}$$

06.42.03.0038.01

$$\text{Subfactorial}(4) = 9$$

06.42.03.0039.01

$$\text{Subfactorial}\left(\frac{9}{2}\right) = \frac{511\,i}{16} + \frac{945\sqrt{\pi}\ \text{erfc}(i)}{32\,e}$$

06.42.03.0040.01

$$\text{Subfactorial}(5) = 44$$

06.42.03.0041.01

$$\text{Subfactorial}\left(\frac{11}{2}\right) = \frac{5589\,i}{32} + \frac{10\,395\sqrt{\pi}\ \text{erfc}(i)}{64\,e}$$

06.42.03.0042.01

$$\text{Subfactorial}(6) = 265$$

06.42.03.0043.01

$$\text{Subfactorial}\left(\frac{13}{2}\right) = \frac{72\,721\,i}{64} + \frac{135\,135\sqrt{\pi}\ \text{erfc}(i)}{128\,e}$$

06.42.03.0044.01

$$\text{Subfactorial}(7) = 1854$$

06.42.03.0045.01

$$\text{Subfactorial}\left(\frac{15}{2}\right) = \frac{1\,090\,687\,i}{128} + \frac{2\,027\,025\,\sqrt{\pi}\ \text{erfc}(i)}{256\,e}$$

06.42.03.0046.01

$$\text{Subfactorial}(8) = 14\,833$$

06.42.03.0047.01

$$\text{Subfactorial}\left(\frac{17}{2}\right) = \frac{18\,541\,935\,i}{256} + \frac{34\,459\,425\,\sqrt{\pi}\ \text{erfc}(i)}{512\,e}$$

06.42.03.0048.01

$$\text{Subfactorial}(9) = 133\,496$$

06.42.03.0049.01

$$\text{Subfactorial}\left(\frac{19}{2}\right) = \frac{352\,296\,253\,i}{512} + \frac{654\,729\,075\,\sqrt{\pi}\ \text{erfc}(i)}{1024\,e}$$

06.42.03.0050.01

$$\text{Subfactorial}(10) = 1\,334\,961$$

General characteristics

Domain and analyticity

$!\,z$ is an analytical function of z which is defined in the whole complex z-plane.

06.42.04.0001.01

$$z \longrightarrow \text{Subfactorial}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

06.42.04.0002.01

$$\text{Subfactorial}(\bar{z}) = \overline{!\,z}$$

Periodicity

No periodicity

Poles and essential singularities

The function $!\,z$ has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

06.42.04.0003.01

$$\mathcal{Sing}_z(\text{Subfactorial}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function $!\,z$ does not have branch points.

06.42.04.0004.01

$$\mathcal{BP}_z(\text{Subfactorial}(z)) = \{\}$$

Branch cuts

The function $!z$ does not have branch cuts.

06.42.04.0005.01

$$\mathcal{BC}_z(\text{Subfactorial}(z)) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

06.42.06.0001.01

$$\text{Subfactorial}(z) \propto \text{Subfactorial}(z_0) + \frac{1}{e}\left(-(-1)^{z_0}\, {}_2\tilde{F}_2(z_0+1, z_0+1; z_0+2, z_0+2; 1)\,\Gamma(z_0+1)^2 + \psi(z_0+1)\,\Gamma(z_0+1) - i\,\pi\,\Gamma(z_0+1, 0, -1)\right)(z-z_0) + \frac{1}{2\,e}\left(-\pi^2\,\Gamma(z_0+1, -1) + \Gamma(z_0+1)\left(\psi(z_0+1)^2 + \pi^2 + \psi^{(1)}(z_0+1)\right) + \frac{1}{(z_0+1)^3}\left(\left({}_3F_3(z_0+1, z_0+1, z_0+1; z_0+2, z_0+2, z_0+2; 1) - i\,\pi\,(z_0+1)\,{}_2F_2(z_0+1, z_0+1; z_0+2, z_0+2; 1)\right)\right)(2\,(-1)^{z_0})\right)\right)(z-z_0)^2 + \ldots \;/;\; (a \to a_0)$$

