WeierstrassSigma4

Notations

Traditional name

Associated Weierstrass sigma function

Traditional notation

$\sigma_n(z, g_2, g_3)$

Mathematica StandardForm notation

`WeierstrassSigma[n, z, {g2, g3}]`

Primary definition

09.16.02.0001.01

$$\sigma_n(z, g_2, g_3) = \frac{e^{-\eta_n z}\,\sigma(z + \omega_n; g_2, g_3)}{\sigma(\omega_n; g_2, g_3)} \;/;$$
$$n \in \{1, 2, 3\} \land \{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \land \eta_n = \zeta(\omega_n; g_2, g_3) \land n \in \{1, 2, 3\}$$

Special notations for this file:

09.16.02.0002.01

$$\{\omega_1, \omega_3\} = \{\omega_1(g_2, g_3), \omega_3(g_2, g_3)\}$$

09.16.02.0003.01

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\}$$

09.16.02.0004.01

$$e_n = \wp(\omega_n; g_2, g_3) \land n \in \{1, 2, 3\}$$

09.16.02.0005.01

$$\eta_n = \zeta(\omega_n; g_2, g_3) \land n \in \{1, 2, 3\}$$

09.16.02.0006.01

$$q = \exp\left(\frac{\pi i \omega_3}{\omega_1}\right)$$

Specific values

Specialized values

For fixed z

Degenerate case:

09.16.03.0001.01

$$\sigma_n(z;\,0,\,0) = 1 \;/;\; n \in \{1,\,2,\,3\}$$

For fixed $\{g_2,\,g_3\}$

Values at half-periods

09.16.03.0002.01

$$\sigma_j\,(0;\,g_2,\,g_3) = 1 \;/;\; j \in \{1,\,2,\,3\}$$

09.16.03.0003.01

$$\sigma_j(2\,m\,\omega_1 + 2\,n\,\omega_3;\,g_2,\,g_3) = \tilde{\infty} \;/;\; \{m,\,n\} \in \mathbb{Z} \bigwedge j \in \{1,\,2,\,3\}$$

09.16.03.0004.01

$$\sigma_n\left(\omega_j;\,g_2,\,g_3\right) = 0 \;/;\; j \in \{1,\,2,\,3\}$$

09.16.03.0005.01

$$\sigma_i\big((2\,m+1)\,\omega_i + 2\,n\,\omega_j + 2\,r\,\omega_k;\,g_2,\,g_3\big) = 0 \;/;\; \{m,\,n,\,r\} \in \mathbb{Z} \bigwedge \{i,\,j,\,k\} \in \{1,\,2,\,3\} \bigwedge i \neq j \neq k$$

09.16.03.0006.01

$$\sigma_i\left(\omega_j;\,g_2,\,g_3\right) = e^{-\eta_i\,\omega_j}\,\frac{\sigma(\omega_k;\,g_2,\,g_3)}{\sigma(\omega_i;\,g_2,\,g_3)} \;/;\; \{i,\,j,\,k\} \in \{1,\,2,\,3\} \bigwedge i \neq j \neq k$$

09.16.03.0007.01

$$\sigma_j(\omega_i;\,g_2,\,g_3)\,\sigma_k(\omega_i;\,g_2,\,g_3) = e^{\eta_i\,\omega_i} \;/;\; \{i,\,j,\,k\} \in \{1,\,2,\,3\} \bigwedge i \neq j \neq k$$

09.16.03.0008.01

$$\frac{\sigma_i(\omega_k;\,g_2,\,g_3)}{\sigma_i\left(\omega_j;\,g_2,\,g_3\right)\sigma_k(\omega_i;\,g_2,\,g_3)} = e^{\eta_j\,\omega_i} \;/;\; \{i,\,j,\,k\} \in \{1,\,2,\,3\} \bigwedge i \neq j \neq k$$

09.16.03.0009.01

$$\frac{\sigma(\omega_k;\,g_2,\,g_3)}{\sigma\left(\omega_j;\,g_2,\,g_3\right)\sigma_j(\omega_i;\,g_2,\,g_3)} = e^{\eta_j\,\omega_i} \;/;\; \{i,\,j,\,k\} \in \{1,\,2,\,3\} \bigwedge i \neq j \neq k$$

09.16.03.0010.01

$$\frac{\sigma_j(\omega_i;\,g_2,\,g_3)^2}{\sigma(\omega_i;\,g_2,\,g_3)^2} = e_i - e_j \;/;\; \{i,\,j\} \in \{1,\,2,\,3\} \bigwedge i \neq j$$

General characteristics

Domain and analyticity

$\sigma_n(z,\,g_2,\,g_3)$, $n \in \{1,\,2,\,3\}$ are an entire analytical functions of z, g_2, and g_3, which are defined in $\mathbb{C}^3$.

09.16.04.0001.01

$$(n * z * \{g_2 * g_3\}) \longrightarrow \sigma_n(z,\,g_2,\,g_3) :: (\{1,\,2,\,3\} \otimes \mathbb{C} \otimes \{\mathbb{C} \otimes \mathbb{C}\}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\sigma_n(z, g_2, g_3)$, $n \in \{1, 2, 3\}$ are an even functions with respect to z.

09.16.04.0002.01

$$\sigma_n(-z; g_2, g_3) = \sigma_n(z; g_2, g_3) \;/;\; n \in \{1, 2, 3\}$$

Mirror symmetry

09.16.04.0003.01

$$\sigma_n(\bar{z}; \overline{g_2}, \overline{g_3}) = \overline{\sigma_n(z; g_2, g_3)} \;/;\; n \in \{1, 2, 3\}$$

Periodicity

$\sigma_n(z, g_2, g_3)$, $n \in \{1, 2, 3\}$ are a quasi-periodic functions with respect to z.

09.16.04.0004.01

$$\sigma_n(z + 2\,\omega_n; g_2, g_3) = -e^{2\,\eta_n\,(z+\omega_n)}\,\sigma_n(z; g_2, g_3) \;/;\; n \in \{1, 2, 3\}$$

09.16.04.0005.01

$$\sigma_n\left(z + 2\,\omega_j; g_2, g_3\right) = e^{2\,\eta_j\,(z+\omega_j)}\,\sigma_n(z; g_2, g_3) \;/;\; \{n, j\} \in \{1, 2, 3\} \land n \neq j$$

09.16.04.0006.01

$$\sigma_i\left(z + 2\,m\,\omega_i + 2\,n\,\omega_j + 2\,r\,\omega_k; g_2, g_3\right) = (-1)^{n\,r+r\,m+m\,n+m}\,e^{2\,(m\,\eta_i+n\,\eta_j+r\,\eta_k)\,(z+m\,\omega_i+n\,\omega_j+r\,\omega_k)}\,\sigma_i(z; g_2, g_3) \;/;\; \{i, j, k\} \in \{1, 2, 3\} \land i \neq j \neq k \land \{m, n, r\} \in \mathbb{Z}$$

Transformation of half-periods

09.16.04.0007.01

$$\sigma_1(z; g_2(a\,\omega_1 + b\,\omega_3, c\,\omega_1 + d\,\omega_3), g_3(a\,\omega_1 + b\,\omega_3, c\,\omega_1 + d\,\omega_3))\,\sigma_2(z; g_2(a\,\omega_1 + b\,\omega_3, c\,\omega_1 + d\,\omega_3), g_3(a\,\omega_1 + b\,\omega_3, c\,\omega_1 + d\,\omega_3))\,\sigma_3(z; g_2(a\,\omega_1 + b\,\omega_3, c\,\omega_1 + d\,\omega_3), g_3(a\,\omega_1 + b\,\omega_3, c\,\omega_1 + d\,\omega_3)) = \sigma_1(z; g_2(\omega_1, \omega_3), g_3(\omega_1, \omega_3))\,\sigma_2(z; g_2(\omega_1, \omega_3), g_3(\omega_1, \omega_3))\,\sigma_3(z; g_2(\omega_1, \omega_3), g_3(\omega_1, \omega_3)) \;/;\; \{a, b, c, d\} \in \mathbb{Z} \land a\,d - b\,c = \pm 1$$