06.42.06.0002.01

$$\text{Subfactorial}(z) \propto \text{Subfactorial}(z_0) + \frac{1}{e}\left(-(-1)^{z_0}\, {}_2\tilde{F}_2(z_0+1, z_0+1; z_0+2, z_0+2; 1)\,\Gamma(z_0+1)^2 + \psi(z_0+1)\,\Gamma(z_0+1) - i\,\pi\,\Gamma(z_0+1, 0, -1)\right)(z-z_0) + \frac{1}{2\,e}\left(-\pi^2\,\Gamma(z_0+1, -1) + \Gamma(z_0+1)\left(\psi(z_0+1)^2 + \pi^2 + \psi^{(1)}(z_0+1)\right) + \frac{1}{(z_0+1)^3}\left(\left({}_3F_3(z_0+1, z_0+1, z_0+1; z_0+2, z_0+2, z_0+2; 1) - i\,\pi\,(z_0+1)\,{}_2F_2(z_0+1, z_0+1; z_0+2, z_0+2; 1)\right)\right)(2\,(-1)^{z_0})\right)\right)(z-z_0)^2 + O\left((z-z_0)^3\right)$$

06.42.06.0003.01

$$\text{Subfactorial}(z) = \frac{1}{e}\sum_{k=0}^{\infty}\frac{1}{k!}\left((-1)^{z_0}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}(k-j)!\,\Gamma(z_0+1)^{k-j+1}\,(\pi\,i)^j\,{}_4\tilde{F}_4\left(a_1, a_2, \ldots, a_{n-j+1}; a_1+1, a_2+1, \ldots, a_{n-j+1}+1; 1\right) + \Gamma^{(k)}(z_0+1)\right)(z-z_0)^k \;/;\; a_1 = a_2 = \ldots = a_{k+1} = z_0+1 \bigwedge k \in \mathbb{N}$$

06.42.06.0004.01

$$\text{Subfactorial}(z) \propto \text{Subfactorial}(z_0)\,(1 + O(z-z_0))$$

Asymptotic series expansions

06.42.06.0005.01

$$\text{Subfactorial}(z) \propto e^{-z-1} \sqrt{2\pi} \sqrt{z}\, z^z \left(1 + \frac{1}{12\,z} + \frac{1}{288\,z^2} + O\left(\frac{1}{z^3}\right)\right) + \frac{(-1)^z}{z} \left(1 - \frac{2}{z} + \frac{5}{z^2} + O\left(\frac{1}{z^3}\right)\right) /;\ (|z| \to \infty)$$

06.42.06.0006.01

$$\text{Subfactorial}(z) \propto \frac{1}{e\,z} \left(z\,z! + (-1)^z \sum_{k=0}^{\infty} (-1)^k z^{-k} + (-1)^z \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^k (j+2)^k z^{-k}}{(j+1)!}\right) /;\ (|z| \to \infty)$$

06.42.06.0007.01

$$\text{Subfactorial}(z) \propto \frac{1}{e} \left(\left(\frac{1}{2} \csc(z\,\pi)\right)^{\left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor} e^{-z} \sqrt{2\pi}\, z \left(e^{\pi i \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor} z\right)^{z-\frac{1}{2}} e^{\sum_{k=0}^{\infty} \frac{B_{2k+2}}{2(k+1)(2k+1)z^{2k+1}}} - (1-e)(-1)^z - (-1)^z \sum_{k=0}^{\infty} \sum_{j=1}^{\infty} \frac{(-j)^k z^{-k}}{j!}\right) /;$$

$$\neg (z \in \mathbb{Z} \land z < 1) \land (|z| \to \infty)$$

06.42.06.0008.01

$$\text{Subfactorial}(z) \propto \frac{1}{e} \left((-1)^z (e-1) + z\,\Gamma(z) - (-1)^z \sum_{k=0}^{\infty} \sum_{j=1}^{\infty} \frac{(-j)^k z^{-k}}{j!}\right) /;\ (|z| \to \infty)$$

06.42.06.0009.01

$$\text{Subfactorial}(z) \propto \sqrt{2\pi}\, e^{-z-1} z^{z+\frac{1}{2}} \left(1 + O\left(\frac{1}{z}\right)\right) + \frac{(-1)^z}{z} \left(1 + O\left(\frac{1}{z}\right)\right) /;\ (|z| \to \infty)$$