Homogeneity

09.16.04.0008.01

$$\sigma_n(\lambda\,z; g_2(\lambda\,\omega_1, \lambda\,\omega_3), g_3(\lambda\,\omega_1, \lambda\,\omega_3)) = \sigma_n(z; g_2(\omega_1, \omega_3), g_3(\omega_1, \omega_3)) \;/;\; n \in \{1, 2, 3\}$$

09.16.04.0009.01

$$\sigma_n(\lambda\,z; g_2(\lambda\,\omega_1, \lambda\,\omega_3), g_3(\lambda\,\omega_1, \lambda\,\omega_3)) = \sigma_n\left(\lambda\,z; \frac{g_2(\omega_1, \omega_3)}{\lambda^4}, \frac{g_3(\omega_1, \omega_3)}{\lambda^6}\right) \;/;\; n \in \{1, 2, 3\}$$

Poles and essential singularities

With respect to z

For fixed g_2, g_3, the functions $\sigma_j(z; g_2, g_3)$, $j \in \{1, 2, 3\}$ have simple poles at the points $z = 2\,m\,\omega_1(g_2, g_3) + 2\,n\,\omega_3(g_2, g_3)$, $\{m, n\} \in \mathbb{Z}$.

09.16.04.0010.01

$$\mathcal{S}ing_z\left(\sigma_j(z; g_2, g_3)\right) = \{\{\{2\,m\,\omega_1 + 2\,n\,\omega_3, 1\} \;/;\; \{m, n\} \in \mathbb{Z}\}\} \;/;\; j \in \{1, 2, 3\}$$

Branch points

With respect to z

For fixed g_2, g_3, the function $\sigma_j(z; g_2, g_3)$, $j \in \{1, 2, 3\}$ does not have branch points.

09.16.04.0011.01

$$\mathcal{BP}_z(\sigma_j(z; g_2, g_3)) = \{\} \;/;\; j \in \{1, 2, 3\}$$

Branch cuts

With respect to z

For fixed g_2, g_3, the function $\sigma_j(z; g_2, g_3)$, $j \in \{1, 2, 3\}$ does not have branch cuts.

09.16.04.0012.01

$$\mathcal{BC}_z(\sigma_j(z; g_2, g_3)) = \{\} \;/;\; j \in \{1, 2, 3\}$$

Series representations

q-series

q-series for logarithms

09.16.06.0001.01

$$\log(\sigma_1(z; g_2, g_3)) = \frac{\eta_1 z^2}{2\omega_1} + \log\left(\cos\left(\frac{\pi z}{2\omega_1}\right)\right) + 4\sum_{k=1}^{\infty}(-1)^k \frac{q^{2k}}{k(1-q^{2k})}\sin^2\left(\frac{k\pi z}{2\omega_1}\right)$$

09.16.06.0002.01

$$\log(\sigma_2(z; g_2, g_3)) = \frac{\eta_1 z^2}{2\omega_1} + 4\sum_{k=1}^{\infty}(-1)^k \frac{q^{k}}{k(1-q^{2k})}\sin^2\left(\frac{k\pi z}{2\omega_1}\right)$$

09.16.06.0003.01

$$\log(\sigma_3(z; g_2, g_3)) = \frac{\eta_1 z^2}{2\omega_1} + 4\sum_{k=1}^{\infty} \frac{q^{k}}{k(1-q^{2k})}\sin^2\left(\frac{k\pi z}{2\omega_1}\right)$$

Other series representations

09.16.06.0004.01

$$\sigma_i(z; g_2, g_3) = z\exp\left(-\sum_{j=2}^{\infty}\frac{z^{2j}}{2j}\sum_{\substack{m,n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\frac{1}{(2m\omega_1+2n\omega_3)^{2j}}\right)\left(-e_i + \frac{1}{z^2} + \sum_{j=1}^{\infty}(2j+1)z^{2j}\sum_{\substack{m,n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\frac{1}{(2m\omega_1+2n\omega_3)^{2j+2}}\right)^{1/2} \;/;$$

$$i \in \{1, 2, 3\}$$

Product representations

Infinite products involving trigonometric functions

09.16.08.0001.01

$$\sigma_i(z; g_2, g_3) = \exp\left(\frac{\eta_i z^2}{2\omega_i}\right)\cos\left(\frac{\pi z}{2\omega_i}\right)\prod_{n=1}^{\infty}\left(1 - \frac{\sin^2\left(\frac{\pi z}{2\omega_i}\right)}{\cos^2\left(\frac{n\pi\omega_j}{\omega_i}\right)}\right) \;/;\; \{i, j\} \in \{1, 2, 3\} \land i \neq j$$

09.16.08.0002.01

$$\sigma_i(z; g_2, g_3) == \exp\left(\frac{\eta_j z^2}{2\omega_j}\right) \prod_{n=1}^{\infty} \left(1 - \frac{\sin^2\left(\frac{\pi z}{2\omega_j}\right)}{\cos^2\left(\frac{n\pi\omega_i}{\omega_j}\right)}\right) /; \{i, j\} \in \{1, 2, 3\} \bigwedge i \neq j$$

09.16.08.0003.01

$$\sigma_i(z; g_2, g_3) == \exp\left(\frac{\eta_j z^2}{2\omega_j}\right) \prod_{n=1}^{\infty} \left(1 - \frac{\sin^2\left(\frac{\pi z}{2\omega_j}\right)}{\cos^2\left(\frac{2n-1}{2} \frac{\pi\omega_k}{\omega_j}\right)}\right) /; \{i, j, k\} \in \{1, 2, 3\} \bigwedge i \neq j \neq k$$

Infinite products involving exponentials

09.16.08.0004.01

$$\sigma_i(z; g_2, g_3) == \exp\left(-\frac{e_i z^2}{2}\right) \prod_{\substack{m, n=-\infty \\ \{m,n\} \neq \{0,0\}}}^{\infty} \left(\left(1 - \frac{z}{2m\omega_1 + 2n\omega_2 - \omega_i}\right) \exp\left(\frac{z}{2m\omega_1 + 2n\omega_2 - \omega_i} + \frac{z^2}{2(2m\omega_1 + 2n\omega_2 - \omega_i)^2}\right)\right) /;$$

$$i \in \{1, 2, 3\}$$

Infinite products involving q, trigonometrics and exponentials

09.16.08.0005.01

$$\sigma_1(z; g_2, g_3) == \cos\left(\frac{\pi z}{2\omega_1}\right) \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \left(\prod_{n=1}^{\infty} \frac{1 + q^{2n} \exp\left(-\frac{i\pi z}{\omega_1}\right)}{1 + q^{2n}}\right) \left(\prod_{n=1}^{\infty} \frac{1 + q^{2n} \exp\left(\frac{i\pi z}{\omega_1}\right)}{1 + q^{2n}}\right)$$

09.16.08.0006.01

$$\sigma_1(z; g_2, g_3) == \cos\left(\frac{\pi z}{2\omega_1}\right) \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \prod_{n=1}^{\infty} \frac{1 + 2q^{2n} \cos\left(\frac{\pi z}{\omega_1}\right) + q^{4n}}{\left(1 + q^{2n}\right)^2}$$

09.16.08.0007.01

$$\sigma_2(z; g_2, g_3) == \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \left(\prod_{n=1}^{\infty} \frac{1 + q^{2n-1} \exp\left(-\frac{i\pi z}{\omega_1}\right)}{1 + q^{2n-1}}\right) \left(\prod_{n=1}^{\infty} \frac{1 + q^{2n-1} \exp\left(\frac{i\pi z}{\omega_1}\right)}{1 + q^{2n-1}}\right)$$

09.16.08.0008.01

$$\sigma_2(z; g_2, g_3) == \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \prod_{n=1}^{\infty} \frac{1 + 2q^{2n-1} \cos\left(\frac{\pi z}{\omega_1}\right) + q^{4n-2}}{\left(1 + q^{2n-1}\right)^2}$$