Integral representations

On the real axis

06.42.07.0001.01

$$\text{Subfactorial}(z) = \int_0^{\infty} (t-1)^z e^{-t}\, dt /;\ \operatorname{Re}(z) > -1$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.42.16.0001.01

$$\text{Subfactorial}(z+1) = (z+1)\,\text{Subfactorial}(z) - (-1)^z$$

06.42.16.0002.01

$$\text{Subfactorial}(z-1) = \frac{\text{Subfactorial}(z) - (-1)^z}{z}$$

06.42.16.0003.01

$$\text{Subfactorial}(z+n) = (z+1)_n\, \text{Subfactorial}(z) + (-1)^{n+z} \sum_{k=0}^{n-1} (-n-z)_k /;\ n \in \mathbb{N}$$

06.42.16.0004.01

$$\text{Subfactorial}(z-n) = \frac{(-1)^n}{(-z)_n}\,\text{Subfactorial}(z) + (-1)^{z-n}\sum_{k=0}^{n-1}\frac{(-1)^k}{(z-n+1)_{k+1}}\ /;\ n\in\mathbb{N}$$

Identities

Recurrence identities

Consecutive neighbors

06.42.17.0001.01

$$\text{Subfactorial}(z) = \frac{\text{Subfactorial}(z+1)+(-1)^z}{z+1}$$

06.42.17.0002.01

$$\text{Subfactorial}(z) = z\,\text{Subfactorial}(z-1)+(-1)^z$$

06.42.17.0003.01

$$\text{Subfactorial}(z) = \frac{\text{Subfactorial}(z+1)}{z} - \text{Subfactorial}(z-1)$$

Distant neighbors

06.42.17.0004.01

$$\text{Subfactorial}(z) = \frac{(-1)^n}{(-n-z)_n}\,\text{Subfactorial}(n+z) + (-1)^z\sum_{k=0}^{n-1}\frac{(-1)^k}{(z+1)_{k+1}}\ /;\ n\in\mathbb{N}$$

06.42.17.0005.01

$$\text{Subfactorial}(z) = (z-n+1)_n\,\text{Subfactorial}(z-n) + (-1)^z\sum_{k=0}^{n-1}(-z)_k\ /;\ n\in\mathbb{N}$$

Differentiation

Low-order differentiation

06.42.20.0001.01

$$\frac{\partial\,\text{Subfactorial}(z)}{\partial z} = \frac{1}{e}\,G_{2,3}^{3,0}\!\left(-1\,\middle|\,\begin{matrix}1,\,1\\0,\,0,\,z+1\end{matrix}\right) + i\,\pi\,\text{Subfactorial}(z)$$

06.42.20.0002.01

$$\frac{\partial\,\text{Subfactorial}(z)}{\partial z} = \frac{1}{e}\left(-(-1)^z\,{}_2\tilde{F}_2(z+1,\,z+1;\,z+2,\,z+2;\,1)\,\Gamma(z+1)^2 + \psi(z+1)\,\Gamma(z+1) - i\,\pi\,\Gamma(z+1,\,0,\,-1)\right)$$

06.42.20.0003.01

$$\frac{\partial^2\,\text{Subfactorial}(z)}{\partial z^2} = \frac{1}{e}\left(2\,i\,\pi\,G_{2,3}^{3,0}\!\left(-1\,\middle|\,\begin{matrix}1,\,1\\0,\,0,\,z+1\end{matrix}\right) + 2\,G_{3,4}^{4,0}\!\left(-1\,\middle|\,\begin{matrix}1,\,1,\,1\\0,\,0,\,0,\,z+1\end{matrix}\right)\right) - \pi^2\,\text{Subfactorial}(z)$$

06.42.20.0004.01

$$\frac{\partial^2 \text{Subfactorial}(z)}{\partial z^2} = \frac{1}{e}\Bigg(-\pi^2\,\Gamma(z+1,\,-1) + \frac{1}{(z+1)^3}\left((2\,(-1)^z)\left({}_3F_3(z+1,\,z+1,\,z+1;\,z+2,\,z+2,\,z+2;\,1) - i\,\pi\,(z+1)\,{}_2F_2(z+1,\,z+1;\,z+2,\,z+2;\,1)\right)\right) + \Gamma(z+1)\left(\psi(z+1)^2 + \pi^2 + \psi^{(1)}(z+1)\right)\Bigg)$$

Symbolic differentiation

06.42.20.0005.01

$$\frac{\partial^n \text{Subfactorial}(z)}{\partial z^n} = \frac{1}{e}\left(\Gamma^{(n)}(z+1) - \sum_{k=0}^{\infty}\frac{(-1)^{n-k}\,\Gamma(n+1,\,-\pi\,i\,(k+z+1))}{(k+z+1)^{n+1}\,k!}\right) /;\, n \in \mathbb{N}$$

06.42.20.0006.01

$$\frac{\partial^n \text{Subfactorial}(z)}{\partial z^n} = (\pi\,i)^n\,\text{Subfactorial}(z) + \frac{n!}{e}\sum_{k=1}^{n}\frac{(\pi\,i)^{n-k}}{(n-k)!}\,G_{k+2,k+2}^{k+2,0}\left(-1\,\middle|\,\begin{array}{c}1,\,1,\,\ldots,\,1\\ 0,\,0,\,\ldots,\,0,\,z+1\end{array}\right) /;\, n \in \mathbb{N}$$

06.42.20.0007.01

$$\frac{\partial^n \text{Subfactorial}(z)}{\partial z^n} = \frac{1}{e}\left((-1)^z\sum_{j=0}^{n}(-1)^{n-j}\binom{n}{j}(n-j)!\,\Gamma(z+1)^{n-j+1}\,(\pi\,i)^j\;{}_{n-j+1}\tilde{F}_{n-j+1}(z+1,\,\ldots,\,z+1;\,z+2,\,\ldots,\,z+2;\,1) + \Gamma^{(n)}(z+1)\right) /;$$

$$a_1 = a_2 = \ldots = a_{n+1} = z+1 \bigwedge n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *a*

06.42.20.0008.01

$$\frac{\partial^\alpha \text{Subfactorial}(z)}{\partial z^\alpha} = z^{-\alpha}\int_0^\infty (t-1)^z\,(z\log(t-1))^\alpha\,Q(-\alpha,\,0,\,z\log(t-1))\,e^{-t}\,dt /;\, \text{Re}(z) > -1$$

Integration

Indefinite integration

06.42.21.0001.01

$$\int \text{Subfactorial}(z)\,dz = \int_0^\infty \frac{(t-1)^z\,e^{-t}}{\log(t-1)}\,dt /;\, \text{Re}(z) > -1$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_1\tilde{F}_1$

06.42.26.0001.01

$$\text{Subfactorial}(z) = \frac{\Gamma(z+1)}{e} \left(1 + (-1)^z \, {}_1\tilde{F}_1(z+1; z+2; 1)\right) /; \neg (-z \in \mathbb{Z} \land -z > 0)$$

Involving ${}_1F_1$

06.42.26.0002.01

$$\text{Subfactorial}(z) = \frac{1}{e} \left(\Gamma(z+1) + \frac{(-1)^z}{z+1} \, {}_1F_1(z+1; z+2; 1)\right) /; \neg (-z \in \mathbb{Z} \land -z > 0)$$

Involving hypergeometric U

06.42.26.0003.01

$$\text{Subfactorial}(z) = U(-z, -z, -1) /; \neg (z \in \mathbb{Z} \land z > 1)$$

Through other functions

Involving some hypergeometric-type functions

06.42.26.0004.01

$$\text{Subfactorial}(z) = \frac{\Gamma(z+1) + \Gamma(z+1, -1, 0)}{e} /; \neg (-z \in \mathbb{Z} \land -z > 0)$$

06.42.26.0005.01

$$\text{Subfactorial}(z) = \frac{\Gamma(z+1)}{e} (1 + Q(z+1, -1, 0)) /; \neg (-z \in \mathbb{Z} \land -z > 0)$$

06.42.26.0006.01

$$\text{Subfactorial}(z) = \frac{\Gamma(z+1)}{e} Q(z+1, -1)$$

06.42.26.0007.01

$$\text{Subfactorial}(z) = \frac{(-1)^{z-1}}{e} E_{-z}(-1)$$

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