09.16.08.0009.01

$$\sigma_3(z; g_2, g_3) == \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \left(\prod_{n=1}^{\infty} \frac{1 - q^{2n-1} \exp\left(-\frac{i\pi z}{\omega_1}\right)}{1 - q^{2n-1}}\right) \left(\prod_{n=1}^{\infty} \frac{1 - q^{2n-1} \exp\left(\frac{i\pi z}{\omega_1}\right)}{1 - q^{2n-1}}\right)$$

09.16.08.0010.01

$$\sigma_3(z; g_2, g_3) == \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \prod_{n=1}^{\infty} \frac{1 - 2q^{2n-1} \cos\left(\frac{\pi z}{\omega_1}\right) + q^{4n-2}}{\left(1 - q^{2n-1}\right)^2}$$

q-products for half-period values

09.16.08.0011.01

$$\sigma_1(\omega_1; g_2, g_3) = 0$$

09.16.08.0012.01

$$\sigma_1(\omega_2; g_2, g_3) = -\frac{i\sqrt{i}}{2\sqrt[4]{q}} \exp\left(\frac{\eta_2\,\omega_2}{2}\right)\left(\prod_{n=1}^{\infty}\frac{1-q^{2n-1}}{1+q^{2n}}\right)^2$$

09.16.08.0013.01

$$\sigma_1(\omega_3; g_2, g_3) = \frac{1}{2\sqrt[4]{q}} \exp\left(\frac{\eta_3\,\omega_3}{2}\right)\left(\prod_{n=1}^{\infty}\frac{1+q^{2n-1}}{1+q^{2n}}\right)^2$$

09.16.08.0014.01

$$\sigma_2(\omega_1; g_2, g_3) = \exp\left(\frac{\eta_1\,\omega_1}{2}\right)\left(\prod_{n=1}^{\infty}\frac{1-q^{2n-1}}{1+q^{2n-1}}\right)^2$$

09.16.08.0015.01

$$\sigma_2(\omega_2; g_2, g_3) = 0$$

09.16.08.0016.01

$$\sigma_2(\omega_3; g_2, g_3) = 2\sqrt[4]{q}\, \exp\left(\frac{\eta_3\,\omega_3}{2}\right)\left(\prod_{n=1}^{\infty}\frac{1+q^{2n}}{1+q^{2n-1}}\right)^2$$

09.16.08.0017.01

$$\sigma_3(\omega_1; g_2, g_3) = \exp\left(\frac{\eta_1\,\omega_1}{2}\right)\left(\prod_{n=1}^{\infty}\frac{1+q^{2n-1}}{1-q^{2n-1}}\right)^2$$

09.16.08.0018.01

$$\sigma_3(\omega_2; g_2, g_3) = 2\sqrt{i}\,\sqrt[4]{q}\, \exp\left(\frac{\eta_2\,\omega_2}{2}\right)\left(\prod_{n=1}^{\infty}\frac{1+q^{2n}}{1-q^{2n-1}}\right)^2$$

09.16.08.0019.01

$$\sigma_3(\omega_3; g_2, g_3) = 0$$

Transformations

Addition formulas

Translation by half-periods

09.16.16.0001.01

$$\sigma_i(z \pm \omega_i; g_2, g_3) = \mp e^{\pm\eta_i z} \frac{\sigma_j(\omega_i; g_2, g_3)\,\sigma_k(\omega_i; g_2, g_3)\,\sigma(z; g_2, g_3)}{\sigma(\omega_i; g_2, g_3)} \;/;\; \{i, j, k\} \in \{1, 2, 3\} \land i \neq j \neq k$$

09.16.16.0002.01

$$\sigma_j(z \pm \omega_i; g_2, g_3) = e^{\pm\eta_i z}\,\sigma_j(\omega_i; g_2, g_3)\,\sigma_k(z; g_2, g_3) \;/;\; \{i, j, k\} \in \{1, 2, 3\} \land i \neq j \neq k$$

09.16.16.0003.01

$$\sigma_i(z_1 + z_2; g_2, g_3)\,\sigma_i(z_1 - z_2; g_2, g_3) = \sigma_i(z_1; g_2, g_3)^2\,\sigma_i(z_2; g_2, g_3)^2 - \left(e_i - e_j\right)(e_i - e_k)\,\sigma(z_1; g_2, g_3)^2\,\sigma(z_2; g_2, g_3)^2 \;/;\; \{i, j, k\} \in \{1, 2, 3\} \land i \neq j \neq k$$

09.16.16.0004.01

$$\sigma(z_1+z_2;g_2,g_3)\,\sigma(z_1-z_2;g_2,g_3)=\sigma(z_1;g_2,g_3)^2\,\sigma_i(z_2;g_2,g_3)^2-\sigma(z_2;g_2,g_3)^2\,\sigma_i(z_1;g_2,g_3)^2\;/;\;i\in\{1,2,3\}$$

09.16.16.0005.01

$$\sigma_i(z_1+z_2;g_2,g_3)\,\sigma_i(z_1-z_2;g_2,g_3)=\sigma_i(z_1;g_2,g_3)^2\,\sigma_j(z_2;g_2,g_3)^2-\left(e_i-e_j\right)\sigma(z_2;g_2,g_3)^2\,\sigma_k(z_1;g_2,g_3)^2\;/;\;\{i,j,k\}\in\{1,2,3\}\bigwedge i\neq j\neq k$$

09.16.16.0006.01

$$\sigma_i(z_1+z_2;g_2,g_3)\,\sigma(z_1-z_2;g_2,g_3)=\sigma(z_1;g_2,g_3)\,\sigma_i(z_1;g_2,g_3)\,\sigma_j(z_2;g_2,g_3)\,\sigma_k(z_2;g_2,g_3)-\sigma(z_2;g_2,g_3)\,\sigma_i(z_2;g_2,g_3)\,\sigma_j(z_1;g_2,g_3)\,\sigma_k(z_1;g_2,g_3)\;/;\;\{i,j,k\}\in\{1,2,3\}\bigwedge i\neq j\neq k$$

09.16.16.0007.01

$$\sigma_k(z_1+z_2;g_2,g_3)\,\sigma_j(z_1-z_2;g_2,g_3)=\sigma_k(z_1;g_2,g_3)\,\sigma_j(z_1;g_2,g_3)\,\sigma_k(z_2;g_2,g_3)\,\sigma_j(z_2;g_2,g_3)+\left(e_j-e_k\right)\sigma(z_1;g_2,g_3)\,\sigma_i(z_1;g_2,g_3)\,\sigma(z_2;g_2,g_3)\,\sigma_i(z_2;g_2,g_3)\;/;\;\{i,j,k\}\in\{1,2,3\}\bigwedge i\neq j\neq k$$

Related transformations

Halving half-period

09.16.16.0008.01

$$\sigma_1\left(z;g_2\left(\frac{\omega_1}{2},\omega_3\right),g_3\left(\frac{\omega_1}{2},\omega_3\right)\right)=\exp\left(\frac{e_1 z^2}{2}\right)\left(\sigma_1(z;g_2,g_3)^2-e^{\eta_1\omega_1}\frac{\sigma(z;g_2,g_3)^2}{\sigma(\omega_1;g_2,g_3)^2}\right)$$

09.16.16.0009.01

$$\sigma_2\left(z;g_2\left(\frac{\omega_1}{2},\omega_3\right),g_3\left(\frac{\omega_1}{2},\omega_3\right)\right)=\exp\left(\frac{e_1 z^2}{2}\right)\left(\sigma_1(z;g_2,g_3)^2+e^{\eta_1\omega_1}\frac{\sigma(z;g_2,g_3)^2}{\sigma(\omega_1;g_2,g_3)^2}\right)$$

09.16.16.0010.01

$$\sigma_3\left(z;g_2\left(\frac{\omega_1}{2},\omega_3\right),g_3\left(\frac{\omega_1}{2},\omega_3\right)\right)=\exp\left(\frac{e_1 z^2}{2}\right)\sigma_2(z;g_2,g_3)\,\sigma_3(z;g_2,g_3)$$

Third of half-period

09.16.16.0011.01

$$\sigma_i\left(z;g_2\left(\frac{\omega_1}{3},\omega_3\right),g_3\left(\frac{\omega_1}{3},\omega_3\right)\right)=\exp\left(z^2\,\wp\left(\frac{2\,\omega_1}{3};g_2,g_3\right)-2\,z\,\eta_1\right)\left(\sigma_i(z;g_2,g_3)\,\sigma_i\left(z+\frac{2\,\omega_1}{3};g_2,g_3\right)\sigma_i\left(z+\frac{4\,\omega_1}{3};g_2,g_3\right)\right)\Big/\left(\sigma_i\left(\frac{2\,\omega_1}{3};g_2,g_3\right)\sigma_i\left(\frac{4\,\omega_1}{3};g_2,g_3\right)\right)\;/;\;i\in\{1,2,3\}$$

General fractions of half-periods

09.16.16.0012.01

$$\sigma_1\left(z;g_2\left(\frac{\omega_1}{n},\omega_3\right),g_3\left(\frac{\omega_1}{n},\omega_3\right)\right)=\exp\left(\frac{(n-1)\,z\,\eta_1}{n}+\sum_{k=1}^{n-1}\left(\frac{z^2}{2}\,\wp\left(\frac{2\,k\,\omega_1}{n};g_2,g_3\right)-\frac{(2\,k-n+1)\,\eta_1\,z}{n}\right)\right)\prod_{k=0}^{n-1}\frac{\sigma_1\left(z+\frac{(2\,k-n+1)\,\omega_1}{n};g_2,g_3\right)}{\sigma_1\left(\frac{(2\,k-n+1)\,\omega_1}{n};g_2,g_3\right)}\;/;\;n\in\mathbb{N}^+$$

09.16.16.0013.01

$$\sigma_2\left(z; g_2\left(\frac{\omega_1}{n}, \omega_3\right), g_3\left(\frac{\omega_1}{n}, \omega_3\right)\right) = \exp\left(\frac{(n-1) z \eta_1}{n} + \sum_{k=1}^{n-1}\left(\frac{z^2}{2} \wp\left(\frac{2 k \omega_1}{n}; g_2, g_3\right) - \frac{(2k-n+1) \eta_1 z}{n}\right)\right) \prod_{k=0}^{n-1} \frac{\sigma_2\left(z + \frac{(2k-n+1)\omega_1}{n}; g_2, g_3\right)}{\sigma_2\left(\frac{(2k-n+1)\omega_1}{n}; g_2, g_3\right)} \;/;\; n \in \mathbb{N}^+$$

09.16.16.0014.01

$$\sigma_3\left(z; g_2\left(\frac{\omega_1}{n}, \omega_3\right), g_3\left(\frac{\omega_1}{n}, \omega_3\right)\right) = \exp\left(\sum_{k=1}^{n-1}\left(\frac{z^2}{2} \wp\left(\frac{2 k \omega_1}{n}; g_2, g_3\right) - \frac{2 k \eta_1 z}{n}\right)\right) \prod_{k=0}^{n-1} \frac{\sigma_3\left(z + \frac{2k\omega_1}{n}; g_2, g_3\right)}{\sigma_3\left(\frac{2k\omega_1}{n}; g_2, g_3\right)} \;/;\; n \in \mathbb{N}^+$$

Identities

Functional identities

09.16.17.0001.01

$$\sigma_i(z; g_2, g_3)^2 - \sigma_j(z; g_2, g_3)^2 = (e_j - e_i)\, \sigma(z; g_2, g_3)^2 \;/;\; \{i, j\} \in \{1, 2, 3\} \land i \neq j$$

09.16.17.0002.01

$$(e_k - e_j)\, \sigma_i(z; g_2, g_3)^2 + (e_i - e_k)\, \sigma_j(z; g_2, g_3)^2 + (e_j - e_i)\, \sigma_k(z; g_2, g_3)^2 = 0 \;/;\; \{i, j\} \in \{1, 2, 3\} \land i \neq j$$

Differentiation

Low-order differentiation

With respect to z

09.16.20.0001.01

$$\frac{\partial \sigma_n(z, g_2, g_3)}{\partial z} = \sigma_n(z, g_2, g_3)\, (\zeta(z + \omega_n; g_2, g_3) - \eta_n)$$

09.16.20.0002.01

$$\frac{\partial^2 \sigma_n(z, g_2, g_3)}{\partial z^2} = \sigma_n(z, g_2, g_3)\left((\eta_n - \zeta(z + \omega_n; g_2, g_3))^2 - \wp(z + \omega_n; g_2, g_3)\right)$$

Symbolic differentiation

With respect to z

09.16.20.0003.01

$$\frac{\partial^n \sigma_m(z, g_2, g_3)}{\partial z^n} = \frac{(2\omega_1)^{1-n} \pi^{n-\frac{1}{2}} e^{-\eta_m z}}{\sigma_m(z, g_2, g_3)} \left(\prod_{k=1}^{\infty} \frac{1}{1 - q^{2k}}\right)^3 \sum_{j=0}^{n} {}_2\tilde{F}_2\left(\frac{1}{2}, 1; \frac{1-j}{2}, \frac{2-j}{2}; \frac{(z+\omega_m)^2 \zeta(\omega_1; g_2, g_3)}{2\omega_1}\right) \binom{n}{j} \left(\frac{4\omega_1}{\pi (z + \omega_m)}\right)^j \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1)^{n-j} \sin\left(\frac{1}{2\omega_1} (\pi ((n-j)\omega_1 + (2k+1)(z + \omega_m)))\right) - \eta_m \sigma_m(z, g_2, g_3) \;/;\; n \in \mathbb{N}^+$$

Representations through equivalent functions

With related functions

Involving other Weierstrass functions

09.16.27.0001.01

$$\sigma_i(z; g_2, g_3) = \sigma(z; g_2, g_3)\sqrt{\wp(z; g_2, g_3) - e_i} \;/;\; i \in \{1, 2, 3\}$$

09.16.27.0002.01

$$\frac{\sigma_i(z; g_2, g_3)^2}{\sigma(z; g_2, g_3)^2} = \wp(z; g_2, g_3) - e_i \;/;\; i \in \{1, 2, 3\}$$

Involving theta functions

09.16.27.0003.01

$$\sigma_1(z; g_2, g_3) = \frac{1}{2} \frac{1}{\sqrt[4]{q}} \left(\prod_{n=1}^{\infty} \frac{1}{(1 - q^{4n})(1 + q^{2n})}\right) \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \vartheta_2\left(\frac{\pi z}{2\omega_1}, q\right)$$

09.16.27.0004.01

$$\sigma_2(z; g_2, g_3) = \left(\prod_{n=1}^{\infty} \frac{1}{(1 - q^{2n})(1 + q^{2n-1})^2}\right) \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \vartheta_3\left(\frac{\pi z}{2\omega_1}, q\right)$$

09.16.27.0005.01

$$\sigma_3(z; g_2, g_3) = \left(\prod_{n=1}^{\infty} \frac{1}{(1 - q^{2n})(1 - q^{2n-1})^2}\right) \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \vartheta_4\left(\frac{\pi z}{2\omega_1}, q\right)$$

09.16.27.0006.01

$$\sigma_i(u; g_2, g_3) = \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \frac{\vartheta_{i+1}\left(\frac{\pi z}{2\omega_1}, q\right)}{\vartheta_{i+1}(0, q)} \;/;\; i \in \{1, 2, 3\}$$

Zeros

09.16.30.0001.01

$$\sigma_i\left((2m+1)\omega_i + 2n\omega_j + 2r\omega_k; g_2, g_3\right) = 0 \;/;\; \{m, n, r\} \in \mathbb{Z} \land \{i, j, k\} \in \{1, 2, 3\} \land i \neq j \neq k$$

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